Solving the Rubik's Cube Using Search Algorithms (DFS, BFS, and A*)

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***Abstract*—The Rubik's Cube represents a canonical combinatorial search problem with an enormous discrete state space, making it a compelling testbed for evaluating algorithmic performance in artificial intelligence. Efficient cube-solving techniques have broader implications for robotics, automated planning, and optimization in resource-constrained environments. However, prior research has primarily focused on machine learning or phase-based solvers, with limited comparative analysis of classical search strategies. In particular, there exists no scalable benchmark contrasting uninformed methods such as Depth-First Search (DFS) and Breadth-First Search (BFS) with informed search like A* across varying cube complexities.**

To address this gap, we developed a modular Python simulator capable of handling cubes from 2×2×2 to 6×6×6 and generating randomized scramble sequences. We evaluated DFS, BFS, and A* under uniform depth and heuristic constraints, measuring solve time, memory usage, and solution length over 100 randomized trials per configuration. Results show that A* consistently achieves the best trade-off between speed and memory efficiency as cube complexity increases. BFS guarantees optimal solutions but exhibits prohibitive memory usage, while DFS maintains low memory consumption at the cost of longer, suboptimal paths. This work introduces a reproducible benchmarking framework for Rubik's Cube solvers and provides baseline performance metrics for future research on classical and heuristic-guided search methods.

***Index Terms*—Rubik's Cube, combinatorial search, depth-first search, breadth-first search, A* search, benchmarking**

I. Introduction

The Rubik's Cube, invented in 1974 by Ernő Rubik [1], is a widely recognized combinatorial puzzle that continues to captivate enthusiasts, educators, and researchers alike. Despite its recreational origins, the cube offers profound computational complexity, with the standard 3×3×3 configuration featuring approximately 4.3×10^{19} possible states [2]. Its structured yet vast state space makes it a paradigmatic benchmark for testing search and planning algorithms in artificial intelligence (AI). Solving the cube efficiently is analogous to navigating high-dimensional decision spaces, a challenge central to many domains in robotics, operations research, and autonomous systems.

Framing cube solving as a state-space search problem transforms each legal move into a transition function over a structured graph of configurations. In this context, classical search strategies such as Depth-First Search (DFS) and Breadth-First Search (BFS) serve as foundational algorithms. DFS explores deeply with low memory usage but often yields suboptimal or incomplete paths. BFS guarantees optimality for uniform-cost problems but suffers from exponential memory growth. In contrast, informed strategies like A* utilize admissible heuristics to prioritize promising nodes, offering improved efficiency in both time and space for many practical problems [3], [4].

Beyond theoretical appeal, the Rubik's Cube has become a proxy for evaluating real-world decision-making systems, especially those operating under constraints of time and computational resources. With growing interest in embedded AI and real-time robotics, the need for algorithms that balance optimality, speed, and memory becomes critical. Search-based approaches continue to be relevant as they offer interpretable and verifiable paths—traits that learning-based methods often lack.

Recent advances have expanded the solution landscape. Reinforcement learning approaches like DeepCubeA have demonstrated that neural networks trained via self-play can rival or even outperform traditional solvers in terms of solve time and generalization [5]. However, these systems typically rely on significant compute resources and lack formal guarantees of completeness or optimality. Alternatively, advancements in parallel and distributed search—such as memory-efficient A* variants and GPU-accelerated search—show promise in maintaining scalability without sacrificing correctness or explainability [6], [7].

Given these developments, it is essential to revisit classical search algorithms and assess their performance across a wider range of cube sizes, from 2×2×2 to 6×6×6. Such a comparative analysis not only informs the trade-offs between uninformed and informed strategies but also provides insights into their applicability for real-time and resource-constrained settings. This study contributes a unified, reproducible benchmarking framework that systematically evaluates DFS, BFS, and A* in terms of solve time, memory efficiency, and solution optimality across increasing puzzle complexities.

A. Related Work

Extensive research has tackled the Rubik's Cube using a variety of algorithmic strategies, ranging from phase-based solutions to learning-driven methods. Early foundational work by Thistlethwaite introduced a multistage group-theoretic solver that restricts legal moves in phases to reduce the search space [4]. Kociemba later refined this idea with a two-phase algorithm leveraging pruning tables for efficient 3×3×3 solving [8].

