Odd-Frequency Pairing in Floquet Topological Superconductors

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Anomalous Green Function

Anomalous Green function(Cooper pairing amplitude):

$$F_{\sigma_1,\sigma_2}(t,x) = \langle \mathcal{T} c_{\sigma_1}(t,x) c_{\sigma_2}(0,0)\rangle$$

Anomalous Green's function in frequency-momentum space:

$$F_{\sigma_1,\sigma_2}(\omega,k) = \int F_{\sigma_1,\sigma_2}(t,x) e^{-i\omega t + ikx} \mathrm{d}x\mathrm{d}t$$

Fermi statistics

$$F_{\sigma_2,\sigma_1}(-\omega,-k) = -F_{\sigma_1,\sigma_2}(\omega,k)$$

Symmetry Classes

Fermi statistics

$$F_{\sigma_2,\sigma_1}(-\omega,-k) = -F_{\sigma_1,\sigma_2}(\omega,k)$$

[RevModPhys.**91**.045005] [JPSJ.**81**.011013]

$\omega \to -\omega$	$k \to -k$	$(\sigma_1,\sigma_2) \to (\sigma_2,\sigma_1)$	Examples
Even	Even (s-wave, d-wave)	Singlet	conventional BCS, Cuprate
Even	Odd (p-wave)	Triplet	$Sr_2 RuO_4$, UTe_2
Odd	Even (s-wave, d-wave)	Triplet	unconfirmed in bulk
Odd	Odd (p-wave)	Singlet	unconfirmed in bulk

Table: All possible pairing symmetries allowed by Fermi statistics

Generation of Odd-frequency Pairing

1. space translation symmetry breaking (odd-ω pairing at the edge)
   superconducting junctions [PhysRevB.**76**.054522]

2. spin rotation symmetry breaking(odd-ω pairing at the edge)
   superconductor/ferromagnet junctions [PhysRevLett.**86**.4096]

3. Orbital index hybridization (odd-ω pairing in the bulk)
   multi-orbit superconductor [andp.201900298]

4. time-translation symmetry breaking (odd-ω pairing in the bulk)
   periodically-driven superconductors [PhysRevB.**103**.104505]

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Majorana Zero Modes (MZM)

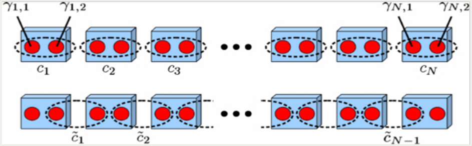
Topological superconductors are characterized by having Majorana fermion edge states at zero energy.

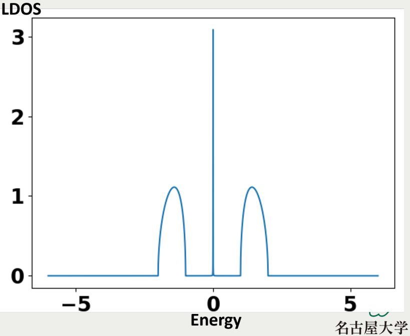
$$\gamma^{\dagger}(E) = \gamma(-E) \Longrightarrow E = 0$$

Majorana fermions are their own anti-particles.

$$H = \sum_i t(c_i^{\dagger} c_{i+1} + \mathrm{h.c.}) + i\Delta(c_i^{\dagger} c_{i+1}^{\dagger} - c_i c_{i+1})$$

[A Yu Kitaev 2001 Phys.-Usp. **44** 131]





SABS and Odd-Frequency Pairing

Bulk symmetry	SABS	Edge symmetry
ESE	absent	ESE(OSO)
ESE	present	OSO(ESE)
ETO	present	OTE(ETO)
ETO	absent	ETO(OTE)

ESE (Even-frequency spin-singlet even-parity)
ETO (Even-frequency spin-triplet odd-parity)
OTE (Odd-frequency spin-triplet even-parity)
OSO (Odd-frequency spin-singlet odd-parity

SABS

$$\Updownarrow$$

odd-ω pairing $\propto \frac{1}{\omega}$

Bulk-Boundary Correspondence for Chiral symmetric Systems

Chiral symmetry: $\Gamma H + H\Gamma = 0$ with $\Gamma^\dagger = \Gamma$ and $\Gamma^2 = 1$.

