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1.

To derive the price elasticity function $\epsilon(p)$ for the given logit price-response function

$$d(p) = \frac{De^{-(a+bp)}}{1+e^{-(a+bp)}}$$

we need to determine the elasticity of demand with respect to price. The elasticity function $\epsilon(p)$ is defined as:

$$\epsilon(p) = \frac{d'(p) \cdot p}{d(p)}$$

where $d'(p)$ is the derivative of $d(p)$ with respect to p.

Step 1: Differentiate $d(p)$ with respect to p

Using the quotient rule for derivatives, we have:

$$d'(p) = \frac{D \cdot \frac{d}{dp}\left(e^{-(a+bp)}\right) \cdot \left(1+e^{-(a+bp)}\right) - D \cdot e^{-(a+bp)} \cdot \frac{d}{dp}\left(1+e^{-(a+bp)}\right)}{\left(1+e^{-(a+bp)}\right)^2}$$

Since $\frac{d}{dp}\left(e^{-(a+bp)}\right) = -be^{-(a+bp)}$ and $\frac{d}{dp}(1+e^{-(a+bp)}) = -be^{-(a+bp)}$, we get:

$$d'(p) = \frac{D \cdot \left(-be^{-(a+bp)}\right) \cdot \left(1+e^{-(a+bp)}\right) - D \cdot e^{-(a+bp)} \cdot \left(-be^{-(a+bp)}\right)}{\left(1+e^{-(a+bp)}\right)^2}$$

Simplifying the terms:

$$d'(p) = \frac{-bDe^{-(a+bp)}(1+e^{-(a+bp)}) + bDe^{-2(a+bp)}}{(1+e^{-(a+bp)})^2}$$

Factoring out $-bDe^{-(a+bp)}$, we have:

$$d'(p) = \frac{-bDe^{-(a+bp)}\left(1+e^{-(a+bp)} - e^{-(a+bp)}\right)}{\left(1+e^{-(a+bp)}\right)^2}$$

This simplifies further to:

$$d'(p) = \frac{-bDe^{-(a+bp)}}{\left(1+e^{-(a+bp)}\right)^2}$$

Step 2: Substitute into $\epsilon(p)$

Now, substitute $d(p)$ and $d'(p)$ into the elasticity formula:

$$\epsilon(p) = \frac{d'(p)\cdot p}{d(p)} = \frac{\left(\frac{-bDe^{-(a+bp)}}{\left(1+e^{-(a+bp)}\right)^2}\right)\cdot p}{\frac{De^{-(a+bp)}}{1+e^{-(a+bp)}}}$$

Simplify by canceling terms:

$$\epsilon(p) = -bp \cdot \frac{1}{1+e^{-(a+bp)}}$$

Thus, the price elasticity function is:

$$\epsilon(p) = -\frac{bp}{1+e^{-(a+bp)}}$$

2. A bookstore owner currently sells a popular writing software package for $129.99, which costs them $89.00 wholesale. They've calculated the price elasticity of demand for this software to be 3.9 at the current sales price. To maximize net contribution, should the owner increase, decrease, or keep the price the same?

If the elasticity estimates of 3.9 remains consistent within a price range of $15.00 above or below the current price, what is the optimal price point for the owner to set?

Solution:

Given:

Current price, P=$129.99

Wholesale cost, C=$89.00

Elasticity of demand, E=3.9

A) Since the price elasticity of demand greater than 1, demand is elastic. For elastic demand, decreasing the price increases total revenue, as the percentage increase in demanded quantity outweighs the percentage decrease in price. A price reduction would increase the net contribution in this case.

So, the bookstore owner should **decrease the price** to maximize net contribution.

B) To determine the optimal price within the price range of $15, we can use the relationship between elasticity, cost, and optimal price for elastic demand.

The elasticity condition can be reversed to provide the condition for a particular price to maximize contribution.

The optimal price p* can be found by this formula.

p* = C* (ε(p*)/ε(p*)-1) = 89.00*(3.9/(3.9-1) = 89.00 *3.9/2.9 = $119.69

The optimal price calculation gives us approximately **$119.69**. This price adjustment within the elastic range would maximize net contribution by lowering the price from the current level, resulting in increased demand and a higher net contribution.

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3. A bicycle manufacturing company can produce electric bikes at an incremental cost of $400 each. The company's market research has identified a logit price-response function for the upcoming month's sales, where $d(p)=De-(a+bp)/(1+e-(a+bp))$, characterized by parameters D = 5,000, b = 0.005. The market research team has also identified that the demand for electric bikes is 2,500 when the price is $1,400. At what price should the company set its electric bikes to maximize net contribution?

Based on this optimal pricing strategy, how many electric bikes is the company expected to sell in the month?

Solution:

Given data :

Incremental cost per bike, C=$400

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Demand function parameters: D=5000 and b=0.005

Demand function as a function of price: $d(p)=De^{-(a+bp)}/(1+e^{-(a+bp)})$

Known demand is 2500 bikes when price p=$1400

(A) Find the price that maximizes net contribution.

