Problem 1

a). $(x, y, z) = (2m, 3m, 4m)$ $f = 24\ mm$

$\qquad = (2000mm, 3000mm, 4000mm)$ $x' = f\ \dfrac{x}{z}$

$x' = 24mm \cdot \dfrac{2000\ mm}{4000\ mm} = 12mm$ $y' = f\ \dfrac{y}{z}$

$y' = 24mm \cdot \dfrac{3000mm}{4000\ mm} = 18mm$

$\boxed{\begin{aligned} x' &= 12mm \\ y' &= 18mm \end{aligned}}$

b). $(x, y, z) = (6m, 9m, 12m)$ $f = 24mm$

$\qquad = (6000\ mm, 9000mm, 12000mm)$ $x' = f\ \dfrac{x}{z}$

$\qquad\qquad\qquad\qquad\qquad\qquad y' = f\ \dfrac{y}{z}$

$x' = 24mm \cdot \dfrac{6000mm}{12000mm} = 12mm$

$\boxed{\begin{aligned} x' &= 12mm \\ y' &= 18mm \end{aligned}}$

$y' = 24mm \cdot \dfrac{9000\ mm}{12000\ mm} = 18mm$

C). $(x, y, z) = (-2m, -1m, 7m)$ $f = 24mm$

$= (-2000mm, -1000mm, 7000mm)$ $x' = f \dfrac{x}{z}$

$y' = f \dfrac{y}{z}$

$x' = 24mm \cdot \dfrac{-2000mm}{7000mm} = -6.86mm$

$y' = 24mm \cdot \dfrac{-1000mm}{7000mm} = -3.43mm$

$$\boxed{\begin{array}{l} x' = -6.86mm \\ y' = -3.43mm \end{array}}$$

D). $(x, y, z) = (0m, 0m, 7m)$ $f = 24mm$

$= (0mm, 0mm, 7000mm)$ $x' = f \cdot \dfrac{x}{z}$

$y' = f \cdot \dfrac{y}{z}$
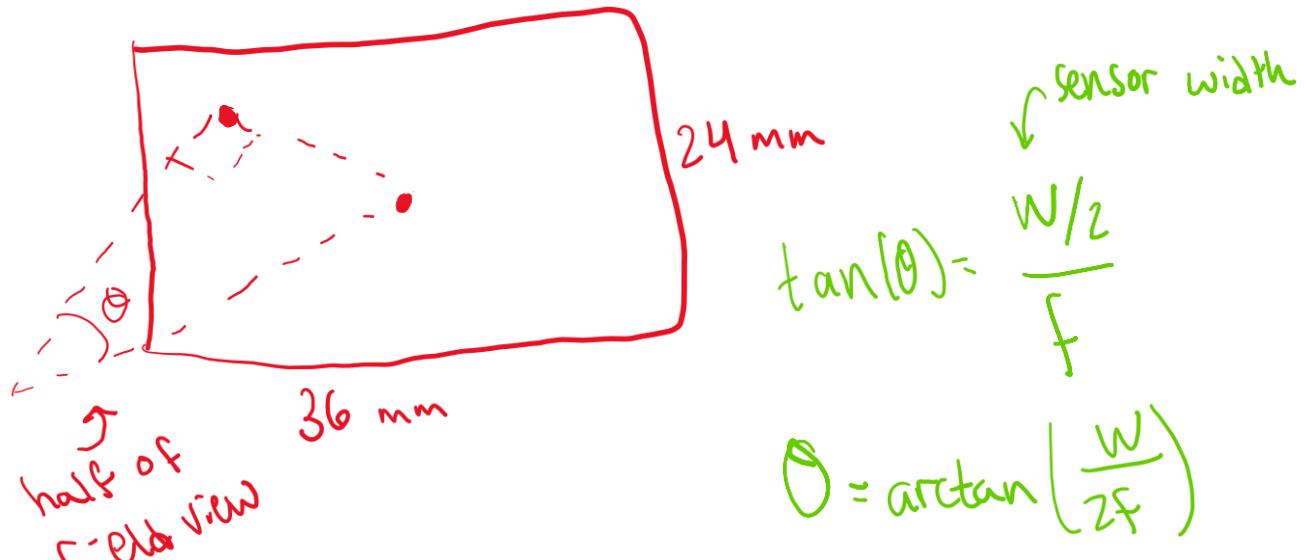
$x' = 24mm \cdot \dfrac{0mm}{7000mm} = 0mm$

$y' = 24mm \cdot \dfrac{0mm}{7000mm} = 0mm$

$$\boxed{\begin{array}{l} x' = 0mm \\ y' = 0mm \end{array}}$$

e).

