وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 05
MOSFET Small Signal Model

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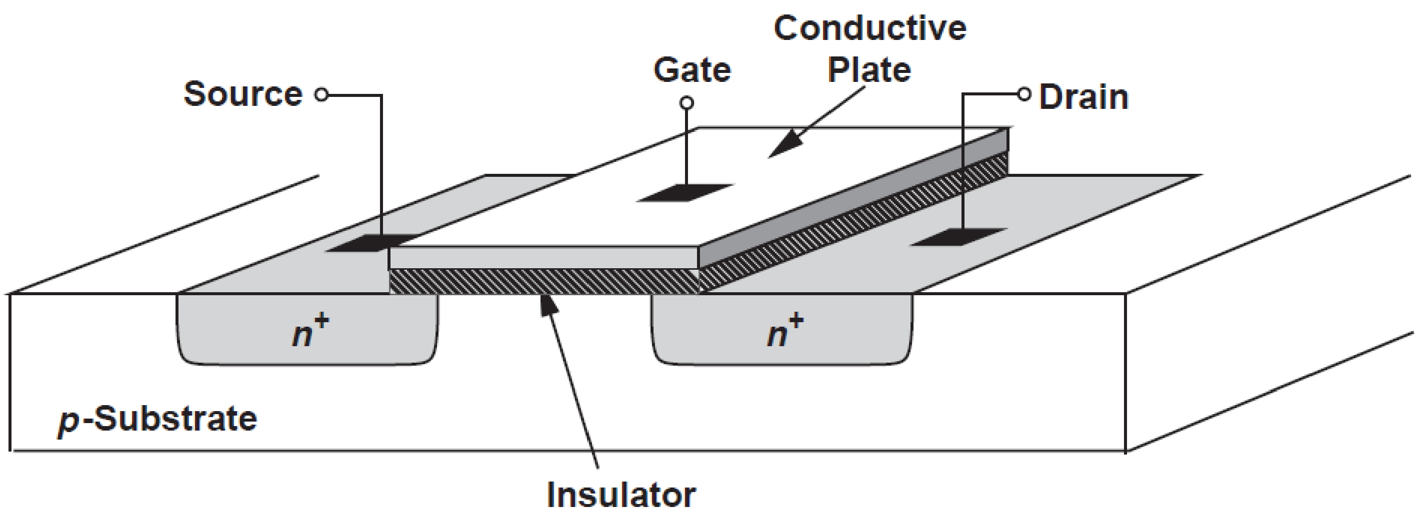
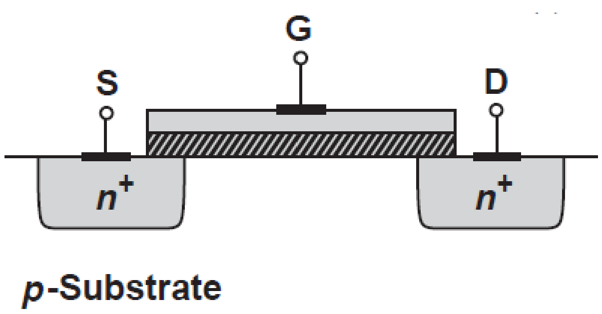
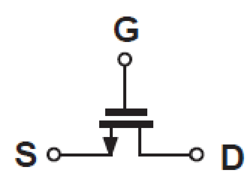
N-Channel MOSFET Structure

- MOSFET: Metal-oxide-semiconductor field-effect transistor
- Three-terminal device: Gate (G), Source (S), and Drain (D)
- Substrate/Bulk/Body (S/B) can be treated as a fourth terminal







[Razavi, 2014]

Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_T$

ON

$V_{GS} > V_T$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_T$

$$I_D = \mu C_{ox} \frac{W}{L} \left(V_{ov} V_{DS} - \frac{V_{DS}^2}{2} \right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

