## UFRGS - INSTITUTO DE MATEMÁTICA Departamento de Matemática Pura e Aplicada MAT01168 - Turma D - 2016/1Prova da área IA

1 - 6	7	8	Total

Nome:	Cartão:	

Regras Gerais:

- $\bullet\,$ Não é permitido o uso de calculadoras, telefones ou qualquer outro recurso computacional ou de comunicação.
- Trabalhe individualmente e sem uso de material de consulta além do fornecido.
- Devolva o caderno de questões preenchido ao final da prova.

Regras para as questões abertas:

- Seja sucinto, completo e claro.
- $\bullet\,$  Justifique todo procedimento usado.
- Indique identidades matemáticas usadas, em especial, itens da tabela.
- Use notação matemática consistente.

Identidades:			
$\operatorname{sen}(x) = \frac{e^{ix} - e^{-ix}}{2i}$	$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$		
$\operatorname{senh}(x) = \frac{e^x - e^{-x}}{2}$	$\cosh(x) = \frac{e^x + e^{-x}}{2}$		
$(a+b)^n = \sum_{j=0}^{\infty} \binom{n}{j} a^{n-j}$	$-jb^j$ , $\binom{n}{j} = \frac{n!}{j!(n-j)!}$		
sen(x + y) = sen(x)cos(y) + sen(y)cos(x)			
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$			

Propriedades:

1	Linearidade	$\mathcal{L}\left\{\alpha f(t) + \beta g(t)\right\} = \alpha \mathcal{L}\left\{f(t)\right\} + \beta \mathcal{L}\left\{g(t)\right\}$
2	Transformada da derivada	$\mathcal{L}\left\{f'(t)\right\} = s\mathcal{L}\left\{f(t)\right\} - f(0)$ $\mathcal{L}\left\{f''(t)\right\} = s^2\mathcal{L}\left\{f(t)\right\} - sf(0) - f'(0)$
3	Deslocamento no eixo $s$	$\mathcal{L}\left\{e^{at}f(t)\right\} = F(s-a)$
4	Deslocamento no eixo $t$	$\mathcal{L}\left\{u(t-a)f(t-a)\right\} = e^{-as}F(s)$ $\mathcal{L}\left\{u(t-a)\right\} = \frac{e^{-as}}{s}$
5	Transformada da integral	$\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{F(s)}{s}$
6	Filtragem da Delta de Dirac	$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$
7	Transformada da Delta de Dirac	$\mathcal{L}\left\{\delta(t-a)\right\} = e^{-as}$
8	Teorema da Convolução	$\mathcal{L}\left\{(f*g)(t)\right\} = F(s)G(s),$ onde $(f*g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$
9	Transformada de funções periódicas	$\mathcal{L}\left\{f(t)\right\} = \frac{1}{1 - e^{-sT}} \int_0^T e^{-s\tau} f(\tau) d\tau$
10	Derivada da transformada	$\mathcal{L}\left\{tf(t)\right\} = -\frac{dF(s)}{ds}$
11	Integral da transformada	$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_{s}^{\infty} F(s)ds$

Séries:
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 \cdots,  -1 < x < 1$
$\frac{x}{(1-x)^2} = \sum_{n=1}^{\infty} nx^n = x + 2x^2 + 3x^3 + \dots, -1 < x < 1$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots,  -\infty < x < \infty$
$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1},  -1 < x < 1$
$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1},  -1 < x < 1$
$sen(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!},  -\infty < x < \infty$
$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!},  -\infty < x < \infty$
$senh(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!},  -\infty < x < \infty$
$\cosh(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!},  -\infty < x < \infty$
$(1+x)^m = 1 + \sum_{n=1}^{\infty} \frac{m(m-1)\cdots(m-n+1)}{n!} x^n,$
$-1 < x < 1, \ m \neq 0, 1, 2, \dots$

Funções especiais:

