# Quick start for the sommer package

Giovanny Covarrubias-Pazaran 2017-08-23

The sommer package was developed to provide R users a powerful and reliable multivariate mixed model solver. The package is focused in problems of the type p > n (more random effect levels than observations). This package allows the user to fit mixed models with the advantage of specifying the variance-covariance structure for the random effects, and specify heterogeneous variances, and obtain other parameters such as BLUPs, BLUEs, residuals, fitted values, variances for fixed and random effects, etc.

The purpose of this quick start guide is to show the flexibility of the package under certain common scenarios:

- 1) Univariate homogeneous variance models
- 2) Univariate heterogeneous variance models
- 3) Multivariate homogeneous variance models
- 4) Multivariate heterogeneous variance models

## Background

The core of the package are the mmer2 (formula-based) and mmer (matrix-based) functions which solve the mixed model equations. The functions are an interface to call the NR Direct-Inversion Newton-Raphson (Tunnicliffe 1989; Gilmour et al. 1995; Lee et al. 2015) or the EMMA efficient mixed model association algorithm (Kang et al. 2008).

Since version 2.0 sommer can handle multivariate models. These have the form:

$$Y = X\beta + Zu + \epsilon$$

with:

$$\mathbf{Y} = \left[ \begin{array}{c} y_1 \\ y_2 \\ \dots \\ y_t \end{array} \right]$$

$$\mathbf{X} = \left[ \begin{array}{ccc} X & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & X \end{array} \right]$$

$$\mathbf{V} = \begin{bmatrix} Z_1 G_1 Z_1' + \dots + Z_1 R_1 Z_1' & \dots & Z_1 H_1 Z_t' + \dots + Z_1 S_1 Z_t' \\ & \dots & \dots & \dots \\ Z_t H_1 Z_1' + \dots + Z_t S_1 Z_1' & \dots & Z_t G_1 Z_t' + \dots + Z_t R_1 Z_t' \end{bmatrix}$$

for 't' traits, where G are H are variance and covariance matrices among random effects for the "t" trait, and R and S are variance and covariance matrices among residuals. Here  $R=S=I\sigma_{\epsilon}$ , where I is an identity matrix. We can specify the covariance matrices. BLUPs will also be corrected for such covariances usually leading to more accurate predictions.

In the following section we will go on quick examples with the same dataset of corn hybrids tested in 4 different environments.

## 1) Univariate homogeneous variance models

This type of models refer to single response models where a variable of interest (i.e. genotypes) needs to be analyzed as interacting with a 2nd random effect (i.e. environments), but you assume that across environments the genotypes have the same variance component. This is the so-called compound simmetry (CS) model.

```
library(sommer)
data(example)
head(example)
##
                   Name
                           Env Loc Year
                                          Block Yield
                                                        Weight
## 33
      Manistee (MSL292-A) CA.2013 CA 2013 CA.2013.1
                                                   4 -1.904711
             CO02024-9W CA.2013 CA 2013 CA.2013.1
## 65
                                                   5 -1.446958
## 66
      Manistee (MSL292-A) CA.2013 CA 2013 CA.2013.2
                                                   5 -1.516271
##
  67
               MSL007-B CA.2011 CA 2011 CA.2011.2
                                                   5 -1.435510
## 68
              MSR169-8Y CA.2013 CA 2013 CA.2013.1
                                                   5 -1.469051
## 103
             AC05153-1W CA.2013 CA 2013 CA.2013.1
                                                   6 -1.307167
ans1 <- mmer2(Yield~Env,
            random= ~ Name + Env:Name,
            rcov= ~ units,
            data=example, silent = TRUE)
summary(ans1)
  Multivariate Linear Mixed Model fit by REML
  *********** sommer 3.0 **********
##
          logLik
                     AIC
                             BIC Method Converge
## Value -20.14537 46.29075 55.95182
                                           TRUE
  Variance-Covariance components:
##
                     VarComp VarCompSE Zratio
## Name.Yield-Yield
                       3.682
                                1.691
## Env:Name.Yield-Yield
                       5.173
                                       3.459
                                1.495
## units.Yield-Yield
                       4.366
                                0.647
  ## Fixed effects:
##
## $Yield
##
              Estimate Std. Error
## (Intercept) 16.496351 0.6855001 24.064695
## EnvCA.2012 -5.776759 0.7558178 -7.643057
## EnvCA.2013 -6.380479 0.7960514 -8.015159
##
##
  Groups and observations:
##
          Observ Groups
## Name
             185
                     41
## Env:Name
             185
                    123
## Use the '$' sign to access results and parameters
```

