

Kernighans Algorithm

count of set bits in a number

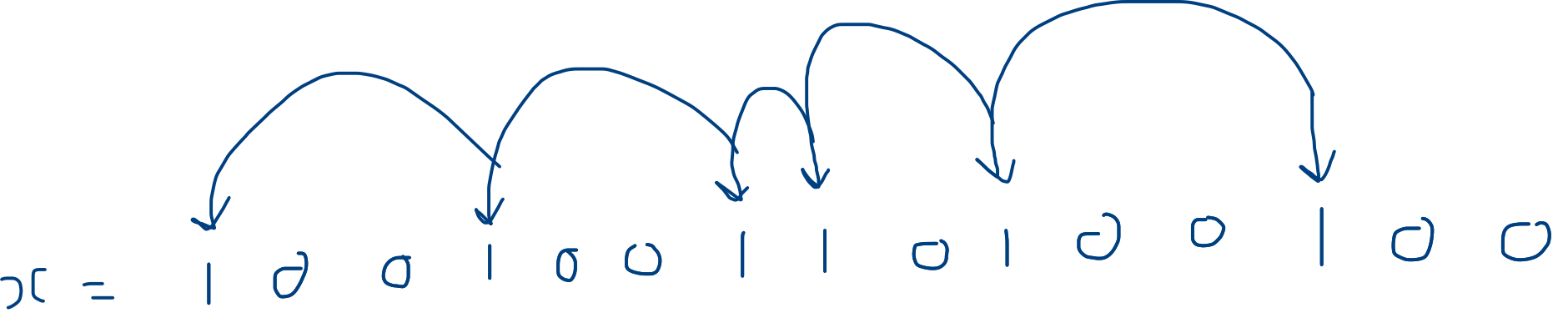
$x = 1\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 0$

$ans = 6$

Brute
Force

↓
0 0 1 0 0 1 0 0 1 1 0 1 0 0 1 0 0

$ans = 1 + 1 + 1 + 1 + 1 + 1$



$$\begin{aligned}
 w &= 1 + 1 + 1 + 1 + 1 + 1 \\
 &= 6
 \end{aligned}$$

$$x = \begin{array}{cccccccccccccc} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{array}$$

$$\underline{x_{\text{mem}}} = \begin{array}{cccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

$$x - x_{\text{mem}} = \begin{array}{cccccccccccccc} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccccccccccccc} x - x_{\text{mem}} & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ & 0 & & & 0 & & & 0 & 0 & & & & & & & \end{array}$$

$$cm = 0 + 1 + 1$$

Sum of Values at Indices With K Set Bits

0	1	2	3	4
5	10	1	5	2

Count Set Bit

Kth smallest algo

K=1

0	→	00	
1	→	01	
2	→	10	
3	→	11	2
4	→	100	1

arr[1] + arr[2] + arr[4]

$$10 + 1 + 2 = 13$$

Decode XORed Array

Sample Input 0

3
1
2
3
1

Sample Output 0

1
0
2
1

$$\text{arr} = \begin{matrix} & 0 & 1 & 2 & 3 \\ & 1 & 0 & 2 & 1 \end{matrix}$$

$$\begin{aligned} \text{arr}[0] &= \text{arr}[0] \wedge \text{arr}[1] & \text{arr}[1] &= \text{arr}[1] \wedge \text{arr}[2] & \text{arr}[2] &= \text{arr}[2] \wedge \text{arr}[3] \\ \hline & 1 & \hline & 2 & \hline & 3 \end{aligned}$$

$$\text{first} = 1$$

Sample Input 0

3
1
2
3
1

Sample Output 0

1
0
2
1

$$\begin{array}{cccc} & 0 & 1 & 2 & 3 \\ \text{arr} = & 1 & 0 & 2 & 1 \end{array}$$

$$\text{enc}(0) = \frac{\text{arr}[0] \wedge \text{arr}[1]}{1} \quad \frac{\text{arr}[1] \wedge \text{arr}[2]}{2} \quad \frac{\text{arr}[2] \wedge \text{arr}[3]}{3}$$