3 1	tulições especials.		
Função Gamma	$\Gamma(k) = \int_0^\infty x^{k-1} e^{-x} dx$		
Propriedade da Função Gamma	$\Gamma(k+1) = k\Gamma(k),  k > 0$ $\Gamma(n+1) = n!,  n \in \mathbb{N}$		
Função de Bessel modificada de ordem $\nu$	$I_{\nu}(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(m+\nu+1)} \left(\frac{x}{2}\right)^{2m+\nu}$		
Função de Bessel de ordem 0	$J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!^2} \left(\frac{x}{2}\right)^{2m}$		
Integral seno	$\operatorname{Si}(t) = \int_0^t \frac{\operatorname{sen}(x)}{x} dx$		

Integrais:

$$\int xe^{\lambda x} dx = \frac{e^{\lambda x}}{\lambda^2} (\lambda x - 1) + C$$

$$\int x^2 e^{\lambda x} dx = e^{\lambda x} \left( \frac{x^2}{\lambda} - \frac{2x}{\lambda^2} + \frac{2}{\lambda^3} \right) + C$$

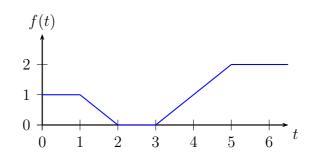
$$\int x^n e^{\lambda x} dx = \frac{1}{\lambda} x^n e^{\lambda x} - \frac{n}{\lambda} \int x^{n-1} e^{\lambda x} dx + C$$

$$\int x \cos(\lambda x) dx = \frac{\cos(\lambda x) + \lambda x \sin(\lambda x)}{\lambda^2} + C$$

$$\int x \sin(\lambda x) dx = \frac{\sin(\lambda x) - \lambda x \cos(\lambda x)}{\lambda^2} + C$$

