NCERT Discrete - 11.9.3.11

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Question 11.9.3.11:
Evaluate $\sum_{k=1}^{11}(2 + 3^k)$.
Solution:

variable	value	description
$x(0)$	3	first term of the GP
r	3	common ratio of the GP
$x(n)$	$3^n u(n)$	n^{th} term of the GP
$y(n)$	$\frac{x(0)(r^{n+1}-1)}{r-1}u(n)$	Sum of the n term of the GP

TABLE 0

INPUT PARAMETERS

$$x(n) = 2 + 3^n u(n) \tag{1}$$

$$X(z) = \frac{2}{1 - z^{-1}} + \frac{1}{1 - 3z^{-1}}, \quad |z| > 1 \tag{2}$$

$$y(n) = x(n) * u(n) \tag{3}$$

$$Y(z) = X(z) U(z) \tag{4}$$

$$= \frac{2}{(1 - z^{-1})^2} + \left(\frac{1}{(1 - 3z^{-1})(1 - z^{-1})}\right), \quad |z| > 1 \tag{5}$$

$$= \frac{2}{(1 - z^{-1})^2} + \left(\frac{1}{2}\right)\left(\left(\frac{3}{1 - 3z^{-1}}\right) - \left(\frac{1}{1 - z^{-1}}\right)\right) \tag{6}$$

Using Contour Integration to find the inverse Z-transform,

$$y(10) = \frac{1}{2\pi j} \oint_C Y(z) z^9 \, dz \tag{7}$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{2}{(1 - z^{-1})^2} + \left(\frac{1}{2}\right)\left(\frac{3}{1 - 3z^{-1}} - \frac{1}{1 - z^{-1}}\right)\right) \tag{8}$$

$$R = \frac{1}{(m - 1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \tag{9}$$

For R_1 , $m = 2$:

$$R_1 = \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz}\left((z - 1)^2 \frac{2z^{10}}{(z - 1)^2}\right) \tag{10}$$

$$= 2 \lim_{z \to 1} \frac{d}{dz}(z^{10}) \tag{11}$$

$$= 20 \tag{12}$$

For R_2 , $m = 1$:

$$R_2 = \frac{1}{(0)!} \lim_{z \to 3} \frac{1}{2}\left(z - 3\frac{3z^{11}}{z - 3}\right) \tag{13}$$

$$= \frac{1}{2}3^{12} \tag{14}$$

$$= 265720.5 \tag{15}$$

For R_3 , $m = 1$:

$$R_3 = \frac{1}{(0)!} \lim_{z \to 1}\left(\frac{1}{2}\left(z - 1\frac{z^{11}}{z - 1}\right)\right) \tag{16}$$

$$= \frac{1}{2} \lim_{z \to 1} z^{11} \quad\quad = 0.5 \tag{17}$$

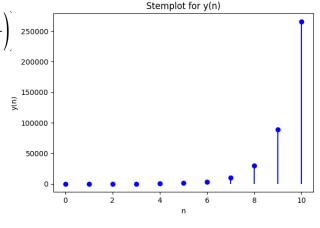$$R_1 + R_2 + R_3 = 265741 \tag{18}$$

$$\implies y(10) = 265741 \tag{19}$$

Fig. 0. stem plot