1. Show that the following statements are true:

a. $n(n-1)/2$ is $O(n^2)$

Solution:

$= (n * n + n * -1)/2$

$= (n^2 – n)/2$

Ignoring constant value using barometer theorem

$= n^2 – n$

Since, for all $n >= 0$, n^2 would return the maximum value.

Hence, $n(n-1)/2$ is $O(n^2)$

b. $\max(n^3, 10\ n^2)$ is $O(n^3)$

Solution:

Ignoring constant value using barometer theorem

$= \max(n^3, n^2)$

$= n^3$

Hence, $\max(n^3, 10\ n^2)$ is $O(n^3)$

c. i^k is $O(n^{k+1})$ for integer k

Solution:

Since k is positive integer

Lets $i = 1$ then, $i^k = 1^k$

$i = 2$ then, $i^k = 2^k$

$i = 3$ then, $i^k = 3^k$

Similarly,

$i = n$ then, $i^k = n^k$

Adding all values,

$1^k + 2^k + 3^k + ... + n^k$

Since, using series formula

n^{k+1}

Hence, i^k is $O(n^{k+1})$ for integer k

d. i^k is $O(n^{k+1})$ for integer k

Solution:

Given, p(x) = kth degree of polynomial

$p(x) = p(p(p(..(p(x))$ // (… is k times)

Then,

$p(n) = p(p(p(…(p(n))$

$= n*n*n….$ k times

$= n^k$

Hence, $p(n) = O(n^k)$

2. Which function grows faster?

a) $n^{\log(n)}$ or $(\log n)^n$

$n^{\log(n)}$	$(\log n)^n$
for n = 1, $1^{\log(1)} = 1^0 = 1$	for n = 1, $(\log 1)^1 = 0^1 = 1$
for n = 2, $2^{\log(2)} = 2^{0.3010} = 1.2319$	for n = 2, $(\log 2)^2 = (0.3010)^2 = 0.0906$
for n = 3, $3^{\log(3)} = 3^{0.4771} = 1.6890$	for n = 3, $(\log 3)^3 = (0.4771)^3 = 0.1085$
for n = 4, $4^{\log(4)} = 4^{0.6020} = 2.3037$	for n = 4, $(\log 4)^4 = (0.6020)^4 = 0.1313$
for n = 5, $5^{\log(5)} = 5^{0.6989} = 3.0797$	for n = 5, $(\log 5)^5 = (0.6989)^5 = 1.667$

Hence, function $\mathbf{n^{\log(n)}}$ will grow faster.

b) $\log (n^k)$ or $(\log n)^k$

$\log (n^k)$	$(\log n)^k$
for n = 1, k = 5 $\log (1)^5 = \log 1 = 0$	for n = 1, k = 5 $(\log 1)^5 = 0^5 = 1$
for n = 2, k = 5 $\log (2)^5 = \log(32) = 1.5051$	for n = 2, k = 5 $(\log 2)^5 = (0.3010)^5 = 0.0024$
for n = 3, k = 5 $\log (3)^5 = \log(243) = 2.3856$	for n = 3, k = 5 $(\log 3)^5 = (0.4771)^5 = 0.024$
for n = 4, k = 5 $\log (4)^5 = \log(1024) = 3.0102$	for n = 4, k = 5 $(\log 4)^5 = (0.6020)^5 = 0.0790$
for n = 5, k = 5 $\log (5)^5 = \log(3125) = 3.4948$	for n = 5, k = 5 $(\log 5)^5 = (0.6989)^5 = 1.667$

Hence, function $\mathbf{\log (n^k)}$ will grow faster.

c) $n^{\log \log \log n}$ or $(\log n)!$

$n^{\log \log \log n}$	$(\log n)!$
for n = 1 $1^{\log \log \log 1}$ = undefined	for n = 1 $(\log 1)! = 0! = 1$
for n = 2 $2^{\log \log \log 2}$ = undefined	for n = 2 $(\log 2)! = (0.3010)! = 0.8973$

for n = 3 $3^{\log\log\log 3}$ = undefined for n = 4 $4^{\log\log\log 4}$ = undefined for n = 5 $5^{\log\log\log 5}$ = undefined	for n = 3 (log 3)! = (0.4771)! = 0.8857 for n = 4 (log 4)! = (0.6020)! = 0.8937 for n = 5 (log 5)! = (0.6989)! = 0.9084

According to logs undefined value receives when taking log of negative numbers, here the function $n^{\log\log\log n}$ returns negative value and that makes error in output hence function is not valid. Even though value of negative log would me higher in negative which when exponential with any number it return minimum value.

Hence, function **(log n)!** will grow faster.

d) n^{n} or n!

n^n	n!
for n = 1 $1^1 = 1$ for n = 2 $2^2 = 4$ for n = 3 $3^3 = 27$ for n = 4 $4^4 = 254$ for n = 5 $5^5 = 3125$	for n = 1 1! = 1 for n = 2 2! = 2 for n = 3 3! = 6 for n = 4 4! = 24 for n = 5 5! = 120

Hence, function $\mathbf{n^n}$ will grow faster.

3.
Solution:

f1(n) <= c1g1(n) and f2(n) <= c2g2(n) for large n.
Thus,
f1(n) + f2(n) <= c1g1(n) + c2g2(n)
f1(n) + f2(n) <= c1 max (g1(n), g2(n)) + c2 max(g1(n), g2(n))
f1(n) + f2(n) <= (c1 + c2) max (g1(n), g2(n)) + c2 max(g1(n), g2(n))

4. Prove or disprove: Any positive n is $O(n/2)$

Solution:

Let us consider $n = n/2$

Implies, $2n = n$

$2n - n = 0$

$n * (2-1) = 0$

$n * 1 = 0$

$n = 0$

The above statement is only valid for $n = 0$.

Hence, the given statement i.e. n is $O(n/2)$ disproves for any number.

5. Prove or disprove: 3^n is $O(2^n)$.

Solution:

Let us consider $3^n = 2^n$.

Now taking log both sides,

$\log(3^n) = \log(2^n)$

Using logarithm power rule i.e. $\log(x^y) = y \cdot \log(x)$

$n \log(3) = n \log(2)$

$n \log(3) - n \log(2) = 0$

$n (\log (3) - \log (2)) = 0$

$n = 0$

The above statement is only valid for $n = 0$.

Hence, the given statement i.e. 3^n is $O(2^n)$ disproves for any number.