$$\text{first} = 1 = \text{arr}[0]$$

$$\text{encoded}[i] = \text{arr}[i] \wedge \text{arr}[i + 1]$$

XOR
reversible
property

$$\text{arr}[i + 1] = \text{encoded}[i] \wedge \text{arr}[i]$$

$$\underline{1}$$

$$\frac{0}{1} \quad \text{encoded}[i] \wedge \text{arr}[i]$$

$$\frac{2 \wedge 0}{1 \wedge 1} = 2$$

$$\underline{3 \wedge 2 = 1}$$

$x \in R \Rightarrow$

$$a \wedge 0 = a$$

$$a \wedge \bar{a} = 0$$

$$a \wedge b \wedge \bar{a} = b$$

Rewrite
=

$$\begin{array}{l} a \wedge b = c \\ b \wedge c = a \end{array}$$

$$1 \wedge 2 = 3$$

$$2 \wedge 3 = 1$$

$$4 \wedge 0 = 4$$

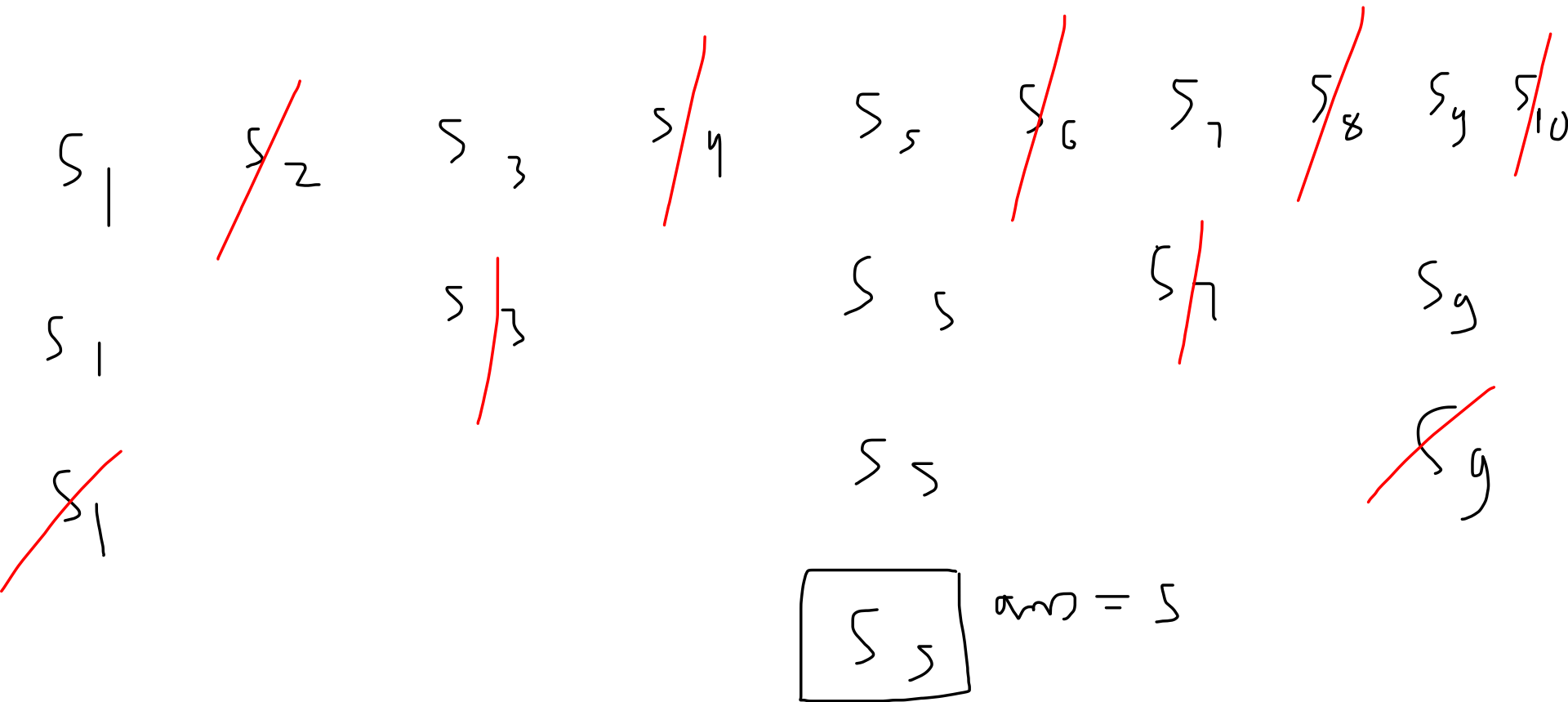
$$4 \wedge 4 = 0$$

$$4 \wedge 3 \wedge 4 = 3$$

Josephus Special

$$n = 10$$

soldiers are in circle



$$\text{Step 1} = n = 2^r + l$$

r = highest possible power of 2 to make n

$$\text{ans} = 2^l + 1 = \text{this soldier survived}$$

$$5 = 2^2 + 1$$

$$7 = 2^2 + 3$$

$$10 = 2^3 + 2$$

$$\text{Step 2} \quad \text{ans} = 2 \times 2 + 1 = 5$$

$$n=10 = 2^3 + 2 \rightarrow a_n = 2 \times 2 + 1$$

why? $2p+1$

Iteration 1

- 1 \rightarrow 0 0 0 1
- ~~2 \rightarrow 0 0 1 0~~
- 3 \rightarrow 0 0 1 1
- ~~4 \rightarrow 0 1 0 0~~
- 5 \rightarrow 0 1 0 1
- ~~6 \rightarrow 0 1 1 0~~
- 7 \rightarrow 0 1 1 1