abela de transformadas de Laplace: 29 $\sqrt{s-a} - \sqrt{s-b}$ $\frac{1}{2\sqrt{\pi t^3}}(e^{bt} - e^{at})$ $\frac{1}{1} = \frac{1}{s}$ 1 30 $\frac{1}{\sqrt{s+a\sqrt{s+b}}}$ $e^{\frac{-(a+b)t}{2}}I_0\left(\frac{a-b}{2}t\right)$ 2 $\frac{1}{s^2}$ 1 31 $\frac{1}{\sqrt{s^2+a^2}}$ 30 $\frac{1}{\sqrt{s+a\sqrt{s+b}}}$ 31 $\frac{1}{\sqrt{s^2+a^2}}$ 31 $\frac{1}{\sqrt{\pi t}}e^{at}(1+2at)$ 32 $\frac{s}{(s-a)^{\frac{3}{2}}}$ $\frac{1}{\sqrt{\pi t}}e^{at}(1+2at)$ 33 $\frac{1}{s^n}$ , $(n=1,2,3,)$ $\frac{t^{n-1}}{(n-1)!}$ 32 $\frac{s}{(s-a)^{\frac{3}{2}}}$ $\frac{1}{\sqrt{\pi t}}e^{at}(1+2at)$ 33 $\frac{1}{(s^2-a^2)^k}$ , $(k>0)$ $\frac{\sqrt{\pi}}{\Gamma(k)}\left(\frac{t}{2a}\right)^{k-\frac{1}{2}}I_{k-\frac{1}{2}}(at)$ 5 $\frac{1}{s^{\frac{3}{2}}}$ , $2\sqrt{\frac{t}{\pi}}$ 34 $\frac{1}{s^2}e^{\frac{t}{s}}$ , $(k>0)$ $J_0(2\sqrt{kt})$ 6 $\frac{1}{s^k}$ , $(k>0)$ $\frac{t^{k-1}}{\Gamma(k)}$ 35 $\frac{1}{\sqrt{s}}e^{-\frac{k}{s}}$ $\frac{1}{\sqrt{\pi t}}\cos(2\sqrt{kt})$ 7 $\frac{1}{s-a}$ $e^{at}$ 36 $\frac{1}{s^{\frac{1}{2}}}e^{\frac{k}{s}}$ $\frac{1}{\sqrt{\pi t}}\sin(2\sqrt{kt})$ 8 $\frac{1}{(s-a)^2}$ $te^{at}$ 37 $e^{-k\sqrt{s}}$ , $(k>0)$ $\frac{k}{2\sqrt{\pi t^3}}e^{-\frac{k^2}{4t}}$ 9 $\frac{1}{(s-a)^n}$ , $(n=1,2,3,)$ $\frac{1}{(n-1)!}t^{n-1}e^{at}$ 38 $\frac{1}{s}\ln(s)$ $-\ln(t)-\gamma$ , $(\gamma\approx0,5772)$ 10 $\frac{1}{(s-a)^k}$ , $(k>0)$ $\frac{1}{\Gamma(k)}t^{k-1}e^{at}$ 39 $\ln\left(\frac{s-a}{s-b}\right)$ $\frac{1}{t}\left(e^{bt}-e^{at}\right)$ 11 $\frac{1}{(s-a)(s-b)}$ , $(a\neq b)$ $\frac{1}{a-b}\left(e^{at}-e^{bt}\right)$ 40 $\ln\left(\frac{s^2+w^2}{s^2}\right)$ 2						
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$					$F(s) = \mathcal{L}\{f(t)\}\$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Tabela	•	2(1) 2-1(-1(1)	29	$\sqrt{s-a} - \sqrt{s-b}$	$\frac{1}{2\sqrt{\pi t^3}}(e^{bt}-e^{at})$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				30	1	• ***
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	1	<u>-</u> s	1		$\sqrt{s+a}\sqrt{s+b}$	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	2	$\frac{1}{s^2}$		31		$J_0(at)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	3	$\frac{1}{s^n}$ , $(n = 1, 2, 3,)$		32	$\frac{s}{(s-a)^{\frac{3}{2}}}$	$\frac{1}{\sqrt{\pi t}}e^{at}(1+2at)$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	4	$\frac{1}{\sqrt{s}}$ ,		33	$\frac{1}{(s^2 - a^2)^k}, \qquad (k > 0)$	$\frac{\sqrt{\pi}}{\Gamma(k)} \left(\frac{t}{2a}\right)^{k-\frac{1}{2}} I_{k-\frac{1}{2}}(at)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	5	$\frac{1}{\frac{3}{s^2}}$	$2\sqrt{\frac{t}{\pi}}$	34	$\frac{1}{2}e^{-\frac{k}{s}} \qquad (k > 0)$	$J_0(2\sqrt{kt})$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	6			35	$\frac{1}{\sqrt{s}}e^{-\frac{k}{s}}$	$\frac{1}{\sqrt{\pi t}}\cos(2\sqrt{kt})$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	7	1		36	$\frac{1}{s^{\frac{3}{2}}}e^{\frac{k}{s}}$	$\frac{1}{\sqrt{\pi t}} \operatorname{senh}(2\sqrt{kt})$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	8	±	$te^{at}$	37	$e^{-k\sqrt{s}}, \qquad (k>0)$	$\frac{k}{2\sqrt{\pi t^3}}e^{-\frac{k^2}{4t}}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	9	, ,	$\frac{1}{(n-1)!