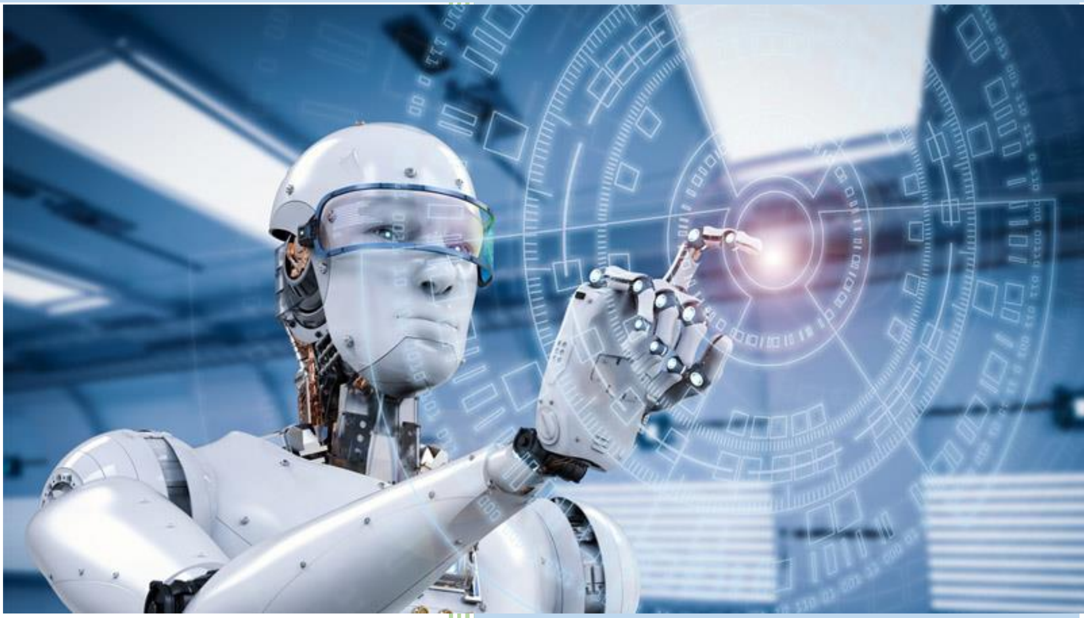
Model Formulations - Case Studies

# Linear Programming



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## Case study 01

The National Farmers' Union requested assistance from Advanced Engineering Mathematics Research Center, University of Moratuwa to identify the optimum land utilization plan for the dominant crop types within the Anuradhapura Irrigation Division in such a way that the overall profit of the total yield is maximized while being subjected to potential limitations. Assume that you are assigned as the data scientist to consult them to achieve the union's objective. The case study is as follows.

Anuradhapura Irrigation Division splits the total farmland area into Thanthirimale, Pulmude, Rambewa, Tirappane, and Rajanganaya divisions for administrative purposes. Black gram, sesame, and big onion are the dominant crop categories in those areas during the coming season and farmers are getting ready for cultivation. The Anuradhapura Irrigation Department has just published its seasonal water allotment, with all five divisions receiving 7,400 acre-feet for cultivation purposes. Each division can only absorb a specified amount of irrigation during the season as specified by the Engineering and Land Utilization division. Thanthirimale division - 2000 acres of farmlands with 3200 acre-feet irrigation limit, Pulmude division - 2300 acres of farmlands with 3400 acre-feet irrigation limit, Rambewa division - 600 acres of farmlands with 800 acre-feet irrigation limit, Tirappane division - 1100 acres of farmlands with 500 acre-feet irrigation limit, Rajanganaya - 500 acres of farmlands with 600 acre-feet irrigation limit. Each crop cultivated in the divisions needs a minimum amount of water requirement per acre, and there is an estimated sales barrier for each crop indicated as follows. Black gram - Maximum sales of 110,000 bushels and 1.6 acre-feet water needed per acre, Sesame - Maximum sales of 1800 tons and 2.9 acre-feet water needed per acre, big onion - Maximum sales of 2200 tons and 3.5 acre-feet water needed per acre. Further at least 800 tons of Sesame must be cultivated to maintain the demand for foreign export. The National Farmers' Union estimate is that they could sell black gram at a net profit of Rs 2,000 per bushel, sesame at Rs 40,000 per ton, and big onion at Rs 50,000 per ton. One acre of land yields an average of 1.5 tons of sesame and 2.2 tons of big onion. The black gram yield is approximately 50 bushels per acre. Further, the water authority informs the farmers that they may obtain an additional allotment of 600 acre-feet of water for an additional fee of Rs 6,000,000 during this season.

