Odd-Frequency Pairing in Periodically-Driven Systems

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Odd-Frequency pairing

Anomalous Green Function

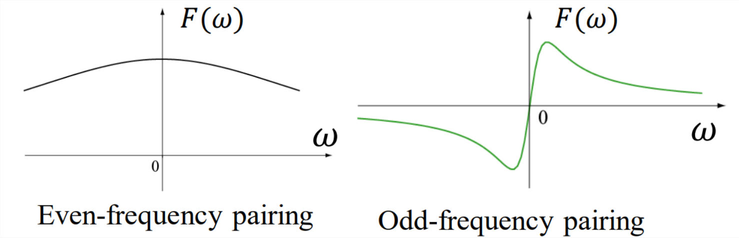
An important quantity in superconductors is the anomalous Green function

$$F_{\sigma_1,\sigma_2;\eta_1,\eta_2}(t,x) = \langle \mathcal{T} c_{\sigma_1,\eta_1}(t,x) c_{\sigma_2,\eta_2}(0,0) \rangle$$

Due to Fermi statistics, we have:

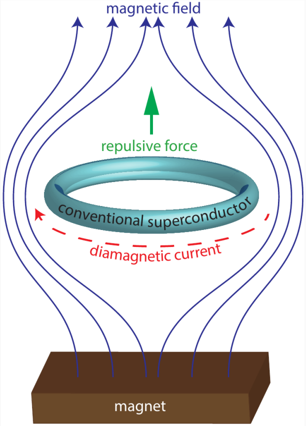
$$F_{\sigma_2,\sigma_1;\eta_2,\eta_1}(-t,-x) = -F_{\sigma_1,\sigma_2;\eta_1,\eta_2}(t,x)$$

Frequency dependence of anomalous Green's function [RevModPhys.**91**.045005]

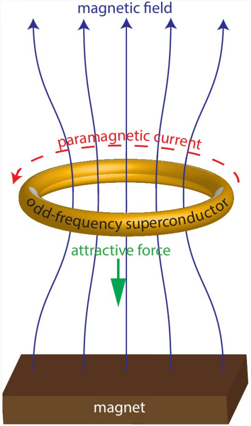


Paramagnetic Meissner effect

Meissner effect
(Even frequency pairing)



Paramagnetic Meissner effect
(Odd frequency pairing)



[PhysRevB.**95**.184506]

Topological Superconductors

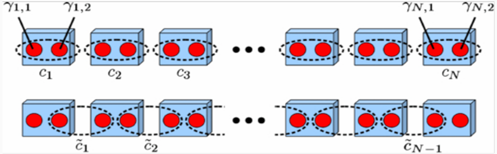
Majorana Bound states

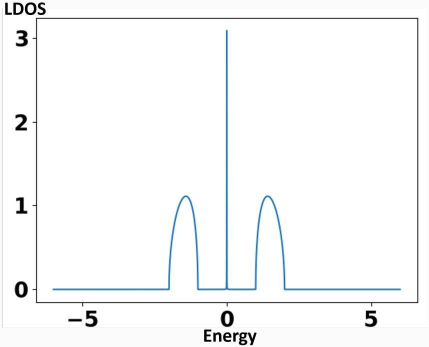
Topological superconductors are characterized by having Majorana fermion states located at the boundary. Majorana fermions are real(not complex) fermions that are their own anti-particles.

$$\gamma^\dagger = \gamma$$

Because of the particle-hole symmetry in superconductors, these Majorna fermions must be localized exactly at zero energy.

