Sliding mode Control of Marine Autonomous Vessels

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1 Introduction

The project is about the system modelling,state space representation of underwater autonomous vessels having 6 degree of freedom and station keeping of
autonomous surface vessel having 3 degree of freedom using sliding mode control
The report consists of two parts -

1. System modelling and linearisation of Underwater Autonomous vehicle
2. Station-keeping of autonomous surface vessels with sliding mode control

Autonomous Surface Vessels will have 6 D.O.F Surge, Sway, Roll , Heave, Yaw and Pitch

D.O.F	Motion	Force/Moment	Linear/Angular velocity	Positions/Euler Angles
1	Surge	X	u	x
2	Sway	Y	V	y
3	Pitch	M	Q	θ
4	Heave	Z	w	z
5	Roll	K	p	ϕ
6	Yaw	N	r	ψ

2 System Equations

The total force and moment equation is given as :-

Surge:

m[$\dot{u}$ -vr + wq - $x_G(q^2 + r^2)$ +y_G(pq - $\dot{r}$] + z_G(pr + $\dot{q}$)]=
X_{res} + $X_{|u|u}$u|u| + $X_{\dot{u}}\dot{u}$ + X_{wq}wq +X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop}

Sway:

m[$\dot{v}$ - wp +ur - $y_G(r^2+p^2)$ +z_G(qr -) + x$_G(pq + \dot{r})] = Y_{res} + Y_{|v|v}v|v| + Y_{|r|r}r|r| + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} +$
$Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r$

Heave:

$$m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + \quad y_G(rq + \dot{p})] = Z_{res} + Z_{|w|w}w|w|$$
$$+Z_{q|q|}q|q| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_s}u^2\delta_s$$

Roll:

$$I_x\dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} - uq + vp) -$$
$$z_G(\dot{v} - wp + ur)] = K_{res} + K_{p|p|}p|p| + K_{\dot{p}}\dot{p} + K_{prop}$$

Pitch:

$$I_y\dot{q} + (I_x - I_z)rp + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + \quad vp)] = M_{res} + M_{w|w|}w|w| + M_{q|q|}q|q| + M_{\dot{w}}\dot{w} + M$$
$$M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw + M_{uu\delta_s}u^2\delta_s$$

Yaw:

$$I_z\dot{r} + (I_y - I_z)pq + m[x_G(\dot{v} - wp + ur) -$$
$$y_G(\dot{u} - vr + wq)] = N_{res} + N_{v|v|}v|v| + N_{r|r|}r|r| + N_{\dot{v}}\dot{v} +$$
$$N_r\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + N_{uu\delta_r}u^2\delta_r \tag{1}$$

Equation (1) can be summarized in matrix form as follows :

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m - Y_{\dot{v}} & 0 & -mz_G & 0 & mx_G - Y_{\dot{r}} \\ 0 & 0 & m - Z_{\dot{w}} & my_G & -mx_G - Z_{\dot{q}} & 0 \\ 0 & -mz_G & my_G & I_x - K_{\dot{p}} & 0 & 0 \\ mz_G & 0 & -mx_G - M_{\dot{w}} & 0 & I_y - M_{\dot{q}} & 0 \\ -my_G & mx_G - N_{\dot{v}} & 0 & 0 & 0 & I_z - N_{\dot{r}} \end{bmatrix}^{-1} \begin{bmatrix} \Sigma X \\ \Sigma Y \\ \Sigma Z \\ \Sigma K \\ \Sigma M \\ \Sigma N \end{bmatrix} \tag{2}$$

3 Linearisation

The nonlinear system of AUV in equation (2) model can be linearied with Jacobian approach where the nonlinear AUV system in general as follows