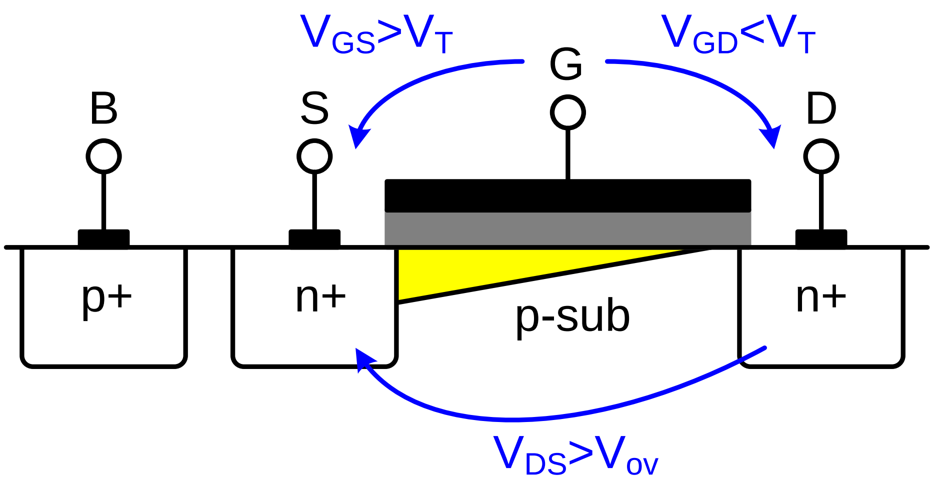
$V_{GD} \leq V_T$

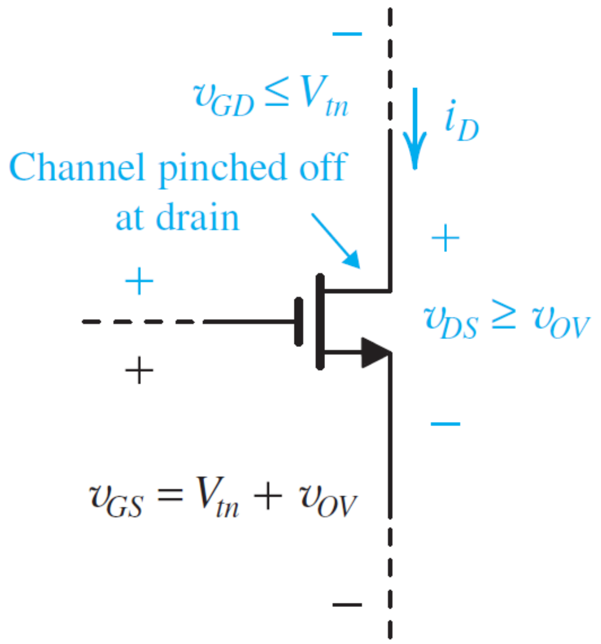
$$I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} V_{ov}^2 (1 + \lambda V_{DS})$$

MOSFET in Saturation

- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L} \cdot V_{ov}^2(1 + \lambda V_{DS})$$



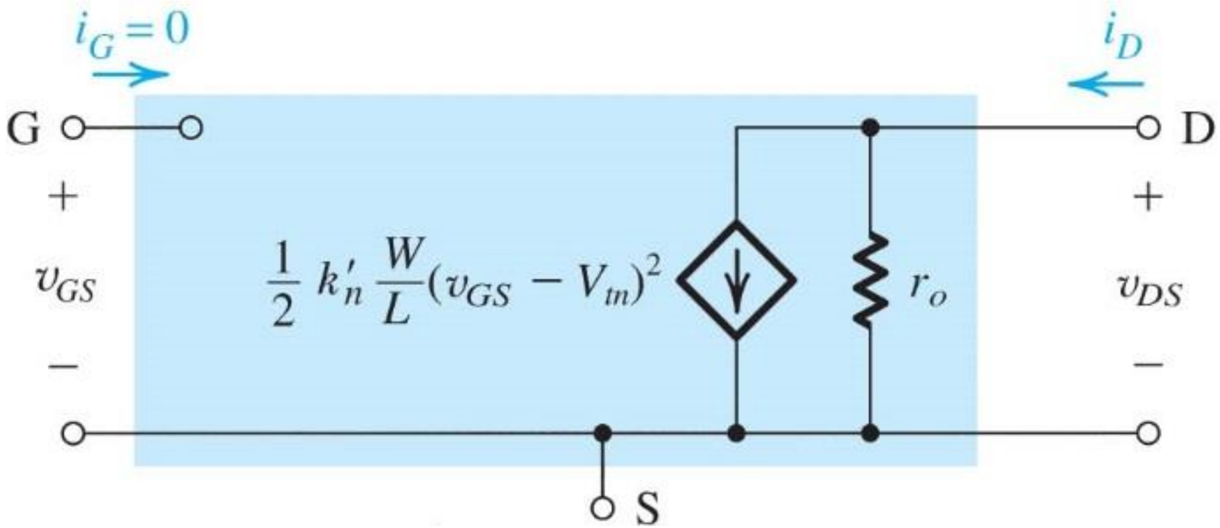


 [Sedra/Smith, 2015]

