

1 - 6	7	8	Total

Nome: \_\_\_\_\_ Cartão: \_\_\_\_\_

Regras Gerais:

- Não é permitido o uso de calculadoras, telefones ou qualquer outro recurso computacional ou de comunicação.
- Trabalhe individualmente e sem uso de material de consulta além do fornecido.
- Devolva o caderno de questões preenchido ao final da prova.

Regras para as questões abertas:

- Seja sucinto, completo e claro.
- Justifique todo procedimento usado.
- Indique identidades matemáticas usadas, em especial, itens da tabela.
- Use notação matemática consistente!

Propriedades das transformadas de Fourier: considere a notação  $F(w) = \mathcal{F}\{f(t)\}$ .

1. Linearidade	$\mathcal{F}\{\alpha f(t) + \beta g(t)\} = \alpha \mathcal{F}\{f(t)\} + \beta \mathcal{F}\{g(t)\}$
2. Transformada da derivada	Se $\lim_{t \rightarrow \pm\infty} f(t) = 0$ , então $\mathcal{F}\{f'(t)\} = iw \mathcal{F}\{f(t)\}$ Se $\lim_{t \rightarrow \pm\infty} f(t) = \lim_{t \rightarrow \pm\infty} f'(t) = 0$ , então $\mathcal{F}\{f''(t)\} = -w^2 \mathcal{F}\{f(t)\}$
3. Deslocamento no eixo $w$	$\mathcal{F}\{e^{at} f(t)\} = F(w + ia)$
4. Deslocamento no eixo $t$	$\mathcal{F}\{f(t - a)\} = e^{-iaw} F(w)$
5. Transformada da integral	Se $F(0) = 0$ , então $\mathcal{F}\left\{\int_{-\infty}^t f(\tau) d\tau\right\} = \frac{F(w)}{iw}$
6. Teorema da modulação	$\mathcal{F}\{f(t) \cos(w_0 t)\} = \frac{1}{2} F(w - w_0) + \frac{1}{2} F(w + w_0)$
7. Teorema da Convolução	$\mathcal{F}\{(f * g)(t)\} = F(w)G(w)$ , onde $(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau) d\tau$ $(F * G)(w) = 2\pi \mathcal{F}\{f(t)g(t)\}$
8. Conjugação	$\overline{F(w)} = F(-w)$
9. Inversão temporal	$\mathcal{F}\{f(-t)\} = F(-w)$
10. Simetria ou dualidade	$f(-w) = \frac{1}{2\pi} \mathcal{F}\{F(t)\}$
11. Mudança de escala	$\mathcal{F}\{f(at)\} = \frac{1}{ a } F\left(\frac{w}{a}\right)$ , $a \neq 0$
12. Teorema da Parseval	$\int_{-\infty}^{\infty}  f(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty}  F(w) ^2 dw$
13. Teorema da Parseval para Série de Fourier	$\frac{1}{T} \int_0^T  f(t) ^2 dt = \sum_{n=-\infty}^{\infty}  C_n ^2$

Séries e transformadas de Fourier:

	Forma trigonométrica	Forma exponencial
Série de Fourier	$f(t) = \frac{a_0}{2} + \sum_{n=1}^N [a_n \cos(w_n t) + b_n \sin(w_n t)]$ <p>onde <math>w_n = \frac{2\pi n}{T}</math>, <math>T</math> é o período de <math>f(t)</math></p> $a_0 = \frac{2}{T} \int_0^T f(t) dt = \frac{2}{T} \int_{-T/2}^{T/2} f(t) dt,$ $a_n = \frac{2}{T} \int_0^T f(t) \cos(w_n t) dt = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos(w_n t) dt,$ $b_n = \frac{2}{T} \int_0^T f(t) \sin(w_n t) dt = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin(w_n t) dt$	$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{i w_n t},$ <p>onde <math>C_n = \frac{a_n - i b_n}{2}</math></p>
Transformada de Fourier	$f(t) = \frac{1}{\pi} \int_0^{\infty} (A(w) \cos(wt) + B(w) \sin(wt)) dw, \text{ para } f(t) \text{ real},$ <p>onde <math>A(w) = \int_{-\infty}^{\infty} f(t) \cos(wt) dt</math> e <math>B(w) = \int_{-\infty}^{\infty} f(t) \sin(wt) dt</math></p>	$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w) e^{i w t} dw,$ <p>onde <math>F(w) = \int_{-\infty}^{\infty} f(t) e^{-i w t} dt</math></p>