## 2) Univariate heterogeneous variance models

Very often in multi-environment trials, the assumption that the genetic variance or the residual variance is the same across locations may be too naive. Because of that, specifying a general genetic component and a location specific genetic variance is the way to go. This require a CS+DIAG model.

```
data(example)
head(example)
                          Env Loc Year
##
                                         Block Yield
                  Name
                                                      Weight
##
  33
      Manistee (MSL292-A) CA.2013 CA 2013 CA.2013.1
                                                  4 -1.904711
##
  65
             CO02024-9W CA.2013
                              CA 2013 CA.2013.1
                                                  5 -1.446958
      Manistee(MSL292-A) CA.2013
  66
                              CA 2013 CA.2013.2
                                                  5 -1.516271
## 67
              MSL007-B CA.2011
                              CA 2011 CA.2011.2
                                                  5 -1.435510
## 68
              MSR169-8Y CA.2013 CA 2013 CA.2013.1
                                                  5 -1.469051
## 103
             AC05153-1W CA.2013 CA 2013 CA.2013.1
                                                  6 -1.307167
ans1 <- mmer2(Yield~Env,
            random= ~Name + at(Env):Name,
            rcov= ~ at(Env):units,
            data=example, silent = TRUE)
summary(ans1)
  _____
      Multivariate Linear Mixed Model fit by REML
  ************ sommer 3.0 **********
  _____
                            BIC Method Converge
##
          logLik
                     AIC
  Value -15.42982 36.85964 46.52071
                                   MNR
  ______
  Variance-Covariance components:
##
                         VarComp VarCompSE Zratio
## Name.Yield-Yield
                           2.962
                                   1.4963 1.980
## CA.2011:Name.Yield-Yield
                          10.148
                                   4.5108
                                         2.250
## CA.2012:Name.Yield-Yield
                           1.879
                                   1.8699
                                          1.005
## CA.2013: Name. Yield-Yield
                           6.629
                                   2.5027
                                          2.649
## CA.2013:units.Yield-Yield
                           2.560
                                   0.6398
                                          4.001
## CA.2011:units.Yield-Yield
                           4.943
                                   1.5246
                                         3.242
## CA.2012:units.Yield-Yield
                           5.725
                                   1.3119 4.364
  ## Fixed effects:
##
## $Yield
##
              Estimate Std. Error
                                 t value
## (Intercept) 16.507678 0.8268665 19.964138
## EnvCA.2012
            -5.816890 0.8575814 -6.782902
## EnvCA.2013 -6.412433 0.9356490 -6.853460
  ______
## Groups and observations:
##
              Observ Groups
                185
## Name
                       41
## CA.2011:Name
                185
                       41
## CA.2012:Name
                185
                       41
```

## CA.2013:Name

185

As you can see the special function at or diag can be used to indicate that there's a different variance for the genotypes in each environment. Same was done for the residual. The difference between at and diag is that the at function can be used to specify the levels or specific environments where the variance is different.

