User manual for PIBLUP v1.0

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Introduction

PIBLUP is an application program for solving mixed model equations (MME), especially for very large ones. The core algorithm is preconditioned conjugate gradient (PCG) with preconditioner of block diagonal of MME, by iteration on data (IOD).

To take full advantage of modern multicore and multiprocessor systems, two parallel computing strategies are applied in PIBLUP. Firstly, Intel Math Kernel Library (MKL) is employed (https://software.intel.com/en-us/intel-mkl) to accelerate mathematical operations. Secondly, we used shared memory programming introduced in Message Passing Interface Standard (MPI) Version 3.0. In a practical application, users can control both the Intel MKL threading and the number of MPI processes to optimally utilize computational resources.

PIBLUP is written in C. The program will be actively maintained and new functions will be incorporated. If you have any questions and suggestions, please do not hesitate to contact us.

Dependencies

1. MPI

Users should install one of MPI implementation. MPICH (http://www.mpich.org/downloads/) is suggested.

2. MKL

Users do need to install MKL (https://software.intel.com/en-us/intel-mkl).

Parameter file for application

PIBLUP is driven by the parameter file. The parameter file consists of keywords that are fixed and followed by values. Keywords underlined are mandatory.

Description of keywords used in the parameter file:

Keyword	Function (F)/Syntax (S)	
DATAFILE	F: specifies name of the file with phenotypes	
	S: DATAFILE fn	
	where: fn = name of data file	
	Full path name if the file is not in the current	
	directory	
	Format of data file see: <u>Associated files</u>	
NVAR	F: specifies the number of variables in data file	
	S: NVAR #var	
	where: #var = number of variables, i.e., number of	
	columns in data file	
<u>NTRAITS</u>	F: specifies the number of traits to be analyzed	
	S: NTRAITS #traits	
	where: #traits = number of traits to be analyzed	

<u>TRAITS</u>	F: specifies the position(s) of observations in data file,	
	indicated by column number(s)	
	S: TRAITS tr1 tr2	
	where: tr1 = the column number of observations for the	
	1 st trait	
	tr2 = the column number of observations for the	
	2 nd trait, if it exists	
MISSING	F: specifies the value should be regarded as missing for	
	real variables	
	S: MISSING miss	
	where: miss = reals equaling to this value are regarded as	
	missing	
<u>NEFFECTS</u>	F: specifies the number of effects, except for residual, in a	
	model for each trait	
	S: NEFFECTS #tr1 #tr2	
	where: $\#tr1 = number of effects for the 1st trait$	
	$\#\text{tr}2 = \text{number of effects for the } 2^{\text{nd}} \text{ trait}$	
WEIGHT	F: specifies the position(s) of weight on observations for	
	each trait, if used. More details see: Weighted analysis	
	S: WEIGHT tr1 tr2	
	where: tr1 = the column number of weight for the 1st trait	
	tr2 = the column number of weight for the 2 nd trait	
	A zero (0) stands for weight is not applied for this trait	
<u>EFFECTS</u>	F: specifies effects, except for residual, in the model for	
	each trait	
	S: EFFECTS	
	Followed by lines with effects specified in a fixed	
	format for traits	
	Format of effects see: <u>Description of effects</u>	
·		

cov	F: specifies (co)variance matrix for each random effect,	
COV		
	except for residual. If the model includes random effects,	
	COV must be specified.	
	S: COV #ran	
	Followed by (co)variance matrix	
	where: #ran = the unique CV number of the random	
	effect, which is specified in EFFECTS keyword section	
	Format of (co)variance matrix see: (Co)variance and	
	<u>residual matrices</u>	
RCOV	F: specifies the residual (co)variance matrix	
	S: RCOV	
	Followed by residual (co)variance matrix	
	Format of residual matrix see: (Co)variance and residual	
	<u>matrices</u>	
COVFILE	F: specifies associated files storing or used to calculate	
	(inverse) kinship matrix, for each random effect, except	
	for residual. If not specified, an identity matrix will be	
	used.	
	S: COVFILE	
	Followed by the corresponding information in a fixed	
	format	
	Format see: Description of COVFILE	
MAXROUND	F: specifies the maximum round of iteration, default =	
	10,000	
	S: MAXROUND #n	
	where: n = maximum round of iteration	
CONV	F: specifies the convergence criterion, default = 1e-13	
	S: CONV value	
	where: v = convergence criterion	
<u> </u>	I	