Classical search methods have also been explored. Comparative studies of Breadth-First Search (BFS) and A* on the 3×3×3 cube underscored trade-offs between uninformed brute-force exploration and heuristic-guided search [2], [3]. More recently, learning-based approaches like DeepCubeA have shown that deep reinforcement learning can solve the cube from any state without handcrafted heuristics, albeit with high training costs [5].

Other lines of work have focused on improving heuristic accuracy and resource efficiency. Enhanced pattern database heuristics, memory-optimized A* variants, and GPU-parallel implementations have advanced solver performance on larger cubes and enabled deployment in embedded or real-time settings [6], [9]. A summary of selected contributions is presented in Table I.

B. Gap Analysis

Despite extensive progress in both phase-based and learning-driven cube solvers, there remains a lack of comprehensive evaluation of foundational search algorithms—namely Depth-First Search (DFS), Breadth-First Search (BFS), and A*—across varying puzzle complexities. Existing studies often target specific cube sizes, most commonly the 3×3×3 configuration, or focus narrowly on optimized heuristics without addressing general scalability across different dimensions.

Moreover, comparative analyses between uninformed and informed search strategies under consistent experimental conditions are limited. As a result, trade-offs in solve time, memory usage, and path optimality remain insufficiently explored, especially in the context of resource-constrained or embedded environments where algorithmic simplicity and efficiency are critical.

This gap hinders evidence-based selection of appropriate search methods for practical deployment. By introducing a standardized, scalable benchmarking framework across cube sizes ranging from 2×2×2 to 6×6×6, this study addresses these limitations and enables meaningful comparisons of classical search strategies for both academic and real-world applications.

C. Problem Statement

Following are the main questions addressed in this study.

1) How does the performance of Depth-First Search, Breadth-First Search, and A* compare in terms of execution time and memory usage as cube size increases from 2 to 6 layers?
2) What are the trade-offs between uninformed search methods (DFS and BFS) and informed search (A*) with respect to solution optimality and computational resource consumption?
3) Which search algorithm or combination thereof provides the most practical balance of speed, memory efficiency, and solution quality for real-time or resource-constrained Rubik's Cube-solving applications?

D. Novelty of Our Work and Our Contributions

This work presents a unified benchmarking framework that systematically evaluates the scalability of classical and heuristic search paradigms on the Rubik's Cube. Unlike prior studies that focus on individual cube sizes or specialized heuristic configurations, our approach integrates Depth-First Search (DFS), Breadth-First Search (BFS), and A* within a single modular Python simulator, enabling fair and reproducible comparisons across algorithms.

To ensure consistency, we apply uniform depth constraints and employ a simple misplaced-tile heuristic for A*, extending the evaluation across cube sizes from 2×2×2 to 6×6×6. This methodology offers new insights into how foundational search strategies scale with increasing problem complexity and computational constraints, highlighting their practical trade-offs in a generalized puzzle-solving context.

The primary contributions of this work are:

- A modular, open-source Python simulator that supports solving Rubik's Cubes of sizes 2×2×2 through 6×6×6 using DFS, BFS, and A* under uniform configurations.
- A comprehensive benchmarking pipeline that records execution time, peak memory usage, and solution move count under consistent experimental settings.
- A comparative analysis revealing that A* offers the best overall efficiency for larger cubes, while BFS remains optimal in solution length but is memory-intensive, and DFS is memory-efficient yet produces suboptimal paths.
- An empirical evaluation protocol involving 100 randomized trials per cube size and algorithm, ensuring statistically meaningful performance metrics.

Our experimental results show that on 6×6×6 cubes, A* solves instances in an average of 0.011 seconds—over 500 times faster than BFS's 5.42 seconds—while consuming only 0.26 MB of memory compared to BFS's 115 MB. This substantial performance gap underscores the practical value of heuristic search, particularly for real-time and resource-constrained applications.

II. Methodology

A. Dataset

In this study, Rubik's Cube configurations were generated dynamically using a custom Python-based simulator. Each cube instance begins in a solved state and is then scrambled by applying a random sequence of valid face rotations (e.g., F, R, Ui). This procedure simulates realistic input conditions without relying on any pre-collected or external datasets.