Bulk: Winding number W

$$W = \frac{i}{4\pi}\int_{-\pi}^{\pi} \mathrm{d}k\, \mathrm{Tr}\left\{\Gamma h^{-1}(k)\partial_k h(k)\right\} \in \mathbb{Z}$$

Boundary: Number of edge modes N

$$N = N_+ - N_-, \qquad \begin{cases} N_+ & \text{Number of positive eigenvalues of } \Gamma. \\ N_- & \quad '' \quad \text{negetive} \quad '' \end{cases}$$

Bulk-Boundary Correspondence:

$$W = N$$

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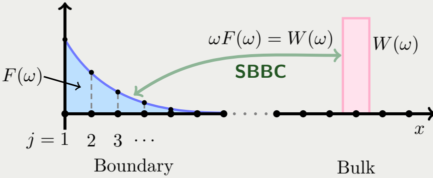
Spectral Bulk-Boundary Correspondence

Bulk: extended winding number $W(\omega)$

$$W(\omega) = \frac{i}{4\pi}\int_{-\pi}^{\pi} \mathrm{d}k\, \mathrm{Tr}\{\Gamma g(\omega,k)\partial_k g(\omega,k)\}$$

Boundary: Odd-ω pairing amplitude accumulated at the edge

$$F(\omega) = \mathrm{Tr}\{\Gamma G(\omega)\}$$



SBBC:

$$\omega F(\omega) = W(\omega)$$

Summary

Bulk

Topological Invariant

Bulk-Boundary correspondence

[PhysRevB.**78**.195125]

Spectral Bulk-Boundary correspondence

[PhysRevB.**99**.184512]

Boundary

Edge modes (Majorana Fermions)

Boundary

Odd-frequency pairing at the edge

[PhysRevB.**87**.104513]

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Periodically-driven Rashba nanowire

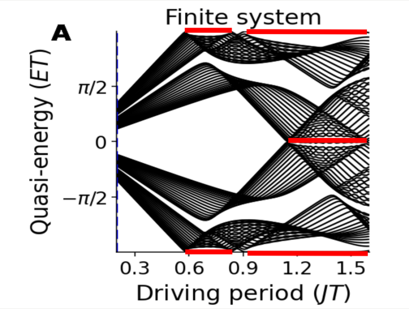
$$H(t) = \left(\frac{k^2}{2m} - \mu(t)\right)\eta_z + \lambda k \sigma_z + B\eta_z\sigma_x + \Delta\eta_y\sigma_y$$

$$\mu(t) = \begin{cases} \mu_1 & nT < t < (n+\frac{1}{2})T \\ \mu_2 & (n+\frac{1}{2})T < t < (n+1)T \end{cases} \qquad n \in \mathbb{Z}$$

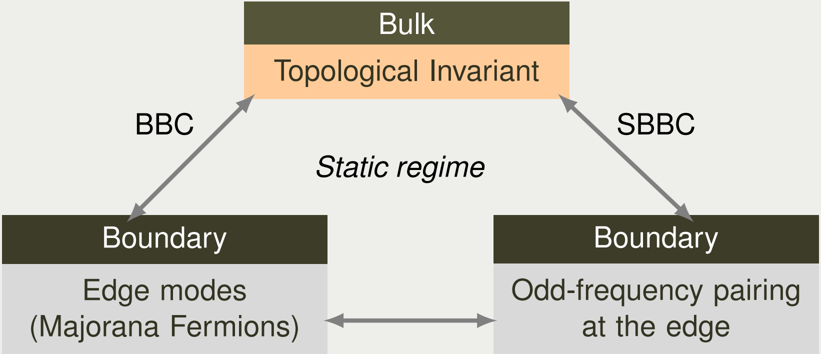
Non-zero energy Majorana edge modes are possible due to periodic driving.

$$\gamma^\dagger(E) = \gamma(-E+2n\pi) \Longrightarrow E = 0, \pi$$

[PhysRevLett.**106**.220402]



Aim


Bulk
Topological Invariant
BBC
SBBC
Static regime
Boundary
Edge modes
(Majorana Fermions)
Boundary
Odd-frequency pairing
at the edge