Tabela de integrais definidas:

1. $\int_0^\infty e^{-ax} \cos(mx) dx = \frac{a}{a^2 + m^2} \quad (a > 0)$	2. $\int_0^\infty e^{-ax} \sin(mx) dx = \frac{m}{a^2 + m^2} \quad (a > 0)$
3. $\int_0^\infty \frac{\cos(mx)}{a^2 + x^2} dx = \frac{\pi}{2a} e^{-ma} \quad (a > 0, m \geq 0)$	4. $\int_0^\infty \frac{x \sin(mx)}{a^2 + x^2} dx = \frac{\pi}{2} e^{-ma} \quad (a \geq 0, m > 0)$
5. $\int_0^\infty \frac{\sin(mx) \cos(nx)}{x} dx = \begin{cases} \frac{\pi}{2}, & n < m \\ \frac{\pi}{4}, & n = m, \\ 0, & n > m \end{cases} \quad \begin{matrix} (m > 0, \\ n > 0) \end{matrix}$	6. $\int_0^\infty \frac{\sin(mx)}{x} dx = \begin{cases} \frac{\pi}{2}, & m > 0 \\ 0, & m = 0 \\ -\frac{\pi}{2}, & m < 0 \end{cases}$
7. $\int_0^\infty e^{-r^2 x^2} dx = \frac{\sqrt{\pi}}{2r} \quad (r > 0)$	8. $\int_0^\infty e^{-a^2 x^2} \cos(mx) dx = \frac{\sqrt{\pi}}{2a} e^{-\frac{m^2}{4a^2}} \quad (a > 0)$
9. $\int_0^\infty x e^{-ax} \sin(mx) dx = \frac{2am}{(a^2 + m^2)^2} \quad (a > 0)$	10. $\int_0^\infty e^{-ax} \sin(mx) \cos(nx) dx = \frac{m(a^2 + m^2 - n^2)}{(a^2 + (m - n)^2)(a^2 + (m + n)^2)} \quad (a > 0)$
11. $\int_0^\infty x e^{-ax} \cos(mx) dx = \frac{a^2 - m^2}{(a^2 + m^2)^2} \quad (a > 0)$	12. $\int_0^\infty \frac{\cos(mx)}{x^4 + 4a^4} dx = \frac{\pi}{8a^3} e^{-ma} (\sin(ma) + \cos(ma))$
13. $\int_0^\infty \frac{\sin^2(mx)}{x^2} dx =  m  \frac{\pi}{2}$	14. $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-z^2} dz$
15. $\int_0^\infty \frac{\sin^2(ax) \sin(mx)}{x} dx = \begin{cases} \frac{\pi}{4}, & (0 < m < 2a) \\ \frac{\pi}{8}, & (0 < 2a = m) \\ 0, & (0 < 2a < m) \end{cases}$	16. $\int_0^\infty \frac{\sin(mx) \sin(nx)}{x^2} dx = \begin{cases} \frac{\pi m}{2}, & (0 < m \leq n) \\ \frac{\pi n}{2}, & (0 < n \leq m) \end{cases}$
17. $\int_0^\infty x^2 e^{-ax} \sin(mx) dx = \frac{2m(3a^2 - m^2)}{(a^2 + m^2)^3} \quad (a > 0)$	18. $\int_0^\infty x^2 e^{-ax} \cos(mx) dx = \frac{2a(a^2 - 3m^2)}{(a^2 + m^2)^3} \quad (a > 0)$
19. $\int_0^\infty \frac{\cos(mx)}{(a^2 + x^2)^2} dx = \frac{\pi}{4a^3} (1 + ma) e^{-ma} \quad \begin{matrix} (a > 0, \\ m \geq 0) \end{matrix}$	20. $\int_0^\infty \frac{x \sin(mx)}{(a^2 + x^2)^2} dx = \frac{\pi m}{4a} e^{-ma} \quad (a > 0, m > 0)$
21. $\int_0^\infty \frac{x^2 \cos(mx)}{(a^2 + x^2)^2} dx = \frac{\pi}{4a} (1 - ma) e^{-ma} \quad \begin{matrix} (a > 0, \\ m \geq 0) \end{matrix}$	22. $\int_0^\infty x e^{-a^2 x^2} \sin(mx) dx = \frac{m\sqrt{\pi}}{4a^3} e^{-\frac{m^2}{4a^2}} \quad (a > 0)$