24 mm

36 mm

half of
field view

sensor width

$$\tan(\theta) = \frac{W/2}{f}$$

$$\theta = \arctan\left(\frac{W}{2f}\right)$$

Full field of view = 2θ

$$\text{Horizontal} = 2 \cdot \arctan\left(\frac{W}{2f}\right) = 2 \cdot \arctan\left(\frac{36 \text{ mm}}{2 \cdot 24 \text{ mm}}\right)$$

$$\approx \boxed{73.74°}$$

$$\text{Vertical} = 2 \cdot \arctan\left(\frac{H}{2f}\right) = 2 \cdot \arctan\left(\frac{24 \text{ mm}}{2 \cdot 24 \text{ mm}}\right)$$

$$\approx \boxed{53.14°}$$

Problem 2

a). Since the flo is planar and the image plane is parallel to e floor due to the camera being pointed downwards i e perspective projection maps straight lines in 3D to straight lines in the image plane. When switce ing between the two the plane intersects the floo in a straight line creating four intersections with preserved right angles. Because of this the portion of the floor is rectangular.

B). Point of projection $1m = 1000mm$

Sensor size = $36mm \times 24mm$

$f = 24mm$

$$x' = \frac{d}{f} \cdot x$$

$$y' = \frac{d}{f} \cdot y$$

$\frac{1000mm}{24mm} = 41.67mm$ ← Scale factor

$36mm \cdot 41.67 = 1500mm = 1.5m$

$24mm \cdot 41.67 = 1000mm = 1.0m$

Dimensions: $1.5m \times 1.0m$

Problem 3

$$a_1 x + b_1 y + c_1 = 0$$
$$a_2 x + b_2 y + c_2 = 0$$

a). $\quad a_1 x + b_1 y = -c_1 \quad \rightarrow \quad b_2 a_1 x + b_2 b_1 y = -b_2 c_1$

$\quad\quad a_2 x + b_2 y = -c_2 \quad\quad -(b_1 a_2 x + b_1 b_2 y = b_1 c_2)$

$$(a_1 b_2 - a_2 b_1) x = -(b_2 c_1 - b_1 c_2)$$

$$\boxed{x = \frac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1}}$$

$\quad\quad a_1 x + b_1 y = -c_1 \quad\quad -(a_2 a_1 x + a_2 b_1 y = -a_2 c_1)$

$\quad\quad a_2 x + b_2 y = -c_2 \quad \longrightarrow \quad a_1 a_2 x + a_1 b_2 y = -a_1 c_2$

$$(a_1 b_2 - a_2 b_1) y = -(a_1 c_2 - a_2 c_1)$$

$$\boxed{y = \frac{a_2 c_1 - a_1 c_2}{a_1 b_2 - a_2 b_1}}$$

b). $a_1 = 3, \ b_1 = 4, \ c_1 = 5, \ a_2 = 6, \ b_2 = 7, \ c_2 = 8$

$$X = \frac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1} = \frac{(4)(8) - (7)(5)}{(3)(7) - (6)(4)} = \frac{32 - 35}{-3} = 1$$

$$Y = \frac{a_2 c_1 - a_1 c_2}{a_1 b_2 - a_2 b_1} = \frac{(6)(5) - (3)(8)}{(3)(7) - (6)(4)} = \frac{30 - 24}{-3} = -2$$

Point of intersection : $(1, -2)$

c). Determinant $= \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$

if Det $= 0$, then parallel

$a_1 b_2 - a_2 b_1 \rightarrow (3)(7) - (6)(4) = 21 - 24 = -3$

$-3 \neq 0,$ not parallel