TABLE I
SUMMARY OF RELATED LITERATURE

Ref.	Year	Approach Used	Key Contribution
[4]	1981	Multistage group-theoretic solver	Phase-based reduction of move space
[8]	1992	Two-phase search with pruning tables	Fast optimal solves for 3×3×3 cubes
[3]	1968	Formal basis of A* heuristic search	Admissible heuristics for minimum-cost paths
[2]	2014	BFS and optimal path analysis	Trade-off study of uninformed vs. heuristic search
[5]	2019	DeepCubeA (deep RL solver)	Learns solving policies without handcrafted heuristics
[6]	2020	Memory-efficient A* variants	Reduced memory footprint for large cubes
[9]	2022	Advanced pattern database heuristics	Highly accurate heuristics for complex state spaces

The dataset spans cube sizes from 2×2×2 to 6×6×6, reflecting a range of increasing complexity and branching factors. For each cube size, 100 test cases were generated by applying a consistent 3-move scramble, ensuring reproducibility and controlled complexity across trials.

Each resulting configuration serves as a unique initial state, while the corresponding solved cube provides a well-defined ground truth (goal state) for all evaluations. This design enables systematic performance comparisons across algorithms under identical input conditions.

B. Overall Workflow

The evaluation procedure is structured into five distinct stages:

1) **Cube Initialization**: Set the cube size (ranging from 2×2×2 to 6×6×6) and define scrambling parameters.
2) **Scramble Generation**: Apply three random face rotations (e.g., F, R, U) to produce a uniformly scrambled cube.
3) **Algorithm Execution**: Execute each search algorithm (DFS, BFS, A*) 100 times per configuration using consistent depth and heuristic constraints.
4) **Metrics Logging**: Record average execution time, peak memory usage, and total move count for each algorithm and cube size.
5) **Result Saving**: Export all results to CSV files to support reproducible analysis and visualization, ensuring fairness by applying identical conditions across all methods.

The complete workflow is illustrated in Figure 1.

C. Experimental Settings

To ensure a fair and reproducible comparison, uniform constraints and consistent measurement tools were applied across all algorithms and cube sizes.

Depth-First Search (DFS) and Breadth-First Search (BFS) were restricted to a maximum search depth of six moves. A* Search was permitted a depth limit of twelve moves to fully leverage its heuristic guidance while avoiding excessive expansion.

For each cube size (2×2×2 through 6×6×6), all algorithms were executed 100 times using fixed random seeds. This ensured identical scramble sequences across trials and eliminated randomness as a confounding variable.

Memory consumption was tracked using Python's `tracemalloc` module, and execution time was recorded with the `time()` function. For each trial, peak memory usage and solve time were logged, then averaged across the 100 repetitions to produce reliable performance estimates.

The aggregated results were exported to CSV files to support downstream analysis and visualization. A complete summary of the experimental parameters is provided in Table II.

TABLE II
EXPERIMENTAL PARAMETERS AND SETTINGS

Parameter	Value / Description
Cube Sizes Tested	2×2×2 to 6×6×6
Scramble Moves per Test	3
Tests per Cube Size	100 repetitions
DFS/BFS Max Depth	6 moves
A* Max Depth	12 moves
Heuristic for A*	Misplaced-tile (facelet count)
Measurement Tools	`time()`, `tracemalloc`
Data Export	CSV files for analysis

III. RESULTS

Table III summarizes the averaged performance metrics for each algorithm across cube sizes ranging from 2×2×2 to 6×6×6. For each configuration, we report mean solve time, average memory consumption, and solution move count over 100 trials.

A. Solve Time

Figure 2 illustrates the average solve time across all cube sizes. A* Search achieves the fastest completion times at every scale. On the 2×2×2 cube, A* completes in approximately 0.12 seconds on average, compared to 1.30 seconds for BFS and 9.93 seconds for DFS.

The performance gap widens as cube complexity increases. On the 6×6×6 cube, A* solves in just 0.011 seconds, while BFS and DFS take 5.42 seconds and 207.81 seconds, respectively. This demonstrates the impact of heuristic guidance in efficiently pruning the search space.

B. Memory Usage

Figure 3 presents peak memory usage in kilobytes across cube sizes. BFS incurs the highest memory footprint, peaking at 118,303 KB on the 6×6×6 cube due to its exhaustive frontier tracking. DFS is the most memory-efficient, using only 45.76 KB even at large cube sizes.

