وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 12
The Five-Transistor (5T) OTA

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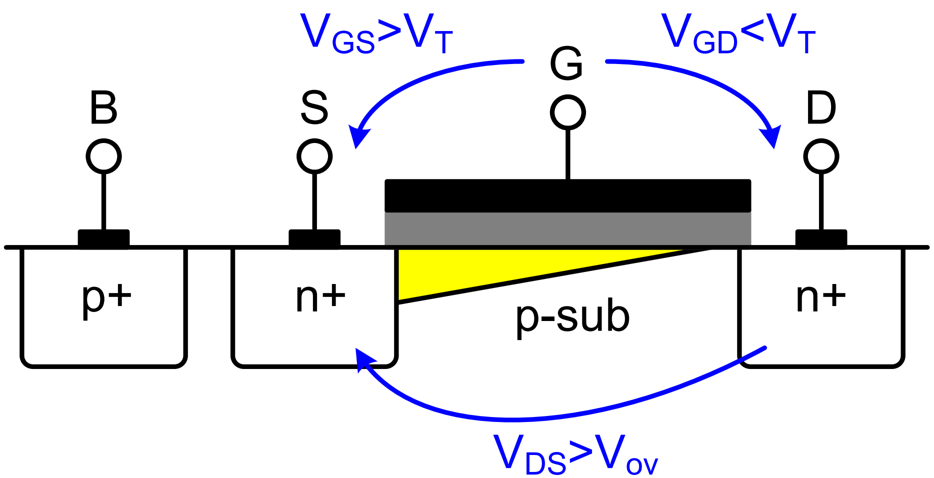
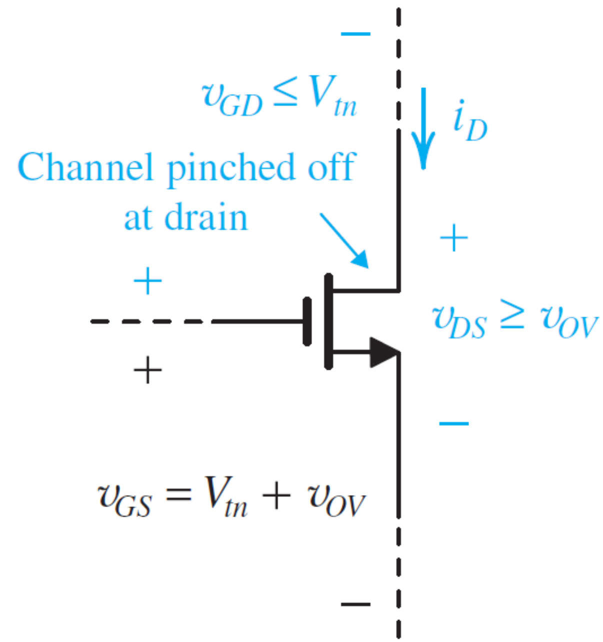
MOSFET in Saturation

- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2 (1 + \lambda V_{DS})$$





[Sedra/Smith, 2015]

Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_T$

ON

$V_{GS} > V_T$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_T$

$$I_D = \mu C_{ox} \frac{W}{L} \left(V_{ov} V_{DS} - \frac{V_{DS}^2}{2} \right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_T$