Convergence indicator is the relative average difference
between the right-hand and left-hand sides (Misztal, et
al., 1987)

Description of effects

The effects for all the traits are specified in EFFECTS keyword section. One line for each trait containing the trait number (starts from 1) followed by information of effects in the model. Information of each effect, except for residual, is expressed as a string consisting of several parts, with delimiters of a comma (,), a slash (/), or a hyphen (-), e.g., 4/RR,1,1-3. Its syntax is as follows:

Tuno	Syntax				
Туре	Part 1	Part 2	Part 3	Part 4	Part 5
Fixed effect	Column	F			
Fixed	number of	FR	column		
regression	effect		number it		
			nested in, if		
			so, else, 0		
Random		R	CV* matrix	number in	
effect			number	CV matrix	
				(default = 1)	
Random		RR	column	CV number	number in
regression			number it		CV matrix
			nested in		

*CV: (co)variance

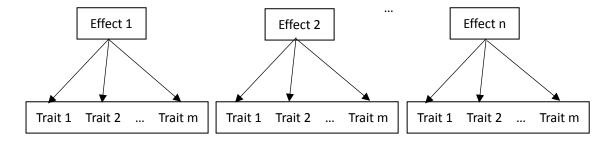
Note: CV matrix number should be coded from 1 continuously.

(Co)variance and residual matrices

In parameter file, (co)variance and residual matrices should be listed as full square matrices.

(Co)variance matrix

In multivariate analysis, if the (co)variance matrix consists of *n* correlated effects and *m* traits, the order of effects and traits is as follows:



Residual matrix

The order of traits in residual matrix should be the same as those specified in TRAITS keyword section.

Description of COVFILE

In parameter file, the COVFILE consists of several columns. Associated files and information used to construct kinship matrices for random effects are specified in this section. The format of associated files see: <u>Associated files</u>. The syntax for different types of kinship matrices is as follows:

Type of kinship matrix	Syntax
Additive genetic relationship	COVFILE #cv PED fped inbreed upg
based on pedigree (A)	where: #cv = (co)variance matrix number it
	corresponds to
	fped = name of pedigree file
	Full path name if the file is not in the
	current directory
	inbreed = 0, if inbreed is not considered;

	else, 1	
	upg = 0, if unknown parent group is not	
	considered; else, 1	
Genomic relationship based	COVFILE #cv INVG fmarker ngid nmarker diag_g	
on genotype file (G)	where: #cv = as in kinship matrix A	
	fmarker = name of genotype file	
	Full path name if the file is not in the	
	current directory	
	ngid = number of genotyped individuals	
	nmarker = number of markers	
	diag_g = a small number added to diagonal	
	of G , e.g., 0.02	
Hybrid relationship matrix	COVFILE #cv HYBRID fped fmarker inbreed upg	
(H) of A and G	ngid nmarker diag_g adj t w ₁ w ₂	
	where: #cv, fped, inbreed, upg = as those in kinship	
	matrix A	
	fmarker, ngid, nmarker, diag_g = as those in	
	kinship matrix G	
	adj = 1, adjust G according to (Christensen,	
	et al., 2012); 0, do not adjust	
	t, w_1 , w_2 = those in inverse of H matrix:	
	$\mathbf{H}^{-1} = \mathbf{A}^{-1} + \begin{bmatrix} 0 & 0 \\ 0 & t [(1 - w_1)\mathbf{G} + w_1\mathbf{A}_{22}]^{-1} - w_2\mathbf{A}_{22}^{-1} \end{bmatrix}$	
Kinship matrix built by the	COVFILE #cv INV finv	
users	where: #cv = as in kinship matrix A	
	finv = name of file containing the inverse of	
	kinship matrix	

Associated files

Associated files include data file, pedigree file, genotype file and kinship file, if they are used. Files should be free-format ASCII files, with delimiters of one or several spaces/tabs. Classified variables can include alpha-numeric characters. Identifications of individuals should be consistent among different files.