}t^{n-1}e^{at}$	38	$\frac{1}{s}\ln(s)$	$-\ln(t) - \gamma, \qquad (\gamma \approx 0, 5772)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	10	$\frac{1}{(s-a)^k}, \qquad (k>0)$	$\frac{1}{\Gamma(k)}t^{k-1}e^{at}$	39	(8 0)	$\frac{1}{t}\left(e^{bt} - e^{at}\right)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	11	$\frac{1}{(s-a)(s-b)}, \qquad (a \neq b)$		40	( 3 /	$\frac{2}{t}\left(1-\cos(wt)\right)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	7 12			41	$\ln\left(\frac{s^2 - a^2}{s^2}\right)$	$\frac{2}{t}\left(1-\cosh(at)\right)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	13	1		42	$\tan^{-1}\left(\frac{w}{s}\right)$	$\frac{1}{t}\operatorname{sen}(wt)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	14	S	$\cos(wt)$	43	$\frac{1}{s}\cot^{-1}(s)$	$\mathrm{Si}\left(t ight)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	15	$\frac{1}{s^2 - a^2}$	$\frac{1}{a}\operatorname{senh}(at)$			Onda quadrada
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	16	$\frac{s}{s^2 - a^2}$	, ,	44	$\frac{1}{s} \tanh\left(\frac{as}{2}\right)$	$f(t) = \begin{cases} 1, & 0 < t < a \\ -1, & a < t < 2a \end{cases}$
$ \frac{19}{s(s^2 + w^2)} \frac{1}{w^2} (1 - \cos(wt)) $ $ \frac{1}{s^2(s^2 + w^2)} \frac{1}{w^3} (wt - \sin(wt)) $ $ \frac{1}{s^2(s^2 + w^2)^2} \frac{1}{2w^3} (\sin(wt) - wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{2w} \sin(wt) - wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{2w} \sin(wt) - wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{2w} (\sin(wt) + wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{(s^2 + w^2)^2} \frac{1}{2w^3} (\cos(at) - \cos(bt)) $ $ \frac{s}{(s^2 + a^2)(s^2 + b^2)}, $ $ \frac{1}{(a^2 \neq b^2)} \frac{1}{b^2 - a^2} (\cos(at) - \cos(bt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{(1 - e^{-\frac{\pi}{w}s})} $ $ \frac{1}{(s^2 + w^2)^2} (1 - e^{-\frac{\pi}{w}s}) $ $ \frac{s}{(s^2 + w^2)^2} (1 - e^{-\frac{\pi}$	17		$\frac{1}{w}e^{at}\operatorname{sen}(wt)$			f(t+2a) = f(t),  t > 0
$ \frac{19}{s(s^2 + w^2)} \frac{1}{w^2} (1 - \cos(wt)) $ $ \frac{1}{s^2(s^2 + w^2)} \frac{1}{w^3} (wt - \sin(wt)) $ $ \frac{1}{s^2(s^2 + w^2)^2} \frac{1}{2w^3} (\sin(wt) - wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{2w} \sin(wt) - wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{2w} \sin(wt) - wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{2w} (\sin(wt) + wt \cos(wt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{(s^2 + w^2)^2} \frac{1}{2w^3} (\cos(at) - \cos(bt)) $ $ \frac{s}{(s^2 + a^2)(s^2 + b^2)}, $ $ \frac{1}{(a^2 \neq b^2)} \frac{1}{b^2 - a^2} (\cos(at) - \cos(bt)) $ $ \frac{s}{(s^2 + w^2)^2} \frac{1}{(1 - e^{-\frac{\pi}{w}s})} $ $ \frac{1}{(s^2 + w^2)^2} (1 - e^{-\frac{\pi}{w}s}) $ $ \frac{s}{(s^2 + w^2)^2} (1 - e^{-\frac{\pi}$	18	$\frac{s-a}{(s-a)^2+w^2}$	, ,			Onda triangular
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	19		$\frac{1}{w^2}(1-\cos(wt))$			( t