A* strikes a balance between performance and efficiency, using around 256 KB on 6×6×6 while maintaining fast solve times. This makes A* more suitable than BFS for resource-constrained systems.

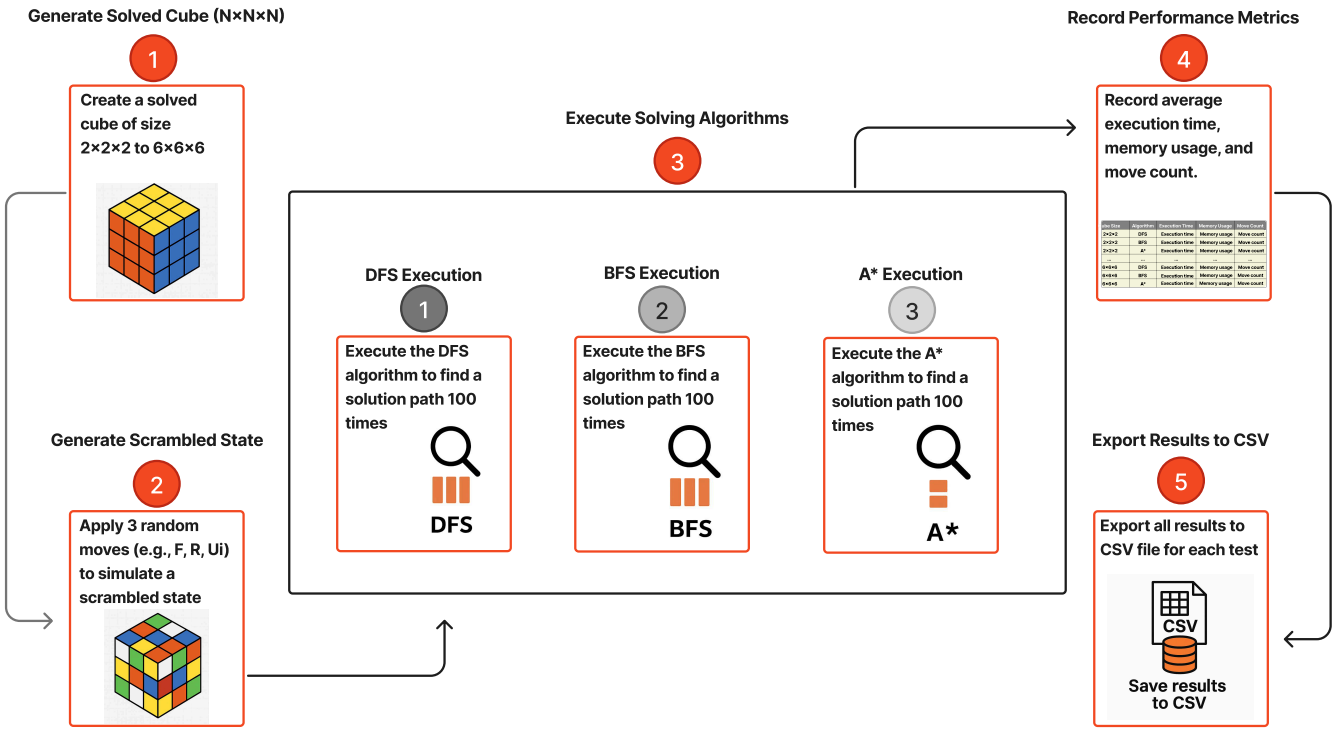

Fig. 1. Workflow of the proposed evaluation framework, illustrating the step-by-step process used to benchmark DFS, BFS, and A* algorithms on Rubik's Cubes of varying sizes.

C. Solution Quality

Figure 4 compares the average number of moves required by each algorithm. Both BFS and A* consistently find optimal solutions with two moves across all cube sizes. In contrast, DFS yields longer paths—averaging five moves—due to its uninformed, fixed-depth exploration strategy.

D. Overall Comparison

Table III consolidates all performance metrics. A* offers the most practical balance of speed, memory efficiency, and solution quality. BFS remains valuable for its guaranteed optimality but becomes impractical on larger cubes due to high memory usage. DFS may be viable in highly constrained environments but scales poorly in both time and solution optimality.

