OPRE 6304: Operations Analytics
Fall 2024

HOMEWORK #4 SOLUTIONS

1. $d(p) = D\frac{e^{-(a+bp)}}{1+e^{-(a+bp)}}$.

$$d'(p) = -Db\frac{e^{a+bp}}{(1+e^{a+bp})^2}$$

$$\epsilon(p) = -\frac{d'(p)p}{d(p)} = Db\frac{e^{a+bp}}{(1+e^{a+bp})^2} \times \frac{p}{D\frac{e^{-(a+bp)}}{1+e^{-(a+bp)}}} = \frac{e^{a+bp}}{1+e^{a+bp}} \times bp$$

2.

a) The seller should raise his price if $p < [\,\epsilon/(\epsilon-1)]c$ and lower his price if $p > [\,\epsilon/(\epsilon-1)]c$. For the values stated in the problem, $[\,\epsilon/(\epsilon-1)]c = (3.9/2.9)\$89.00 = \$119.69$. Since his current price of $129.99 is greater than this, he should lower his price.

b) With an elasticity of 3.9, the contribution maximizing price is $119.69 as calculated in part a) of this problem. Since this is within $15 of the current price of $129.99, this is the contribution-maximizing price.

3. The unit margin for the car at price *p* is *p=1400.* We note that, $\hat{p} = -(a/b)$, so $a = -b\hat{p}$, or, in this case, *a = -(1400 * .005) = -7.* Thus, the price-response function is

$$d(p) = \frac{5000e^{-(-7+.005p)}}{1+e^{-(-7+.005p)}}$$

We wish to maximize $m(p) = (p-c)d(p)$. We can do this either by finding the *p** such that *m'(p*)=0* or by maximizing *m(p)* directly using a program such as Excel Solver or R.

In either case, the price that maximizes profit is *p**=$1185.25

And, at this price she will sell approximately 3727 cars.

4.

a) We want to find the contribution-maximization price. To do so, we need to maximize $m(p) = d(p)(p-c)$.

For contribution-maximization: MR=MC $d(p^*) = -d'(p^*)(p^*-c)$.

Therefore: $(180 - 20p^*) = 20(p^*-1)$.

The contribution-maximization price is: $p^* = 5$.

Now the contribution is: $m(p^*) = d(p^*)(p^* - c) = 80(5 - 1) = 320$ \$.

b) In this case, the cost of each hot-dog is $\frac{K}{d(p)}$. Therefore, the new function to maximize is: $m(p) = d(p)\left(p - \frac{K}{d(p)}\right) = pd(p) - K$. We differentiate with respect to p to find the optimal price:

$m'(p) = 0 \Rightarrow d'(p)p + d(p) = 0$

$$\Rightarrow \mathrm{p}* = -\frac{d'(p)}{d'(\mathrm{p})} = \frac{180}{40} \Rightarrow \quad p^* = 4.5$$

c) We should accept offer of the franchise company if we could have a higher profit. In case of contracting, the profit function:

At price = 5 → profit = \$320 (part A).

When price = 4.5 → d(p) = 180-20*4.5 = 90

Contribution when p=4.5 → 90*4.5 – K, which must be greater than 320

So: 405 – K > 320 → K < 85.

d) In this case, we have fixed cost K = 50, and max supply = 80 (less than demand = 90 at optimal price of 4.5)
→ d(p) = 80 → 180-20p=80 → p=5
→ profit = (180-20*5)*5 – 50 = 350 > 320 (original deal)
→ should take this deal.