

$$5 \qquad 2 \qquad 10$$

$$1. \qquad n \qquad A \qquad r(A) \quad n-1 \qquad A^* \quad 0$$

$$\qquad r(A) \quad n-1 \qquad n \qquad 0 \qquad A^* \quad 0$$

$$2. \qquad n-s \qquad A \quad s \quad n \qquad B \qquad AB \quad 0 \qquad r(A) \quad r(B) \quad s$$

$$3.$$

$$\begin{array}{c} 1 \\ 1 \quad 1 \end{array}$$

$$4. \qquad n \qquad A \qquad A^T \qquad A \qquad X \quad x_1 \qquad x_n^T$$

$$X^TAX \quad 0$$

$$X^TAX \qquad X^TAX \quad (X^TAX)^T \quad X^TA^TX \quad X^TAX$$

$$5. \qquad AX \quad b \qquad X \quad A^{-1}b$$

$$A$$

$$1. \qquad \begin{array}{c} 5 \qquad 4 \qquad 20 \\ \left| \begin{array}{ccc} x & 3 & 1 \\ y & 0 & 1 \\ z & 2 & 1 \end{array} \right| \quad 1 \quad \left| \begin{array}{ccc} x & 1 & y \\ \frac{3}{2} & 0 & 1 \\ 1 & 1 & 1 \end{array} \right| \quad -\frac{1}{2}- \end{array}$$

$$2. \qquad 4 \qquad A \qquad 2 \qquad A^* \qquad \underline{0}$$

$$3 \qquad A \qquad r \quad m \quad n \qquad , \quad B \quad n \quad p \qquad , \qquad AB \quad 0, \quad B \qquad \underline{\hspace{1cm}} \\ 0 \quad r(B) \quad \min\{n-r,p\} \underline{\hspace{1cm}}$$

$$4. \qquad A \quad \begin{array}{ccc} 1 & 0 & 0 \\ 2i & 1 & 0 \\ 0 & 2i & 1 \end{array} , i \quad \sqrt{1} \qquad (A-2E)^{-1}(A-2E) \quad \underline{\hspace{1cm}} \quad \begin{array}{ccc} 3 & 0 & 0 \\ 8i & 3 & 0 \\ 4 & 8i & 3 \end{array}$$

$$5 \qquad n \qquad A,B \qquad AXB \quad C \qquad X \quad \underline{\hspace{1cm}} A^{-1}CB^{-1} \underline{\hspace{1cm}}$$

$$A \quad m \quad n \qquad r(AA^T) \quad r(A) \quad (\quad 10 \quad)$$

$$r(AA^T) = r(A)$$

$$r(AA^T) = r(A) \qquad m - r(AA^T) = m - r(A) = m - r(A^T)$$

$$X_0 \perp N(AA^T) \qquad (AA^T)X_0 = 0$$

$$0 = X_0^T(AA^T)X_0 = (X_0^TA)(A^TX_0) = (A^TX_0)^T(A^TX_0)$$

$$A^TX_0 = 0 \qquad X_0 \perp N(A^T)$$

$$N(AA^T) = N(A^T) \qquad m - r(AA^T) = \dim N(AA^T) = \dim N(A^T) = m - r(A^T)$$

$$r(AA^T) = r(A)$$

$$1$$

$$D_n$$

$$D_n \left| \begin{array}{cc} 3 & 2 \\ 1 & 3 \end{array} \right| \qquad \qquad \qquad \left| \begin{array}{cc} 3 & 2 \\ 1 & 3 \end{array} \right|$$

$$(-1)(-1)^{2n-1} \left| \begin{array}{cc} 3 & 2 \\ 1 & 3 \end{array} \right| \qquad \qquad \qquad \left| \begin{array}{cc} 3 & 0 \\ 1 & 2 \end{array} \right|_{n-1}$$

$$S_{n-1} = D_n - D_{n-1} = 2(D_{n-1} - D_{n-2}) = 2S_{n-2}$$

$$D_n = D_{n-1} + S_{n-1} = 2^{n-2}S_1 = 2^{n-2}(D_2 - D_1) = 2^n$$

$$D_n = 2^{n-1} - 1$$

$$\begin{array}{ccccccc} & & 2 & 0 & 2 & & 1 & 0 & 0 \\ 2 & P & 0 & 1 & 0 & ,Q & 0 & 1 & 0 & ,AP & PQ & A^{100} & (& 10 &) \\ & & 0 & 0 & 1 & & 0 & 0 & 2 & & & & & & \end{array}$$

$$\begin{array}{ccccccc} & & 2 & 0 & 2 & & & & \frac{1}{2} & 0 & 1 \\ P & 0 & 1 & 0 & & P & & P^{-1} & 0 & 1 & 0 & & AP & PQ \\ & 0 & 0 & 1 & & & & & 0 & 0 & 1 & & & \end{array}$$

