

NCERT Discrete - 11.9.3.11

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Question 11.9.3.11:

Evaluate $\sum_{k=1}^{11} (2 + 3^k)$.

Solution:

variable	value	description
$x(0)$	5	first term
r	3	common ratio of the GP
$x(n)$	$2 + 3^{n+1}u(n)$	n^{th} term

TABLE 0
INPUT PARAMETERS

From Table 0

$$x(n) = (2 + 3^{n+1})u(n) \quad (1)$$

$$x(n) = (2 + 3^n \cdot 3)u(n) \quad (2)$$

$$X(z) = \frac{2}{(1 - z^{-1})} + \frac{3}{(1 - 3z^{-1})}, \quad |z| > 3 \quad (3)$$

$$y(n) = x(n) * u(n) \quad (4)$$

$$Y(z) = X(z)U(z) \quad (5)$$

$$= \frac{2}{(1 - z^{-1})^2} + \left(\frac{3}{(1 - 3z^{-1})(1 - z^{-1})} \right), \quad |z| > 3 \quad (6)$$

Using Contour Integration to find the inverse Z-transform,

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1} dz \quad (7)$$

$$y(10) = \frac{1}{2\pi j} \oint_C \left(\frac{2z^9}{(1 - z^{-1})^2} + \frac{3z^9}{(1 - 3z^{-1})(1 - z^{-1})} \right) dz \quad (8)$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{2z^9}{(1 - z^{-1})^2} + \frac{3}{2} \left(\frac{z^{11}}{z - 3} - \frac{z^{11}}{z - 1} \right) \right) dz \quad (9)$$

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (10)$$

For R_1 , $m = 2$:

$$R_1 = \frac{1}{(1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \frac{2z^{11}}{(z-1)^2} \right) \quad (11)$$

$$= 2 \lim_{z \rightarrow 1} \frac{d}{dz} (z^{11}) \quad (12)$$

$$= 22 \quad (13)$$

For R_2 , $m = 1$:

$$R_2 = \frac{1}{(1-1)!} \frac{3}{2} \lim_{z \rightarrow 3} \left((z-3) \frac{z^{11}}{(z-3)} \right) \quad (14)$$

$$= \frac{3}{2} \cdot 3^{11} \quad (15)$$

$$= 265720.5 \quad (16)$$

For R_3 , $m = 1$:

$$R_3 = \frac{1}{(1-1)!} \frac{3}{2} \lim_{z \rightarrow 1} \left((z-1) \frac{z^{11}}{(z-1)} \right) \quad (17)$$

$$= \frac{3}{2} \lim_{z \rightarrow 1} z^{11} \quad (18)$$

$$= \frac{3}{2} \quad (19)$$

$$R_1 + R_2 - R_3 = 265741 \quad (20)$$

$$\Rightarrow y(10) = 265741 \quad (21)$$

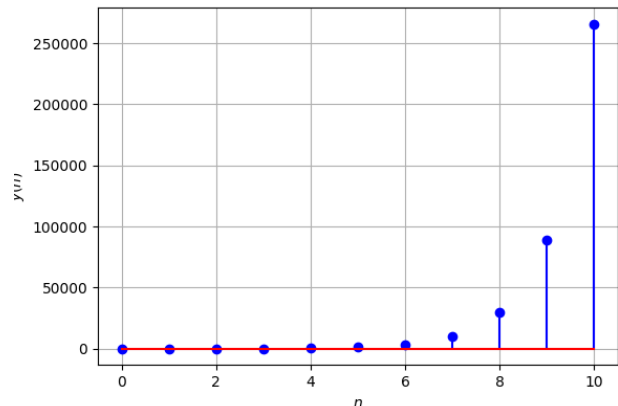


Fig. 0. stem plot