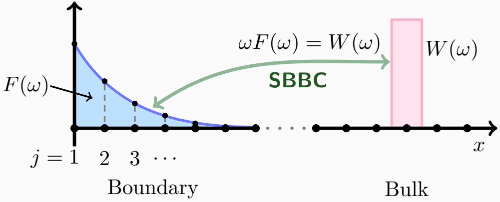
$$H = \sum_i t(c_i^\dagger c_{i+1} + \mathrm{h.c.}) + i\Delta\left(c_i^\dagger c_{i+1}^\dagger - c_i c_{i+1}\right)$$





Spectral Bulk Boundary correspondence

For chiral symmetric systems, we have $\{H, \Gamma\} = 0$



[PhysRevB.**99**.184512]

Winding number

$$w(z) = \frac{1}{2i} \mathrm{Tr}_k \left[\Gamma g(z,k) \partial_k g^{-1}(z,k) \right]$$

Odd-frequency correlation at the edge

$$F(z) = \mathrm{Tr}_{\mathrm{edge}} \left[\Gamma G(z) \right]$$

$$G(z) = (z - H)^{-1}, \; F(-z) = -F(z).$$

Generalized Spectral Bulk Boundary correspondence

For non-chiral symmetric systems, we have $H = H_c + H_{ac}$
where

$$\{\Gamma, H_{ac}\} = [\Gamma, H_c] = 0$$

Winding number

$$w(z) = \frac{1}{2i} \mathrm{Tr}_k \left[\Gamma g(z,k) i [g^{-1}(z,k), \hat{X}] \right]$$

Odd-frequency correlation at the edge

[PhysRevB.**104**.165125]

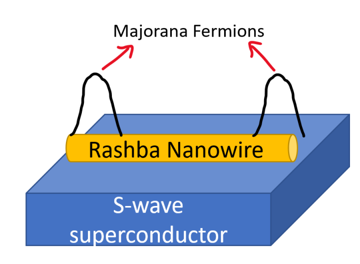
$$F(z) = \mathrm{Tr}_{\mathrm{edge}} \left[\Gamma (1 - H_c) G(z) \right]$$
$$G(z) = (z - H)^{-1}, \; F(-z) = -F(z).$$

Rashba nanowire

The model hamiltonian is given by

$$H = \left(\frac{k^2}{2m} - \mu\right)\eta_z + \lambda k \sigma_z + B\eta_z\sigma_x + \Delta\eta_y\sigma_y$$

where σ_i, η_i are Pauli matrices acting on spin and particle-hole degrees of freedom respectively



[NaturePhys **8**,887–895(2012)]

Figure 1: Schematic diagram of Rashba nanowire

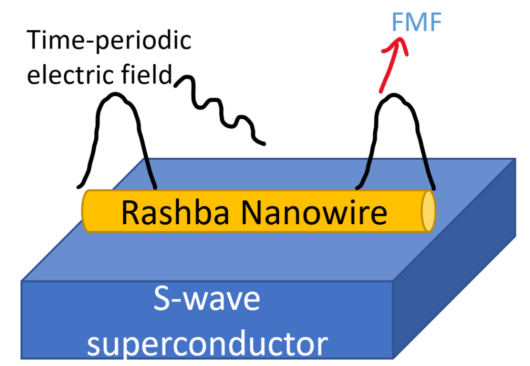
Periodically-driven Rashba nanowire

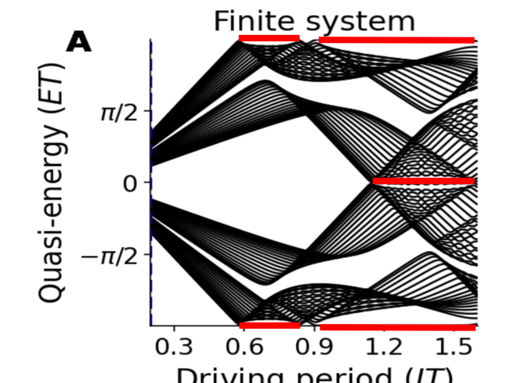
We introduce a time-periodic chemical potential

$$\mu(t) = \begin{cases} \mu_1 & nT < t < (n+\frac{1}{2})T \\ \mu_2 & (n+\frac{1}{2})T < t < (n+1)T \end{cases}. \quad n \in \mathcal{Z}$$

[PhysRevLett.**106**.220402]