Data file

Data file containing observations to be analyzed together with class variables and covariables for each model must be provided. One record in data file corresponds to an observation.

Integer inputs consist of all classified variables, followed by real inputs, which consist of the traits to be analyzed and all necessary covariables and weight variables. In integer input, a zero (0) is treated as missing, whereas real inputs equaling to the value "miss" specified at the MISSING keyword section. Except for the requirement integer variables should be before real variables, there is no restriction on the order of variables within integer variables and real variables. The data file can contain variables that are not used.

Pedigree file

A pedigree file can be used to define an additive genetic relationship. If unknown parent group (UPG) is used, the file consists of four columns corresponding to four variables:

1: ID	Identification of the individual that enters into random effect	
	(genetic effect) in the model	
2: Sire ID	Identification of sire, or genetic group code for unknown sire	
3: Dam ID	Identification of dam, or genetic group code for unknown dam	
4: UPG	genetic group the ID belongs to, if UPG is used. Inbreeding	
	coefficients are calculated according to Aguilar and Misztal (2008)	

If UPG is not used, the file should consist of the first 3 columns.

Genotype file

A genotype file is used to construct a genomic relationship matrix. Genotypes of all

markers for one individual is listed in a row. IDs should be listed in the first column, followed by genotype code for each marker. Genotypes are codes as 0 and 2 for the two homozygotes and 1 for the heterozygote. There are no missing values for genotypes, and genotypes cannot be left as missing. Markers with genotypes listed in genotype file will be used without filtering.

Kinship file

If a kinship matrix constructed by the user would be used, the kinship file containing the **inverse** of the kinship matrix should be specified. The full inverse matrix should be listed row by row in the following format: IdRow IdCol value, where IdRow and IdCol stand for identifications corresponding to row and column, respectively.

Weighted analysis

WEIGHT keyword is optional. A zero (0) indicates no weight is not applied. If weight is applied on at least one trait, WEIGHT must be specified.

The weight modifies the residual (co)variance to $V=\frac{1}{w}V_e$, where w is the weight, and V_e is residual (co)variance matrix specified in RCOV keyword section. In multivariate analysis, residual correlations are kept constant when applying weights.

References:

Aguilar, I. and Misztal, I. Technical note: Recursive algorithm for inbreeding coefficients assuming nonzero inbreeding of unknown parents. *J Dairy Sci* 2008;91(4):1669-1672.

Christensen, O.F., et al. Single-step methods for genomic evaluation in pigs. *Animal*: an international journal of animal bioscience 2012;6(10):1565-1571.

Misztal, I., Gianola, D. and Schaeffer, L.R. Extrapolation and Convergence Criteria with Jacobi and Gauss-Seidel Iteration in Animal-Models. *J Dairy Sci* 1987;70(12):2577-2584.

Running and Solution file

Running

Before running the program, please read <u>dependencies</u> first.

Run the program with command:

PIBLUP m parfile

where:

PIBLUP = full path name if not in the current directory, or add its directory into your PATH

m = number of processors to use (for MPI)

parfile = name of parameter file, full path name if not in the current directory

Solution file

File containing solution is named name_of_parameter_file.sol. Solution file contains 6 columns:

Column	Definition	
1	Trait number	
2	Effect number (useless for users)	
3	Effect column in data file	
4	Class code	
	= 0, if it is fixed regression and not	
	nested within fixed effects	
	= class code where it nested, else	
5	= Column number nests in, if it nests	
	within an effect	
	= 0, else	

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6	Solution

Examples for parameter file

Examples are those in example directory.

