UFRGS – INSTITUTO DE MATEMÁTICA Departamento de Matemática Pura e Aplicada MAT01168 - Turma C - 2016/1Prova da área IA

1 - 6	7	8	Total

Nome:	Cartão:	

Regras Gerais:

- $\bullet\,$ Não é permitido o uso de calculadoras, telefones ou qualquer outro recurso computacional ou de comunicação.
- Trabalhe individualmente e sem uso de material de consulta além do fornecido.
- Devolva o caderno de questões preenchido ao final da prova.

Regras para as questões abertas:

- Seja sucinto, completo e claro.
- $\bullet\,$ Justifique to do procedimento usado.
- Indique identidades matemáticas usadas, em especial, itens da tabela.
- Use notação matemática consistente.

I	d	en	ti	d	a	d	es	

$\operatorname{sen}(x) = \frac{e^{ix} - e^{-ix}}{2i}$	$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$
$\operatorname{senh}(x) = \frac{e^x - e^{-x}}{2}$	$\cosh(x) = \frac{e^x + e^{-x}}{2}$
$(a+b)^n = \sum_{j=0}^{\infty} \binom{n}{j} a^{n-j}$	$-jb^j$, $\binom{n}{j} = \frac{n!}{j!(n-j)!}$
$\operatorname{sen}(x+y) = \operatorname{sen}(x)$	$\cos(y) + \sin(y)\cos(x)$

$$\operatorname{sen}(x+y) = \operatorname{sen}(x)\operatorname{cos}(y) + \operatorname{sen}(y)\operatorname{cos}(x)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

Propr	riedades:	
1	Linearidade	$\mathcal{L}\left\{\alpha f(t) + \beta g(t)\right\} = \alpha \mathcal{L}\left\{f(t)\right\} + \beta \mathcal{L}\left\{g(t)\right\}$
2	Transformada da derivada	$\mathcal{L}\left\{f'(t)\right\} = s\mathcal{L}\left\{f(t)\right\} - f(0)$ $\mathcal{L}\left\{f''(t)\right\} = s^2\mathcal{L}\left\{f(t)\right\} - sf(0) - f'(0)$
3	Deslocamento no eixo s	$\mathcal{L}\left\{e^{at}f(t)\right\} = F(s-a)$
4	Deslocamento no eixo t	$\mathcal{L}\left\{u(t-a)f(t-a)\right\} = e^{-as}F(s)$ $\mathcal{L}\left\{u(t-a)\right\} = \frac{e^{-as}}{s}$
5	Transformada da integral	$\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{F(s)}{s}$
6	Filtragem da Delta de Dirac	$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$
7	Transformada da Delta de Dirac	$\mathcal{L}\left\{\delta(t-a)\right\} = e^{-as}$
8	Teorema da Convolução	$\mathcal{L}\left\{(f*g)(t)\right\} = F(s)G(s),$ onde $(f*g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$
9	Transformada de funções periódicas	$\mathcal{L}\left\{f(t)\right\} = \frac{1}{1 - e^{-sT}} \int_0^T e^{-s\tau} f(\tau) d\tau$
10	Derivada da transformada	$\mathcal{L}\left\{tf(t)\right\} = -\frac{dF(s)}{ds}$
11	Integral da transformada	$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_{s}^{\infty} F(s)ds$

Séries:

. :	Séries:
	$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 \cdots, -1 < x < 1$
	$\frac{x}{(1-x)^2} = \sum_{n=1}^{\infty} nx^n = x + 2x^2 + 3x^3 + \dots, -1 < x < 1$
	$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, -\infty < x < \infty$
	$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}, -1 < x < 1$
	$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, -1 < x < 1$
	$sen(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, -\infty < x < \infty$
	$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, -\infty < x < \infty$
	$senh(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}, -\infty < x < \infty$
	$\cosh(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}, -\infty < x < \infty$
	$(1+x)^m = 1 + \sum_{n=1}^{\infty} \frac{m(m-1)\cdots(m-n+1)}{n!} x^n,$
	$-1 < x < 1, \ m \neq 0, 1, 2, \dots$

Funções especiais:

Função Gamma	$\Gamma(k) = \int_0^\infty x^{k-1} e^{-x} dx$
Propriedade da Função Gamma	$\Gamma(k+1) = k\Gamma(k), k > 0$ $\Gamma(n+1) = n!, n \in \mathbb{N}$
Função de Bessel modificada de ordem ν	$I_{\nu}(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(m+\nu+1)} \left(\frac{x}{2}\right)^{2m+\nu}$
Função de Bessel de ordem 0	$J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!^2} \left(\frac{x}{2}\right)^{2m}$
Integral seno	$\operatorname{Si}(t) = \int_0^t \frac{\operatorname{sen}(x)}{x} dx$

Integrais:

megras.
$\int xe^{\lambda x} dx = \frac{e^{\lambda x}}{\lambda^2}(\lambda x - 1) + C$
$\int x^2 e^{\lambda x} dx = e^{\lambda x} \left(\frac{x^2}{\lambda} - \frac{2x}{\lambda^2} + \frac{2}{\lambda^3} \right) + C$
$\int x^n e^{\lambda x} dx = \frac{1}{\lambda} x^n e^{\lambda x} - \frac{n}{\lambda} \int x^{n-1} e^{\lambda x} dx + C$
$\int x \cos(\lambda x) dx = \frac{\cos(\lambda x) + \lambda x \sin(\lambda x)}{\lambda^2} + C$
$\int x \operatorname{sen}(\lambda x) dx = \frac{\operatorname{sen}(\lambda x) - \lambda x \operatorname{cos}(\lambda x)}{\lambda^2} + C$