Aim

1. Does this relationship hold in the time-periodic regime?
2. How do Majorana π modes affect this relationship?

Boundary		Boundary
Edge modes (Majorana Fermions)	$\longleftrightarrow$	Odd-frequency pairing at the edge

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Model

Step function model:
For the sake of simplicity, we consider a step function-like driving protocol (i.e. binary drive). In this driving protocol, the hamiltonian is given by:

$$H(\tau) = \begin{cases} H_1 & nT < \tau < (n+\frac{1}{2})T \\ H_2 & (n+\frac{1}{2})T < \tau < (n+1)T \end{cases}$$

i.e. the hamiltonian alternates between two different hamiltonians as a function of time

Floquet Theorem

$$H(t+T) = H(t)$$

One-period propagator:

$$U(T) = \mathcal{T} e^{-i\int_0^T H(t')\mathrm{d}t'} \equiv e^{-iH_f T}$$

Let $|u_j\rangle$ be an eigenstate of $U(T)$ with eigenvalue $e^{-i\epsilon_j T}$
The propagator at any time t can be written as

$$U(t) = U(t)\exp\{iH_f t\}\exp\{-iH_f t\} = P(t)\exp\{-iH_f t\}$$

The following state satisfies Schrodinger equation

$$|\psi_j(t)\rangle = U(t)|u_j\rangle = e^{-i\epsilon_j(t-t_0)}|u_j(t)\rangle$$

$$\text{with} \quad |u_j(t)\rangle = P(t)|u_j\rangle, \qquad \epsilon_j \in \left\{-\frac{\Omega}{2}, \frac{\Omega}{2}\right\}$$

Effective hamiltonian and quasienergy spectrum

We consider the one-period propagator

$$U_T = \mathcal{T} \exp\left\{ -i \int_0^T H(\tau) \mathrm{d}\tau \right\}$$

The hamiltonian alternates between two different hamiltonians H_1 and H_2. Thus, we can write U_T as

$$U_T = e^{-iH_2 T/2} e^{-iH_1 T/2}$$

The effective hamiltonian is given by

$$H_f = i \log U_T / T$$

The quasienergies are the eigenvalues of the operator H_f

Green's function

We define the Green's function as

$$\mathcal{G}_f^{R(A)}(\omega) = (\omega \pm i\delta - H_f)^{-1} = \begin{bmatrix} G_f^{R(A)}(\omega) & F_f^{R(A)}(\omega) \\ -F_f^{R(A)\dagger}(\omega) & -G_f^{R(A)\dagger}(\omega) \end{bmatrix}$$

The even and odd frequency pairing amplitudes are given by

$$F_\pm(\omega) = \frac{1}{2}\Big(F_f^R(\omega) \pm F_f^A(-\omega)\Big)$$

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Alternating chemical potential

Alternating Kitaev chain:
Consider Kitaev chain with time-periodic chemical potential.

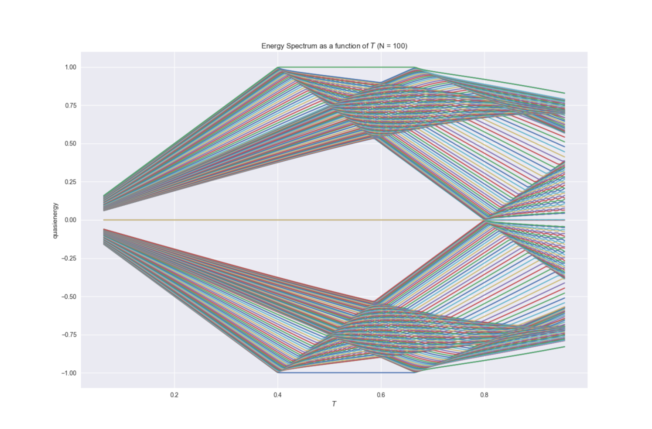
$$H(\tau) = \sum_i \psi_i^\dagger(-\mu(\tau)\tau_z)\psi_i + \psi_i^\dagger(-t\tau_z + i\Delta\tau_y)\psi_{i+1} + h.c.$$

with

$$\mu(\tau) = \begin{cases} \mu_1 & nT < \tau < (n+\frac{1}{2})T \\ \mu_2 & (n+\frac{1}{2})T < \tau < (n+1)T \end{cases} n \in \mathbb{Z}$$

