NCERT Discrete - 11.9.3.11

EE23BTECH11037 - M Esha*

Question 11.9.3.11: Evaluate $\sum_{k=1}^{11}(2+3^k)$.

Solution:

variable	value	description
$x(0)$	3	first term of the geometric progession
r	3	common ratio of the geometeric progression
$x(n)$	$3^n u(n)$	n^{th} term of the geometric progession
$y(n)$	$\frac{x(0)(r^{n+1}-1)}{r-1}u(n)$	Sum of the n term of the geometric progression

TABLE 0
INPUT PARAMETERS

$$\sum_{k=1}^{11}(2+3^k) = \sum_{k=1}^{11}2 + \sum_{k=1}^{11}3^k \tag{1}$$

$$= 2k + \sum_{k=0}^{10}3^{k+1} \tag{2}$$

$$\tag{3}$$

Applying Z-transform:

$$X(z) = x(0)\left(\frac{1}{1-rz^{-1}}\right), \quad |rz^{-1}| < 1 \tag{4}$$

$$y(n) = x(n) * u(n) \tag{5}$$

$$Y(z) = X(z)\,U(z) \tag{6}$$

$$= 3\left(\frac{1}{1-3z^{-1}}\right)\left(\frac{1}{1-z^{-1}}\right), \quad |z| > 3 \tag{7}$$

$$= \left(\frac{3}{2}\right)\left(\left(\frac{3}{1-3z^{-1}}\right) - \left(\frac{1}{1-z^{-1}}\right)\right) \tag{8}$$

$$\frac{1}{1-rz^{-1}} \overset{z^{-1}}{\longleftrightarrow} r^n u(n), \quad |z| > r \tag{9}$$

$$y(n) = 3\left(\frac{3^{n+1}-1}{3-1}\right)u(n) \tag{10}$$

$$y(n) = 3\left(\frac{3^{11}-1}{3-1}\right) \tag{11}$$

$$y(n) = \left(\frac{3^{12}-1}{2}\right) \tag{12}$$

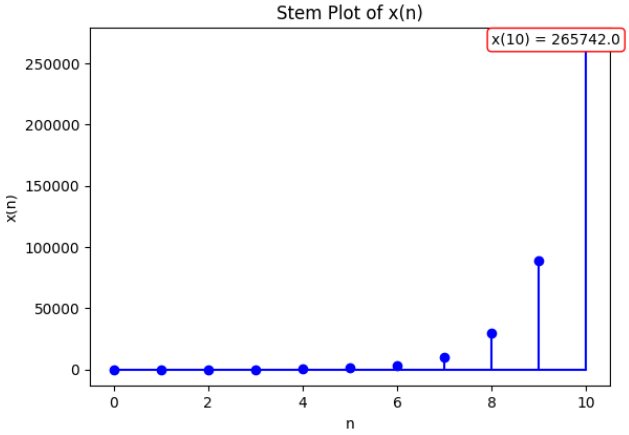

Fig. 0. stem plot