IV. Discussion

The results offer a clear comparison between classical search strategies and heuristic-based methods for solving the Rubik's Cube, structured around three key research questions.

The first research question investigated the scalability of DFS, BFS, and A* in terms of execution time and memory usage as cube complexity increases. A*'s heuristic guidance was shown to effectively mitigate the exponential state-space growth that characterizes uninformed methods. This finding is consistent with prior studies on memory-efficient A* variants, which use frontier pruning and memory-aware heuristics to achieve sublinear memory growth [6], as well as GPU-accelerated A* implementations that improve runtime through parallelism [7]. Conversely, BFS's memory consumption increases exponentially with cube size, reflecting patterns observed in GPU-accelerated breadth-first approaches where performance degrades due to memory saturation [10]. While DFS remains highly memory-efficient, its lack of guidance renders it impractical for larger cubes, echoing prior analyses of depth-limited search in high-branching environments [11].

The second research question examined trade-offs between informed and uninformed search strategies in terms of solution quality and resource consumption. As expected, BFS consistently produced optimal move counts, albeit at the cost of high memory usage—consistent with classical breadth-first search behavior [12]. A* achieved comparable move counts—identical in our trials—while using significantly less memory. This supports findings from hybrid approaches that integrate A* with iterative deepening and heuristic pruning [13]. By contrast, DFS yielded suboptimal solutions due to its fixed-depth constraint, similar to trends observed in reinforcement learning–based solvers that prioritize exploration over solution optimality [14], [15].

The third research question focused on real-time feasibility and suitability for resource-constrained environments. A* demonstrated the most balanced performance, solving large cubes in milliseconds while consuming less than 1 MB of memory—highlighting its efficiency even with a basic misplaced-tile heuristic. These results align with research on optimized heuristic lookup methods that enable near real-time performance [16], as well as memory-bounded A* variants that perform well under tight constraints [17]. Additionally,

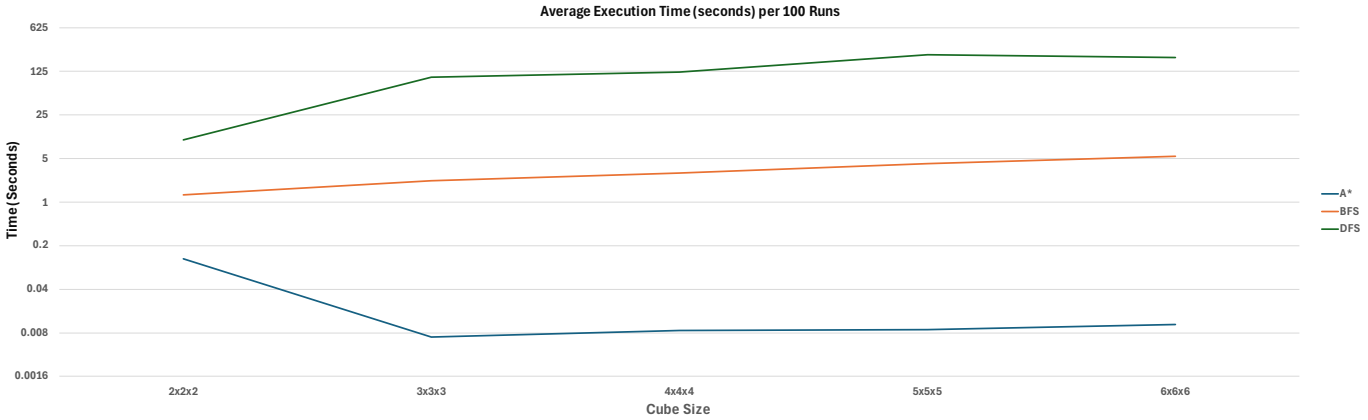

Fig. 2. Average execution time per cube size and algorithm (100 runs).