abela de transformadas de Laplace: 29 $\sqrt{s-a} - \sqrt{s-b}$ $\frac{1}{2\sqrt{\pi t^3}}(e^{bt} - e^{at})$ $\frac{1}{1} = \frac{1}{s}$ 1 30 $\frac{1}{\sqrt{s+a\sqrt{s+b}}}$ $e^{\frac{-(a+b)t}{2}}I_0\left(\frac{a-b}{2}t\right)$ 2 $\frac{1}{s^2}$ 1 31 $\frac{1}{\sqrt{s^2+a^2}}$ 30 $\frac{1}{\sqrt{s+a\sqrt{s+b}}}$ 31 $\frac{1}{\sqrt{s^2+a^2}}$ 31 $\frac{1}{\sqrt{\pi t}}e^{at}(1+2at)$ 32 $\frac{s}{(s-a)^{\frac{3}{2}}}$ $\frac{1}{\sqrt{\pi t}}e^{at}(1+2at)$ 33 $\frac{1}{s^n}$, $(n=1,2,3,)$ $\frac{t^{n-1}}{(n-1)!}$ 32 $\frac{s}{(s-a)^{\frac{3}{2}}}$ $\frac{1}{\sqrt{\pi t}}e^{at}(1+2at)$ 33 $\frac{1}{(s^2-a^2)^k}$, $(k>0)$ $\frac{\sqrt{\pi}}{\Gamma(k)}\left(\frac{t}{2a}\right)^{k-\frac{1}{2}}I_{k-\frac{1}{2}}(at)$ 5 $\frac{1}{s^{\frac{3}{2}}}$, $2\sqrt{\frac{t}{\pi}}$ 34 $\frac{1}{s^2}e^{\frac{t}{s}}$, $(k>0)$ $J_0(2\sqrt{kt})$ 6 $\frac{1}{s^k}$, $(k>0)$ $\frac{t^{k-1}}{\Gamma(k)}$ 35 $\frac{1}{\sqrt{s}}e^{-\frac{k}{s}}$ $\frac{1}{\sqrt{\pi t}}\cos(2\sqrt{kt})$ 7 $\frac{1}{s-a}$ e^{at} 36 $\frac{1}{s^{\frac{1}{2}}}e^{\frac{k}{s}}$ $\frac{1}{\sqrt{\pi t}}\sin(2\sqrt{kt})$ 8 $\frac{1}{(s-a)^2}$ te^{at} 37 $e^{-k\sqrt{s}}$, $(k>0)$ $\frac{k}{2\sqrt{\pi t^3}}e^{-\frac{k^2}{4t}}$ 9 $\frac{1}{(s-a)^n}$, $(n=1,2,3,)$ $\frac{1}{(n-1)!}t^{n-1}e^{at}$ 38 $\frac{1}{s}\ln(s)$ $-\ln(t)-\gamma$, $(\gamma\approx0,5772)$ 10 $\frac{1}{(s-a)^k}$, $(k>0)$ $\frac{1}{\Gamma(k)}t^{k-1}e^{at}$ 39 $\ln\left(\frac{s-a}{s-b}\right)$ $\frac{1}{t}\left(e^{bt}-e^{at}\right)$ 11 $\frac{1}{(s-a)(s-b)}$, $(a\neq b)$ $\frac{1}{a-b}\left(e^{at}-e^{bt}\right)$ 40 $\ln\left(\frac{s^2+w^2}{s^2}\right)$ 2						
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$					$F(s) = \mathcal{L}\{f(t)\}\$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Tabela	•	2(1) 2-1(-1(1)	29	$\sqrt{s-a} - \sqrt{s-b}$	$\frac{1}{2\sqrt{\pi t^3}}(e^{bt}-e^{at})$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				30	1	• ***
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	1	<u>-</u> s	1		$\sqrt{s+a}\sqrt{s+b}$	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	2	$\frac{1}{s^2}$		31		$J_0(at)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	3	$\frac{1}{s^n}$, $(n = 1, 2, 3,)$		32	$\frac{s}{(s-a)^{\frac{3}{2}}}$	$\frac{1}{\sqrt{\pi t}}e^{at}(1+2at)$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	4	$\frac{1}{\sqrt{s}}$,		33	$\frac{1}{(s^2 - a^2)^k}, \qquad (k > 0)$	$\frac{\sqrt{\pi}}{\Gamma(k)} \left(\frac{t}{2a}\right)^{k-\frac{1}{2}} I_{k-\frac{1}{2}}(at)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	5		$2\sqrt{\frac{t}{\pi}}$	34	$\frac{1}{2}e^{-\frac{k}{3}} \qquad (k > 0)$	$J_0(2\sqrt{kt})$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	6		-	35	$\frac{1}{\sqrt{s}}e^{-\frac{k}{s}}$	$\frac{1}{\sqrt{\pi t}}\cos(2\sqrt{kt})$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	7	1		36	$\frac{1}{s^{\frac{3}{2}}}e^{\frac{k}{s}}$	$\frac{1}{\sqrt{\pi t}} \operatorname{senh}(2\sqrt{kt})$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	8	±	te^{at}	37	$e^{-k\sqrt{s}}, \qquad (k>0)$	$\frac{k}{2\sqrt{\pi t^3}}e^{-\frac{k^2}{4t}}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	9	, ,	$\frac{1}{(n-1)!