Large Signal Model

- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2 (1 + \lambda V_{DS})$$

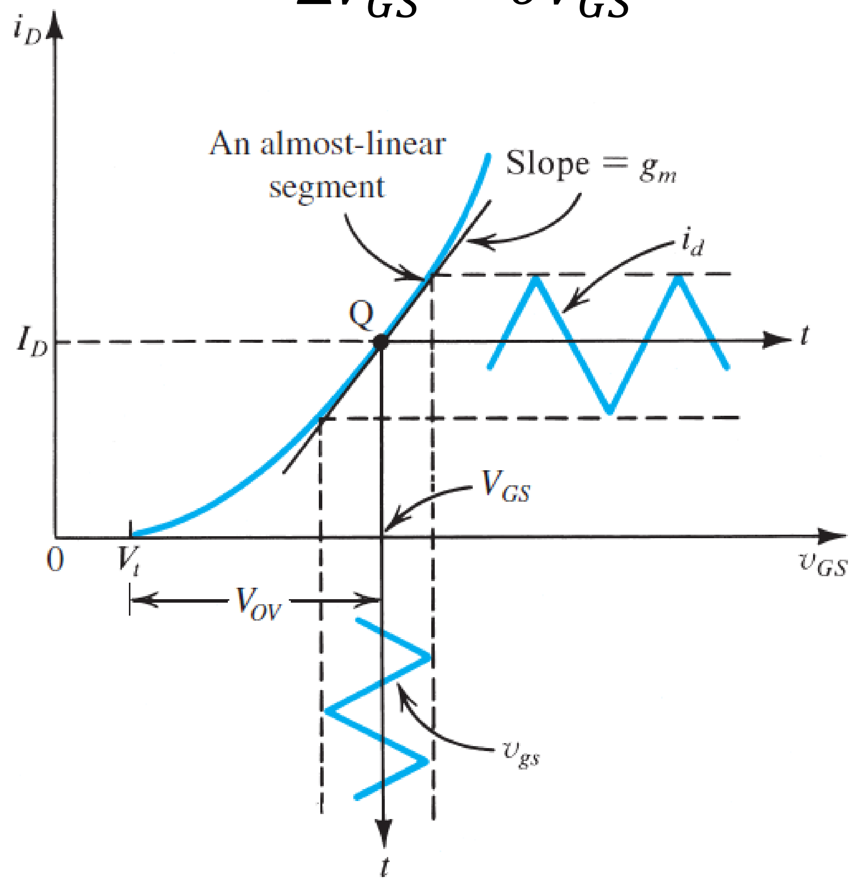


[Sedra/Smith, 2015]

Small Signal Approximation

- The transistor is a VCCS
- Transconductance: how well it converts the voltage to a current

$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{\partial I_D}{\partial V_{GS}}$$

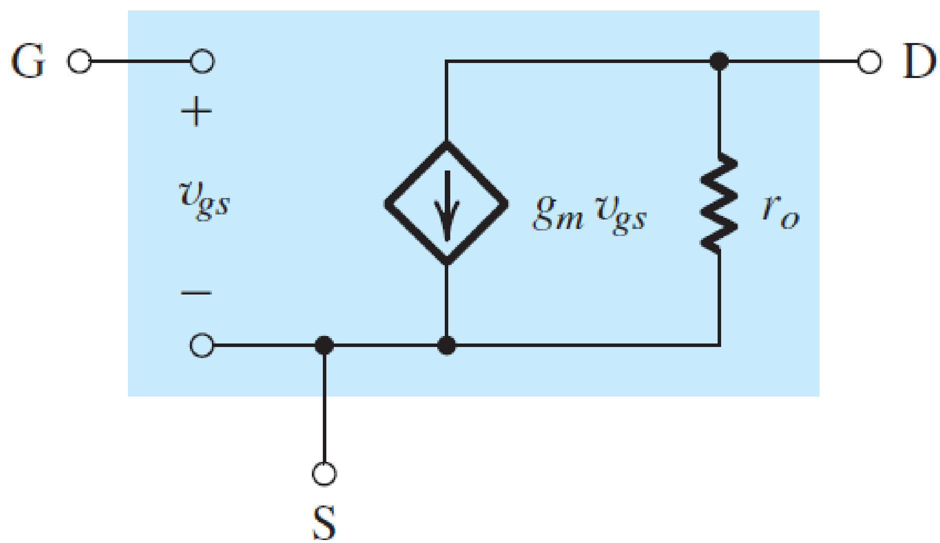


[Sedra/Smith, 2015]

Small Signal Model

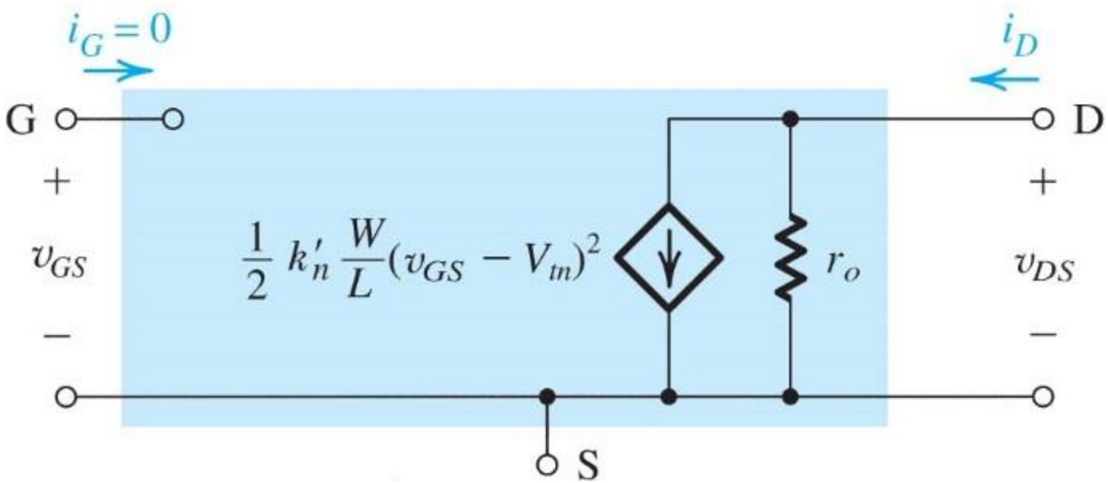
$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{\partial I_D}{\partial V_{GS}}$$