### 3) Multivariate homogeneous variance models

Currently there's a great push for multi-response models. This is motivated by the correlation that certain variables hide and that could benefit in the prediction perspective. In sommer to specify multivariate models the response requires the use of the cbind() function in the response, and the us(trait), diag(trait), or at(trait) functions in the random part of the model.

```
data(example)
head(example)
                                                Block Yield
##
                              Env Loc Year
                     Name
                                                                Weight
## 33
       Manistee (MSL292-A) CA.2013 CA 2013 CA.2013.1
                                                          4 -1.904711
##
               CD02024-9W CA.2013 CA 2013 CA.2013.1
                                                          5 -1.446958
  65
##
  66
       Manistee (MSL292-A) CA.2013 CA 2013 CA.2013.2
                                                          5 -1.516271
## 67
                 MSL007-B CA.2011 CA 2011 CA.2011.2
                                                          5 -1.435510
## 68
                MSR169-8Y CA.2013 CA 2013 CA.2013.1
                                                          5 -1.469051
## 103
               AC05153-1W CA.2013 CA 2013 CA.2013.1
                                                          6 -1.307167
ans1 <- mmer2(cbind(Yield, Weight) ~ Env,
              random= ~ us(trait):Name + us(trait):Env:Name,
              rcov= ~ us(trait):units,
              data=example, silent = TRUE)
summary(ans1)
```

```
##
     Multivariate Linear Mixed Model fit by REML
  *********** sommer 3.0 **********
##
                            BIC Method Converge
         logLik
                    AIC
## Value 167.0252 -322.0505 -298.5695
  _____
  Variance-Covariance components:
##
                     VarComp VarCompSE Zratio
## Name.Yield-Yield
                      3.7091
                              1.68159
                                     2.206
## Name.Yield-Weight
                      0.9071
                              0.37954
                                     2.390
## Name.Weight-Weight
                      0.2244
                              0.08777
                                     2.556
## Env:Name.Yield-Yield
                      5.0922
                              1.47905
                                     3.443
## Env:Name.Yield-Weight
                      1.0269
                              0.30773
                                     3.337
## Env:Name.Weight-Weight
                      0.2101
                              0.06663
                                     3.153
## units.Yield-Yield
                      4.3838
                              0.64953
                                     6.749
## units.Yield-Weight
                      0.9078
                              0.14148
                                     6.416
## units.Weight-Weight
                      0.2280
                              0.03378 6.750
  ______
## Fixed effects:
##
## $Yield
##
             Estimate Std. Error
## (Intercept) 14.741985 0.6783206 21.733063
```

```
## EnvCA.2012 -3.199172 0.7474097 -4.280347
## EnvCA.2013
             -4.003349 0.7850509 -5.099477
##
## $Weight
##
               Estimate Std. Error
                                   t value
## (Intercept)
              0.5847374 0.1497090
                                  3.905826
## EnvCA.2012 -0.9711517
                        0.1592564 -6.098038
## EnvCA.2013 -1.1643244 0.1681079 -6.926052
##
##
  ______
  Groups and observations:
##
           Observ Groups
## Name
             185
                     41
## Env:Name
             185
                    123
## Use the '$' sign to access results and parameters
```

You may notice that we have added the us(trait) behind the random effects. This is to indicate the structure that should be assume in the multivariate model. The diag(trait) used behind a random effect (i.e. Name) indicates that for the traits modeled (Yield and Weight) there's no a covariance component and should not be estimated, whereas us(trait) assumes that for such random effect, there's a covariance component to be estimated (i.e. covariance between Yield and Weight for the random effect Name). Same applies for the residual part (rcov).

## 4) Multivariate heterogeneous variance models

##

This is just an extension of the univariate heterogeneous variance models but at the multivariate level. This would be a CS+DIAG multivariate model:

```
data(example)
head(example)
##
                   Name
                            Env Loc Year
                                            Block Yield
                                                          Weight
      Manistee (MSL292-A) CA.2013
                                CA 2013 CA.2013.1
## 33
                                                     4 -1.904711
##
  65
              C002024-9W CA.2013
                                CA 2013 CA.2013.1
                                                     5 -1.446958
##
  66
      Manistee (MSL292-A) CA.2013 CA 2013 CA.2013.2
                                                     5 -1.516271
## 67
               MSL007-B CA.2011
                                CA 2011 CA.2011.2
                                                     5 -1.435510
## 68
               MSR169-8Y CA.2013
                                CA 2013 CA.2013.1
                                                     5 -1.469051
## 103
              AC05153-1W CA.2013 CA 2013 CA.2013.1
                                                     6 -1.307167
ans1 <- mmer2(cbind(Yield, Weight) ~ Env,</pre>
             random= ~ us(trait):Name + us(trait):at(Env):Name,
             rcov= ~ us(trait):at(Env):units,
             data=example, silent = TRUE)
summary(ans1)
  ______
##
        Multivariate Linear Mixed Model fit by REML
  ************ sommer 3.0 ***********
##
          logLik
                               BIC Method Converge
                      AIC
## Value 177.8154 -343.6309 -320.1498
                                      MNR
                                              TRUE
  _____
## Variance-Covariance components:
```

