

# ASSIGNMENT - 1

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SUB : DESIGN AND ANALYSIS OF ALGORITHM

Sub code : CSA 0688

Solve

$$a) x(n) = x(n-1) + 5 \quad \text{for } n > 1 \quad x(1) = 0$$

$$b) x(n) = 3x(n-1) \quad \text{for } n > 1 \quad x(1) = 4$$

$$x(n) = x(n-1) + 5 \rightarrow ①$$

Sub  $x(n-1)$  in ①

$$x(n) = (x(n-2) + 5) + 5$$

$$= x(n-2) + 10 \rightarrow ②$$

Sub  $x(n-2)$  in ②

$$x(n) = (x(n-3) + 10) + 5$$

$$= x(n-3) + 15 \rightarrow ③$$

$$x(k) = x(n-k) + (n+4)$$

$$n-k = 0$$

$$n = k$$

$$= x(n-n) + (n+4)$$

$$= x(0) + n+4$$

$$= 1 + n + 4$$

$$= 5$$

$$\Rightarrow O(n) \rightarrow \text{Linear}$$

$$x(n) = x(n-1) + 1$$

$$x(n-1) = x(n-1-1) + 1$$

$$x(n-2) + 1$$

$$x(n-2) = x(n-2-1) + 1$$

$$x(n-3) + 1$$

$$5) x(n) = 3x(n-1) + D \text{ for } n \geq 1 \quad x(0) = 4$$

$$x(n) = 3x(n-1) + D \quad \textcircled{1}$$

sub  $x(n-1)$  in  $\textcircled{1}$

$$\begin{aligned} x(n) &= 3(3x(n-2) + D) \\ &= 4x(n-2) + 3D \end{aligned} \quad \textcircled{2}$$

$$\begin{aligned} x(n) &= 9 \cdot 3x(n-3) \\ &= 27x(n-3) \end{aligned} \quad \textcircled{3}$$

$$x(k) = k \cdot 3x(n-k)$$

$$n-k=0$$

$$n=k$$

$$= n \cdot 3x(n-k)$$

$$= n \cdot 30$$

$$= O(n) \rightarrow \text{Linear}$$

$$x(n) = 3x(n-1)$$

$$\begin{aligned} x(n-1) &= 3x(n-2) \\ &= 3x(n-3) \end{aligned}$$

$$\begin{aligned} x(n-2) &= 3x(n-3) \\ &= 3x(n-3) \end{aligned}$$

$$2. a) T(n) = T(n/2) + 1$$

$$b) T(n) = T(n/3) + T(2n/3) + cn$$

$$a) T(n) = T(n/2) + 1 \rightarrow n = 2^k$$

$$T(n) = T(n/2) + 1$$

$$T(n) = T(n/4) + 1 + 1$$

$$= T(n/4) + 2$$

$$T(n) = T(n/8) + 1 + 2$$

$$= T(n/16) + 3$$

$\vdots$   $k$  times

$$T(n) = T(n/2^k) + k$$

$$n/2^k = 1$$

$$2^k = n$$

$$k = \log_2 n$$

$$T(n) = \log_2 n + 1$$

3. Analyze the order of growth

$$f(n) = 2n^2 + 5 \quad g(n) = 7n$$

$$f(n) = 2n^2 + 5$$

$$g(n) = 7n$$

$$= (2+5) \cdot n^2$$

$$= 7n^2$$

$$c = 7 \quad g(n) = 7n$$

$$f(n) \geq g(n)$$

$$\Rightarrow f(n) = 2g(n)$$

Hence proved.

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Tours of Hanoi: find minimum solution

Tours of Hanoi ( $n$ , source, destination, aux)

if  $n = 1$

move disk from destination.

else

Tours of Hanoi ( $n-1$ , source, destination, aux)

move disk from source to destination

Tours of Hanoi ( $n-1$ , aux, destination, source)

$$\Rightarrow T(n) = 2T(n-1) + 1$$

$$T(n-1) = 2T(n-2) + 1$$

$$T(n) = 2[2T(n-2) + 1] + 1$$

$$= 4T(n-2) + 3$$

$$T(n) = 2^2 T(n-2) + (2^2 - 1)$$

$$= 2^n - 1$$

$$T(n) = O(2^n)$$

$$\Rightarrow T(n) = 3T(n/2) + n^2$$

$$T(n) = T(n/2) + 2^n$$

using Master theorem

$$\Rightarrow T(n) = 3T(n/2) + n^2$$

$$a=3, b=2$$

$$y(n) = n^2$$

$$\Rightarrow n \log_2^3$$

$$f(n) \geq n \log_2^3$$

$$= \Theta(n^2) \parallel$$

b) Master theorem can't be applied

$$T(n) = T(n/2) / 2^n$$