Frequências das notas musicais em Hertz:

Nota \ Escala	1	2	3	4	5	6
Dó	65,41	130,8	261,6	523,3	1047	2093
Dó #	69,30	138,6	277,2	554,4	1109	2217
Ré	73,42	146,8	293,7	587,3	1175	2349
Ré #	77,78	155,6	311,1	622,3	1245	2489
Mi	82,41	164,8	329,6	659,3	1319	2637
Fá	87,31	174,6	349,2	698,5	1397	2794
Fá #	92,50	185,0	370,0	740,0	1480	2960
Sol	98,00	196,0	392,0	784,0	1568	3136
Sol #	103,8	207,7	415,3	830,6	1661	3322
Lá	110,0	220,0	440,0	880,0	1760	3520
Lá #	116,5	233,1	466,2	932,3	1865	3729
Si	123,5	246,9	493,9	987,8	1976	3951

Identidades Trigonômétricas:

$\cos(x) \cos(y) = \frac{\cos(x+y) + \cos(x-y)}{2}$
$\sin(x) \sin(y) = \frac{\cos(x-y) - \cos(x+y)}{2}$
$\sin(x) \cos(y) = \frac{\sin(x+y) + \sin(x-y)}{2}$

Integrais:

$\int x e^{\lambda x} dx = \frac{e^{\lambda x}}{\lambda^2} (\lambda x - 1) + C$
$\int x^2 e^{\lambda x} dx = e^{\lambda x} \left( \frac{x^2}{\lambda} - \frac{2x}{\lambda^2} + \frac{2}{\lambda^3} \right) + C$
$\int x^n e^{\lambda x} dx = \frac{1}{\lambda} x^n e^{\lambda x} - \frac{n}{\lambda} \int x^{n-1} e^{\lambda x} dx + C$
$\int x \cos(\lambda x) dx = \frac{\cos(\lambda x) + \lambda x \sin(\lambda x)}{\lambda^2} + C$
$\int x \sin(\lambda x) dx = \frac{\sin(\lambda x) - \lambda x \cos(\lambda x)}{\lambda^2} + C$

• **Questão 1** (1.0 ponto) Considere a função  $f(t) = 8 \cos^4(t)$ . Calcule os coeficientes da expansão em série de Fourier de  $f(t)$  e assinale na primeira coluna a representação trigonométrica e na segunda a representação exponencial.

☐  $3 + 8 \sum_{n=1}^{\infty} \left( \frac{1}{2n+1} \cos(2nt) + \frac{n}{2n+1} \sin(2nt) \right)$

☐  $3 + \sum_{n=1}^{\infty} \frac{1}{2n} e^{2int}$

☐  $3 + 4 \cos(t) + 2 \cos(2t) + \cos(3t) + \frac{1}{2} \cos(4t)$

☐  $3 + 4 \cos(2t) + \cos(4t)$

☐  $3 + 4 \sin(t) + 2 \sin(2t)$

☐  $\sum_{n=-\infty}^{\infty} \left( \frac{3}{2n+1} - \frac{in}{2n^2+1} \right) e^{2nit}$

☐  $3 + \sum_{n=1}^{\infty} \frac{1}{2n} \cos(2nt)$

☐  $\frac{i}{2} e^{-4it} + 2e^{-2it} + 3 + 2e^{2it} - \frac{i}{2} e^{4it}$

☐  $\frac{1}{2} e^{-4it} + 2e^{-2it} + 3 + 2e^{2it} + \frac{1}{2} e^{4it}$

☐  $\frac{i}{2} e^{-2it} + 2ie^{-it} + 3 - 2ie^{it} - \frac{i}{2} e^{2it}$

• **Questão 2** (1.0 ponto) Considere a função periódica  $f(t) = \sin(4t) + \sin(6t) + \sin(8t)$ . Marque na primeira coluna o período fundamental e, na segunda, a frequência angular fundamental.