1. Animal model with pedigree

```
Model:
```

```
y_{ijk} = sex_i + birth \_mon_j + birth \_weight + a_k + e_{ijk}
```

where:

yijki: weight

sex_i: sex

birth_moni: birth month

birth_weight: birth weight

 a_k : additive genetic effect

eijki: residual

and

$$Var(a) = 0.3, Var(e) = 0.7$$

Data file (data.txt):

(First 10 lines)

Format: ID/birth mon/sex/birth weight/weight

192243 2 2 27.50 147.35

192240 3 2 26.00 124.91

192242 3 2 29.50 142.78

192246 3 1 30.30 143.43

192241 3 1 36.20 147.95

192251 3 2 36.00 154.93

192245 3 2 32.00 142.20

192247 3 1 31.00 148.84

192250 3 1 29.00 123.64

192249 3 2 24.80 140.96

Pedigree (ped.txt):

(First 10 lines)

Format: ID/sire/dam

192243	181	4007
192240	3980	3762
192242	3980	4010
192246	3980	4899
192241	847	3525
192251	4597	4204
192245	3920	3588
192247	3980	4341
192250	4597	4464
192249	3980	184

Parameter file (1.par):

DATAFILE data.txt

NTRAITS 1

TRAITS 5

NEFFECTS 4

NVAR 5

MISSING -99

WEIGHT 0

EFFECTS

1 2/F 3/F 4/FR,0 1/R,1

COV 1

0.3

RCOV

0.7

COVFILE 1 PED ped.txt 0 0

Solution file (1.par.sol):

(First 10 lines)

1 1 2 2 0 43.9714

1 1 2 3 0 26.1468

1 1 2 4 0 15.3176

 $1\ 1\ 2\ 5\ 0\ 29.9166$

 $1\ 1\ 2\ 6\ 0\ 29.0574$

 $1\; 2\; 3\; 2\; 0\; 25.3442$

1 2 3 1 0 22.4064

1 3 4 0 0 2.74704

1 4 1 192243 0 2.49802

1 4 1 192240 0 3.15123

2. Animal model with genotype file

Model:

$$y_i = \mu + a_i + e_i$$

where:

y_{ij}: phenotype (pseudo-phenotype, EBV)

 μ : mean

ai: additive genetic effect

 e_{ij} : residual

and

$$Var(a) = 200231, Var(e) = 1176064$$

Residual variance is heterogeneous.

Data file (data.txt):

(First 10 lines)

Format: ID/ μ /observation/weight

4104	1	-1584.03502	1.94364
4123	1	-1220.60346	0.32261
4125	1	-4367.64766	0.31403
4129	1	-964.11084	0.30168
4130	1	-507.71934	1.90909
4143	1	-4299.13739	0.32762
4149	1	-273.90995	0.30886
4154	1	151.22413	1.92342
4158	1	-535.19628	1.93967
4168	1	-881.81554	1.92867

Genotype file (marker.txt):

(First 10 lines, first 10 columns)

4104 0 0 0 2 0 0 0 0 2

4123000200001

4125 1 2 2 1 2 1 1 1 1

4129011111112

4130 1 2 2 1 1 1 1 1 1

 $4143\ 0\ 0\ 1\ 2\ 1\ 0\ 0\ 0\ 2$

414901111112

4154000200101

4158 1 2 2 1 2 1 1 1 1 1 4168 0 0 0 2 0 0 1 0 1

Parameter file (2.par):

DATAFILE data.txt

NTRAITS 1

TRAITS 3

NEFFECTS 2

NVAR 4

MISSING -99

WEIGHT 4

EFFECTS

1 2/F 1/R,1,1

COV 1

200231

RCOV

1176064

COVFILE 1 INVG marker.txt 2000 20010 0.02

Solution file (2.par.sol):

(First 10 lines)