}t^{n-1}e^{at}$	38	$\frac{1}{s}\ln(s)$	$-\ln(t) - \gamma, \qquad (\gamma \approx 0, 5772)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	10	$\frac{1}{(s-a)^k}, \qquad (k>0)$	$\frac{1}{\Gamma(k)}t^{k-1}e^{at}$	39	(8 0)	$\frac{1}{t}\left(e^{bt} - e^{at}\right)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	11	$\frac{1}{(s-a)(s-b)}, \qquad (a \neq b)$		40	(3 /	$\frac{2}{t}\left(1-\cos(wt)\right)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	7 12			41	$\ln\left(\frac{s^2 - a^2}{s^2}\right)$	$\frac{2}{t}\left(1-\cosh(at)\right)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	13	1		42	$\tan^{-1}\left(\frac{w}{s}\right)$	$\frac{1}{t}\operatorname{sen}(wt)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	14		$\cos(wt)$	43	$\frac{1}{s}\cot^{-1}(s)$	$\mathrm{Si}\left(t ight)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	15	$\frac{1}{s^2 - a^2}$	$\frac{1}{a}\operatorname{senh}(at)$			Onda quadrada
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	16	$\frac{s}{s^2 - a^2}$, ,	44	$\frac{1}{s}\tanh\left(\frac{as}{2}\right)$	$f(t) = \begin{cases} 1, & 0 < t < a \\ -1, & a < t < 2a \end{cases}$
$ \frac{19}{s(s^2 + w^2)} \frac{1}{w^2} (1 - \cos(wt)) $ $ \frac{1}{s^2(s^2 + w^2)} \frac{1}{w^3} (wt - \sin(wt)) $ $ \frac{1}{s^2(s^2 + w^2)^2} \frac{1}{2w^3} (\sin(wt) - wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{2w} \sin(wt) - wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{2w} \sin(wt) - wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{2w} (\sin(wt) + wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{(s^2 + w^2)^2} \frac{1}{2w^3} (\cos(at) - \cos(bt)) $ $ \frac{s}{(s^2 + a^2)(s^2 + b^2)}, $ $ \frac{1}{(a^2 \neq b^2)} \frac{1}{b^2 - a^2} (\cos(at) - \cos(bt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{(1 - e^{-\frac{\pi}{w}s})} $ $ \frac{1}{(s^2 + w^2)^2} (1 - e^{-\frac{\pi}{w}s}) $ $ \frac{s}{(s^2 + w^2)^2} (1 - e^{-\frac{\pi}$	17		$\frac{1}{w}e^{at}\operatorname{sen}(wt)$			f(t+2a) = f(t), t > 0
$ \frac{19}{s(s^2 + w^2)} \frac{1}{w^2} (1 - \cos(wt)) $ $ \frac{1}{s^2(s^2 + w^2)} \frac{1}{w^3} (wt - \sin(wt)) $ $ \frac{1}{s^2(s^2 + w^2)^2} \frac{1}{2w^3} (\sin(wt) - wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{2w} \sin(wt) - wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{2w} \sin(wt) - wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{2w} (\sin(wt) + wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{(s^2 + w^2)^2} \frac{1}{2w^3} (\cos(at) - \cos(bt)) $ $ \frac{s}{(s^2 + a^2)(s^2 + b^2)}, $ $ \frac{1}{(a^2 \neq b^2)} \frac{1}{b^2 - a^2} (\cos(at) - \cos(bt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{(1 - e^{-\frac{\pi}{w}s})} $ $ \frac{1}{(s^2 + w^2)^2} (1 - e^{-\frac{\pi}{w}s}) $ $ \frac{s}{(s^2 + w^2)^2} (1 - e^{-\frac{\pi}$	18	$\frac{s-a}{(s-a)^2+w^2}$, ,			Onda triangular
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	19		$\frac{1}{w^2}(1-\cos(wt))$			(t
$ \frac{s}{(s^{2}+w^{2})^{2}} = \frac{t}{2w} \operatorname{sen}(wt) \\ 23 \frac{s^{2}}{(s^{2}+w^{2})^{2}} = \frac{1}{2w} (\operatorname{sen}(wt) + wt \cos(wt)) \\ 24 \frac{s}{(s^{2}+a^{2})(s^{2}+b^{2})}, \\ (a^{2} \neq b^{2}) = \frac{1}{b^{2}-a^{2}} (\cos(at) - \cos(bt)) \\ 25 \frac{1}{(s^{4}+4a^{4})} = \frac{1}{4a^{3}} [\operatorname{sen}(at) \cosh(at) - \cos(at)] \\ 26 \frac{s}{(s^{4}+4a^{4})} = \frac{1}{2a^{2}} \operatorname{sen}(at) \operatorname{senh}(at)] \\ 27 \frac{1}{(s^{4}-a^{2})} = \frac{1}{2a^{3}} (\operatorname{senh}(at) - \operatorname{sen}(at)) \\ 28 \frac{s}{(s^{4}-a^{4})} = \frac{1}{2a^{2}} (\cosh(at) - \cos(at)) \\ 29 \frac{s}{(s^{4}-a^{4})} = \frac{1}{2a^{2}} (\cosh(at) - \cos(at)) \\ 40 \frac{w}{(s^{2}+w^{2})} \left(1 - e^{-\frac{\pi}{w}s}\right) \\ \frac{w}{(s$	20	$\frac{1}{s^2(s^2+w^2)}$	$\frac{1}{w^3}(wt - \operatorname{sen}(wt))$	45	$\frac{1}{as^2}\tanh\left(\frac{as}{2}\right)$	$f(t) = \begin{cases} a, & 0 < t < a \\ -\frac{t}{a} + 2, & a < t < 2a \end{cases}$