☐ 1

☐ 2

☐ 4

☐  $\pi$

☐  $2\pi$

☐  $4\pi$

☐ 1

☐ 2

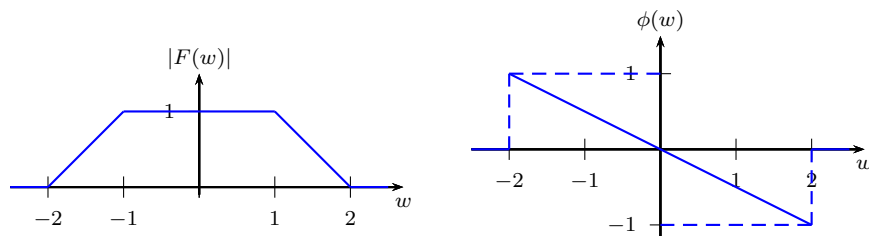
☐ 4

☐  $\pi$

☐  $2\pi$

☐  $4\pi$

• **Questão 3** (1.0 ponto) Considere os diagramas de espectro de magnitude e de fase da transformada de Fourier  $F(w) = \mathcal{F}\{f(t)\}$  dados nos gráficos abaixo:



Assinale na primeira coluna  $F(w)$  e, na segunda, a correta afirmação sobre  $f(t)$ .

☐  $F(w) = \begin{cases} 1, & -1 \leq w \leq 1 \\ w+2, & w < -1 \\ 2-w, & w > 1 \end{cases}$

☐  $F(w) = \begin{cases} 1, & -1 \leq w \leq 1 \\ w+2, & -2 < w < -1 \\ 2-w, & 1 < w < 2 \\ 0, & |w| \geq 2 \end{cases}$

☐  $F(w) = \begin{cases} e^{-iw/2}, & -1 \leq w \leq 1 \\ (w+2)e^{-iw/2}, & -2 < w < -1 \\ (2-w)e^{-iw/2}, & 1 < w < 2 \\ 0, & |w| \geq 2 \end{cases}$

☐  $F(w) = \begin{cases} e^{-iw}, & -1 \leq w \leq 1 \\ (w+2)e^{-iw}, & -2 < w < -1 \\ (2-w)e^{-iw}, & 1 < w < 2 \\ 0, & |w| \geq 2 \end{cases}$

