OPRE 6304 Fall 2024 Homework 3 (80 points)

(Homework 3 will be due on Oct 17th, Thursday, 11:59pm (CT). Please submit a single PDF write-up including answers to all questions on eLearning, and attach the Excel/R script files to your submission)

1. **Quantile Regression (30 points)** We have generated a synthetic dataset that includes the demand and quality scores of a product. The dataset is split into 80% training data in "quantile_train.csv" and 20% test data in "quantile_test.csv. For this problem you will use quantile regression to get the quantile estimates for this dataset.
    a. **(10 points)** First, plot a scatterplot of the data to show the relationship between quality and demand. Then, plot three quantile regression lines on top of the scatterplot at the following quantiles: $\tau = 0.25, 0.50, 0.75$.
    b. **(5 points)** Report the estimated quantile function for each value of τ.

    Suppose the product can be purchased at a cost of $10 and sold at a price of $30, regardless of its quality score.
    c. **(5 points)** What is the optimal ordering quantity if the quality score of the product is 5? Use quantile regression to find the optimal ordering quantity.
    d. **(10 points)** Use quantile regression to find the optimal ordering quantities for the data samples in the test set and report the out-of-sample cost.

2. **Revenue Management for Car Rental Service (20 points)** Town and Country Rental Car has experienced a substantial increase in business volume because of recent fare wars between the major air carriers. Town and Country operate a single office at a major international airport, with a fleet of 60 compact and 30 midsize cars. Recent developments have prompted management to rethink the company's reservation policy. The table below contains data on the rental experience of Town and Country. Midsize-car customers do not choose to rent a compact when no midsize car is available, or the other way around. The discount rate is available to persons who are willing to reserve a car in advance, and there is sufficient demand for the discount rate. The daily demand (of full rate customers) appears to follow a uniform distribution. The current reservation policy is that 40 compact cars are held for customers who are willing to pay the full rate and 25 midsize cars are held for full rate-paying customers. Using revenue management, determine the optimal number of compact and midsize cars to be held for customers paying the full rate.

Car	Full Rate	Discount Rate	Daily Demand (on the day of rental)
Compact	$30	$20	Uniform between [40,60]
Midsize	$45	$30	Uniform between [25,35]

Compact:
C_u = $30 – $20 = $10
C_o = $20
Critical ratio = 10 / (10+20) = 0.333
Should reserve 40 + 0.333 * (60-40), round up, 47 cars

Midsize:
C_u = \$45 – \$30 = \$15
C_o = \$30
Critical ratio = C_u / (C_u+C_o) = 15 / (15+30) = 0.333
Should reserve 25 + 0.333 * (35-25) = 28.3, round up, 29 cars

3. **Hotel Revenue Management (20 points)** You are in charge of the revenue management department at the Richardson's Days Out Hotel, which has a total of 200 rooms. Xpedia (a travel fare aggregator company) is offering to buy your rooms for Valentine's Day at a rate of \$200. On the other hand, you can book the rooms through your own website (which is far less popular than Xpedia) at a rate of \$400. The demand on your own website follows a uniform distribution between 15 and 25, and all the rooms that you did not sell to Xpedia nor booked through your website result unoccupied.

a. How many rooms should you sell to Xpedia? **(6 points)**

$$C_u = 400 - 200 = 200$$
$$C_0 = 200$$
$$\Rightarrow CR = \frac{C_u}{C_u + C_o} = \frac{200}{200 + 200} = \frac{1}{2}$$
$$\Rightarrow Q^* = 15 + \frac{1}{2} * (25 - 15) = 20$$

Notice that 20 is the total number of rooms that you should reserve for bookings through your website. Hence, you should sell 200-20=180 rooms to Xpedia.

Suppose now that, instead of selling rooms to Xpedia, you partner with them and do all the bookings through their website. The problem is that, since Xpedia is an aggregator, many guests tend to cancel their reservations at the last minute, which results in hotel rooms being unoccupied. You see this as a missed revenue opportunity and decide to implement the practice of overbooking. The price of each room is \$200, and if a guest shows up and a room is not available, you have to book a room at the nearby hotel that costs you \$450. Based on historical data, you know that late cancellations happen according to the following distribution:

Number of late cancellations	Probability	Cumulative Probability
0	0.1	0.1
1	0.2	0.3
2	0.4	0.7
3	0.15	0.85
4	0.1	0.95
5	0.05	1

b. How many overbookings should you accept? **(6 points)**

$$C_u = 200$$
$$C_0 = 450 - 200 = 250$$
$$\Rightarrow CR = \frac{C_u}{C_u + C_o} = \frac{200}{200 + 250} = \frac{4}{9} = 0.444$$

Hence, you should take 2 overbookings.

Due to the same late cancellations problem, you decide to reduce the price of your rooms to $150 and request customers to pay a non-refundable fee of $50 at the moment of booking (as a result, customers that show-up only pay $100 on sight). Despite these changes, you estimate that the distribution of late cancelations will be the same as before.

c. How many overbookings should you accept? **(8 points)**

$$C_u = 150 - 50 = 100$$
$$C_0 = 450 - 150 = 300$$
$$\Rightarrow CR = \frac{C_u}{C_u + C_o} = \frac{100}{100 + 300} = \frac{1}{4} = 0.25$$

Hence, you should accept only one overbooking.