$ \frac{s}{(s^{2}+w^{2})^{2}} = \frac{t}{2w} \operatorname{sen}(wt) \\ 23  \frac{s^{2}}{(s^{2}+w^{2})^{2}} = \frac{1}{2w} (\operatorname{sen}(wt) + wt \cos(wt)) \\ 24  \frac{s}{(s^{2}+a^{2})(s^{2}+b^{2})}, \\ (a^{2} \neq b^{2}) = \frac{1}{b^{2}-a^{2}} (\cos(at) - \cos(bt)) \\ 25  \frac{1}{(s^{4}+4a^{4})} = \frac{1}{4a^{3}} [\operatorname{sen}(at) \cosh(at) - \cos(at)] \\ 26  \frac{s}{(s^{4}+4a^{4})} = \frac{1}{2a^{2}} \operatorname{sen}(at) \operatorname{senh}(at)] \\ 27  \frac{1}{(s^{4}-a^{2})} = \frac{1}{2a^{3}} (\operatorname{senh}(at) - \operatorname{sen}(at)) \\ 28  \frac{s}{(s^{4}-a^{4})} = \frac{1}{2a^{2}} (\cosh(at) - \cos(at)) \\ 29  \frac{s}{(s^{4}-a^{4})} = \frac{1}{2a^{2}} (\cosh(at) - \cos(at)) \\ 40  \frac{w}{(s^{2}+w^{2})} \left(1 - e^{-\frac{\pi}{w}s}\right) \\ \frac{w}{(s$	20	$\frac{1}{s^2(s^2+w^2)}$	$\frac{1}{w^3}(wt - \operatorname{sen}(wt))$	45	$\frac{1}{as^2}\tanh\left(\frac{as}{2}\right)$	$f(t) = \begin{cases} a, & \text{of } t < a \\ -\frac{t}{a} + 2, & \text{of } a < t < 2a \end{cases}$
$ \frac{s^{2}}{(s^{2}+w^{2})^{2}} = \frac{1}{2w}(\operatorname{sen}(wt) + wt \operatorname{cos}(wt)) $ 24 $ \frac{s}{(s^{2}+a^{2})(s^{2}+b^{2})},  \frac{1}{b^{2}-a^{2}}(\cos(at) - \cos(bt)) $ 25 $ \frac{1}{(s^{4}+4a^{4})} = \frac{1}{4a^{3}}[\operatorname{sen}(at) \cosh(at) - \cos(at)] $ 26 $ \frac{s}{(s^{4}+4a^{4})} = \frac{1}{2a^{2}} \operatorname{sen}(at) \operatorname{senh}(at)) $ 27 $ \frac{1}{(s^{4}-a^{2})} = \frac{1}{2a^{3}}(\operatorname{senh}(at) - \operatorname{sen}(at)) $ 28 $ \frac{s}{(s^{4}-a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ 46 $ \frac{w}{(s^{2}+w^{2})\left(1-e^{-\frac{\pi}{w}s}\right)} = f(t) = \begin{cases} \operatorname{sen}(wt), & 0 < t < \frac{\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases} $ $ f\left(t + \frac{2\pi}{w}\right) = f(t), & t > 0 \end{cases} $ Retificador de meia onda $ f(t) = \begin{cases} \operatorname{sen}(wt), & 0 < t < \frac{\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases} $ $ f\left(t + \frac{2\pi}{w}\right) = f(t), & t > 0 \end{cases} $ Onda dente de serra $ f(t) = \begin{cases} \operatorname{sen}(wt), & 0 < t < \frac{\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases} $ $ f(t) =  \operatorname{sen}(wt)  $ Onda dente de serra $ f(t) = \frac{t}{a}, & 0 < t < a \end{cases} $	21	, , ,	$\frac{1}{2w^3}(\operatorname{sen}(wt) - wt \cos(wt))$			f(t+2a) = f(t),  t > 0
$ \frac{s}{(s^{2} + w^{2})^{2}} = \frac{\frac{1}{2w}(\operatorname{sen}(wt) + wt \cos(wt))}{\frac{s}{(s^{2} + a^{2})(s^{2} + b^{2})}},  \frac{1}{b^{2} - a^{2}}(\cos(at) - \cos(bt)) $ $ \frac{s}{(s^{2} + a^{2})(s^{2} + b^{2})},  \frac{1}{b^{2} - a^{2}}(\cos(at) - \cos(bt)) $ $ \frac{1}{(s^{4} + 4a^{4})} = \frac{1}{4a^{3}}[\operatorname{sen}(at) \cosh(at) - \cos(at) + \cos(at)] $ $ \frac{s}{(s^{4} + 4a^{4})} = \frac{1}{2a^{2}}(\operatorname{sen}(at) \operatorname{senh}(at)) $ $ \frac{s}{(s^{4} + 4a^{4})} = \frac{1}{2a^{2}}(\operatorname{sen}(at) \operatorname{senh}(at)) $ $ \frac{s}{(s^{4} + 4a^{4})} = \frac{1}{2a^{2}}(\operatorname{sen}(at) \operatorname{senh}(at)) $ $ \frac{s}{(s^{4} + 4a^{4})} = \frac{1}{2a^{2}}(\operatorname{senh}(at) - \operatorname{sen}(at)) $ $ \frac{s}{(s^{4} - a^{2})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{4}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{4}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{4}}(\operatorname{cosh}(at) - \operatorname{cos}(at)) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{4}}(\operatorname{cosh}(at) - \operatorname{cos}(at) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{4}}(\operatorname{cosh}(at) - \operatorname{cos}(at) $ $ \frac{s}{(s^{4} - a^{4})} = \frac{1}{2a^{4}}(\operatorname{cosh}(at) - \operatorname{cos}(at) $ $ \frac{s}{(s^$	22		$\frac{t}{2w}\operatorname{sen}(wt)$			Retificador do maio ando