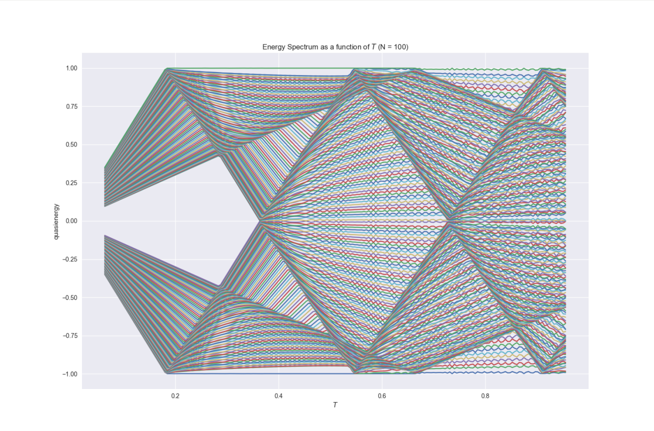
energy vs period - case1: $\mu(\tau) < 2t$

- Majorana zero modes persist for all values of T
- Majorana π modes can be engineered by manipulating the period T.



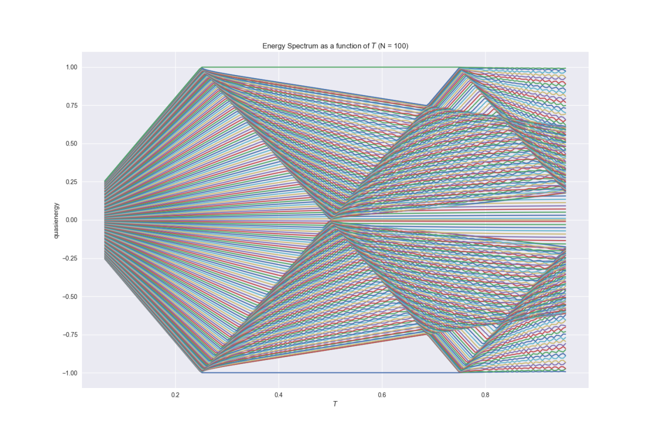
energy vs period - case2: $\mu(\tau) > 2t$

- Majorana zero modes persist for all values of T
- Majorana π modes can be engineered by manipulating the period T.



energy vs period - case3: $\mu_1 < 2t, \mu_2 > 2t$

- Majorana zero modes persist for all values of T
- Majorana π modes can be engineered by manipulating the period T.



Effective hamiltonian at first order

H_f can be obtained using Baker-Campbell-Hausdorff (BCH) formula. Up to second order, we have

$$H_f = \frac{(H_1 + H_2)}{2} - \frac{iT}{8}[H_2, H_1] + \dots$$

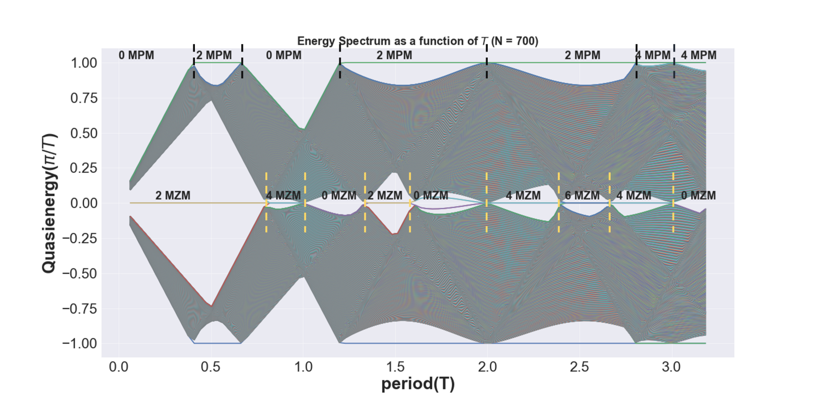
In momentum space, we have:

$$h_f(k) = \Big(-\frac{\mu_1 + \mu_2}{2} - 2w\cos k \Big)\tau_z + 2\Delta \sin k\tau_y + \frac{T\Delta(\mu_2 - \mu_1)}{4}\sin k\tau_x + \dots$$

