TUTORIAL - 4

1) $T(n) = 3T(n/2) + n^2$

Sol $a = 3, \quad b = 2, \quad f(n) = n^2$

Here, a and b are constant and $f(n)$ is a +ve function so, Master's theorem is applicable.

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.58}$ which is $f(n) > n^c$

So, Case 3 is applied here

$\Rightarrow T(n) = \Theta(n^2)$

2) $T(n) = 4T(n/2) + n^2$

Sol $a = 4, \quad b = 2, \quad f(n) = n^2$

Here a and b are constant & $f(n)$ is a +ve function So, Master's theorem is applicable.

$c = \log_b a = \log_2 2^2 = 2\log_2 2 = 2$

$n^c = n^2$ which is $f(n) = n^c$

So, Case 2 is applied here

$\Rightarrow T(n) = \Theta(n^2 \log n)$

3) $T(n) = T(n/2) + 2^n$

$a = 1, \quad b = 2, \quad f(n) = 2^n$

Here a & b are constant & $f(n)$ is a +ve function So, Master's theorem is applicable.

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$ which is $f(n) > n^c$

So, Case 3 is applied here

$\Rightarrow T(n) = \Theta(2^n)$

4) $T(n) = 2^n T(n/2) + n^n$

$a = 2^n, \quad b = 2, \quad f(n) = n^n$

Here a is not constant, as its value is dependent on n
So, Master's theorem is not applicable here.

5) $T(n) = 16T(n/4) + n$

$a = 16, \quad b = 4, \quad f(n) = n$

Here a and b are constant and f(n) is a +ve fn
So, Master's theorem is applicable.

$c = \log_b a = \log_4 16 = \log_4 4^2 = 2\log_4 4 = 2$

$n^c = n^2 \quad$ i.e $f(n) < n^c$

So, Case 1 is applied here

$\Rightarrow T(n) = \Theta(n^2)$

6) $T(n) = 2T(n/2) + n\log n$

$a = 2, \quad b = 2, \quad f(n) = n\log n$

Here a & b are constant and f(n) is a +ve function
So, Master's theorem is applicable.

$c = \log_b a = \log_2 2 = 1$

$n^c = n^1 = n \quad$ i.e $f(n) > n^c$

So Case 3 is applied here

$\Rightarrow T(n) = \Theta(n\log n)$

7) $T(n) = 2T(n/2) + n/\log n$

$a=2, \quad b=2, \quad f(n) = n/\log n$

Here a & b are constant & $f(n)$ is a +ve function

So, ~~Rest~~ $c = \log_b a = \log_2 2 = 1$

$n^c = n^1 = n$

There is a non-polynomial difference b/w $f(n)$ and n^c

So, Master's theorem is not applicable.

8) $T(n) = 2T(n/4) + n^{0.51}$

$a=2, \quad b=4, \quad f(n) = n^{0.51}$

a and b are constant & $f(n)$ is a +ve function

$\therefore$ Master's theorem is applicable.

$c = \log_b a = \log_4 2 = 0.50$

$n^c = n^{0.50}$ i.e $f(n) > n^c$

Here, Case 3 is applicable.

$\Rightarrow T(n) = \Theta(n^{0.51})$

9) $T(n) = 0.5\, T(n/2) + 1/n$

$a = 0.5, \quad b = 2, \quad f(n) = 1/n$

$\because a < 1$

$\therefore$ Master's theorem is not applicable.

10) $T(n) = 16\, T(n/4) + n!$

$a=16, \quad b=4, \quad f(n) = n!$

Here a and b are constant & $f(n)$ is a +ve function

So, Master's theorem is applicable.

$c = \log_b a = \log_4 16 = \log_4 4^2 = 2\log_4 4 = 2$

$n^c = n^2$ i.e $f(n) > n^c$

Here, Case 3 is applied here

$\Rightarrow T(n) = \Theta(n!)$

11) $T(n) = 4T(n/2) + \log n$

$a = 4, \quad b = 2, \quad f(n) = \log n$

Here a and b are constant & f(n) is a +ve function

So, Master's theorem is applicable.