We determine a first.

$d(p)=De^{-(a+bp)}/(1+e^{-(a+bp)})$

$2500 = 5000 * e^{-(a+0.005*1400)}/(1+e^{-(a+0.005*1400)})$

$2500/5000 = e^{-(a+0.005*1400)}/(1+e^{-(a+0.005*1400)})$

$0.5 = e^{-(a+7)} /(1+e^{-(a+7)})$

Solving for a, we get a = -7

Contribution function m(p) = d(p)*(p-c)

Our aim is to find the price p that maximizes the total contribution.

As it is a logit function, we can solve it using Excel.

In Excel, we put the known parameters and expressions for demand function.

D	5000
a	-7
b	0.005
unit cost	400
demand	3,726.53
revenue	4,416,881.89
cost	1,490,610.83
contribution	2,926,271.06
price	1185.25421
number of bikes	3726.52707

D, a, b and unit cost are known to us.

Demand at price p is calculated using formula: $d(p)=De^{-(a+bp)}/(1+e^{-(a+bp)})$

Revenue = Price * demand = p*d(p)

Cost = Unit cost * demand

We find the price p that maximizes total contribution using Solver function in Excel. We go to Data > Solver. We then set the objective to maximize the contribution cell by changing the price cell . We then select Max for optimization, then press Solve.

Using this, we get **the optimal price as $1185.25** for maximum contribution of $2926271.

The Excel sheet is attached herewith.

B) How many bikes expected to sell in one month?

To find the electric bikes expected to sell in the month, we need to divide the revenue by optimal price per bike.

So, number of bikes = Revenue/Optimal price

= 4,416,881.89/1185.25 = 3727(rounding up)

So, at the optimal price of approximately $1,185.25, **the company is expected to sell around 3,727 electric bikes in the month.**

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4. Julia runs a coffee cart in the JSOM. It costs her $1 to produce each cup of coffee, and she observes a linear price-response function d(p) = (180 – 20p)+ for her coffee each day.

a. Find the price per cup that maximizes Julia's net contribution, along with the resulting contribution. (5 points)

b. Suppose the large coffee chain proposes a deal to supply her with unlimited brewed coffee daily for a fixed cost of K dollars per day. If Julia agrees to this arrangement, what would be the optimal price per cup? (10 points)

c. At what fixed cost K values would entering this contract increase Julia's profits? (5 points)

d. The chain now modifies the offer to a fixed daily cost of $50 but limits the supply to a maximum of 80 cups of coffee per day. Should Julia accept this revised proposal? Justify your answer. (10 points)

Given:

Cost per cup C=$1

Price-response function: $d(p)=(180-20p)$

It means demand decreases linearly with price.

(A) Find the price per cup that maximizes Julia's net contribution.

Revenue = $p*d(p)=p*(180-20p)$

Since the cost per cup is $1, the total cost for d(p) cups is $1\times d(p)=180-20p$

Net Contribution = Revenue−Cost

Net Contribution, $m(p) = p*(180-20p)-(180-20p)$

$m(p) = 200p-20p^2-180$

To Maximize the Net Contribution, we differentiate the Net Contribution with respect to p and set it to zero to find the optimal price.

d(Net Contribution)/d(p) = 0

$200-40p = 0$

$p= 5$

So, the optimal price per cup that maximizes Julia's net contribution is $5.

At the optimal price of $5 per cup, Julia's maximum net contribution

= 200*5-20*25-180 = $320.

(B) Optimal Price per Cup with a Fixed Daily Cost K

Julia agrees to a fixed cost K for unlimited coffee.

Net Contribution = Revenue−Cost

Net Contribution=p*(180−20p)−K = 180p-20p^2-K

To Maximize the Net Contribution, we differentiate the Net Contribution with respect to p and set it to zero to find the optimal price.

d(Net Contribution)/d(p) = 0

180-40p = 0

p= 4.5

So, the optimal price per cup that maximizes Julia's net contribution is $4.50.

(C) Julia's maximum net contribution without the contract was $320.

For the contract to increase Julia's profits, her net contribution with the fixed cost K should be greater than this amount.

Net Contribution, m(p) = p×(180−20p)−K = 180p-20p^2-K

For price per cup at $4.50, m(p) = 180*4.5-20*4.5^2 -K = 405-K

Now, 405-K > 320

So, K < 85

So, the fixed cost value for K should be less than 85 to increase Julia's profits compared to the previous one.

(D) With a fixed cost of $50 and a maximum daily supply of 80 cups, we first determine the demand at Optimal Price of $5.

d(p)=d(5)= 180-20p = 180−20×5 = 80 cups, which matches the maximum supply.

Revenue= p*d(p)= 5×80=400

The fixed daily cost is $50

Net Contribution = 400−50 = $350

Without the contract, her maximum net contribution was $320.

With the new contract, her net contribution is $350.

So, Julia should accept the revised proposal because it increases her net contribution to $350 compared to her previous maximum of $320.

Julia can set the price to be $5 per cup and accept the supply of 80 cups.