$(a^{2} \neq b^{2})$ $\frac{1}{(s^{4} + 4a^{4})}$ $\frac{1}{4a^{3}}[\operatorname{sen}(at) \operatorname{cosh}(at) \operatorname{cos}(at) \operatorname{senh}(at)]$ $\frac{s}{(s^{4} + 4a^{4})}$ $\frac{1}{2a^{2}} \operatorname{sen}(at) \operatorname{senh}(at)$ $\frac{1}{2a^{2}} \operatorname{sen}(at) \operatorname{senh}(at)$ $\frac{1}{(s^{4} - a^{2})}$ $\frac{1}{2a^{3}}(\operatorname{senh}(at) - \operatorname{sen}(at))$ $\frac{1}{2a^{3}}(\operatorname{senh}(at) - \operatorname{sen}(at))$ $\frac{1}{2a^{3}}(\operatorname{cosh}(at) - \operatorname{cos}(at))$ $\frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at))$ $\frac{1}{2a^{2}}(\operatorname{cosh}(at) - \operatorname{cos}(at))$ $\frac{1}{as^{2}} - \frac{e^{-as}}{s(1 - e^{-as})}$ $f(t) = \frac{t}{a},  0 < t < a$	23	$\frac{s^2}{(s^2+w^2)^2}$	$\frac{1}{2w}(\operatorname{sen}(wt) + wt \cos(wt))$			<i></i>
$ \frac{1}{4a^3}[\operatorname{sen}(at) \cosh(at) - \cos(at) \operatorname{senh}(at)] - \cos(at) \operatorname{senh}(at)] = \frac{1}{2a^2} \operatorname{sen}(at) \operatorname{senh}(at) = \frac{1}{2a^2} \operatorname{sen}(at) \operatorname{senh}(at) = \frac{1}{2a^2} \operatorname{sen}(at) \operatorname{senh}(at) = \frac{1}{2a^2} \operatorname{sen}(at) \operatorname{senh}(at) = \frac{1}{2a^3} \operatorname{senh}(at) - \operatorname{sen}(at) = \frac{1}{2a^3} \operatorname{senh}(at) - \operatorname{senh}(at) =$	24	, , , ,	$\frac{1}{b^2 - a^2} (\cos(at) - \cos(bt))$	46	$\frac{w}{(s^2+w^2)\left(1-e^{-\frac{\pi}{w}s}\right)}$	$\overset{\bullet}{u}  w$
$ \frac{s}{(s^4 + 4a^4)} \qquad \frac{1}{2a^2} \operatorname{sen}(at) \operatorname{senh}(at)) \qquad 47 \qquad \frac{w}{s^2 + w^2} \operatorname{coth}\left(\frac{\pi s}{2w}\right) \qquad \operatorname{hethicator de office complete} $ $ \frac{1}{(s^4 + 4a^4)} \qquad \frac{1}{2a^2} \operatorname{sen}(at) \operatorname{senh}(at)) \qquad 48 \qquad \frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})} \qquad f(t) = \frac{t}{a},  0 < t < a $ $ \frac{t}{(t)} =  \operatorname{sen}(wt)  $ Onda dente de serra $ f(t) = \frac{t}{a},  0 < t < a $	25	$\frac{1}{(s^4 + 4a^4)}$	$4a^{\circ}$			
27 $\frac{1}{(s^4 - a^2)}$ $\frac{1}{2a^3}(\operatorname{senh}(at) - \operatorname{sen}(at))$ Onda dente de serra  28 $\frac{s}{(s^4 - a^4)}$ $\frac{1}{2a^2}(\cosh(at) - \cos(at))$ 48 $\frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}$ $f(t) = \frac{t}{a},  0 < t < a$	26	$\frac{s}{(s^4 + 4a^4)}$	1	47	$\frac{w}{s^2 + w^2} \coth\left(\frac{\pi s}{2w}\right)$	•
28 $\frac{s}{(s^4 - a^4)}$ $\frac{1}{2a^2}(\cosh(at) - \cos(at))$ 48 $\frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}$ $f(t) = \frac{t}{a},  0 < t < a$	27	1	$\frac{1}{2a^3}(\operatorname{senh}(at) - \operatorname{sen}(at))$			Onda dente de serra
1	28	s		48	$\frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}$	$f(t) = \frac{t}{a}, \qquad 0 < t < a$
		· /			, , ,	f(t) = f(t - a),  t > a