VarComp VarCompSE Zratio

```
## Name.Yield-Yield
                             3.32291
                                       1.45386 2.2856
## Name.Yield-Weight
                             0.79475
                                       0.32648 2.4343
## Name.Weight-Weight
                             0.19103
                                       0.07509 2.5442
## CA.2011:Name.Yield-Yield
                                       4.00992 2.1695
                             8.69943
## CA.2011:Name.Yield-Weight
                             1.77753
                                       0.83835 2.1203
## CA.2011:Name.Weight-Weight
                             0.35939
                                       0.17885 2.0094
## CA.2012: Name. Yield-Yield
                             2.57327
                                       1.95113 1.3189
## CA.2012: Name. Yield-Weight
                             0.33267
                                       0.39866 0.8345
## CA.2012:Name.Weight-Weight
                             0.03842
                                       0.08600 0.4467
## CA.2013:Name.Yield-Yield
                             5.46657
                                       2.16184 2.5287
## CA.2013:Name.Yield-Weight
                             1.34662
                                       0.50455 2.6689
## CA.2013: Name. Weight-Weight
                             0.32893
                                       0.12203 2.6954
## CA.2013:units.Yield-Yield
                             2.56131
                                       0.63996 4.0023
## CA.2013:units.Yield-Weight
                             0.44569
                                       0.12645 3.5246
## CA.2013:units.Weight-Weight 0.12232
                                       0.03057 4.0009
## CA.2011:units.Yield-Yield
                             4.93845
                                       1.52314 3.2423
## CA.2011:units.Yield-Weight
                             0.99446
                                       0.32150 3.0932
## CA.2011:units.Weight-Weight 0.23982
                                       0.07394 3.2433
## CA.2012:units.Yield-Yield
                             5.73841
                                       1.31504 4.3637
## CA.2012:units.Yield-Weight
                             1.27999
                                       0.30150 4.2454
## CA.2012:units.Weight-Weight 0.31804
                                       0.07285 4.3657
## -----
## Fixed effects:
##
## $Yield
##
               Estimate Std. Error
                                    t value
                        0.7889029 18.377621
## (Intercept) 14.498157
  EnvCA.2012
              -3.009537
                        0.8264035 -3.641728
## EnvCA.2013 -3.731629 0.8754507 -4.262524
##
## $Weight
##
                Estimate Std. Error
                                     t value
## (Intercept)
               0.5746062
                         0.1682642
                                    3.414905
## EnvCA.2012
              -0.9334404
                         0.1697663 -5.498384
  EnvCA.2013
              -1.1375574
                        0.1914161 -5.942851
##
  ______
## Groups and observations:
##
               Observ Groups
## Name
                  185
## CA.2011:Name
                  185
                         41
## CA.2012:Name
                         41
                  185
## CA.2013:Name
                  185
                         41
## -----
## Use the '$' sign to access results and parameters
```

Any number of random effects can be specified with different structures.