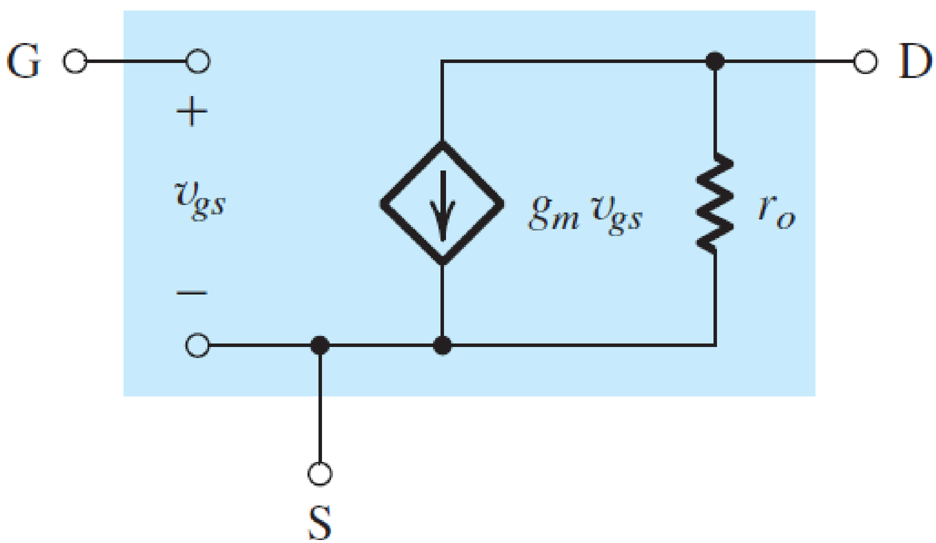
$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}}$$



[Sedra/Smith, 2015]

Large Signal vs Small Signal Model





[Sedra/Smith, 2015]

Transconductance

- The transistor is a VCCS
- Transconductance: how well it converts the voltage to a current

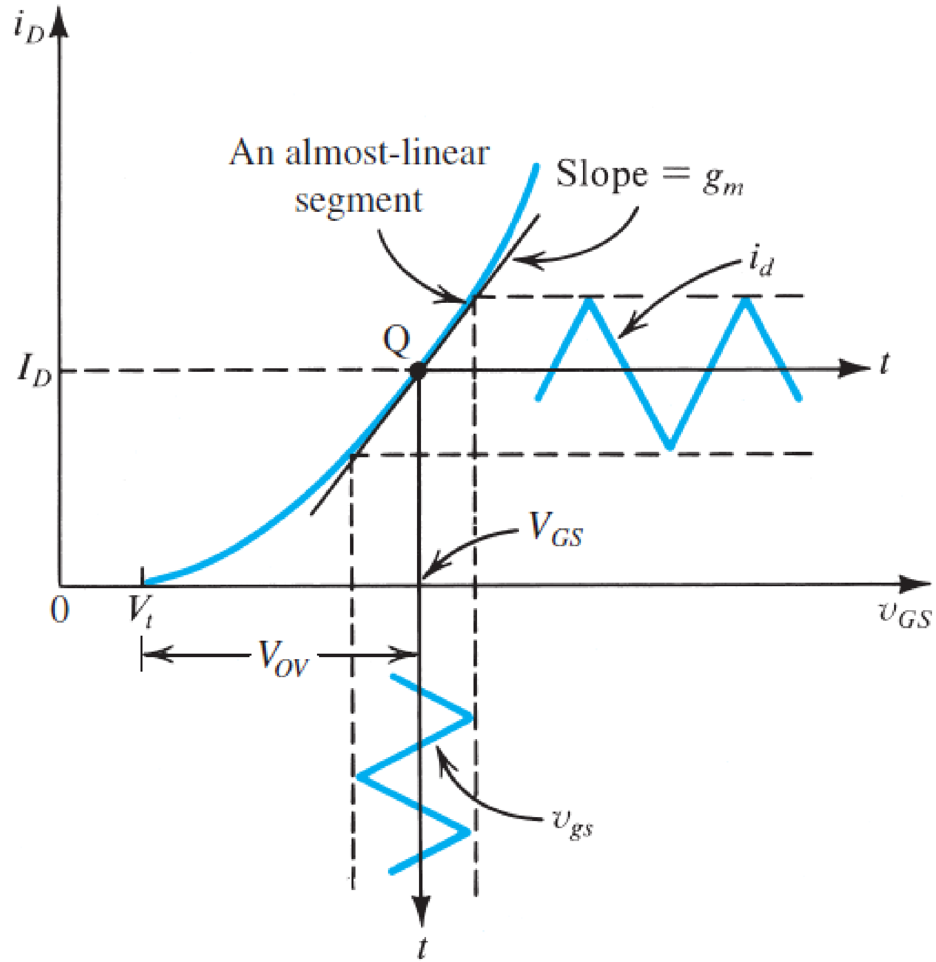
$$I_D \approx \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2$$

$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{\partial I_D}{\partial V_{GS}} = \frac{\partial I_D}{\partial V_{ov}}$$

$$= \mu C_{ox} \frac{W}{L} V_{ov}$$

$$= \sqrt{\mu C_{ox} \frac{W}{L} \cdot 2 I_D}$$

$$= \frac{2 I_D}{V_{ov}}$$



[Sedra/Smith, 2015]

Transconductance

$$I_D \approx \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} V_{ov} = \sqrt{\mu C_{ox} \frac{W}{L} \cdot 2 I_D} = \frac{2 I_D}{V_{ov}}$$

