NCERT Discrete - 11.9.3.11

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Question 11.9.3.11:
Evaluate $\sum_{k=1}^{11}(2+3^k)$.

Solution:

variable	value	description
$x(0)$	5	first term
r	3	common ratio of the GP
$x(n)$	$2+3^{n+1}u(n)$	n^{th} term

TABLE 0
INPUT PARAMETERS

From Table 0

$$x(n) = \left(2+3^{n+1}\right)u(n) \tag{1}$$

$$x(n) = \left(2+3^{n}\cdot 3\right)u(n) \tag{2}$$

$$X(z) = \frac{2}{(1-z^{-1})} + \frac{3}{(1-3z^{-1})}, \quad 1<|z|<3 \tag{3}$$

$$y(n) = x(n) * u(n) \tag{4}$$

$$Y(z) = X(z)\,U(z) \tag{5}$$

$$= \frac{2}{(1-z^{-1})^2} + \left(\frac{3}{(1-3z^{-1})(1-z^{-1})}\right), \quad 1<|z|<3 \tag{6}$$

Using Contour Integration to find the inverse Z-transform,

$$y(n) = \frac{1}{2\pi j}\oint_C Y(z)\,z^{n-1}\,dz \tag{7}$$

$$y(10) = \frac{1}{2\pi j}\oint_C \left(\frac{2z^9}{(1-z^{-1})^2} + \frac{3z^9}{(1-3z^{-1})(1-z^{-1})}\right)dz \tag{8}$$

$$= \frac{1}{2\pi j}\oint_C \left(\frac{2z^9}{(1-z^{-1})^2} + \frac{3}{2}\left(\frac{z^{11}}{z-3}\right) - \frac{z^{11}}{z-1}\right)dz \tag{9}$$

$$R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{10}$$

For R_1 , $m=2$:

$$R_1 = \frac{1}{(1)!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2\frac{2z^{11}}{(z-1)^2}\right) \tag{11}$$

$$= 2\lim_{z\to 1}\frac{d}{dz}(z^{11}) \tag{12}$$

$$= 22 \tag{13}$$

For R_2 , $m=1$:

$$R_2 = \frac{1}{(1-1)!}\frac{3}{2}\lim_{z\to 3}\left((z-3)\frac{z^{11}}{(z-3)}\right) \tag{14}$$

$$= \frac{3}{2}\cdot 3^{11} \tag{15}$$

$$= 265720.5 \tag{16}$$

For R_3 , $m=1$:

$$R_3 = \frac{1}{(1-1)!}\frac{3}{2}\lim_{z\to 1}\left((z-1)\frac{z^{11}}{(z-1)}\right) \tag{17}$$

$$= \frac{3}{2}\lim_{z\to 1} z^{11} \tag{18}$$

$$= \frac{3}{2} \implies 1.5 \tag{19}$$

$$R_1 + R_2 - R_3 = 265741 \tag{20}$$

$$\implies y(10) = 265741 \tag{21}$$

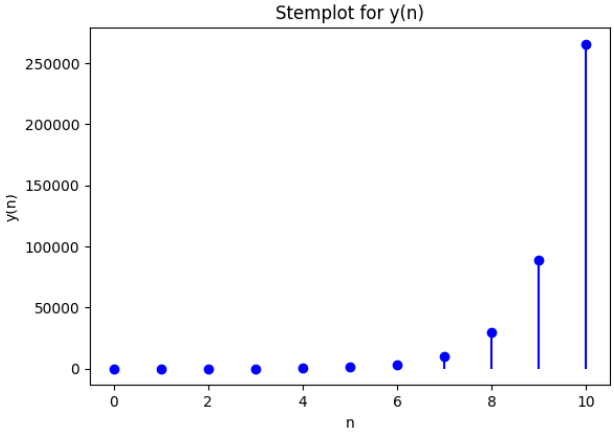

Fig. 0. stem plot