Tutorial - 2

1.
```
Void fun (int n) {
        int j=1, i=0;
        while (i<n)
        {
                i = i+j;
                j++;
        }
}
```

=> Values after execution

1st time → $i = 1$

2nd time → $i = 1+2$

3rd time → $i = 1+2+3$

4th time → $i = 1+2+3+4$

for i^{th} time → $i = (1+2+3+ --- i) < n$

$$= \frac{i(i+1)}{2} < n$$

$$= i^2 < n$$

$$= i = \sqrt{n}$$

Time Complexity $= O(\sqrt{n})$

2.
```
int fib (int n)
{
        if(n<=1)
            return n;
        return fib (n-1) + fib(n-2);
}
```

Recurrance Relation

$$F(n) = F(n-1) + F(n-2)$$

Let $T(n)$ denote the time complexity of $F(n)$.

In $F(n-1)$ and $F(n-2)$ time will be $T(n-1)$ and $T(n-2)$. We have one more addition to sum our results

For $n > 1$

$$T(n) = T(n-1) + T(n-2) + 1 \quad - (1)$$

For $n=0$ & $n=1$, no addition occours

$$\therefore \; T(0) = T(1) = 0$$

$$\text{Let } T(n-1) \simeq T(n-2) \quad - (2)$$

Adding (2) in (1)

$$T(n) = T(n-1) + T(n-1) + 1$$

$$= 2 \times T(n-1) + 1$$

Using backward substitution

$$T(n-1) = 2 \times T(n-2) + 1$$

$$T(n) = 2 \times [2 \times T(n-2) + 1] + 1$$

$$= 4T(n-2) + 3$$

We can substitute

$$T(n-2) = 2 \times T(n-3) + 1$$

$$T(n) = 8 \times T(n-3) + 7$$

General equation —

$$T(n) = 2^k \times T(n-k) + (2^k - 1) \quad - (3)$$

for $T(0)$

$$n - k = 0 \Rightarrow k = n$$

Substituting values in (3)

$$T(n) = 2^n \times T(0) + 2^n - 1$$

$= 2^n + 2^n - 1$

$$\boxed{T(n) = O(2^n)}$$

Space complexity $= O(N)$

The function calls are executed sequentially. Sequential execution grantees that the stack size will never exced the depth of cells for first $F(n-1)$ it will create N stack.

3.(i) $O(n \log n)$

```
#include<iostream>
using namespace std;
int partition (int arr[], int s, int e)
{
        int pivot = arr[s];
        int count = 0;
        for (int i=s; i<=e; i++)
        {
                if (arr[i] <= pivot)
                        count ++;
        }
        int pivot_ind = s+ count;
        swap (arr [pivot_ind], arr[s]);
        int i=s , j=e;
        while (i< pivot_ind && j> pivot_ind)
        {
```

```
    while (arr[i] <= pivot)
        i++;
    while (arr[j] > pivot)
        j--;
    if( i<pivot_ind && j>pivot_ind )
    {
        swap(arr[i++], arr[j--]); }
  }
  return pivot_ind;
}
void quick (int arr[], int s, int e)
{
    if (s==e)
        return;
    int p= partition (arr, s, e);
    quicksort (arr, s, p-1);
    quicksort (arr, p+1, e);
}
int main()
{
    int arr[]={6,8,5,2,1}
    int n=5;
    quicksort (arr, 0, n-1);
    return 0;
}
```

11) $O(N^3)$

```
int main()
{
    int n=10;
    for (int i=0; i<n; i++)
```

```
        {
            for (int j=0; j<n; j++)
            {
                for (int k=0; k<n; k++)
                {
                        printf(" *");
                }
            }
    }
    return 0;
}
```

iii) O(log log n)

```
int countPrimes(int n)
{
        if(n<2)
            return 0;
        bool * non-prime = new bool[n];
        non-prime[i]= true;
        int num nonPrime = 1;
        for (int i=2; i<n; i++)
        {
            if(nonPrime[i])
                continue;
            int j=i+2;
            while (j<n)
            {
                if(!nonprime[j])
                {
                    nonPrime[j]=true;
```

```
            numnonprime++;
        }
            j+=i;
    }
  }
  return (n-1) - numnonPrime;
}
```