This will produce long range pairing and hopping at higher orders of T

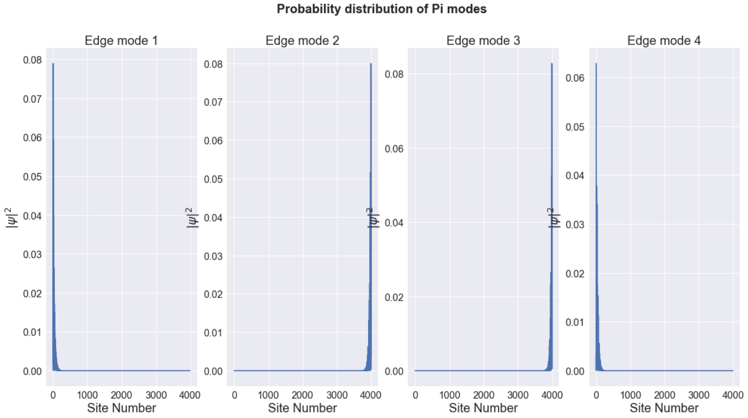
Quasienergies vs period

The following graph shows the quasienergies (in units of π/T) as a function of period for $\mu_1 = 0$, $\mu_2 = t$, $\Delta = t$, $N_{sites} = 700$.



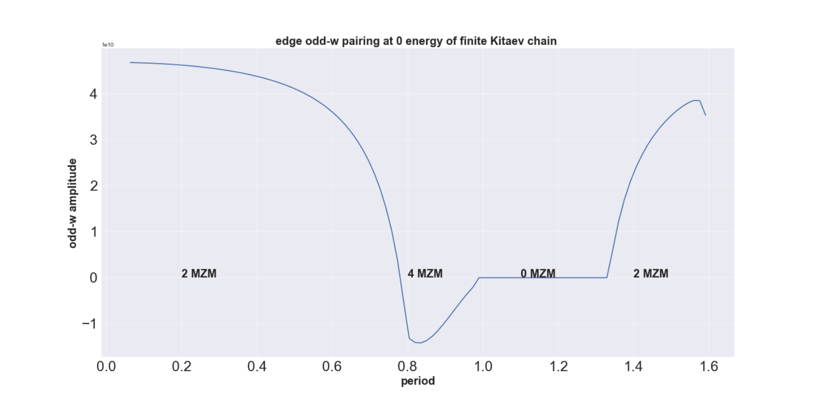
Multi-Majorana edge states

The number of both Majorana π and zero states at the edge can be tuned by changing the period T. At $T = 3.1$, we observe 4 Majorana π-modes localized at the edge.

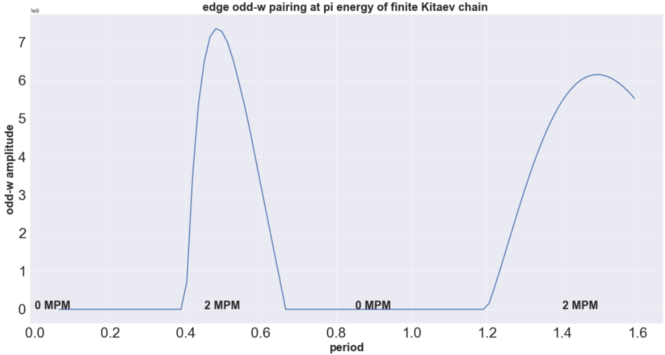


Odd anomalous Green's function near $\omega = 0$

The odd anomalous Green's function was plotted as a function of period for a system with 600 lattice sites.



Odd anomalous Green's function near $\omega = \pi$



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Static Kitaev chain at the special limit

Static Kitaev chain has two special limits in the parameter space of μ, t, Δ

- Case1: Topological sweet spot ($\mu = 0, t = \Delta$)

$$H = i\Delta \sum_{n=1}^{N-1} \gamma_{2n}\gamma_{2n+1}$$

- Case2: Trivial sweet spot ($\mu \neq 0, t = \Delta = 0$)

$$H = \frac{i\mu}{2} \sum_{n=1}^{N} \gamma_{2n-1}\gamma_{2n}$$

Floquet Kitaev chain at the special limit

consider the following Kitaev chain:

$$H(\tau) = \begin{cases} H_1 = i\Delta \sum_{n=1}^{N-1} \gamma_{2n}\gamma_{2n+1} & nT < \tau < (n+\frac{1}{2})T \\ \\ H_2 = \frac{i\mu}{2} \sum_{n=1}^{N} \gamma_{2n-1}\gamma_{2n} & (n+\frac{1}{2})T < \tau < (n+1)T \end{cases}$$