1 1 2 1 0 152.594

1 2 1 4104 0 -692.656

1 2 1 4123 0 -461.149

1 2 1 4125 0 -826.65

1 2 1 4129 0 -369.614

1 2 1 4130 0 -760.704

1 2 1 4143 0 -550.76 1 2 1 4149 0 -355.458

1 2 1 4154 0 -218.863

1 2 1 4158 0 -195.657

3. Random regression test-day model with pedigree and genotype file

Model:

$$y_{tik} = \sum_{k=1}^{3} \varphi_{itk} \beta_k + \sum_{k=1}^{3} \varphi_{itk} a_{ik} + \sum_{k=1}^{3} \varphi_{itk} p e_{ik} + e_{tik}$$

where:

y_{tik}: test-day record of individual i made on time t

 φ_{itk} : the kth parameter of Wilmink function (W0, W1, W2)

 β_k : fixed regression coefficients

 a_{ik} : random regression coefficients for additive genetic effect

peik: random regression coefficients for permanent environmental effect

etik: residual

and

$$Var(a) = \begin{bmatrix} 5.8089184 & -9.75E - 03 & -8.7098898 \\ -9.75E - 03 & 9.81E - 05 & 1.72E - 02 \\ -8.7098898 & 1.72E - 02 & 18.045758 \end{bmatrix},$$

$$Var(pe) = \begin{bmatrix} 11.71613 & -3.29E - 02 & -9.0165899 \\ -3.29E - 02 & 1.90E - 04 & 2.38E - 02 \\ -9.0165899 & 2.38E - 02 & 23.941236 \end{bmatrix},$$

Var(e) = 3.984657

Data file (data.txt):

(First 10 lines)

Format: a/pe/W0/W1/W2/phe

4101	4101	1	25 2.865E-01	42.37
4101	4101	1	60 4.979E-02	41.88
4101	4101	1	86 1.357E-02	47.22
4101	4101	1	119 2.606E-03	43.05
4101	4101	1	150 5.531E-04	42.49
4101	4101	1	172 1.841E-04	40.27
4101	4101	1	196 5.545E-05	37.48
4101	4101	1	230 1.013E-05	38.65
4101	4101	1	264 1.851E-06	31.87
4101	4101	1	286 6.160E-07	31.94

Pedigree (ped.txt):

(First 10 lines)

Format: ID/sire/dam

410100

4102 0 0

4103 0 0

4104 0 0

4105 0 0

4106 0 0

4107 0 0

4108 0 0

```
4109 0 0
4110 0 0
```

Genotype file (marker.txt):

```
(First 10 lines, first 10 columns)
4104 0 0 0 2 0 0 0 0 2
4123 0 0 0 2 0 0 0 0 1
4125 1 2 2 1 2 1 1 1 1
4129 0 1 1 1 1 1 1 1 1 2
4130 1 2 2 1 1 1 1 1 1
4143 0 0 1 2 1 0 0 0 2
4149 0 1 1 1 1 1 1 1 1 2
4154 0 0 0 2 0 0 1 0 1
4158 1 2 2 1 2 1 1 1 1
4168 0 0 0 2 0 0 1 0 1
```

Parameter file (3.par):

DATAFILE data.txt

NTRAITS 1

TRAITS 6

NEFFECTS 9

NVAR 6

MISSING -99

EFFECTS

1 3/FR,0 4/FR,0 5/FR,0 1/R,1 2/R,2 4/RR,1,1-2 5/RR,1,1-3 4/RR,2,2-2 5/RR,2,2-3

COV 1

5.8089184-9.75E-03-8.7098898-9.75E-039.81E-051.72E-02-8.70988981.72E-0218.045758

COV 2

11.71613 -3.29E-02 -9.0165899 -3.29E-02 1.90E-04 2.38E-02 -9.0165899 2.38E-02 23.941236

RCOV

3.984657

COVFILE 1 HYBRID ped.txt marker.txt 1 0 2000 20010 0.01 1 1 0.05 1

Solution file (3.par.sol):

(First 10 lines)

1 1 3 0 0 44.9568

1 2 4 0 0 -0.05023

1 3 5 0 0 -27.9127

1 4 1 4101 0 1.59294

15441011-0.00189875

1 6 5 4101 1 -1.79034

```
1 4 1 4102 0 -0.0292396
1 5 4 4102 1 0.00748955
1 6 5 4102 1 1.23103
1 4 1 4103 0 0.681633
```