$ \frac{s^{2}}{(s^{2}+w^{2})^{2}} = \frac{1}{2w}(\operatorname{sen}(wt) + wt \operatorname{cos}(wt)) $ 24 $ \frac{s}{(s^{2}+a^{2})(s^{2}+b^{2})}, \frac{1}{b^{2}-a^{2}}(\cos(at) - \cos(bt)) $ 25 $ \frac{1}{(s^{4}+4a^{4})} = \frac{1}{4a^{3}}[\operatorname{sen}(at) \cosh(at) - \cos(at)] $ 26 $ \frac{s}{(s^{4}+4a^{4})} = \frac{1}{2a^{2}} \operatorname{sen}(at) \operatorname{senh}(at)) $ 27 $ \frac{1}{(s^{4}-a^{2})} = \frac{1}{2a^{3}}(\operatorname{senh}(at) - \operatorname{sen}(at)) $ 28 $ \frac{s}{(s^{4}-a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ 46 $ \frac{w}{(s^{2}+w^{2})\left(1-e^{-\frac{\pi}{w}s}\right)} = f(t) = \begin{cases} \operatorname{sen}(wt), & 0 < t < \frac{\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases} $ $ f\left(t + \frac{2\pi}{w}\right) = f(t), & t > 0 \end{cases} $ Retificador de meia onda $ f(t) = \begin{cases} \operatorname{sen}(wt), & 0 < t < \frac{\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases} $ $ f\left(t + \frac{2\pi}{w}\right) = f(t), & t > 0 \end{cases} $ Onda dente de serra $ f(t) = \begin{cases} \operatorname{sen}(wt), & 0 < t < \frac{\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases} $ Onda dente de serra $ f(t) = \begin{cases} \operatorname{sen}(wt), & 0 < t < \frac{\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases} $ Onda dente de serra $ f(t) = \begin{cases} \operatorname{sen}(wt), & 0 < t < \frac{\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases} $ $ f(t) = \begin{cases} \operatorname{sen}(wt), & 0 < t < \frac{\pi}{w} < t < \frac{2\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases} $ $ f(t) = \begin{cases} \operatorname{sen}(wt), & 0 < t < \frac{\pi}{w} < t < \frac{2\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} < t < \frac{2\pi}{w} \end{aligned} $ $ f(t) = \begin{cases} \operatorname{sen}(wt), & 0 < t < \frac{\pi}{w} < $	21	, , ,	$\frac{1}{2w^3}(\operatorname{sen}(wt) - wt \cos(wt))$			f(t+2a) = f(t), t > 0
$ \frac{s}{(s^{2} + w^{2})^{2}} = \frac{\frac{1}{2w}(\operatorname{sen}(wt) + wt \cos(wt))}{\frac{s}{(s^{2} + a^{2})(s^{2} + b^{2})}}, \frac{1}{b^{2} - a^{2}}(\cos(at) - \cos(bt)) $ $ \frac{s}{(s^{2} + a^{2})(s^{2} + b^{2})}, \frac{1}{b^{2} - a^{2}}(\cos(at) - \cos(bt)) $ $ \frac{1}{(s^{4} + 4a^{4})} = \frac{\frac{1}{4a^{3}}[\operatorname{sen}(at) \cosh(at) \cos(at) \operatorname{senh}(at)]}{-\cos(at) \operatorname{senh}(at)} $ $ \frac{s}{(s^{4} + 4a^{4})} = \frac{1}{2a^{2}} \operatorname{sen}(at) \operatorname{senh}(at)) $ $ \frac{s}{(s^{4} + 4a^{4})} = \frac{1}{2a^{2}} \operatorname{sen}(at) \operatorname{senh}(at)) $ $ \frac{s}{(s^{4} + 4a^{4})} = \frac{1}{2a^{2}} \operatorname{sen}(at) \operatorname{senh}(at)) $ $ \frac{s}{(s^{4} - a^{2})} = \frac{1}{2a^{3}} (\operatorname{senh}(at) - \operatorname{sen}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}} (\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}} (\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}} (\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}} (\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}} (\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}} (\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}} (\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}} (\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{2})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $ $ \frac{s}{(s^{4} - a^{4})} = \frac{s}{(s^{4} - a^{4})} $	22		$\frac{t}{2w}\operatorname{sen}(wt)$			Retificador do maio ando