It follows that the one-period propagator is given by

$$\begin{aligned} U_T &= \prod_{n=1}^{N-1} \exp\left\{\frac{\Delta T}{2}\gamma_{2n}\gamma_{2n+1}\right\} \prod_{n=1}^{N} \exp\left\{\frac{\mu T}{4}\gamma_{2n-1}\gamma_{2n}\right\} \\ &= \prod_{n=1}^{N}\left[\cos\left(\frac{\Delta T}{2}\right) + \sin\left(\frac{\Delta T}{2}\right)\gamma_{2n}\gamma_{2n+1}\right] \\ &\quad \prod_{n=1}^{N}\left[\cos\left(\frac{\mu T}{4}\right) + \sin\left(\frac{\mu T}{4}\right)\gamma_{2n-1}\gamma_{2n}\right] \end{aligned}$$

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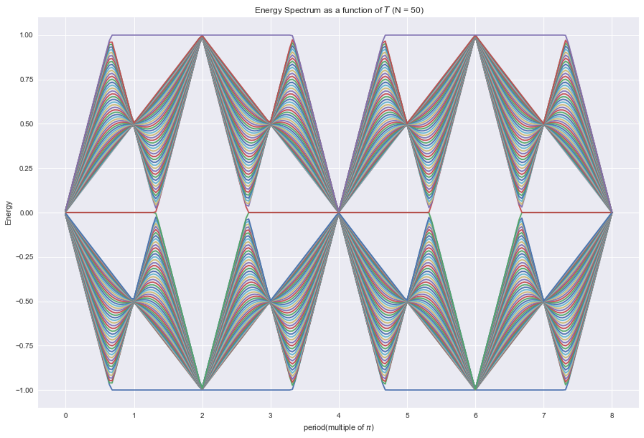
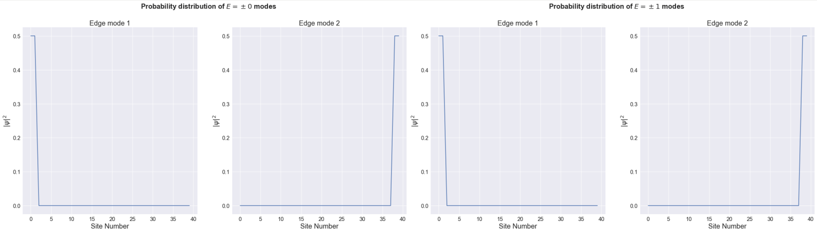
Quasienergies vs period

Quasienergies (in units of π/T) of Floquet Kitaev chain as a function of period for $\mu = \Delta = 1$, $N_{sites} = 20$.



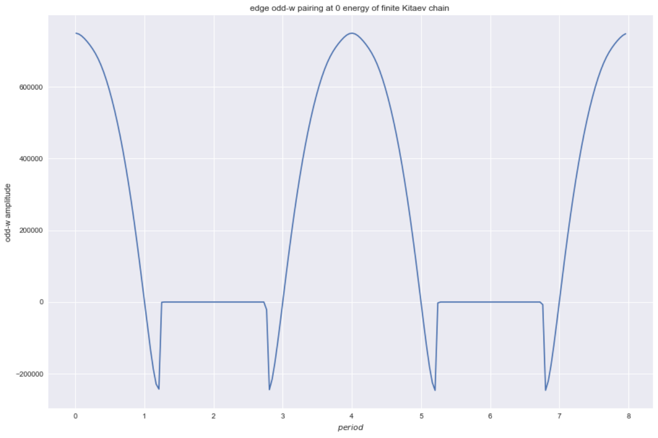
Sharply localized Majorana edge states

At the special limit $T = \pi$, we observe sharply localized edge modes at the edge.