4. Model with both additive genetic and epigenetic effects

Model:

$$y_i = a_i + epi_i + e_i$$

where:

y_i: observation

ai: additive genetic effect

epii: epigenetic effect

ei: residual

and

$$Var(a) = 1581628.4, Var(epi) = 1617241.9, Var(e) = 1536088.1$$

Data file (data.txt):

(First 10 lines)

Format: a/epi/observation

1 1 2.458E+04

2 2 2.680E+04

3 3 2.420E+04

4 4 2.264E+04

5 5 2.677E+04

6 6 2.836E+04

7 7 2.356E+04

8 8 2.310E+04

9 9 2.971E+04

10 10 2.464E+04

File containing inverse of kinship matrix for additive genetic effect (add.inv.txt):

(First 10 lines)

Format: ID1/ID2/value

1 1 11.9502

2 1 10.7577

2 2 12.5347

```
3 1 10.7502
```

3 2 10.6436

3 3 18.5077

4 1 10.7565

4 2 10.7163

4 3 3.66267

4 4 18.4615

File containing inverse of kinship matrix for epigenetic effect (epi.inv.txt):

(First 10 lines)

Format: ID1/ID2/value

1 1 1.13624

2 1 -0.00286483

2 2 1.1676

3 1 -0.000616532

3 2 -0.0366526

3 3 3.81434

4 1 -0.00147606

4 2 -0.0191144

4 3 -3.32862

4 4 3.80128

Parameter file (4.par):

DATAFILE data.txt

NTRAITS 1

TRAITS 3

NEFFECTS 2

NVAR 3

MISSING -99

WEIGHT 0

EFFECTS

1 1/R,1 2/R,2

COV 1

1581628.4

COV 2

1617241.9

RCOV

1536088.1

COVFILE 1 INV add.inv.txt

COVFILE 2 INV epi.inv.txt

Solution file (4.par.sol):

(First 10 lines)

1 1 1 1 0 1901.36

11120-6838.82

1 1 1 3 0 -10705.1

11140-10760.5

1 1 1 5 0 2104.78

 $1\ 1\ 1\ 6\ 0\ 2625.57$

11170-9187.34

11180-9194.34

111902458.65

1 1 1 10 0 2580.03

5. Multivariate random regression test-day model with pedigree

Model:

$$y_{tmijkl} = s_i + c_j + \sum_{k=1}^{4} \varphi_{jtk} \beta_{jk} + \sum_{k=1}^{4} \varphi_{ltk} a_{lk} + \sum_{k=1}^{3} \varphi_{ltk} pe_{lk} + e_{tmijkl}$$

where:

 y_{tmijkl} : test-day record for trait m (tr1, tr2, tr3, tr4, tr5, tr6, tr7, tr8 and tr9) of individual j made on time t

si: herd test season

 c_j : calving age and season

 φ_{itk} : the kth Legendre polynomial (L1, L2, L3, L4)

 $oldsymbol{eta}_{\it{jk}}$: fixed regression coefficients nest within calving age and season

 a_{lk} : random regression coefficients for additive genetic effect

pelk: random regression coefficients for permanent environmental effect

etmijlk: residual

and

$$Var(a) = mat1, Var(pe) = mat2, Var(e) = mat3$$

Data file (data.txt):

(First 10 lines)

Format: a/pe/s/c/L1/L2/L3/L4/tr1/tr2/t3/tr4/tr5/tr6/tr7/tr8/tr9

7683 1 1 1 0.11 -0.77 -0.24 0.74 34 0.65 1.02 -99 -99 -99 -99 -99 -99

17677 2 2 2 0.32 -0.63 -0.65 0.30 21.4 0.62 0.61 -99 -99 -99 -99 -99 -99

10294 3 3 1 0.26 -0.68 -0.55 0.45 24.8 0.99 0.79 -99 -99 -99 -99 -99 -99

Pedigree (ped.txt):