$$\dot{x}(t) = f(x(t), u(t),\ t)$$
$$y(t) = g(x(t), u(t),\ t)$$

$$J_{x1} = \begin{bmatrix} \frac{\partial \Sigma X}{\partial u} & \frac{\partial \Sigma X}{\partial v} & \frac{\partial \Sigma X}{\partial w} & \frac{\partial \Sigma X}{\partial p} & \frac{\partial \Sigma X}{\partial q} & \frac{\partial \Sigma X}{\partial r} \\ \frac{\partial \Sigma Y}{\partial u} & \frac{\partial \Sigma Y}{\partial v} & \frac{\partial \Sigma Y}{\partial w} & \frac{\partial \Sigma Y}{\partial p} & \frac{\partial \Sigma Y}{\partial q} & \frac{\partial \Sigma Y}{\partial r} \\ \frac{\partial \Sigma Z}{\partial u} & \frac{\partial \Sigma Z}{\partial v} & \frac{\partial \Sigma Z}{\partial w} & \frac{\partial \Sigma Z}{\partial p} & \frac{\partial \Sigma Z}{\partial q} & \frac{\partial \Sigma Z}{\partial r} \\ \frac{\partial \Sigma K}{\partial u} & \frac{\partial \Sigma K}{\partial v} & \frac{\partial \Sigma K}{\partial w} & \frac{\partial \Sigma K}{\partial p} & \frac{\partial \Sigma K}{\partial q} & \frac{\partial \Sigma K}{\partial r} \\ \frac{\partial \Sigma M}{\partial u} & \frac{\partial \Sigma M}{\partial v} & \frac{\partial \Sigma M}{\partial w} & \frac{\partial \Sigma M}{\partial p} & \frac{\partial \Sigma M}{\partial q} & \frac{\partial \Sigma M}{\partial r} \\ \frac{\partial \Sigma N}{\partial u} & \frac{\partial \Sigma N}{\partial v} & \frac{\partial \Sigma N}{\partial w} & \frac{\partial \Sigma N}{\partial p} & \frac{\partial \Sigma N}{\partial q} & \frac{\partial \Sigma N}{\partial r} \end{bmatrix}$$

Jacobian matrix thus obtained after linearisation are:

$$J_{u2} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m - Y_{\dot{v}} & 0 & -mz_G & 0 & mx_G - Y_{\dot{r}} \\ 0 & 0 & m - Z_{\dot{w}} & my_G & -mx_G - Z_{\dot{q}} & 0 \\ 0 & -mz_G & my_G & I_x - K_{\dot{p}} & 0 & 0 \\ mz_G & 0 & -mx_G - M_{\dot{w}} & 0 & I_y - M_{\dot{q}} & 0 \\ -my_G & mx_G - N_{\dot{v}} & 0 & 0 & 0 & I_z - N_{\dot{r}} \end{bmatrix}^{-1} J_{u1}$$

$$J_{u1} = \begin{bmatrix} \frac{\partial\Sigma X}{\partial X_{prop}} & \frac{\partial\Sigma X}{\partial\delta_r} & \frac{\partial\Sigma X}{\partial\delta_s} & \frac{\partial\Sigma X}{\partial K_{prop}} & \frac{\partial\Sigma X}{\partial\delta_s} & \frac{\partial\Sigma X}{\partial\delta_r} \\ \frac{\partial\Sigma Y}{\partial X_{prop}} & \frac{\partial\Sigma Y}{\partial\delta_r} & \frac{\partial\Sigma Y}{\partial\delta_s} & \frac{\partial\Sigma Y}{\partial K_{prop}} & \frac{\partial\Sigma Y}{\partial\delta_s} & \frac{\partial\Sigma Y}{\partial\delta_r} \\ \frac{\partial\Sigma Z}{\partial X_{prop}} & \frac{\partial\Sigma Z}{\partial\delta_r} & \frac{\partial\Sigma Z}{\partial\delta_s} & \frac{\partial\Sigma Z}{\partial K_{prop}} & \frac{\partial\Sigma Z}{\partial\delta_s} & \frac{\partial\Sigma Z}{\partial\delta_r} \\ \frac{\partial\Sigma K}{\partial X_{prop}} & \frac{\partial\Sigma K}{\partial\delta_r} & \frac{\partial\Sigma K}{\partial\delta_s} & \frac{\partial\Sigma K}{\partial K_{prop}} & \frac{\partial\Sigma K}{\partial\delta_s} & \frac{\partial\Sigma K}{\partial\delta_r} \\ \frac{\partial\Sigma M}{\partial X_{prop}} & \frac{\partial\Sigma M}{\partial\delta_r} & \frac{\partial\Sigma M}{\partial\delta_s} & \frac{\partial\Sigma M}{\partial K_{prop}} & \frac{\partial\Sigma M}{\partial\delta_s} & \frac{\partial\Sigma M}{\partial\delta_r} \\ \frac{\partial\Sigma N}{\partial X_{prop}} & \frac{\partial\Sigma N}{\partial\delta_r} & \frac{\partial\Sigma N}{\partial\delta_s} & \frac{\partial\Sigma N}{\partial K_{prop}} & \frac{\partial\Sigma N}{\partial\delta_s} & \frac{\partial\Sigma N}{\partial\delta_r} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{uu\delta_r}u^2 & 0 & 0 & 0 & Y_{uu\delta_r}u^2 \\ 0 & 0 & Z_{uu\delta_s}u^2 & 0 & Z_{uu\delta_s}u^2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & M_{uu\delta_s}u^2 & 0 & M_{uu\delta_s}u^2 & 0 \\ 0 & N_{uu\delta_r}u^2 & 0 & 0 & 0 & N_{uu\delta_r}u^2 \end{bmatrix}$$

