OPRE 6304 Spring 2024 Homework 2 (80 points)

(Homework 2 will be due on Sept 26th, Sunday, 11:59pm (CT). Please submit a single PDF write-up including answers to all questions on eLearning, and attach the Excel/R script files to your submission)

1. Multiple Linear Regression for AR model (30 points)

An **autoregressive (AR) model** is when the demand at time t is regressed on previous values from the same demand time series, i.e.

$$D_t = \beta_0 + \beta_1 D_{t-1} + \cdots + \beta_k D_{t-k} + \epsilon_t.$$

Here, k is the **order** of an autoregression model, for example, the AR model with order 2 (written as AR(2)) is

$$D_t = \beta_0 + \beta_1 D_{t-1} + \beta_2 D_{t-2} + \epsilon_t.$$

Given historical demand data $D_1, D_2, \dots, D_{202}$, estimate and report $\beta_0, \beta_1, \beta_2$ for AR(2) model using the method of multiple linear regression. You can find observation of $D_1, D_2, \dots, D_{202}$ in the file 'AR2.csv'

2. EOQ (20 points) Richardson's hospital consumers 500 boxes of masks on average each week. The price of each box is $50. The cost of placing an order is $250, and the cost of holding a box for a year is 1% of the value of the box. The demand is normally distributed with a standard deviation of 100 boxes (the mean is 500 as indicated before). Assume 50 weeks per year. The lead time is ½ week.

a. What is the optimal quantity order? **(3 points)**
b. What is the number of weeks that elapse between two consecutive orders? **(2 points)**
c. What is the number of orders placed per year? **(2 points)**
d. What is the total cost of carrying inventory and placing orders? **(3 points)**
e. Suppose that the hospital wants to keep a safety stock of 50 boxes. What would be the re-ordering point? **(2 points)**
f. Suppose that the hospital wants to guarantee a service level of 95%. What would be the re-ordering point? (**3 points**)
g. Due to supply chain issues, the hospital decided to switch its supplier of the masks. Due to the change of supply, the cost of placing an order increase to $260 per order. All the other cost parameters remain the same as those described in the paragraph before part a. What quantity should the hospital order with each order? **(3 points)**
h. How much additional total annual cost of placing orders & carrying inventory should the hospital pay if they switch the supplier (relative to the cost in part f) **(2 points)**

3. Newsvendor with Uniform and Normal Demand (10 points) Suppose you are a print house manager for newspapers. Your cost is 20 cents per newspaper, and you charge the retailer 80 cents per newspaper. The retailer sells to customers at $1 per newspaper. Any unsold newspaper is returned to you for full refund of the wholesale price.

a. If the demand for newspapers is uniformly distributed between 40 and 60, ow many newspapers should you print? **(5 points)**

b. If the demand for newspapers is normally distributed with mean 50 and standard deviation 9, how many newspapers should you print? **(5 points)**

4. Newsvendor with Discrete Demand (20 points) The UTD cafeteria offers scones for $1.5 each from 8 am to 3 pm. The scones are ordered from their supplier at the beginning of each day and are delivered before the store opens. The supplier charges 75 cents per scone. If at 3 pm, some scones are left unsold, the cafeteria sells the remaining scones for 50 cents each. Assume that all leftover scones are always sold by 4 pm when the cafeteria closes. If a customer asks for a scone before 3 pm but the cafeteria has run out, the customer always buys a bag of chips for $1 instead (after 3 pm, the customer does not buy anything instead). Assume the chips are always in stock, and they are purchased from the same supplier for 40 cents each. According to historical data, demand for scones before 3 pm at the cafeteria is variable but can only take values between 40 and 50, with probabilities given in the following table.

Number of customers	Probability
40	0.05
41	0.05
42	0.15
43	0.2
44	0.15
45	0.1
46	0.1
47	0.1
48	0.05
49	0.025
50	0.025

a. Yesterday the cafeteria ordered 45 scones, and 49 customers came wanting to buy a scone between 8 am and 3 pm. What was their total profit (including, if any, the profit on chips sold instead of scones)? **(6 points)**
b. How much is the underage cost C_u? **(3 points)**
c. How much is the overage cost C_o? **(3 points)**
d. What is the optimal number of scones to order at the start of each day? **(4 points)**
e. Given the order quantity in part d, what is the probability that the cafeteria cannot satisfy all the demand for scones before 3 pm? **(4 points)**