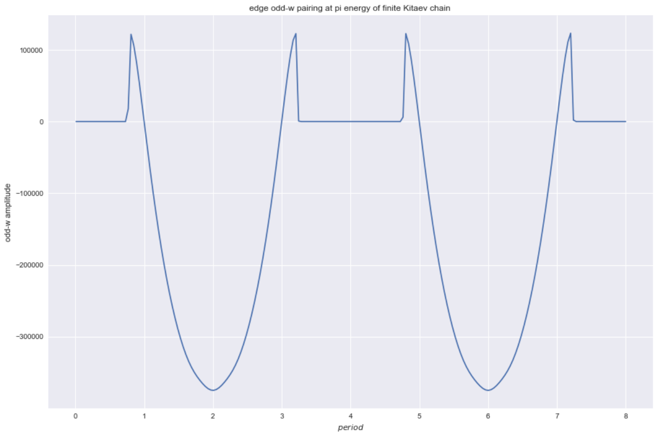
Odd anomalous Green's function near $\omega = 0$

Odd-frequency pairing has a peculiar sign change at $T = \pi \mod 2\pi$

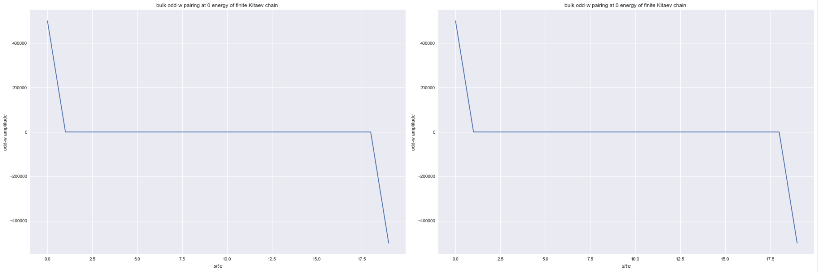


Odd anomalous Green's function near $\omega = \pi$

The same peculiar sign change happens at $\omega = \pi$



Sharply localized odd-ω pairing at the special limit $T = \pi$



Analytical analysis at the special limit: $\Delta T = \mu T = \pi$

Evolution of the system after one period has passed:

$\gamma_{2n-1} \longrightarrow U_T^\dagger \gamma_{2n-1} U_T = \gamma_{2n}$	$\gamma_{2n} \longrightarrow U_T^\dagger \gamma_{2n} U_T = -\gamma_{2n-1}$
$\gamma_{2N-1} \longrightarrow U_T^\dagger \gamma_{2N-1} U_T = \gamma_{2N}$	$\gamma_{2N} \longrightarrow U_T^\dagger \gamma_{2N} U_T = \gamma_{2N-1}$
$\gamma_1 \longrightarrow U_T^\dagger \gamma_1 U_T = -\gamma_2$	$\gamma_2 \longrightarrow U_T^\dagger \gamma_2 U_T = -\gamma_1$

It follows that the eigenstates are:

- Bulk fermion modes at $E = \pm\frac{\pi}{2}$: $\Psi^n_{\pm\pi/2} = \gamma_{2n-1} \pm i\gamma_{2n}$
- Edge Majorana Zero Modes (EMZM):
  - EMZM at the left edge: $\Gamma_0^L = \gamma_1 - \gamma_2$
  - EMZM at the left edge: $\Gamma_0^R = \gamma_{2N-1} + \gamma_{2N}$
- Edge Majorana π Modes (EMPM):
  - EMPM at the left edge: $\Gamma_\pi^L = \gamma_1 + \gamma_2$
  - EMPM at the left edge: $\Gamma_\pi^R = \gamma_{2N-1} - \gamma_{2N}$