$(a^{2} \neq b^{2})$ $\frac{1}{(s^{4} + 4a^{4})}$ $\frac{1}{4a^{3}}[\operatorname{sen}(at) \operatorname{cosh}(at) \operatorname{cos}(at) \operatorname{senh}(at)]$ $\frac{s}{(s^{4} + 4a^{4})}$ $\frac{1}{2a^{2}} \operatorname{sen}(at) \operatorname{senh}(at)$ $\frac{1}{2a^{2}} \operatorname{sen}(at) \operatorname{senh}(at)$ $\frac{1}{(s^{4} - a^{2})}$ $\frac{1}{2a^{3}}(\operatorname{senh}(at) - \operatorname{sen}(at))$ $\frac{1}{2a^{3}}(\operatorname{senh}(at) - \operatorname{sen}(at))$ $\frac{1}{2a^{3}}(\operatorname{cosh}(at) - \operatorname{cos}(at))$ $\frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at))$ $\frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at))$ $\frac{1}{as^{2}} - \frac{e^{-as}}{s(1 - e^{-as})}$ $f(t) = \frac{t}{a}, 0 < t < a$	23	$\frac{s^2}{(s^2+w^2)^2}$	$\frac{1}{2w}(\operatorname{sen}(wt) + wt \cos(wt))$			σ.
$ \frac{1}{4a^3}[\operatorname{sen}(at) \cosh(at) - \cos(at) \operatorname{senh}(at)] - \cos(at) \operatorname{senh}(at)] = \frac{1}{2a^2} \operatorname{sen}(at) \operatorname{senh}(at) = \frac{1}{2a^2} \operatorname{sen}(at) \operatorname{senh}(at) = \frac{1}{2a^2} \operatorname{sen}(at) \operatorname{senh}(at) = \frac{1}{2a^2} \operatorname{sen}(at) \operatorname{senh}(at) = \frac{1}{2a^3} \operatorname{senh}(at) - \operatorname{sen}(at) = \frac{1}{2a^3} \operatorname{senh}(at) - \operatorname{senh}(at) =$	24	, , , ,	$\frac{1}{b^2 - a^2} (\cos(at) - \cos(bt))$	46	$\frac{w}{(s^2+w^2)\left(1-e^{-\frac{\pi}{w}s}\right)}$	$\overset{\bullet}{}$ w w
$ \frac{s}{(s^4 + 4a^4)} \qquad \frac{1}{2a^2} \operatorname{sen}(at) \operatorname{senh}(at)) \qquad 47 \qquad \frac{w}{s^2 + w^2} \operatorname{coth}\left(\frac{\pi s}{2w}\right) \qquad \operatorname{hethicator de office complete} $ $ \frac{1}{(s^4 + 4a^4)} \qquad \frac{1}{2a^2} \operatorname{sen}(at) \operatorname{senh}(at)) \qquad 48 \qquad \frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})} \qquad f(t) = \frac{t}{a}, 0 < t < a $ $ \frac{t}{(t)} = \operatorname{sen}(wt) $ Onda dente de serra $ f(t) = \frac{t}{a}, 0 < t < a $	25	$\frac{1}{(s^4 + 4a^4)}$	$4a^{\circ}$			
27 $\frac{1}{(s^4 - a^2)}$ $\frac{1}{2a^3}(\operatorname{senh}(at) - \operatorname{sen}(at))$ Onda dente de serra 28 $\frac{s}{(s^4 - a^4)}$ $\frac{1}{2a^2}(\cosh(at) - \cos(at))$ 48 $\frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}$ $f(t) = \frac{t}{a}, 0 < t < a$	26	$\frac{s}{(s^4 + 4a^4)}$	1	47	$\frac{w}{s^2 + w^2} \coth\left(\frac{\pi s}{2w}\right)$	•
28 $\frac{s}{(s^4 - a^4)}$ $\frac{1}{2a^2}(\cosh(at) - \cos(at))$ 48 $\frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}$ $f(t) = \frac{t}{a}, 0 < t < a$	27	1	$\frac{1}{2a^3}(\operatorname{senh}(at) - \operatorname{sen}(at))$			Onda dente de serra
1	28	s		48	$\frac{1}{as^2} - \frac{e^{-as}}{s\left(1 - e^{-as}\right)}$	$f(t) = \frac{t}{a}, \qquad 0 < t < a$
		· /		,	, , , , ,	f(t) = f(t - a), t > a