☐ Não há suficiente informação para conhecer  $F(w)$

☐  $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w) e^{iwt} dw$

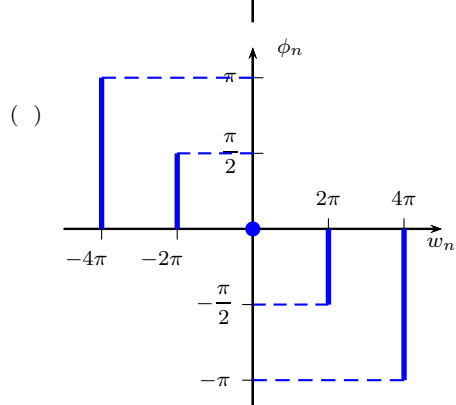
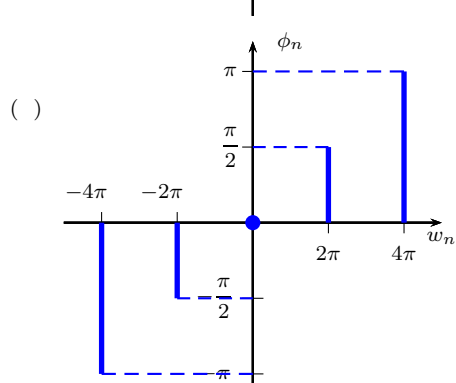
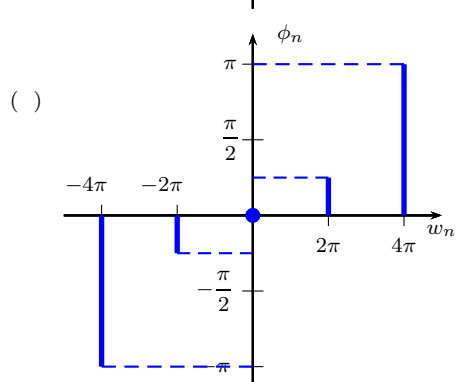
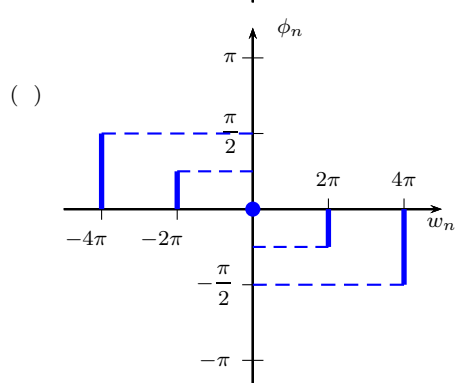
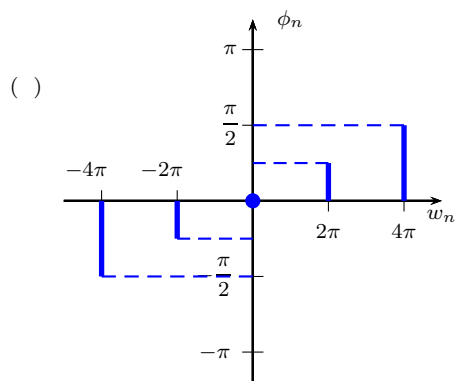
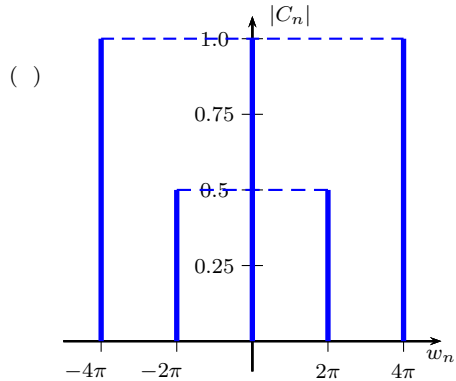
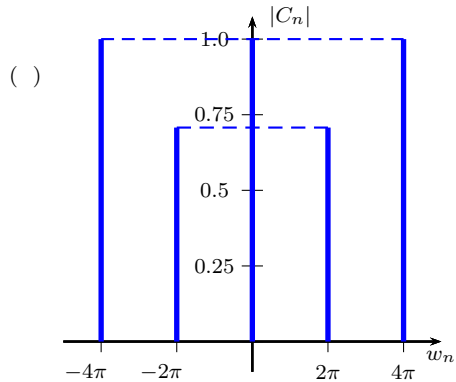
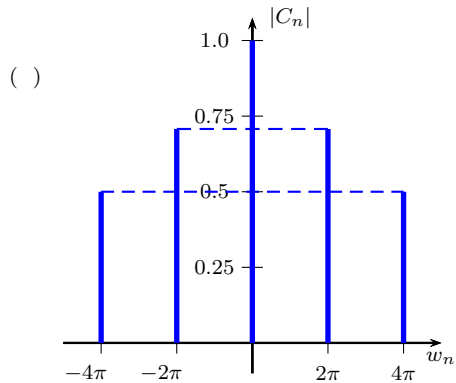
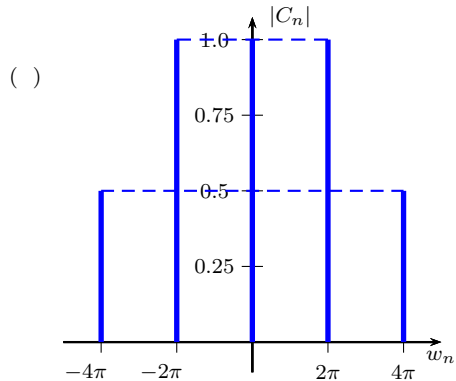
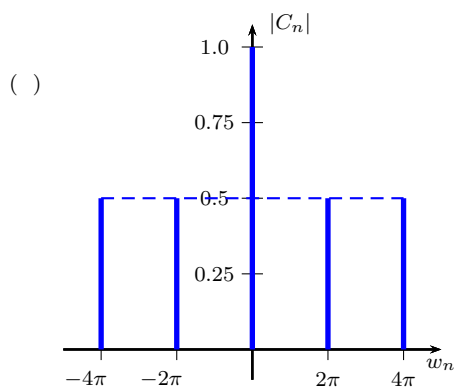
☐  $f(t) = \int_{-\infty}^{\infty} F(w) e^{-iwt} dw$

☐  $f(t) = \begin{cases} e^{-it/2}, & -1 \leq t \leq 1 \\ (t+2)e^{-it/2}, & -2 < t < -1 \\ (2-t)e^{-it/2}, & 1 < t < 2 \\ 0, & |t| \geq 2 \end{cases}$