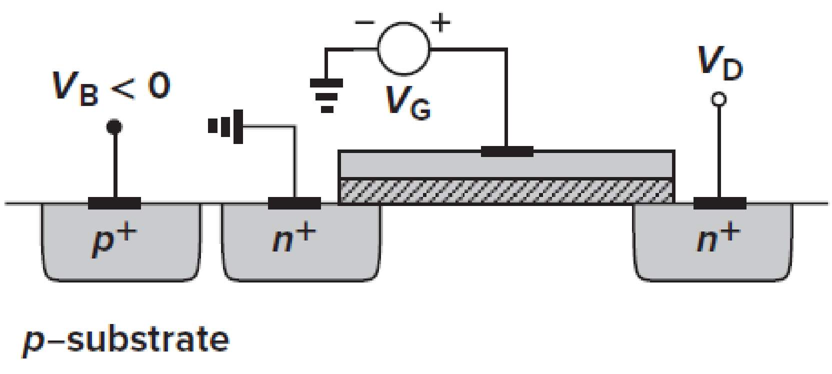
$\frac{W}{L}$ **Constant** $V_{GS}-V_{TH}$ **Variable**	$\frac{W}{L}$ **Variable** $V_{GS}-V_{TH}$ **Constant**	$\frac{W}{L}$ **Variable** $V_{GS}-V_{TH}$ **Constant**
$g_m \propto \sqrt{I_D}$	$g_m \propto I_D$	$g_m \propto \sqrt{\frac{W}{L}}$
$g_m \propto V_{GS}-V_{TH}$	$g_m \propto \frac{W}{L}$	$g_m \propto \frac{1}{V_{GS}-V_{TH}}$

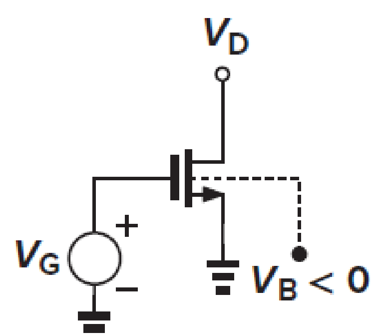
Body Effect

- V_{SB} affects the charge required to invert the channel
  - Increasing V_S or decreasing V_B increases V_{TH}

$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{|2\Phi_F|} \right)$$

  - ϕ_F = surface potential at threshold
    - Depends on doping level and intrinsic carrier concentration n_i
  - γ = *body effect coefficient*
    - Depends on C_{ox} and doping





[Razavi, 2014]

Bulk Transconductance

- The bulk behaves as a second gate that changes the output current

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}}$$

$$g_{mb} = g_m \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}$$
$$= \eta g_m$$

- η is typically $0.1 \rightarrow 0.25$

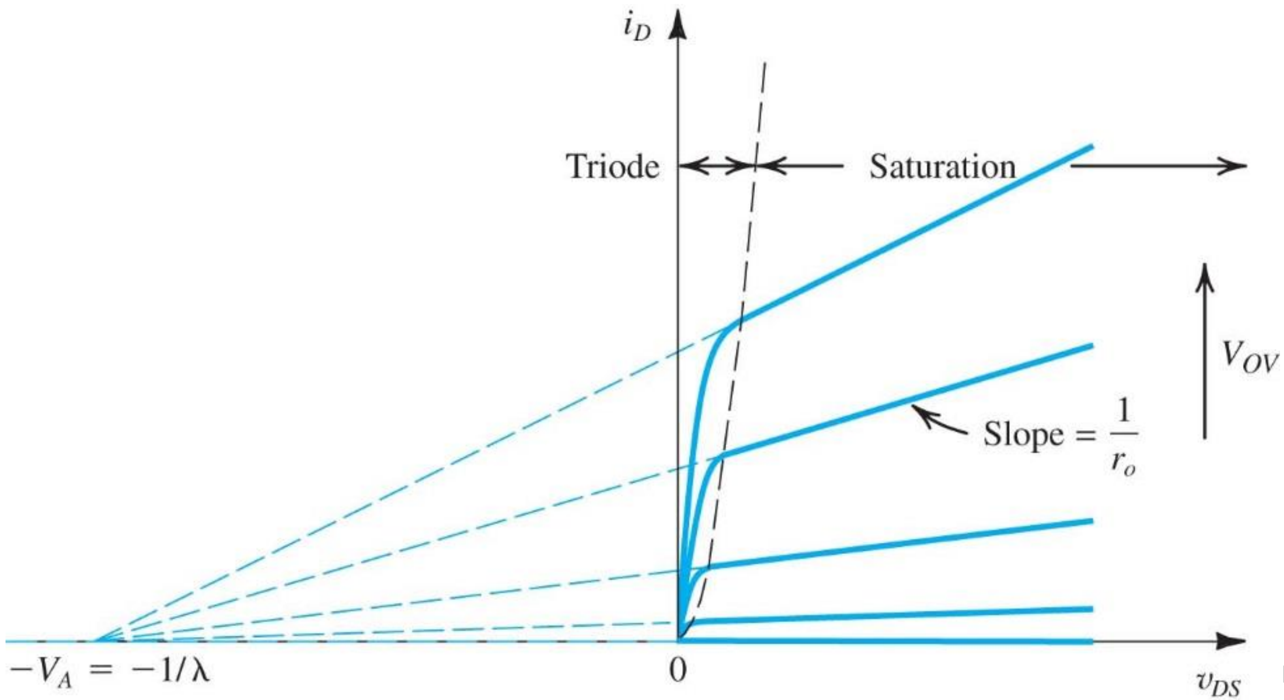
Output Resistance

- I_D increases with V_{DS} in saturation

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2 (1 + \lambda V_{DS})$$

- The VCCS is not ideal → it has output resistance

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}} = \frac{1}{\lambda I_D}, \qquad \boldsymbol{\lambda \propto \frac{1}{L}}$$