#### 5) Including special functions

Several random effects require the use of covariance structures that specify an special relationship among the levels of such random effect. The sommer package includes the g() function to include such known covariance structures:

```
data(example)
head(example)
##
                        Env Loc Year
                                       Block Yield
                 Name
                                                   Weight
## 33 Manistee(MSL292-A) CA.2013 CA 2013 CA.2013.1 4 -1.904711
                                             5 -1.446958
            CO02024-9W CA.2013 CA 2013 CA.2013.1
## 65
## 66 Manistee(MSL292-A) CA.2013 CA 2013 CA.2013.2 5 -1.516271
## 67
            MSL007-B CA.2011 CA 2011 CA.2011.2 5 -1.435510
## 68
            MSR169-8Y CA.2013 CA 2013 CA.2013.1
                                              5 -1.469051
            AC05153-1W CA.2013 CA 2013 CA.2013.1
## 103
                                               6 -1.307167
K[1:4,1:4]
##
                  Manistee (MSL292-A) C002024-9W MSL007-B MSR169-8Y
## Manistee(MSL292-A)
                                1
                                   0
## C002024-9W
                                0
                                         1
                                                 0
                                                         0
## MSL007-B
                                0
                                         0
                                                 1
                                                         0
## MSR169-8Y
                                0
                                                         1
ans1 <- mmer2(Yield ~ Env,</pre>
           random= ~ g(Name) + at(Env):g(Name),
           rcov= ~ at(Env):units,
           G=list(Name=K),
           data=example, silent = TRUE)
summary(ans1)
       Multivariate Linear Mixed Model fit by REML
## ************* sommer 3.0 ***********
logLik AIC
                           BIC Method Converge
## Value -15.42982 36.85964 46.52071 MNR
## -----
## Variance-Covariance components:
                         VarComp VarCompSE Zratio
##
## g(Name).Yield-Yield
                           2.962 1.4963 1.980
## CA.2011:g(Name).Yield-Yield 10.148
                                  4.5108 2.250
## CA.2012:g(Name).Yield-Yield 1.879 1.8699 1.005
## CA.2013:g(Name).Yield-Yield 6.629 2.5027 2.649
## CA.2013:units.Yield-Yield 2.560 0.6398 4.001
## CA.2011:units.Yield-Yield 4.943
                                 1.5246 3.242
## CA.2012:units.Yield-Yield 5.725
                                   1.3119 4.364
## Fixed effects:
##
## $Yield
##
             Estimate Std. Error t value
## (Intercept) 16.507678 0.8268665 19.964138
## EnvCA.2012 -5.816890 0.8575814 -6.782902
## EnvCA.2013 -6.412433 0.9356490 -6.853460
##
## -----
## Groups and observations:
               Observ Groups
## g(Name)
                  185
```

```
## CA.2011:g(Name)
                    185
                           41
                    185
## CA.2012:g(Name)
                           41
## CA.2013:g(Name)
                    185
                           41
## Use the '$' sign to access results and parameters
and for multivariate models:
data(example)
head(example)
##
                   Name
                           Env Loc Year
                                           Block Yield
                                                         Weight
## 33
      Manistee(MSL292-A) CA.2013 CA 2013 CA.2013.1
                                                    4 -1.904711
## 65
             CO02024-9W CA.2013 CA 2013 CA.2013.1
                                                    5 -1.446958
## 66
      Manistee (MSL292-A) CA.2013 CA 2013 CA.2013.2
                                                    5 -1.516271
## 67
               MSL007-B CA.2011 CA 2011 CA.2011.2
                                                    5 -1.435510
## 68
              MSR169-8Y CA.2013 CA 2013 CA.2013.1
                                                    5 -1.469051
## 103
             ACO5153-1W CA.2013 CA 2013 CA.2013.1
                                                    6 -1.307167
K[1:4,1:4]
                    Manistee (MSL292-A) C002024-9W MSL007-B MSR169-8Y
##
## Manistee(MSL292-A)
                                   1
                                              0
                                                      0
                                                               0
## C002024-9W
                                    0
                                              1
                                                      0
                                                               0
## MSL007-B
                                    0
                                              0
                                                               0
                                                      1
## MSR169-8Y
                                    0
                                              0
                                                      0
ans1 <- mmer2(cbind(Yield, Weight) ~ Env,
            random= ~ us(trait):g(Name) + us(trait):at(Env):g(Name),
            rcov= ~ us(trait):at(Env):units,
            G=list(Name=K),
            data=example, silent = TRUE)
summary(ans1)
## -----
         Multivariate Linear Mixed Model fit by REML