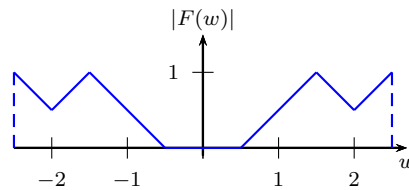
☐  $f(t) = \begin{cases} e^{-it}, & -1 \leq t \leq 1 \\ (t+2)e^{-it}, & -2 < t < -1 \\ (2-t)e^{-it}, & 1 < t < 2 \end{cases}$

☐ Não há suficiente informação para conhecer  $f(t)$

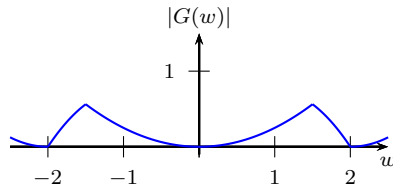
• **Questão 4** (1.0 pontos) Considere a função  $f(t) = 1 + \sin(2\pi t) + \cos(2\pi t) + \sin(4\pi t)$ . Assinale na primeira coluna o diagrama de espectro do módulo e, na segunda, o diagrama de espectro da fase.



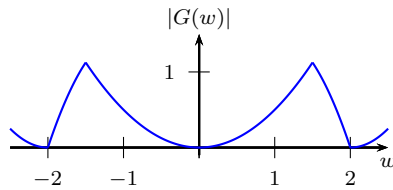
• **Questão 5** (1.0 ponto) Considere o diagrama de espectro de magnitudes da Transformada de Fourier da função  $f(t)$  dados nos gráficos abaixo. Assinale na primeira coluna a alternativa que representa o diagrama de espectro de magnitudes de  $g(t) = \frac{1}{2}f'(t)$  e, na segunda, o diagrama de espectro de magnitudes de  $h(t) = 2f(t) \cos\left(\frac{5}{2}t\right)$ .



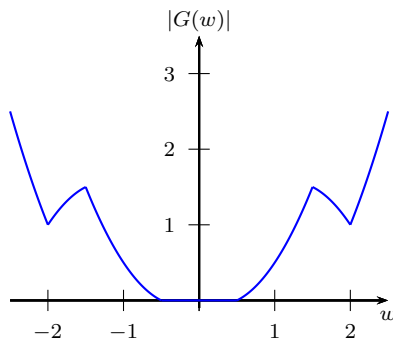
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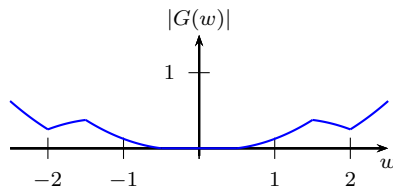
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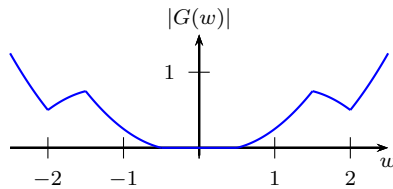
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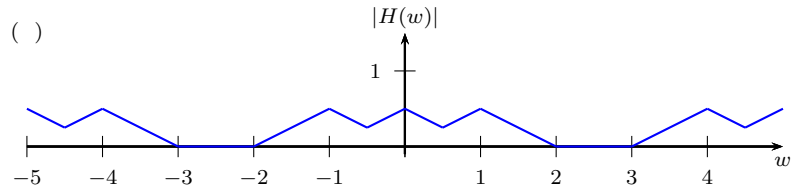
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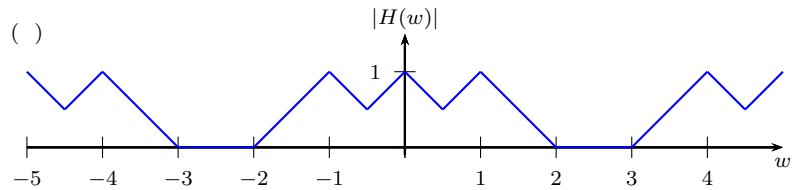
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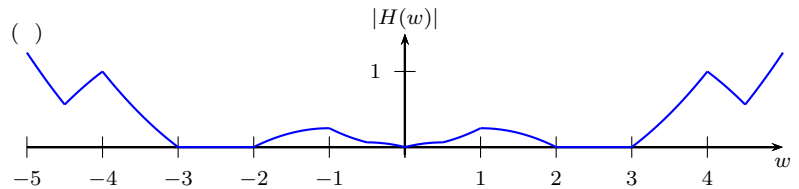
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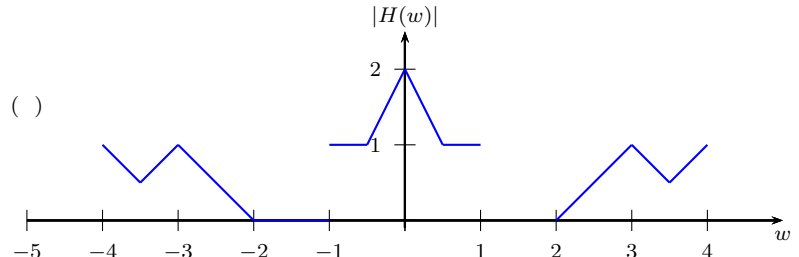
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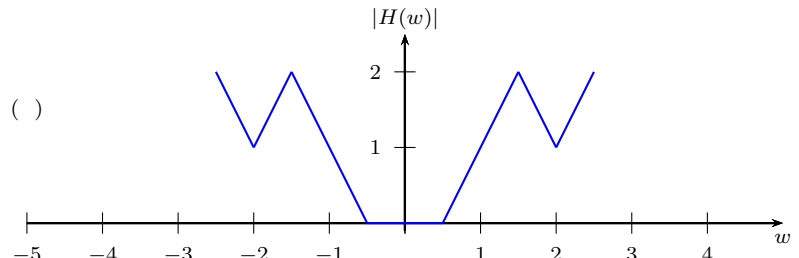
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• **Questão 6** (1.0 ponto) Considere