$$J_{u2} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m - Y_{\dot{v}} & 0 & -mz_G & 0 & mx_G - Y_{\dot{r}} \\ 0 & 0 & m - Z_{\dot{w}} & my_G & -mx_G - Z_{\dot{q}} & 0 \\ 0 & -mz_G & my_G & I_x - K_{\dot{p}} & 0 & 0 \\ mz_G & 0 & -mx_G - M_{\dot{w}} & 0 & I_y - M_{\dot{q}} & 0 \\ -my_G & mx_G - N_{\dot{v}} & 0 & 0 & 0 & I_z - N_{\dot{r}} \end{bmatrix}^{-1} J_{u1}$$

State space representation

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = J_{x2} \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix} + J_{u2} \begin{bmatrix} X_{prop} \\ \delta_r \\ \delta_s \\ K_{prop} \\ \delta_s \\ \delta_r \end{bmatrix}$$

Now we will discuss about station-keeping of ASV using sliding mode control

4 Sliding Mode Control

The basic principle of sliding mode control is to create a sliding surface in the state space of the system and design a control law that drives the system's state trajectory onto the sliding surface and keeps it there. The sliding surface is typically a hyperplane defined in terms of the system state variables, and its slope and intercept are chosen to ensure that the system's state converges to it in a finite time.

Once the system's state is on the sliding surface, the control law is designed to keep it there by generating a sliding mode control signal that drives the system's state along the sliding surface. The sliding mode control signal is typically a discontinuous signal that switches between two or more modes of operation, each of which corresponds to a different control law.

The switching between the modes of operation ensures that the system's state remains on the sliding surface and that the control signal remains robust to disturbances and uncertainties in the system. The control signal is designed to minimize the distance between the system's state and the sliding surface, and to prevent the system's state from leaving the sliding surface by creating a virtual barrier around it.

The reaching conditions are normally established by defining

$$(1/2)S^T S$$

as the Lyapunov function and ensuring that its time derivative is negative. The reaching condition for each surface i may be defined as
$\mathrm{S}_i S_i \leq -\eta_i |S_i|, \qquad \eta_i > 0; \quad i = 1,2$

A robust sliding mode station-keeping controller was designed and implemented on the ASV to mitigate slowly varying environmental disturbances, such as tidal currents, that the system cannot directly measure through its sensors.
The inputs to all of the controllers are the position and heading errors in the body-fixed coordinate frame

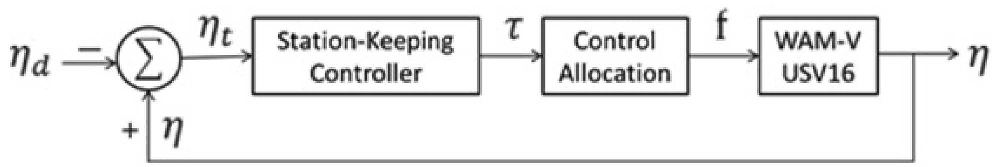

Figure 1: Control flow block diagram for station-keeping

A sliding function is defined for station-keeping

$$\boldsymbol{s} = \dot{\boldsymbol{\eta}}_t + 2\boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\Lambda}^2 \int_0^t \boldsymbol{\eta}_t \, dt.$$

where $\dot{\eta}_t$ represents derivative of earth-fixed tracking error vector , $\wedge$ is the lyapunov exponent gain matrix

For minimizing unmodeled environmental disturbance, reference trajectory is defined as

$$\dot{\boldsymbol{\eta}}_r = \dot{\boldsymbol{\eta}}_d - 2\boldsymbol{\Lambda}\boldsymbol{\eta}_t - \boldsymbol{\Lambda}^2 \int_0^t \boldsymbol{\eta}_t \, dt.$$

where $\dot{\eta}_d$ is derivative of desired state

The control law ensures that if the system deviates from the surface defined , it is forced back to it. Once on the surface, the under-modeled system reduces to an exponentially stable, second-order system. The system's response therefore depends heavily on the choice of the sliding surface.