Because of the periodicity in time, Majorana edge states appear at energies $= 0, \pi$





Floquet Theorem

Floquet theorem

Assume that we have a hamiltonian that is periodic in time

$$H(t+T) = H(t)$$

Floquet theorem states that the eigenstates of this hamiltonian are given by

$$|\psi_j(t)\rangle = e^{-i\epsilon_j(t-t_0)}\,|u_j(t)\rangle$$
$$\text{with} \quad |u_j(t+T)\rangle = |u_j(t)\rangle\,, \qquad \epsilon_j \in \left\{-\frac{\pi}{T}, \frac{\pi}{T}\right\}$$

Proof of Floquet Theorem

Let $H(t+T) = H(t)$. The propagator is

$$U(t) = \mathcal{T}\exp\left\{-i\int_0^t H(t')\mathrm{d}t'\right\}$$

We define the one period propagator $F = U(T) = \exp\{-iH_f T\}$

Let $|u_j\rangle$ be eigenstate of F with eigenvalue $e^{-i\epsilon_j T}$

The propagator can be rewritten as

$$U(t) = U(t)\exp\{iH_f t\}\exp\{-iH_f t\} = P(t)\exp\{-iH_f T\}$$

It follows that the solution to Schrodinger equation is

$$|\psi_j(t)\rangle = e^{-i\epsilon_j(t-t_0)}\,|u_j(t)\rangle$$

$$\text{with} \quad |u_j(t)\rangle = P(t)\,|u_j\rangle\,, \qquad \epsilon_j \in \left\{-\frac{\pi}{T}, \frac{\pi}{T}\right\}$$

Extended Hilbert space

By extending the Hilbert space $\mathcal{H}$ to $L^2[0, T] \otimes \mathcal{H}$, it is possible to turn the time-dependent Schrodinger equation into a time-independent problem. The effective hamiltonian is

$$K(t') = H(t') - i\partial_{t'}$$

The eigenstates are

$$\left|\psi_j(t, t')\right\rangle = e^{i\epsilon_j t} \left|u_j(t')\right\rangle$$

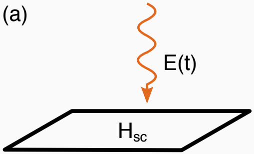
The extended Hilbert space formalism reduces to the time-dependent solution for Schrodinger equation when $t = t'$

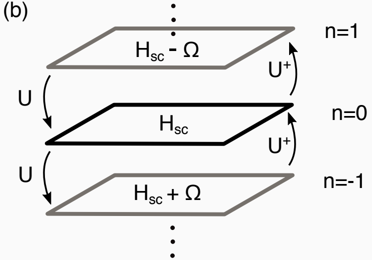
$$\left|\psi_j(t, t' = t)\right\rangle = |\psi_j(t)\rangle$$

Floquet superconductors

Floquet superconductivity

Any time-periodic system can be turned into a static system using Floquet theorem. The static system has infinite number of Floquet bands. These Floquet bands are coupled to each other by the driving force.





$$H_{sc} = \frac{k^2}{2m}\sigma_z + \Delta\sigma_x$$

$$U = \frac{e}{2m}\int_0^T \mathrm{d}t e^{i\Omega t}\vec{A}(t)\cdot\vec{k}\sigma_z$$

[PhysRevB.**103**.104505]

Odd-frequency Pairing symmetries in Floquet superconductors

class	**Floquet index** $(n, m) \to (-m, -n)$	**Frequency** $\omega \to -\omega$	**Spin** $s = \pm 1$	**Momentum** $\boldsymbol{k}_1 \to \boldsymbol{k}_2$
1	Even	Odd	Triplet	Even
2	Even	Odd	Singlet	Odd
3	Odd	Odd	Triplet	Odd
4	Odd	Odd	Singlet	Even

Table 1: All possible odd-frequency pairing in Floquet superconductors allowed by Fermi statistics

[PhysRevB.**103**.104505]

Floquet Topological Superconductors