$$f(x) = \begin{cases} 2 & \text{se } 0 < x < 1, \\ 0 & \text{caso contrário.} \end{cases}$$

Assinale na primeira coluna  $\mathcal{F}\{f(x)\}$  e, na segunda,  $\mathcal{F}\{f(x) \cos(x)\}$ .

( )  $\frac{2 \operatorname{sen}(k)}{k}$

( )  $\frac{2 \operatorname{sen}(k)}{k} + i \frac{2 \cos(k)}{k}$

( )  $\frac{2 \operatorname{sen}(k)}{k} + 2i \left( \frac{\cos(k)}{k} - 1 \right)$

( )  $\frac{2 \operatorname{sen}(k) - 2}{k} + 2i \frac{\cos(k)}{k}$

( )  $\frac{2 \operatorname{sen}(k)}{k} + 2i \frac{\cos(k) - 1}{k}$

( )  $\frac{\operatorname{sen}(k+1)}{k+1} + \frac{\operatorname{sen}(k-1)}{k-1} + i \left( \frac{\cos(k+1) - 1}{k+1} + \frac{\cos(k-1) - 1}{k-1} \right)$

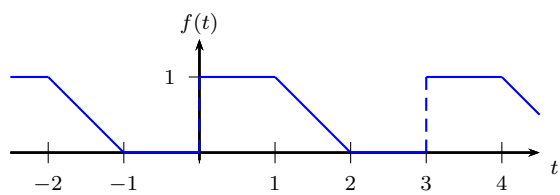
( )  $\frac{\operatorname{sen}(k+1)}{k+1} + i \frac{\cos(k-1) - 1}{k-1}$

( )  $\frac{\operatorname{sen}(k+1)}{k} + \frac{\operatorname{sen}(k-1)}{k} + i \left( \frac{\cos(k+1) - 1}{k} + \frac{\cos(k-1) - 1}{k} \right)$

( )  $\frac{\operatorname{sen}(k-1)}{k-1} + i \frac{\cos(k+1) - 1}{k+1}$

( )  $\frac{\operatorname{sen}(k+1)}{k+1} + \frac{\operatorname{sen}(k+1)}{k+1} + i \left( \frac{\cos(k-1) - 1}{k-1} + \frac{\cos(k-1) - 1}{k-1} \right)$

- **Questão 7** (2.0 ponto) Calcule a série de Fourier para a seguinte função periódica:



- **Questão 8** (2.0 pontos) Resolva o seguinte problema de difusão de calor com velocidade:

$$u_t + 3u_x - u_{xx} = 0$$

$$u(x, 0) = 120\delta(x).$$