- ~~8 \rightarrow 1 0 0 0~~
- 9 \rightarrow 1 0 0 1
- ~~10 \rightarrow 1 0 1 0~~

1 st iteration \rightarrow even soldiers get killed whose
LSB = 0

Soldiers have LSB = 1 get survived

1

iteration 2

all soldiers have bit = 1 get killed

1 → 0 0 0 1

~~2 → 0 0 1 0~~

~~3 → 0 0 1 1~~

~~4 → 0 1 0 0~~

5 → 0 1 0 1

~~6 → 0 1 1 0~~

~~7 → 0 1 1 1~~

~~8 → 1 0 0 0~~

9 → 1 0 0 1

~~10 → 1 0 1 0~~

0 1

iteration 3

all soldiers have bit = 1 get killed

1 → 0 0 0 1
~~2 → 0 0 1 0~~
~~3 → 0 0 1 1~~
~~4 → 0 1 0 0~~
 5 → 0 1 0 1
~~6 → 0 1 1 0~~
~~7 → 0 1 1 1~~
~~8 → 1 0 0 0~~
~~9 → 1 0 0 1~~
~~10 → 1 0 1 0~~

0 1 0 1 = 5

2 1 + 1

10 → 1 0 1 0

$$2^3 + 2 =$$

1 0 0 0
 8

+ 1 0
 2

(10) × 2

$$= 100 + 001$$

$$= 101$$

$$= 5$$

Counting Bits 2

$$r = 5$$

$$O/P = \begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 \end{matrix}$$

Count
set
bit

0	1	1	2	1	2
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Kennigham algo

Single Number |

arr = 4 8 4 8 1 2 1 3 3

ans = 2 \longrightarrow present one time in arr

xor =
 $a \wedge a = 0$
 $a \wedge 0 = a$

arr = ~~4~~ \wedge ~~8~~ \wedge ~~4~~ \wedge ~~8~~ \wedge ~~1~~ \wedge 2 \wedge ~~1~~ \wedge ~~3~~ \wedge ~~3~~
 $2 \wedge 0 = 2 = \text{ans}$