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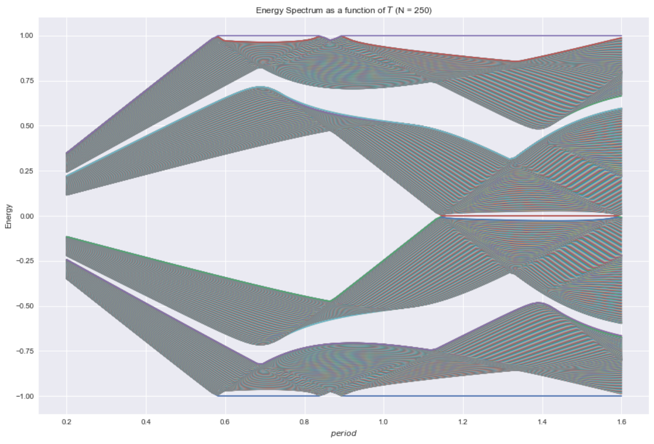
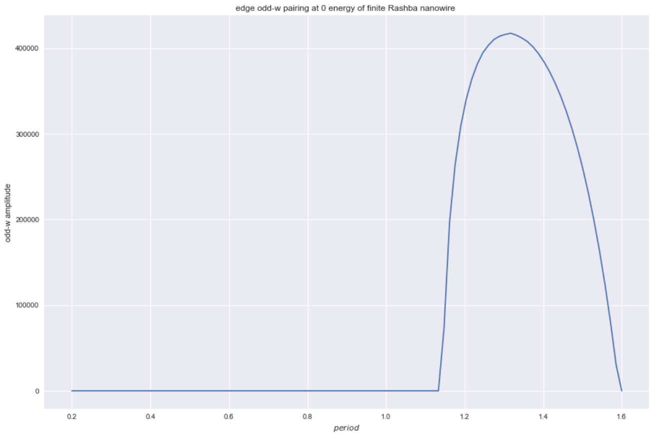
Quasienergies vs period

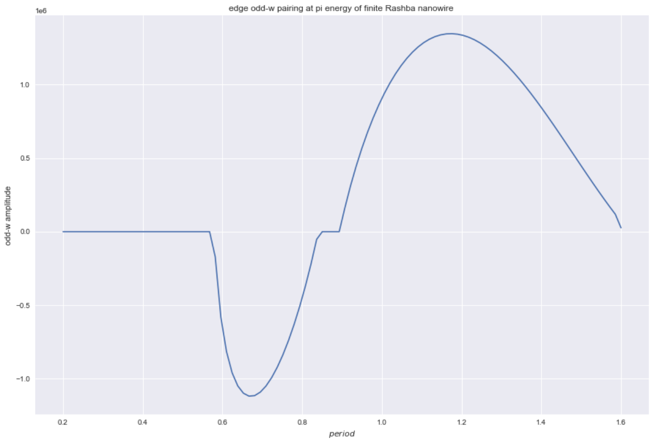
We study the floquet rashba nanowire model proposed by Kitagawa et. al.



odd-ω pairing amplitude near $\omega = 0$ at the edge of rashba nanowire



odd-ω pairing amplitude near $\omega = 0$ at the edge of rashba nanowire



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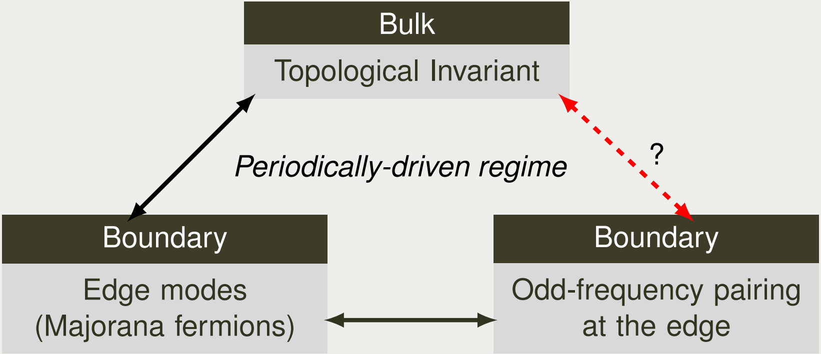
Conclusion

1. Odd-Frequency Pairing amplitude crosses zero whenever we have a gap-closing/gap-opening point in the spectrum.
2. Odd-Frequency Pairing near $\omega = 0(\pi/T)$ is non-zero only when Majorana zero (π) modes are present.
3. Odd-Frequency pairing at the edge is related with the presence of Majorana fermions in periodically-driven topological superconductors.

Future Goals



Possible extensions

1. Effect of strong interaction and random impurities

   .

2. Extension to more realistic models (smoother driving protocols, spin degrees of freedom, etc)

   .

3. Extension to quasiperiodic systems (The basic theory was proposed by Halprin in 2017)