Q1.

A1. Solution Summary:

1. **Opened Distribution Centers (DCs)**:
    - **Austin**
    - **Houston**
    - **Laredo**
2. **Customer Assignments**:
    - **Amarillo** → Assigned to **Austin**
    - **Austin** → Assigned to **Laredo**
    - **Dallas** → Assigned to **Austin**
    - **Houston** → Assigned to **Houston**
    - **Irving** → Assigned to **Austin**
    - **Laredo** → Assigned to **Laredo**
    - **Pasadena** → Assigned to **Laredo**
    - **San Antonio** → Assigned to **Houston**
3. **Total Annual Cost**: **$89,392,207.81**

{Kindly refer to attached python notebook for detailed calculation}

Q2.

A2.

Modified Formulation of CFLP

Decision Variables:

- x_{ij}: Binary variable, 1 if customer i is assigned to warehouse j, 0 otherwise.
- y_j: Binary variable, 1 if warehouse j is opened, 0 otherwise.
- z_j: Binary variable, 1 if warehouse j's capacity is expanded, 0 otherwise.

Parameters:

- d_i: Demand of customer i (in pallets).
- c_{ij}: Transportation cost per pallet from warehouse j to customer i (includes unit costs a_j).
- f_j: Fixed cost of opening warehouse j.
- β_j: Processing cost per pallet at warehouse j.
- w_{ij}: Fixed shipping cost from warehouse j to customer i, regardless of the quantity.
- p_j: Fixed cost of capacity expansion for warehouse j.
- t_j: Threshold beyond which warehouse j requires capacity expansion.
- s_j: Capacity of warehouse j without expansion.
- s'_j: Additional capacity provided if $z_j = 1$.

Objective Function:

$$\text{Minimize} \quad \sum_j f_j y_j + \sum_{i,j} c_{ij} d_i x_{ij} + \sum_j \beta_j \left(\sum_i d_i x_{ij} \right) + \sum_{i,j} w_{ij} x_{ij} + \sum_j p_j z_j$$

Explanation:

- $f_j y_j$: Fixed cost for opening warehouses.
- $c_{ij} d_i x_{ij}$: Transportation costs based on distances and demands.
- $\beta_j \sum_i d_i x_{ij}$: Processing costs at warehouses.
- $w_{ij} x_{ij}$: Fixed shipping costs for customer assignments.
- $p_j z_j$: Capacity expansion costs if required.

Constraints:

1. **Customer Assignment**:

$$\sum_j x_{ij} = 1 \quad \forall i$$

   Each customer i must be assigned to exactly one warehouse.

2. **Warehouse Capacity**:

$$\sum_i d_i x_{ij} \leq s_j y_j + s'_j z_j \quad \forall j$$

   The total demand served by a warehouse j must not exceed its capacity (including expanded capacity if $z_j = 1$).

3. **Capacity Expansion**:

$$z_j \geq \frac{\sum_i d_i x_{ij} - t_j}{s'_j} \quad \forall j$$

   If demand exceeds threshold t_j, the warehouse must expand its capacity ($z_j = 1$).

4. **Binary Variables**:

$$x_{ij} \in \{0,1\}, \quad y_j \in \{0,1\}, \quad z_j \in \{0,1\}$$

Key Changes:

(a) **Shipping Cost**: Added unit costs a_j as part of c_{ij}.

(b) **Processing Cost**: Introduced β_j to account for processing costs at each warehouse.

(c) **Fixed Shipping Cost**: Incorporated w_{ij} into the objective to handle a fixed cost for customer assignments.

(d) **Capacity Expansion Cost**: Added binary variable z_j and fixed cost p_j for capacity expansion, along with constraints to trigger expansion when demand exceeds t_j.