## *********** sommer 3.0 **********
AIC
          logLik
                               BIC Method Converge
## Value 177.8154 -343.6309 -320.1498
                                             TRUE
## ============
## Variance-Covariance components:
##
                              VarComp VarCompSE Zratio
## g(Name).Yield-Yield
                              3.32291
                                      1.45386 2.2856
## g(Name).Yield-Weight
                              0.79475
                                       0.32648 2.4343
                                       0.07509 2.5442
## g(Name).Weight-Weight
                              0.19103
## CA.2011:g(Name).Yield-Yield
                              8.69943
                                       4.00992 2.1695
## CA.2011:g(Name).Yield-Weight 1.77753
                                       0.83835 2.1203
## CA.2011:g(Name).Weight-Weight 0.35939
                                       0.17885 2.0094
## CA.2012:g(Name).Yield-Yield
                              2.57327
                                       1.95113 1.3189
## CA.2012:g(Name).Yield-Weight 0.33267
                                       0.39866 0.8345
## CA.2012:g(Name).Weight-Weight 0.03842
                                       0.08600 0.4467
## CA.2013:g(Name).Yield-Yield
                              5.46657
                                       2.16184 2.5287
## CA.2013:g(Name).Yield-Weight 1.34662
                                       0.50455 2.6689
## CA.2013:g(Name).Weight-Weight 0.32893
                                       0.12203 2.6954
## CA.2013:units.Yield-Yield
                              2.56131
                                       0.63996 4.0023
## CA.2013:units.Yield-Weight
                              0.44569
                                       0.12645 3.5246
```

```
## CA.2013:units.Weight-Weight
                              0.12232
                                        0.03057 4.0009
## CA.2011:units.Yield-Yield
                              4.93845
                                        1.52314 3.2423
## CA.2011:units.Yield-Weight
                              0.99446
                                        0.32150 3.0932
## CA.2011:units.Weight-Weight
                              0.23982
                                        0.07394 3.2433
## CA.2012:units.Yield-Yield
                              5.73841
                                        1.31504 4.3637
## CA.2012:units.Yield-Weight
                              1.27999
                                        0.30150 4.2454
## CA.2012:units.Weight-Weight
                              0.31804
                                        0.07285 4.3657
## Fixed effects:
##
## $Yield
##
               Estimate Std. Error
                                   t value
## (Intercept) 14.498157
                        0.7889029 18.377621
## EnvCA.2012
             -3.009537
                        0.8264035 -3.641728
## EnvCA.2013
             -3.731629 0.8754507 -4.262524
##
## $Weight
##
               Estimate Std. Error
                                    t value
## (Intercept)
              0.5746062
                        0.1682642
                                   3.414905
## EnvCA.2012
             -0.9334404
                         0.1697663 -5.498384
## EnvCA.2013
             -1.1375574
                        0.1914161 -5.942851
##
  ______
## Groups and observations:
##
                 Observ Groups
## g(Name)
                    185
                           41
## CA.2011:g(Name)
                    185
                           41
## CA.2012:g(Name)
                    185
                           41
## CA.2013:g(Name)
                    185
                           41
## Use the '$' sign to access results and parameters
```

Notice that the g() function is applied at the random effect called "Name", and the covariance structure is provided in the argument "G". In the example, we used a diagonal covariance structure for demonstration purposes but any dense covariance matrix can be used.

Other special functions such as and() for overlay models, eig() for an eigen decomposition of the covariance matrix, grp() for customized random effects providing an incidence matrix.

Keep in mind that sommer uses direct inversion (DI) algorithm which can be very slow for large datasets. The package is focused in problems of the type p > n (more random effect levels than observations) and models with dense covariance structures. For example, for experiment with dense covariance structures with low-replication (i.e. 2000 records from 1000 individuals replicated twice with a covariance structure of 1000x1000) sommer will be faster than MME-based software. Also for genomic problems with large number of random effect levels, i.e. 300 individuals (n) with 100,000 genetic markers (p). For highly replicated trials with small covariance structures or n > p (i.e. 2000 records from 200 individuals replicated 10 times with covariance structure of 200x200) asreml or other MME-based algorithms will be much faster and we recommend you to opt for those software.

## Literature

Covarrubias-Pazaran G. 2016. Genome assisted prediction of quantitative traits using the R package sommer. PLoS ONE 11(6):1-15.

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