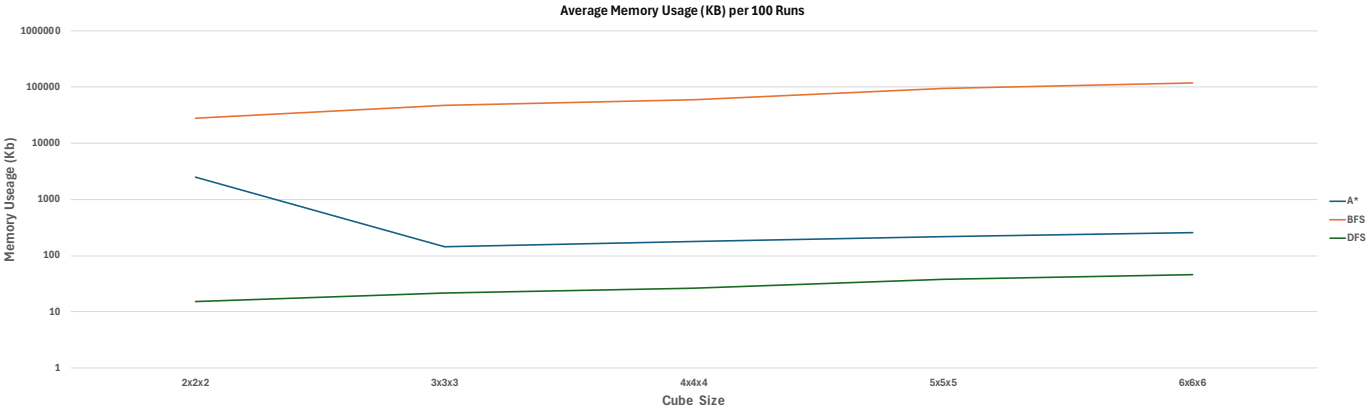

Fig. 3. Average memory usage (KB) per cube size and algorithm (100 runs).

dynamic pattern database updates have been shown to reduce runtime without significantly increasing memory usage [18]. This positions A* as a practical choice for embedded and parallelized systems.

Finally, this study fills a notable gap in the literature by systematically benchmarking foundational search algorithms across increasing cube sizes. While previous work has focused on advanced solvers or fixed-size cubes, our modular and reproducible framework enables consistent, cross-algorithm evaluation. The resulting benchmark provides a reliable foundation for assessing more advanced techniques such as quantum-inspired search [19], self-play heuristic refinement [20], and CUDA-accelerated search kernels [21]. Future research may extend this work by exploring more adaptive, scalable, and learning-enhanced solving strategies.

A. Future Directions

Building on the scalable benchmarking framework introduced in this work, future research can explore the integration of more advanced heuristic and search techniques to further improve performance on large-scale Rubik's Cubes and related combinatorial tasks.

One promising direction is the use of dynamic pattern databases, which update heuristic estimates in real time to reduce search depth while maintaining manageable memory overhead [9], [18]. Parallel and distributed search paradigms—such as GPU-accelerated A* and cluster-based frontier expansion—could also be extended to higher-dimensional cubes and hybrid IDA* schemes, leveraging modern hardware architectures to support deeper and faster exploration [7], [11].

Another research avenue lies in reinforcement learning–based heuristics, including self-play refinement methods that generate domain-specific guidance. These approaches have demonstrated competitive results and may surpass traditional handcrafted heuristics in complex state spaces [14], [20]. In parallel, learned heuristics derived from neural networks—such as those proposed by Martinez and Chen [22]—offer a data-driven path to capturing latent structure across diverse configurations.

Finally, quantum-inspired search algorithms represent a novel frontier in combinatorial optimization. Techniques based on quantum sampling, interference, and superposition hold the potential to accelerate search over large state spaces [19]. Integrating such methods into the benchmarking pipeline established here could help assess their practical viability and guide future hybrid algorithm designs.