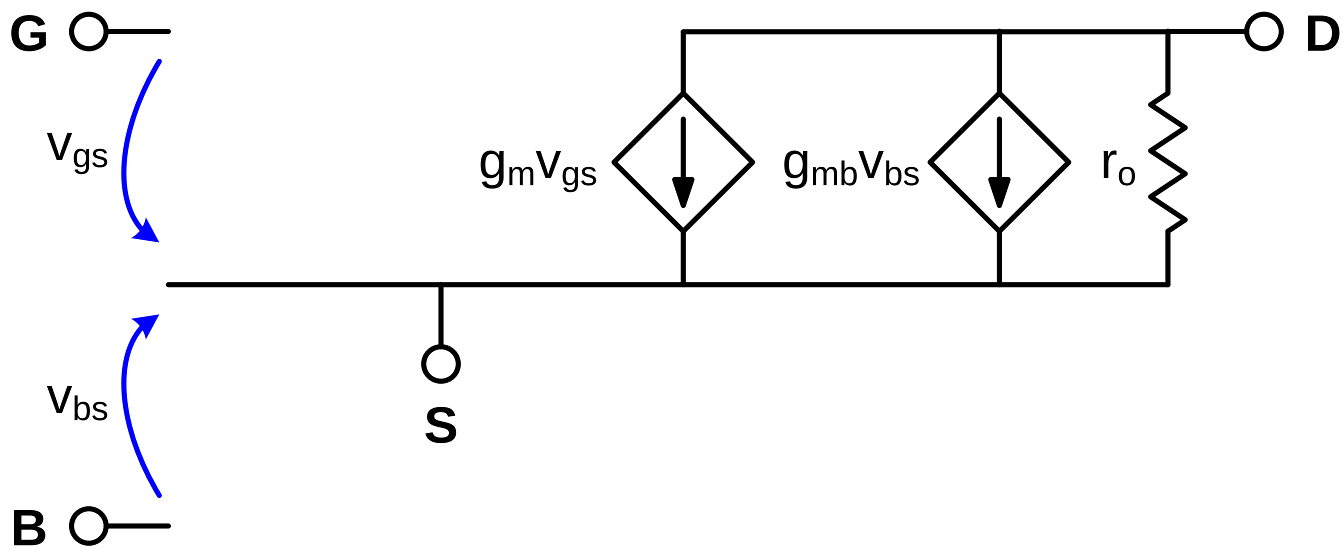
mith, 2015]

Low-Frequency Small-Signal Model

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} V_{ov} = \sqrt{\mu C_{ox} \frac{W}{L} \cdot 2 I_D} = \frac{2 I_D}{V_{ov}}$$

$$g_{mb} = \eta g_m, \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}} = \frac{1}{\lambda I_D}, \qquad \lambda \propto \frac{1}{L}$$



Intrinsic Gain

$$v_{out} = -(g_m v_{in}) r_o$$

$$|A_v| = \left|\frac{v_{out}}{v_{in}}\right| = g_m r_o$$

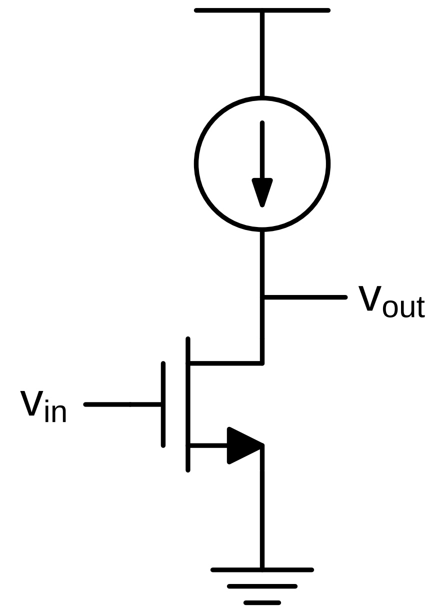
- $g_m r_o$ is the max gain that can be obtained from a single transistor
- Common approximations that we usually use