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2 \log_2^2 = 2$

$n^c = n^2 \quad$ i.e $f(n) < n^c$

Here Case 1 is applicable

$\Rightarrow T(n) = \Theta(n^2)$

12) $T(n) - \text{sqrt}(n) T(n/2) + \log n$

$a = \sqrt{n}, \quad b = 2, \quad f(n) = \log n$

Here, a is not constant.

So, Master's theorem is not applicable.

13) $T(n) = 3T(n/2) + n$

$a = 3, \quad b = 2, \quad f(n) = n$

Here a & b are constant & f(n) is a +ve function.

So, Master's theorem is applicable.

$c = \log_b a = \log_2 3 = 0.158$

$n^c = n^{1.58} \quad$ i.e $f(n) < n^c$

Case 1 is applicable here

$\Rightarrow T(n) = \Theta(n^{0.158})$

14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3, \quad b = 3, \quad f(n) = \sqrt{n}$

Here a & b are constant & f(n) is a +ve function

So, Master's theorem is applicable

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n \quad$ i.e $f(n) < n^c$

$\therefore$ Case 1 is applicable

$\Rightarrow \quad T(n) = \Theta(n)$

15) $T(n) = 4T(n/2) + c.n$

$a = 4, \quad b = 2, \quad f(n) = c.n$

a & b are constant & $f(n)$ is +ve

So, Master's theorem is applicable

$c = \log_b a = \log_2 2^2 = 2 \quad ; \quad n^c = n^2 \quad f(n) < n^c$

Case 1 is applicable here

$\Rightarrow \quad T(n) = \Theta(n^2)$

16) $T(n) = 3T(n/4) + n\log n$

$a = 3, \ b = 4, \quad f(n) = n\log n$

a & b are constant & $f(n)$ is +ve

So, Master's theorem is applicable

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79} \quad ; \quad f(n) > n^c$

$\therefore$ Case 3 is applicable here

$\Rightarrow \quad T(n) = \Theta(n \log n)$

17) $T(n) = 3T(n/3) + n/2$

$a = 3, \ b = 3, \quad f(n) = n/2$

a & b are constant & $f(n)$ is +ve

So, Master's theorem is applicable here

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n \quad f(n) < n^c$

Case 2 is applicable here

$\Rightarrow \quad T(n) = \Theta(n)$

18) $T(n) = 6T(n/3) + n^2 \log n$

$a = 6,\ b = 3,\quad f(n) = n^2 \log n$

a & b are constant & f(n) is +ve

So, master's theorem is applicable here

$c = \log_b a = \log_3 6 = 1.63$

$n^c = n^{1.63} \qquad f(n) > n^c$

Case 3 is applied here

$\Rightarrow T(n) = \Theta(n^2 \log n)$

19) $T(n) = 4T(n/2) + n/\log n$

$a = 4,\ b = 2,\quad f(n) = n/\log n$

a & b are constant & f(n) is +ve

So, master's theorem is applicable here

$c = \log_b a = \log_2 2^2 = 2$

$n^c = n^2 \qquad f(n) < n^c$

Case 1 is applied here

$\Rightarrow T(n) = \Theta(n^2)$

20) $T(n) = 64T(n/8) - n^2 \log n$

$a = 64,\ b = 8,\quad f(n) = -n^2 \log n$

a & b are constant but f(n) is -ve

So, Master theorem is not applicable here

21) $T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3,\quad f(n) = n^2$

a & b are constant & f(n) is +ve

So, Master's theorem is applicable here

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$ i.e. $f(n) > n^c$

Case 3 is applicable here

$\Rightarrow T(n) = \theta(n^2)$

22) $T(n) = T(n/2) + n(2 - \cos n)$

Here, $f(n)$ is not a regular function

So, Master's theorem is not applicable.