$$A \quad PQP^{-1}$$

$$\begin{array}{cccccccccccccccc} A^{100} & PQ^{100}P^{-1} & & 2 & 0 & 2 & 1 & 0 & 0 & ^{100} & \frac{1}{2} & 0 & 1 & & 1 & 0 & 2^{101} & 2 \\ & & & 0 & 1 & 0 & 0 & 1 & 0 & & 0 & 1 & 0 & & 0 & 1 & & 0 \\ & & & 0 & 0 & 1 & 0 & 0 & 2 & & 0 & 0 & 1 & & 0 & 0 & & 2^{100} \end{array}$$

$$(\quad 10 \quad)$$

$$\begin{array}{ccccccc} A & \begin{array}{cc} 2 & 1 \\ 3 & 2 \end{array} ,B & \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 7 & 6 \end{array} ,C & \begin{array}{cc} 1 & 0 \\ 2 & 3 \\ 1 & 1 \end{array} & X & \begin{array}{cc} 0 & B \\ A & 0 \end{array} X & \begin{array}{cc} C & \\ 0 & \end{array} (\end{array}$$

$$10 \quad)$$

$$\begin{array}{ccccccc} A & \begin{array}{cc} 2 & 1 \\ 3 & 2 \end{array} ,B & \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 7 & 6 \end{array} & & A,B \end{array}$$

$$\begin{array}{ccccccc} A^{-1} & \begin{array}{cc} 2 & 1 \\ 3 & 2 \end{array} ,B^{-1} & \begin{array}{ccc} \frac{5}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{7}{3} & \frac{5}{3} & \frac{2}{3} \\ \frac{3}{3} & \frac{3}{3} & \frac{3}{3} \\ 3 & 2 & 1 \end{array} \end{array}$$

$$\begin{array}{ccccccc} 0 & B & & 0 & B^{-1} & & 0 & A^{-1} \\ A & 0 & & A & 0 & & B^{-1} & 0 \end{array}$$

$$X \begin{pmatrix} 0 & B^{-1}C & 0 & A^{-1}C & 0 \\ A & 0 & 0 & B^{-1} & 0 & 0 & B^{-1}C \end{pmatrix}$$

$$A \begin{pmatrix} m & n \\ n & m \end{pmatrix}, \quad B \begin{pmatrix} n & m \\ m & n \end{pmatrix}, \quad ABX=0$$

$$m \leq n \qquad m \leq n \quad r(A) = r(AB) \qquad ABX=0$$

$$\begin{aligned} 1 \quad & A \begin{pmatrix} B & n \\ B & A \end{pmatrix} I \qquad (A-I)^{-1} \begin{pmatrix} B & I \end{pmatrix}^T, \quad A \\ 2 \quad & A \begin{pmatrix} B & n \\ I & AB \end{pmatrix} \qquad I \begin{pmatrix} BA \end{pmatrix} \\ (I-BA)^{-1} \quad & I \begin{pmatrix} B(I-AB)^{-1}A \end{pmatrix} \quad (\quad 10 \quad) \end{aligned}$$

$$1 \qquad (A-I)^{-1} \begin{pmatrix} B & I \end{pmatrix}^T$$

$$I \begin{pmatrix} (A-I)(A-I)^{-1} \quad (A-I)(B-I)^T \quad A(B-I)^T \quad (B-I)^T \quad A(B-I)^T \quad B^T \quad I \end{pmatrix}$$

$$A(B-I)^T \quad B^T$$

$$\begin{matrix} B & A \\ 2 \end{matrix}$$

$$\begin{aligned} (I-BA)(I-B(I-AB)^{-1}A) \quad I \begin{pmatrix} B(I-AB)^{-1}A \quad BA \quad BAB(I-AB)^{-1}A \end{pmatrix} \\ I \begin{pmatrix} BA \quad (B-BAB)(I-AB)^{-1}A \quad I \quad BA \quad B(I-AB)(I-AB)^{-1}A \quad I \end{pmatrix} \\ I \begin{pmatrix} BA \end{pmatrix} \qquad (I-BA)^{-1} \quad I \begin{pmatrix} B(I-AB)^{-1}A \end{pmatrix} \end{aligned}$$

$$A \qquad r(AB) = r(B) \quad (\quad 10 \quad)$$

$$r(AB) = r(B) \qquad r(AB) = r(B) \qquad n \leq r(AB) \leq n \leq r(B)$$

$$n \qquad B$$

$$X_0 \begin{pmatrix} N(AB) \quad ABX_0=0 \quad A \quad N(A)=0 \end{pmatrix}$$

$$BX_0=0 \qquad X_0 \begin{pmatrix} N(B) \quad N(AB) \quad N(B) \end{pmatrix}$$

$$n \leq r(AB) \leq \dim N(AB) \leq \dim N(B) \leq n \leq r(B)$$

$$r(AB) = r(B)$$