$$g_m r_o \gg 1$$

$$r_o \gg \frac{1}{g_m}$$

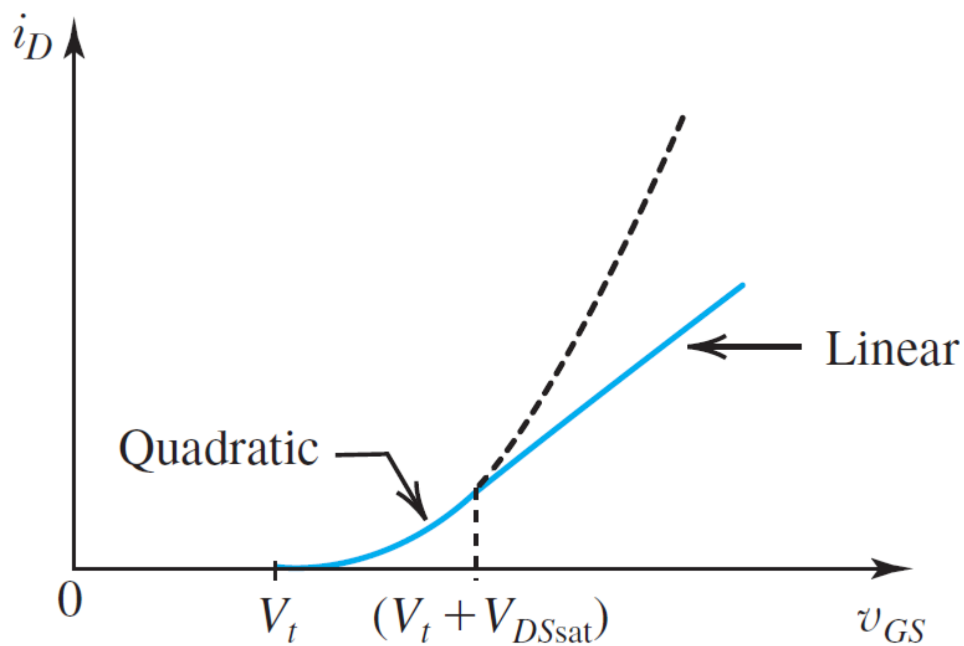
$$g_m + \frac{1}{r_o} \approx g_m$$

$$r_o // \frac{1}{g_m} \approx \frac{1}{g_m}$$



Short Channel Effects: Velocity Saturation

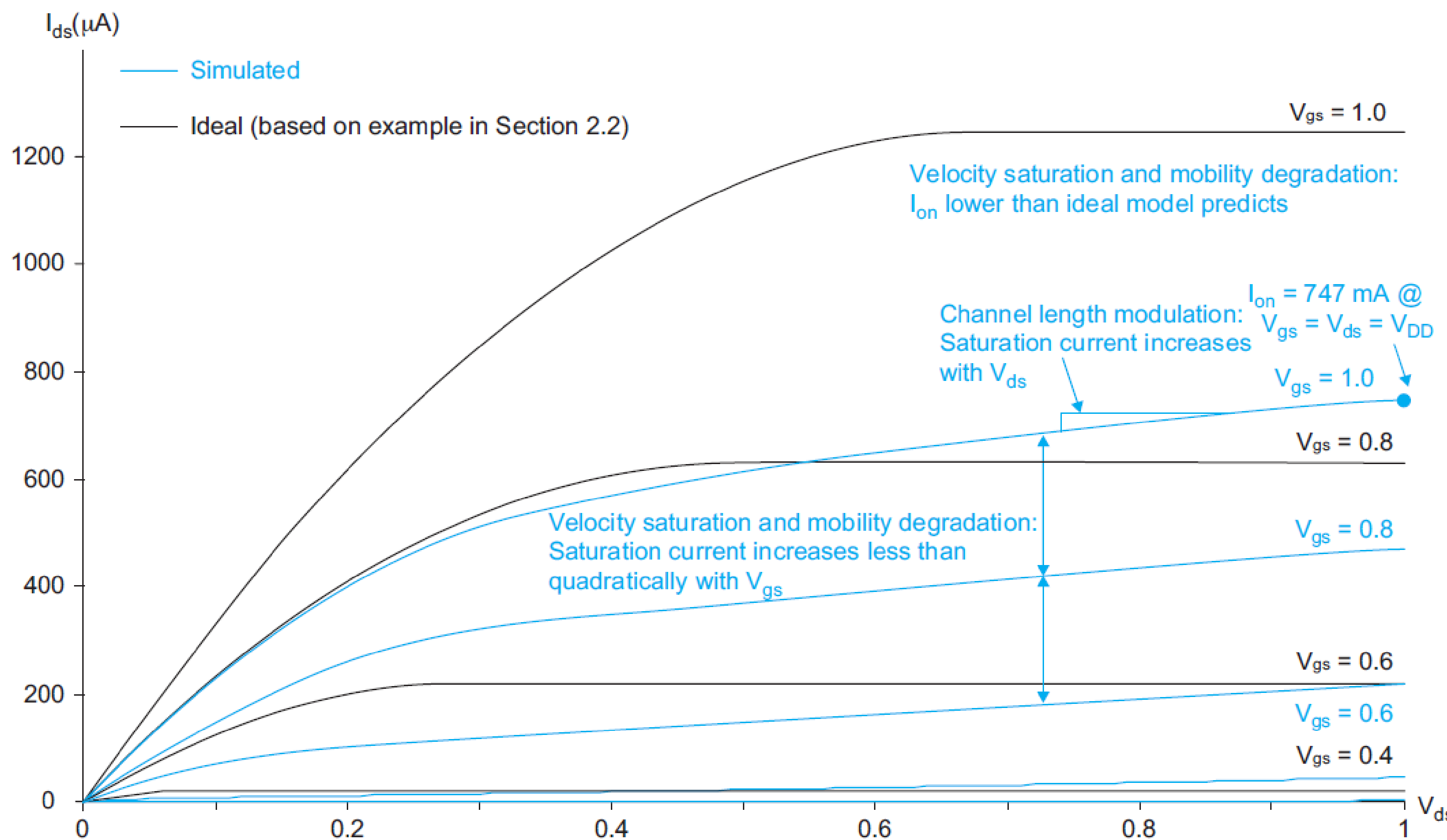
- ID-VGS quadratic: $g_m = \frac{\partial I_D}{\partial V_{GS}} =$ linear → g_m increases with V_{GS}
- ID-VGS linear: $g_m = \frac{\partial I_D}{\partial V_{GS}} =$ constant → g_m saturates