• Questão 1 (1.0 ponto) A transformada inversa de Laplace da função
$$\frac{e^{-3s}}{(s^2-3s+2)}$$
 é

()
$$u(t-3) \left(e^{2t} - e^{t}\right)$$

()
$$u(t-3)\left(2e^{\frac{3}{2}(t-3)}\operatorname{senh}\left(\frac{t-3}{2}\right)\right)$$

$$() e^{2(t-3)} - e^{t-3}$$

$$(\)\ u(t-3)\left(2e^{\frac{3}{2}t}\operatorname{senh}\left(\frac{t}{2}\right)\right)$$

()
$$e^{-3t} \left(e^{2(t-3)} - e^{t-3} \right)$$

$$() e^{-3t} \left(2e^{\frac{3}{2}(t-3)} \operatorname{senh}\left(\frac{t-3}{2}\right) \right)$$

• Questão 2 (1.0 ponto) Sabendo que $\mathcal{L}{f(t)} = F(s)$ e que os limites $\lim_{t\to 0+} \frac{f(t)}{t}$ e $\lim_{t\to 0+} \frac{f(t)}{t^2}$ existem, é correto afirmar que

$$(\) \mathcal{L}\left\{\frac{f(t)}{t^2}\right\} = s^2 \int_s^\infty F(u) du$$

$$(\)\ \mathcal{L}\left\{\frac{f(t)}{t^2}\right\} = s \int_s^\infty F(u) du$$

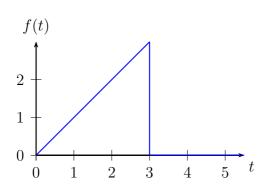
$$(\quad) \ \mathcal{L}\left\{\frac{f(t)}{t^2}\right\} = s^2 F(s)$$

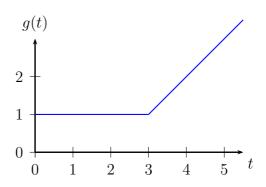
$$(\quad) \ \mathcal{L}\left\{\frac{f(t)}{t^2}\right\} = \frac{1}{s^2}F(s)$$

()
$$\mathcal{L}\left\{\frac{f(t)}{t^2}\right\} = \frac{1}{s} \int_{0}^{\infty} F(u) du$$

$$(\)\ \mathcal{L}\left\{\frac{f(t)}{t^2}\right\} = \int_s^\infty \int_v^\infty F(u) du dv$$

ullet Questão 3 (1.0 ponto) Considere as funções f e g dadas nos gráficos abaixo:





É correto afirmar que

()
$$\mathcal{L}{f(t)g(t)} = \frac{1 - e^{-3s}}{s^2} \in \mathcal{L}{u(t-4)f(t)g(t)} = 0$$

()
$$\mathcal{L}{f(t)g(t)} = \frac{1 - e^{-3s}}{s^2} \in \mathcal{L}{u(t-4)f(t)g(t)} = e^{-4s}$$

()
$$\mathcal{L}{f(t)g(t)} = \frac{1 - e^{-3s}}{s^2} \in \mathcal{L}{u(t-4)f(t)g(t)} = 4e^{-4s}$$