 $\bullet$  Questão 1 (1.0 ponto) Considere a função  $f:\mathbb{R}_+\to\mathbb{R}$  dada no gráfico abaixo:



A transformada de Laplace da função f(t) é

( ) 
$$\frac{1 - e^{-s} + e^{-2s} + e^{-3s} - e^{-5s}}{s^2}$$

$$(\ )\ \frac{1 - e^{-s} + e^{-2s} + e^{-3s} - e^{-5s}}{s}$$

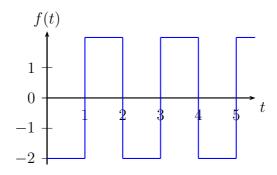
$$(\ )\ \frac{-e^{-s}+e^{-2s}+e^{-3s}-e^{-5s}}{s^2}$$

$$(\ )\ \frac{-e^{-s}+e^{-2s}+e^{-3s}-e^{-5s}}{s}$$

$$(\ )\ \frac{1-e^{-s}+e^{-2s}+e^{-3s}-e^{-5s}}{s^3}$$

$$\left(\ \right) \frac{1-e^{-s}-e^{-5s}}{s^2}$$

 $\bullet$  Questão 2 (1.0 ponto) Considere a função periódica  $f:\mathbb{R}_+\to\mathbb{R}$  dada no gráfico abaixo:



É correto afirmar que

$$(\ )\ 2\frac{e^{-s}}{s}\tanh\left(\frac{s}{2}\right)$$

$$() 2 \left[ \frac{1}{s} + \frac{e^{-s}}{s} \tanh\left(\frac{s}{2}\right) \right]$$

$$(\ )\ 2\left[\frac{e^{-s}-1}{s}+\frac{e^{-s}}{s}\tanh\left(\frac{s}{2}\right)\right]$$

$$(\phantom{x}) \ 2 \left[ \frac{1 - e^{-s}}{s^2} + \frac{e^{-s}}{s^2} \tanh\left(\frac{s}{2}\right) \right]$$

$$(\phantom{a}) \ 2\frac{e^{-s}}{s^2}\tanh\left(\frac{s}{2}\right)$$

$$() 2 \tanh(s)$$

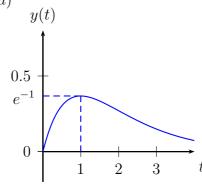
- $\bullet$  Questão 3 (1.0 ponto) A transformada de Laplace da função  $t^4\delta(t-2)$  é
- $(\ )\ 8e^{-2s}$
- $() 16e^{-2s}$
- $() 32e^{-2s}$
- $(\ ) \frac{24e^{-2s}}{s^5}$
- $() \frac{24}{s^5}e^{-2s}$
- $\left(\ \right) \frac{24}{s^5} \frac{e^{-2s}}{s}$
- Questão 4 (1.0 ponto) Considere o circuito RLC regido pela equação

$$\begin{cases} y'' + Ry' + \frac{1}{C}y = \delta(t) \\ y(0) = 0 \\ y'(0) = 0 \end{cases}.$$