[Sedra/Smith, 2015]

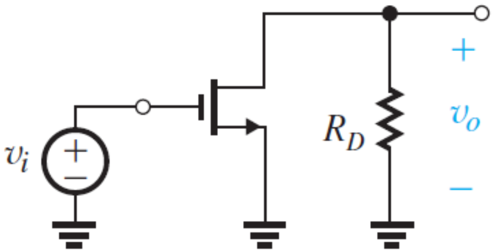
Short Channel Effects: CLM and DIBL

- ID-VDS horizontal: no V_{DS} dependence → ideal current source
- More slope → more V_{DS} dependence → smaller r_o → smaller gain

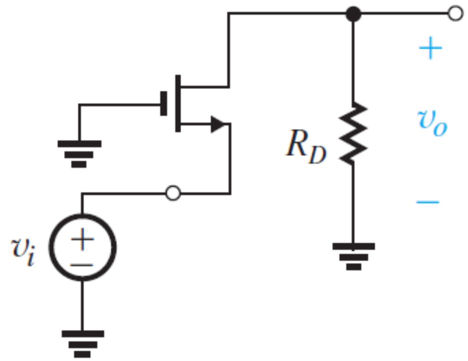


[Weste & Harris]

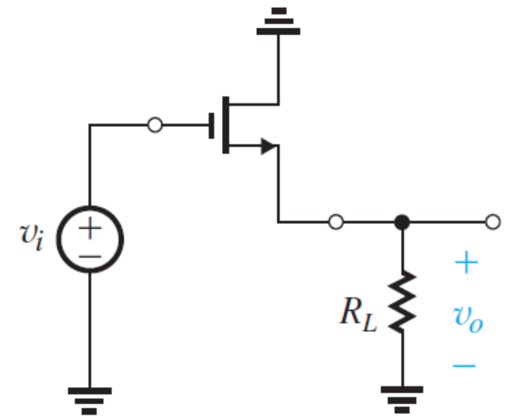
MOSFET Amplifier Configurations



(a) Common Source (CS)



(b) Common Gate (CG)



(c) Common Drain (CD)
or Source Follower

Topology	Input	Output
Common-Source	Gate	Drain
Common-Gate	Source	Drain
Common-Drain (Source-Follower)	Gate	Source

[Sedra/Smith, 2015]

MOSFET Amplifier Analysis Steps

1. DC analysis
   - Coupling and bypass capacitors → open-circuit (o.c)
   - Calculate Q-point and check operation in saturation $(V_{DS} > V_{ov})$
2. Calculate small signal parameters (g_m, r_o)
3. Draw the small signal equivalent circuit
   - DC voltage source → short-circuit (s.c.)
   - DC current source → open-circuit (o.c.)
   - Coupling and bypass capacitors → short-circuit (s.c)
4. Determine the amplifier parameters
   - Input resistance and output resistance
   - Voltage gain and current gain

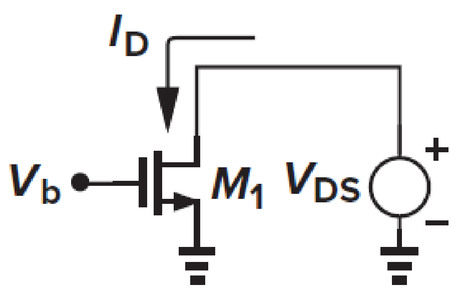
Thank you!

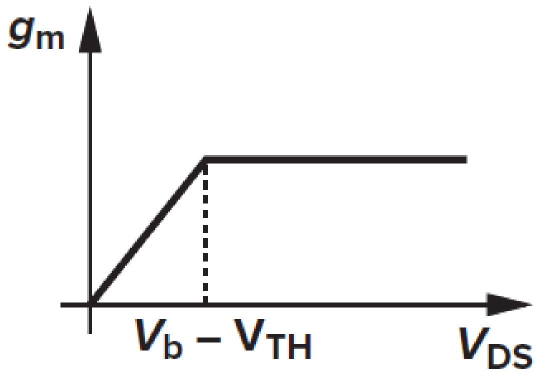
Transconductance

- The transconductance drops in the triode region
  - Thus, for amplifiers, we usually employ MOSFETs in saturation

$$g_m = \frac{\partial}{\partial V_{GS}} \left\{ \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \right] \right\}$$

$$= \mu_n C_{ox} \frac{W}{L} V_{DS}$$





[Razavi, 2017]