4> $T(n) = T(n/4) + T(n/2) + Cn^2$

using master's Theorem

we can assume $T(n/2) >= T(n/4)$

Equation can be rewritten as

$T(n) <= 2T(n/2) + Cn^2$

$T(n) <= O(n^2)$

$T(n) = O(n^2)$

Also

$T(n) >= Cn^2 \Rightarrow T(n) >= O(n^2)$

$T(n) = \Omega(n^2)$

$T(n) = O(n^2)$ and $T(n) = \Omega(n^2)$

$T(n) = O(n^2)$

5>
```
int fun (int n)
{
    for (int i=1; i<=n; i++)
    {
        for (int j=1; j<n; j+=i)
        {
              // some O(1) task
        }
    }
}
```

for i=1, inner loop is executed n times

for i=2, inner loop is executed n/2 times.

for i=3, inner loop is executed n/3 times.

It is forming a series

$$n + \frac{n}{2} + \frac{n}{3} + \cdots + \frac{n}{n}$$

$$n\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$$

$$n \times \sum_{K=1}^{n} \frac{1}{K}$$

$$n \times \log n$$

Time Complexity = $O(n \log n)$

6)
```
for (int i=2; i<=n; i=pow(i,K))
{
    // some O(1) expressions
}
```

with iterations

i take values

for 1st iteration $\rightarrow 2$

for 2nd iteration $\rightarrow 2^K$

for 3rd iteration $\rightarrow (2^K)^K$

⋮

for n iteration $\rightarrow 2^{K \log_K(\log(n))}$

∵ last term must be less than or equal to n

$$2^{k^{\log_k(\log(n))}} = 2^{\log n} = n$$

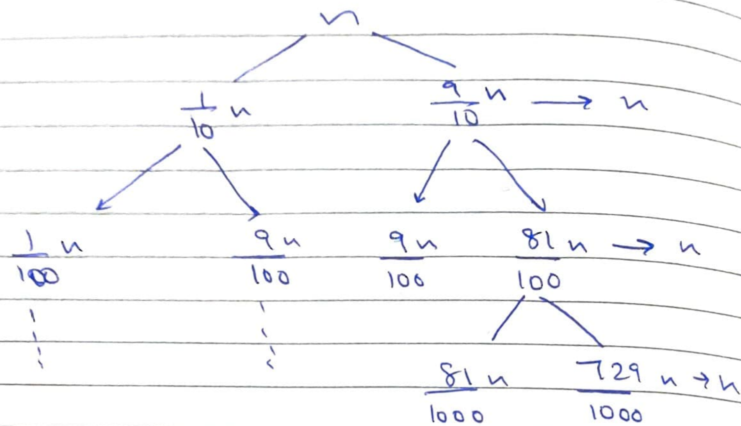
Each iteration takes constant time

∴

Total iteration $= \log_k(\log(n))$

Time Complexity $= O(\log(\log(n)))$

7)



If we split in this manner

Recurrence Relation

$$T(n) = T\left(\frac{9n}{10}\right) + T\left(\frac{n}{10}\right) + O(n)$$

when first branch is of size $9n/10$ & second one is $n/10$. showing the above using recursion tree approach calculating values.

At 1^{st} level, Value $= n$
At 2nd level, value $= \frac{9n}{10} + \frac{n}{10} = n$

Value remains same at all levels i.e n

Time Complexity = Summation of value
$O(n \times \log n)$ (upper bound)
$\Omega(n \log_{10} n)$ (lower bound)

$\Rightarrow \boxed{O(n \log n)}$

8) (a)
$100 < \log(\log n) < \log(n) < \sqrt{n} < n < n\log(n)$

$< \log^{\wedge}2(n) < \log(n!) < n^2 < 2^n < n! < 4^n < 2^{2^n}$

(b) $1 < \log(\log(n)) < \sqrt{\log(n)} < \log(n) < 2\log(n)$
$< \log(2n) < n < n\log(n) < \log(\sqrt{n}) < 2n$
$< 4n < n^2 < n! < 2(2^n)$

(c) $96 < \log_8(n) < n\log_6(n) < \log_2(n)$
$< n\log_2(n) < \log(n!) < 5n < 8n^2 < 7n^3$
$< n! < (8)^{2n}$