()
$$\mathcal{L}{f(t)g(t)} = \frac{1 - e^{-3s} - 3se^{-3s}}{s^2} \in \mathcal{L}{u(t-4)f(t)g(t)} = 0$$

()
$$\mathcal{L}{f(t)g(t)} = \frac{1 - e^{-3s} - 3se^{-3s}}{s^2} \in \mathcal{L}{u(t-4)f(t)g(t)} = e^{-4s}$$

()
$$\mathcal{L}{f(t)g(t)} = \frac{1 - e^{-3s} - 3se^{-3s}}{s^2} \in \mathcal{L}{u(t-4)f(t)g(t)} = 4e^{-4s}$$

 \bullet Questão 4 (1.0 ponto) A transformada de Laplace da função $f(t) = \mathrm{sen}(t) \delta(t-1)$ é

$$(\) \frac{1}{1 - e^{-s}}$$

$$() \frac{\operatorname{sen}(1)}{1 - e^s}$$

$$(\) \frac{e^{-s}}{s^2+1}$$

()
$$sen(1)e^{-s}$$

$$() \frac{\pi}{s^2+1}$$

$$(\)\ 0$$

• Questão 5 (1.0 ponto) Considere o circuito RLC regido pela equação

$$\begin{cases} y'' + Ry' + \frac{1}{C}y = \delta(t) \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$$

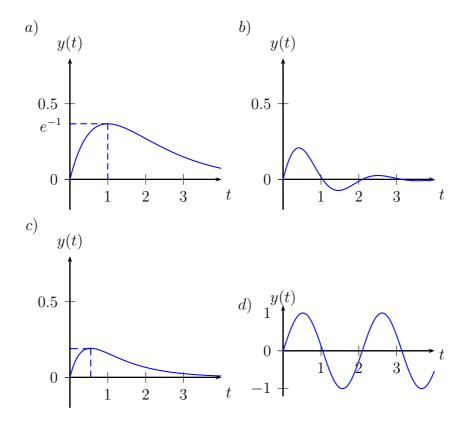
Também considere alguns valores para a capacitância C e a resistência R:

i)
$$R = 0 e C = \frac{1}{9}$$

ii)
$$R = 2 e C = \frac{1}{10}$$

iii)
$$R = 4 e C = \frac{1}{3}$$

iv)
$$R = 2 e C = 1$$



Relacione os itens i), ii), iii) e iv) aos itens a), b), c) e d) [Cada item relacionado corretamente vale 0.25 pontos].

• Questão 6 (1.0 ponto) Marque a opção que apresenta a transformada inversa da função F(s) = 6

$$\frac{\sigma}{(s^2-1)(s+2)}$$

()
$$f(t) = -\frac{6}{5}\cos(t) + \frac{12}{5}\sin(t) + \frac{6}{5}e^{-2t}$$

()
$$f(t) = -3e^t + 2e^{2t} + e^{-t}$$

()
$$f(t) = -e^t - 2e^{-2t} + 3e^{-t}$$

()
$$f(t) = -\frac{6}{5}\cosh(t) + \frac{12}{5}\operatorname{senh}(t) + \frac{6}{5}e^{-2t}$$

()
$$f(t) = e^t + 2e^{-2t} - 3e^{-t}$$

• Questão 7 (2.0 pontos) Considere a função

$$f(t) = \begin{cases} t, & 0 < t < 1; \\ 1, & 1 < t < 2; \\ -1, & t > 2. \end{cases}$$

- a) (0.5) Esboce o gráfico da função f(t).
- b) (0.5) Esboce o gráfico da função g(t) = f'(t).
- c) (1.0) Calcule a transformada de Laplace $F(s) = \mathcal{L}\{f(t)\}$ e $G(s) = \mathcal{L}\{g(t)\}$

 \bullet Questão 8 (2.0 pontos) Considere o seguinte problema de valor inicial para um sistema de equações integro-diferenciais:

$$x'(t) + x(t) = 2y(t)$$
$$x(t) = \int_0^t y(\tau)d\tau + 1$$

com x(0) = 0. Usando a teoria das Transformadas de Laplace, resolve o sistema, obtendo x(t) e y(t). **Obs:** Este sistema apresenta "problemas na origem".