During the consultation process, you are supposed to

1. Identify the problem of the case study, the objective of the study, the decisions to be taken, and the limitations of the problem. Then suggest a suitable optimization technique to formulate the problem with justifications.
2. Formulate the optimization model.
3. Obtain the solution for the optimization problem by using a suitable programing language (R/Python) with appropriate libraries.
4. Identify the optimum cultivation plan (number of acres to be allocated for each crop type in each division) and the maximum expected profit that could be earned from the yield.
5. How do you recommend the acquisition of additional water allotment?

## Case study 02

The ComputeCloud company offers eight cloud services across five data centers: Alpha, Beta, Gamma, Delta, and Epsilon. These data centers can host various types of services: Web Hosting (WH), Database Services (DB), File Storage (FS), Data Analytics (DA), Virtual Private Servers (VPS), Content Delivery Network (CDN), Machine Learning (ML), and Internet of Things (IoT) services. Each data center has limitations on computational power (measured in CPU-GHz), storage capacity (measured in Terabytes - TB), and network bandwidth (measured in Gbps). Each type of service has different requirements for computational power, storage, and bandwidth, as well as different operational costs and revenue potentials. ComputeCloud needs to decide how much of each service to run in each data center to maximize their profit. and ensuring that customer demand and service level agreements (SLAs) are met.

Data Center Capacities

| | Alpha | Beta | Gamma | Delta | Epsilon |
|---|---|---|---|---|---|
| Computational Power (CPU-GHz) | 2500 | 2000 | 2300 | 3000 | 1500 |
| Storage Capacity (Terabytes - TB) | 1200 | 2500 | 1800 | 3500 | 1000 |
| Network Bandwidth (Gbps) | 800 | 1500 | 1000 | 900 | 600 |

Service Requirements (per unit)

| | WH | DB | FS | DA | VPS | CDN | ML | IoT |
|---|---|---|---|---|---|---|---|---|
| Computational Power (CPU-GHz) | 8 | 12 | 4 | 18 | 10 | 5 | 20 | 6 |
| Storage Capacity (Terabytes - TB) | 4 | 8 | 15 | 12 | 6 | 3 | 10 | 2 |
| Network Bandwidth (Gbps) | 2 | 4 | 2 | 8 | 3 | 10 | 5 | 1 |
| Service Profit (per unit) in $ | 90 | 140 | 70 | 190 | 110 | 160 | 210 | 120 |
| Minimum Demand (units) | 120 | 90 | 160 | 75 | 130 | 85 | 60 | 140 |

Interdependent service constraints: Some service may require others to function

- Each unit of Database Services (DB) requires at least half a unit of File Storage (FS)

Shared resource constraints: some services share a common infrastructure

- Web Hosting (WH) and Content Delivery Network (CDN) services share a common load balancer in each center having capacity of 300 units.

Data sovereignty compliance:

- IoT data must be stored and processed within the Alpha or Beta centers only due to data sovereignty laws.

Formulate the optimization model (Linear programming model) for the above case study.

Case study 03

The Union Chemical Sri Lanka operates five production facilities: Alpha, Beta, Gamma, Delta, and Epsilon. These facilities can produce four primary chemicals: Xylenol, Methacrylate, Ethanolamine, and Butanol. The company has outlined the resource allocation for each facility based on their production capabilities, raw material availability and energy constraints for the upcoming production cycle.

Facility capacities, energy limits, and raw material availability are as follows:

| Facility Name | Alpha | Beta | Gamma | Delta | Epsilon |
|---|---|---|---|---|---|
| Production_Capacity_in'000_metric_tons | 35 | 30 | 15 | 25 | 20 |
| Energy limit in '000 MWh | 50 | 45 | 20 | 30 | 25 |
| Raw material A availability in '000 tons | 20 | 18 | 10 | 15 | 12 |
| Raw material B availability in '000 tons | 15 | 20 | 9 | 12 | 14 |
| Raw material C availability in '000 tons | 10 | 12 | 8 | 15 | 10 |

Chemical production details are:

| Chemical type | Xylenol | Methacrylate | Ethanolamine | Butanol |
|---|---|---|---|---|
| Maximum sales_in'000_metric_tons | 70 | 55 | 45 | 50 |
| Energy requirement (MWh/ton) | 0.9 | 1.3 | 1.6 | 1.1 |
| Raw material A requirement (tons/ton) | 2 | 1.5 | 1 | 0.8 |
| Raw material B requirement (tons/ton) | 1 | 2 | 1.2 | 0.6 |
| Raw material C requirement (tons/ton) | 0.5 | 0.8 | 2 | 1.5 |
| Profit (in '000 Rupees) | 6 | 11 | 16 | 9 |

Develop the linear programming model associated with the case study if total chemical production at Beta facility must not exceed twice the total chemical production at Epsilon facility.