(First 10 lines)

Format: ID/sire/dam

1 0 0

2 0 0

3 0 0

4 0 0

5 0 0

6 5 0

7 6 0

8 0 0

9 8 0

10 0 0

Parameter file (5.par):

DATAFILE data.txt

NTRAITS 9

TRAITS 9 10 11 12 13 14 15 16 17

NEFFECTS 15 15 15 15 15 15 15 15 15

NVAR 17

MISSING -99

WEIGHT 0 0 0 0 0 0 0 0 0

EFFECTS

1 3/F 4/F 1/R,1 2/R,2 5/FR,4 6/FR,4 7/FR,4 8/FR,4 5/RR,1,1-2 6/RR,1,1-3 7/RR,1,1-4 8/RR,1,1-5 5/RR,2,2-2 6/RR,2,2-3 7/RR,2,2-4

2 3/F 4/F 1/R,1 2/R,2 5/FR,4 6/FR,4 7/FR,4 8/FR,4 5/RR,1,1-2 6/RR,1,1-3 7/RR,1,1-4 8/RR,1,1-5 5/RR,2,2-2 6/RR,2,2-3 7/RR,2,2-4

3 3/F 4/F 1/R,1 2/R,2 5/FR,4 6/FR,4 7/FR,4 8/FR,4 5/RR,1,1-2 6/RR,1,1-3 7/RR,1,1-4 8/RR,1,1-5 5/RR,2,2-2 6/RR,2,2-3 7/RR,2,2-4

4 3/F 4/F 1/R,1 2/R,2 5/FR,4 6/FR,4 7/FR,4 8/FR,4 5/RR,1,1-2 6/RR,1,1-3 7/RR,1,1-4 8/RR,1,1-5 5/RR,2,2-2 6/RR,2,2-3 7/RR,2,2-4

5 3/F 4/F 1/R,1 2/R,2 5/FR,4 6/FR,4 7/FR,4 8/FR,4 5/RR,1,1-2 6/RR,1,1-3 7/RR,1,1-4 8/RR,1,1-5 5/RR,2,2-2 6/RR,2,2-3 7/RR,2,2-4

6 3/F 4/F 1/R,1 2/R,2 5/FR,4 6/FR,4 7/FR,4 8/FR,4 5/RR,1,1-2 6/RR,1,1-3 7/RR,1,1-4 8/RR,1,1-5 5/RR,2,2-2 6/RR,2,2-3 7/RR,2,2-4

7 3/F 4/F 1/R,1 2/R,2 5/FR,4 6/FR,4 7/FR,4 8/FR,4 5/RR,1,1-2 6/RR,1,1-3 7/RR,1,1-4

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8/RR,1,1-5 5/RR,2,2-2 6/RR,2,2-3 7/RR,2,2-4
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8 3/F 4/F 1/R,1 2/R,2 5/FR,4 6/FR,4 7/FR,4 8/FR,4 5/RR,1,1-2 6/RR,1,1-3 7/RR,1,1-4 8/RR,1,1-5 5/RR,2,2-2 6/RR,2,2-3 7/RR,2,2-4

9 3/F 4/F 1/R,1 2/R,2 5/FR,4 6/FR,4 7/FR,4 8/FR,4 5/RR,1,1-2 6/RR,1,1-3 7/RR,1,1-4 8/RR,1,1-5 5/RR,2,2-2 6/RR,2,2-3 7/RR,2,2-4

COV 1

mat1

COV 2

mat2

RCOV

mat3

COVFILE 1 PED ped.txt 0 0

Solution file (5.par.sol):

(First 10 lines)

1 1 3 1 0 17.4291

2 1 3 1 0 0.465003

3 1 3 1 0 0.582579

1 1 3 2 0 6.84124

2 1 3 2 0 0.157484

3 1 3 2 0 0.216496

1 1 3 3 0 12.0569

2 1 3 3 0 0.392848

3 1 3 3 0 0.381222

1 1 3 4 0 21.9266