NCERT Discrete - 11.9.3.11

EE23BTECH11037 - M Esha*

Question 11.9.3.11:

Evaluate $\sum_{k=1}^{11} (2 + 3^k)$.

Solution:

variable	value	description
<i>x</i> (0)	5	first term
r	3	common ratio of the GP
x(n)	$2 + 3^{n+1}u(n)$	<i>n</i> th term

INPUT PARAMETERS

From Table 0

$$x(n) = (2 + 3^{n+1})u(n)$$
 (1)

$$x(n) = (2 + 3^n \cdot 3) u(n)$$
 (2)

$$X(z) = \frac{2}{(1 - z^{-1})} + \frac{3}{(1 - 3z^{-1})}, \quad |z| > 3$$
 (3)

$$y(n) = x(n) * u(n)$$

$$(4)$$

$$Y(z) = X(z) U(z)$$
 (5)

$$= \frac{2}{(1-z^{-1})^2} + \left(\frac{3}{(1-3z^{-1})(1-z^{-1})}\right), \quad |z| > 3$$
(6)

Using Contour Integration to find the inverse Ztransform,

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) \, z^{n-1} \, dz \tag{7}$$

$$y(10) = \frac{1}{2\pi j} \oint_C \left(\frac{2z^9}{(1-z^{-1})^2} + \frac{3z^9}{(1-3z^{-1})(1-z^{-1})} \right) dz$$
(8)

$$= \frac{1}{2\pi j} \oint_C \left(\frac{2z^9}{(1-z^{-1})^2} + \frac{3}{2} \left(\frac{z^{11}}{z-3} - \frac{z^{11}}{z-1} \right) \right) dz$$

$$R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z) \right) \tag{10}$$

For R_1 , m = 2:

$$R_1 = \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz} \left((z - 1)^2 \frac{2z^{11}}{(z - 1)^2} \right) \tag{11}$$

$$=2\lim_{z\to 1}\frac{d}{dz}(z^{11})$$
 (12)

$$= 22 \tag{13}$$

For R_2 , m = 1:

$$R_2 = \frac{1}{(1-1)!} \frac{3}{2} \lim_{z \to 3} \left((z-3) \frac{z^{11}}{(z-3)} \right)$$
 (14)

$$= \frac{3}{2} \cdot 3^{11} \tag{15}$$

$$= 265720.5$$
 (16)

For R_3 , m = 1:

$$R_3 = \frac{1}{(1-1)!} \frac{3}{2} \lim_{z \to 1} \left((z-1) \frac{z^{11}}{(z-1)} \right)$$
(17)

$$= \frac{3}{2} \lim_{z \to 1} z^{11} \tag{18}$$

$$=\frac{3}{2}\tag{19}$$

$$R_1 + R_2 - R_3 = 265741 \tag{20}$$

$$\implies y(10) = 265741$$
 (21)

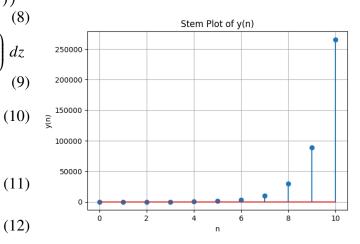


Fig. 0. stem plot