The error feedback control law for station-keeping is defined :

$$\tau = M_1[(J(\eta)^T \ddot{\eta}_r + \dot{J}(\eta)^T \dot{\eta}_r] + C_1(v) J(\eta)^T \dot{\eta}_r + D_1(v) J(\eta)^T \dot{\eta}_r - J(\eta)^T R * sat(E^{-1} * s).$$

the last term in the control law includes the bound on the uncertainties "R" and the boundary layer thickness "E" around the sliding surface "s" system's errors are within specific boundaries dictated by E, the control signal will vary based on s, so that s< $|R|$

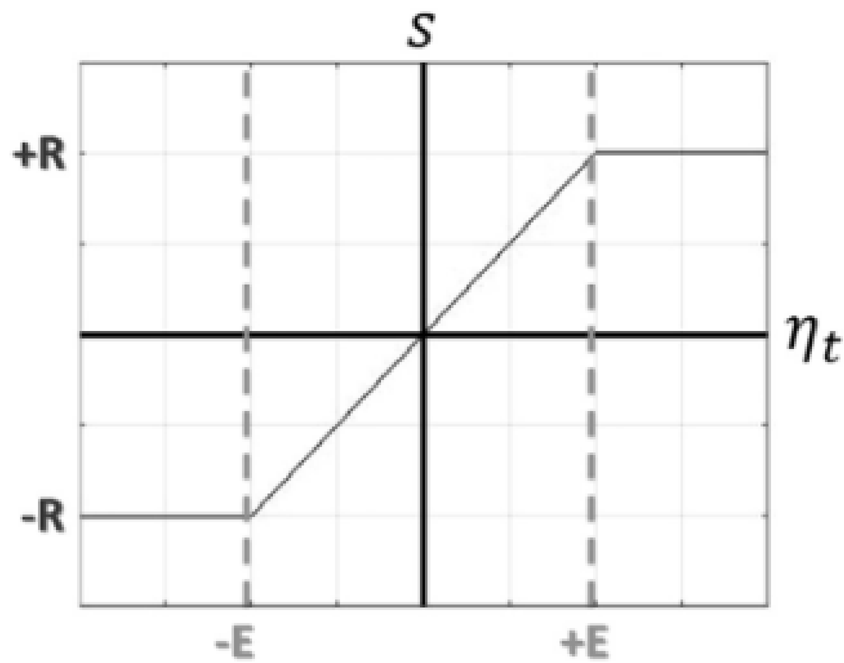

Figure 2: Sliding surface bounded by E & R

The wind forces and moment acting on the vehicle are modeled as:

$$\tau_w = q \begin{bmatrix} C_X(\gamma_{rw})A_{fw} \\ C_Y(\gamma_{rw})A_{Lw} \\ C_N(\gamma_{rw})A_{Lw}Laa \end{bmatrix}$$

where A$_{fw}$, A_{Lw} *are the frontal the frontal and lateral projected windage area*
γ_{rw} *is apparent angle of attack*
q is dynamic pressure

The average wind speed V =2.43rw m/s was taken for simulation ,
the corresponding length scales of the turbulent fluctuations in the wind are expected to be around 78 m

As the length of the vessel is smaller , a single point measurement of the wind speed and direction is taken to be sufficiently representative of the wind characteristics acting across the entire vehicle.

5 Station keeping Procedure

A series of sea trials were performed to test the performance of the station-keeping controllers [2]. Wind and current were therefore the two major causes of environmental disturbance.[1]

Illustration of the desired state (marked red) of the vehicle at location is shown in simulation video . The desired heading of the vehicle was also chosen to reproduce the most friendly scenario.

6 Simulation

Sliding mode controller gave the required thrust so that ship remains stationed at desired position in presence of wind-disturbances Software used is Gazebo with external wind disturbance given , program was written in python. The required thrust was given to vessel as per the disturbance such that ship remains at desired location

The Drive link for the simulation of station keeping is mentioned below :

https://drive.google.com/file/d/1qQiZ3SiBYUiuJCTOZUkJXaJe9KeUpL5A/view?usp=sharing

References

[1] Halil Akçakaya, H Alpaslan Yildiz, Gaye Sağlam, and Fuat Gürleyen. Sliding mode control of autonomous underwater vehicle. In *2009 International Conference on Electrical and Electronics Engineering-ELECO 2009*, pages II–332. IEEE, 2009.

[2] Anthony J Healey and David Lienard. Multivariable sliding mode control for autonomous diving and steering of unmanned underwater vehicles. *IEEE journal of Oceanic Engineering*, 18(3):327–339, 1993.