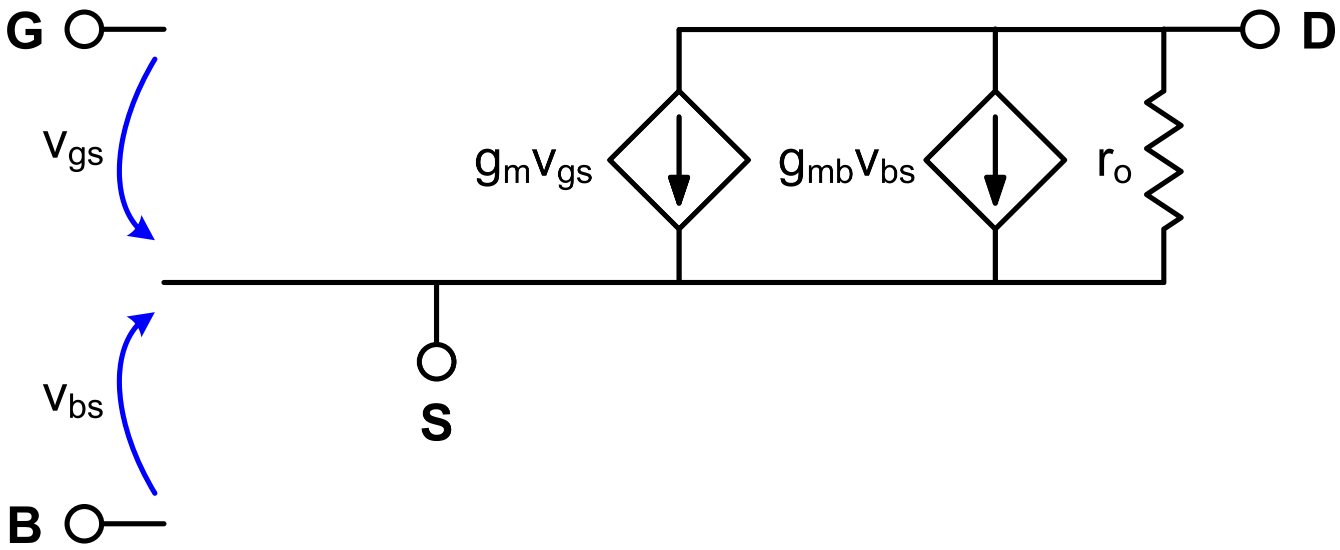
$$I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} V_{ov}^2 (1 + \lambda V_{DS})$$

Low-Frequency Small-Signal Model

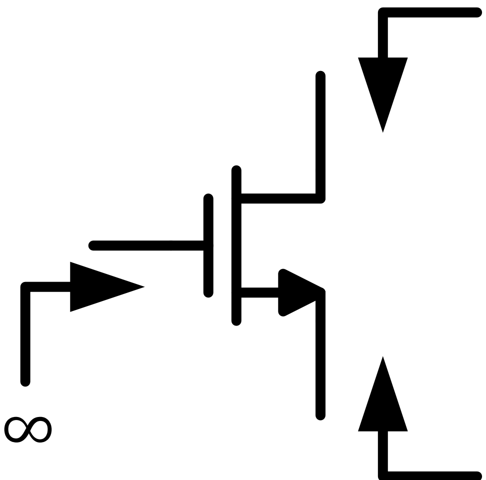
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox}\frac{W}{L}V_{ov} = \sqrt{\mu C_{ox}\frac{W}{L}\cdot 2I_D} = \frac{2I_D}{V_{ov}}$$

$$g_{mb} = \eta g_m, \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}} = \frac{1}{\lambda I_D}, \qquad \lambda \propto \frac{1}{L}$$



Rin/out Shortcuts Summary



$$r_o[1 + (g_m + g_{mb})R_S]$$

H.I.N.