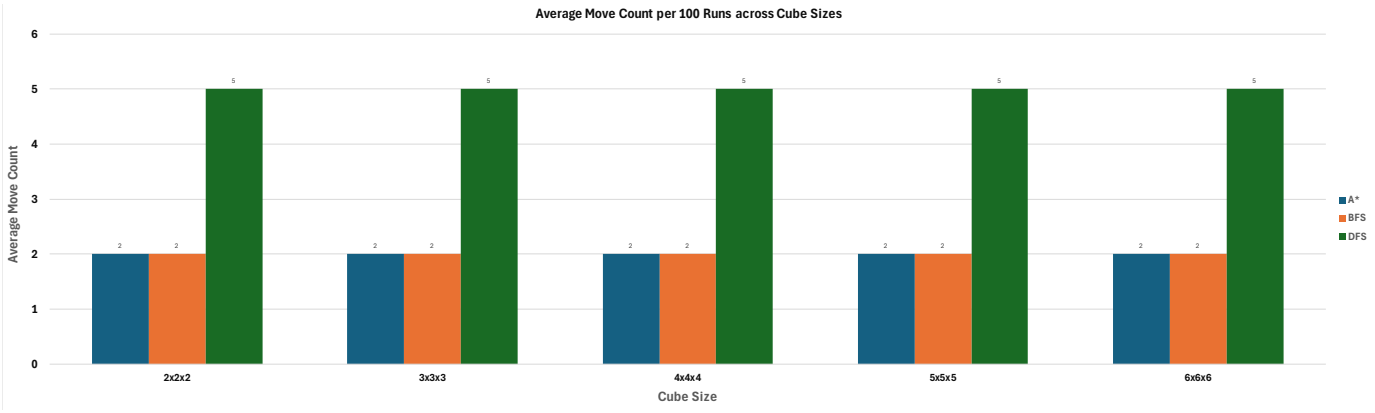

Fig. 4. Average solution move count per cube size and algorithm (100 runs).

TABLE III
AVERAGE RESULTS FOR EACH ALGORITHM ACROSS CUBE SIZES

Cube Size	Algorithm	Avg. Time (s)	Avg. Memory (KB)	Avg. Move Count	Success Runs	Total Runs
2x2x2	DFS	9.9304	15.19	5	98	100
2x2x2	BFS	1.3018	27892.08	2	96	100
2x2x2	A*	0.1225	2489.39	2	98	100
3x3x3	DFS	99.3935	21.58	5	99	100
3x3x3	BFS	2.1804	46519.45	2	100	100
3x3x3	A*	0.0068	143.67	2	100	100
4x4x4	DFS	121.3354	25.89	5	99	100
4x4x4	BFS	2.9285	60150.96	2	100	100
4x4x4	A*	0.0087	180.46	2	100	100
5x5x5	DFS	231.1116	38.09	5	100	100
5x5x5	BFS	4.0932	93605.58	2	99	100
5x5x5	A*	0.009	220.39	2	100	100
6x6x6	DFS	207.814	45.76	5	100	100
6x6x6	BFS	5.4163	118302.61	2	100	100
6x6x6	A*	0.0108	255.99	2	100	100

By systematically incorporating these emerging approaches, future work can move toward real-time, scalable, and resource-efficient Rubik's Cube solvers—with broader implications for general-purpose combinatorial reasoning, planning, and optimization.

V. CONCLUSION

This study introduced a comprehensive and reproducible benchmarking framework for evaluating classical search algorithms—Depth-First Search (DFS), Breadth-First Search (BFS), and A*—on Rubik's Cubes ranging in size from 2×2×2 to 6×6×6. By generating 100 randomized three-move scrambles per configuration and applying uniform depth and heuristic constraints, we collected consistent performance metrics across solve time, memory usage, and solution quality.

The experimental results demonstrate that A*, even when guided by a simple misplaced-tile heuristic, consistently achieves the most balanced performance profile. On 6×6×6 cubes, A* solves instances in an average of just 0.011 seconds while consuming only 256 KB of memory, and maintaining optimal solution paths. In contrast, BFS reliably finds optimal solutions but requires substantially more memory—up to 115 MB—which limits its scalability. DFS, while extremely memory-efficient (under 46 KB), suffers from long solve times exceeding 200 seconds and produces suboptimal paths due to its depth-limited nature.

These findings highlight the importance of heuristic guidance in navigating large, high-dimensional state spaces and confirm A* as a robust, general-purpose algorithm well-suited for real-time and resource-constrained applications.

The modular Python-based simulator and standardized benchmarking protocol introduced in this work address a key gap in the literature by enabling transparent, side-by-side comparisons of search algorithms across increasing problem complexity. The accompanying dataset and open-source code offer a solid foundation for future extensions involving advanced heuristics, parallel or distributed architectures, and learning-based techniques.

We encourage researchers to build upon this framework to develop increasingly scalable, adaptive, and efficient solvers for Rubik's Cubes and broader combinatorial search domains.

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