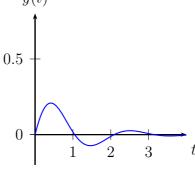
Também considere alguns valores para a capacitância C e a resistência R:

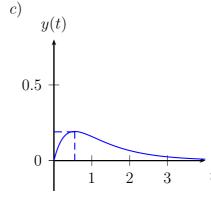
- i) ( )  $R = 0 e C = \frac{1}{9}$
- ii) ( )  $R = 2 e C = \frac{1}{10}$
- iii) ( )  $R = 4 e C = \frac{1}{3}$

iv) ( ) R = 2 e C = 1



b) y(t)





Relacione os itens i), ii), iii) e iv) aos itens a), b), c) e d) [Cada item relacionado corretamente vale 0.25 pontos].

• Questão 5 (1.0 ponto) Dada a equação  $y * y = te^{-t}$ , onde  $f * g = \int_0^t f(t - \tau)g(\tau)d\tau$ , assinale a alternativa que contém apenas soluções da equação.

( ) 
$$y(t) = e^{-t} e y(t) = -e^{-t}$$

( ) 
$$y(t) = J_0(t) e y(t) = J_0(-t)$$

( ) 
$$y(t) = \operatorname{sen}(t) e y(t) = \cos(t)$$

( ) 
$$y(t) = \sqrt{t}e^{-t/2} e y(t) = -\sqrt{t}e^{-t/2}$$

( ) 
$$y(t) = \operatorname{senh}(t) e y(t) = \operatorname{senh}(t)$$

• Questão 6 (1.0 ponto) Considere o circuito RLC regido pela equação

$$\begin{cases} y'' + 3y' + 6y = e^{-t} \\ y(0) = 1 \\ y'(0) = -1 \end{cases}$$

Marque a alternativa que contém  $\mathcal{L}{y(t)} = Y(s)$ :

( ) 
$$Y(s) = \frac{2+s}{s^2+3s+6}$$

( ) 
$$Y(s) = \frac{s^2 + 3s + 3}{(s+1)(s^2 + 3s + 6)}$$

( ) 
$$Y(s) = \frac{s+2}{(s+1)(s^2+3s+6)}$$

( ) 
$$Y(s) = \frac{s^2 + 3s + 3}{(s^2 + 3s + 6)}$$

( ) 
$$Y(s) = \frac{(s+2)(s+1)}{s^2+3s+6}$$

• Questão 7 (2.0 pontos) Considere a função

$$f(t) = \begin{cases} 1-t, & 0 < t < 1; \\ 0, & 1 < t < 2; \\ t-2, & 2 < t < 3; \\ 4-t, & 3 < t < 4; \\ 0, & t > 4. \end{cases}$$

- a) (0.5) Esboce o gráfico da função f(t).
- b) (0.5) Esboce o gráfico da função g(t) = f'(t).
- c) (1.0) Calcule a transformada de Laplace  $F(s) = \mathcal{L}\{f(t)\}$  e  $G(s) = \mathcal{L}\{g(t)\}$

• Questão 8 (2.0 pontos) Um determinado reator promove a oxidação biológica da amônia em nitrito. Considere que a reação pode ser modelada pelo seguinte modelo simplificado:

$$x'(t) = \kappa x(t) + q(t),$$

onde x(t) é a quantidade de amônia no reator, q(t) representa adição dessa substância e  $\kappa$  é uma constante positiva. Suponha que a entrada de amônia acontece periodicamente, isto é

$$q(t) = q_0 \sum_{k=0}^{\infty} \delta(t - kT).$$

Sabendo que x(0) = 0. Usando a teoria das Transformadas de Laplace, encontre uma expressão para x(t).