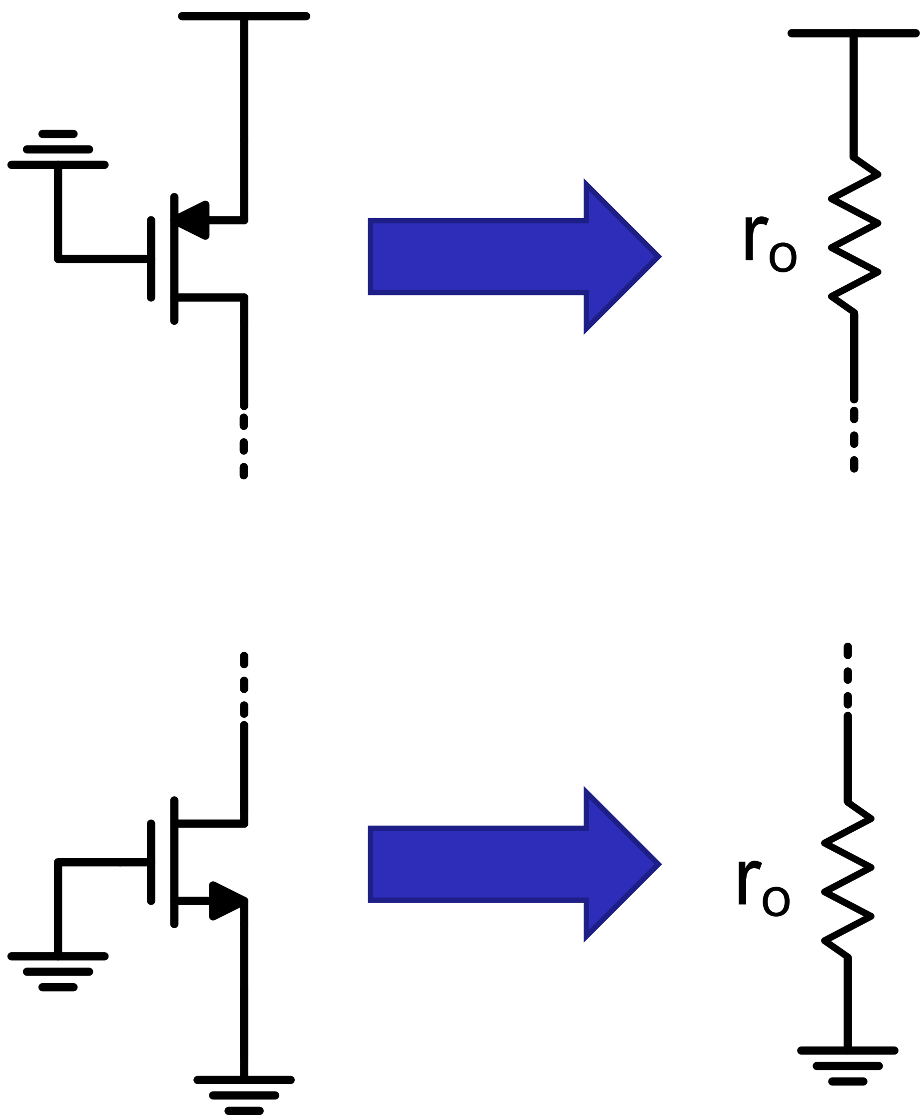
∞

At low frequencies **ONLY**

$$\frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

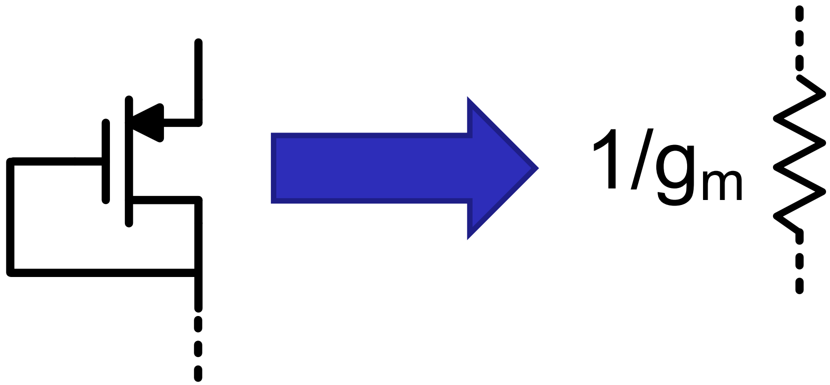
L.I.N.

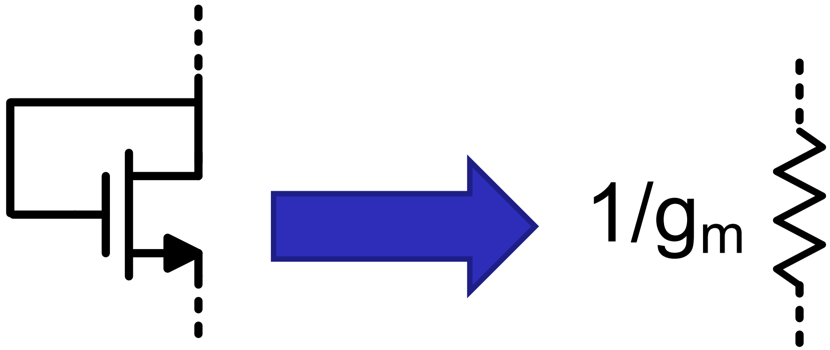
Active Load (Source OFF)



Diode Connected (Source Absorption)

- Always in saturation
- Bulk effect: $g_m \rightarrow g_m + g_{mb}$





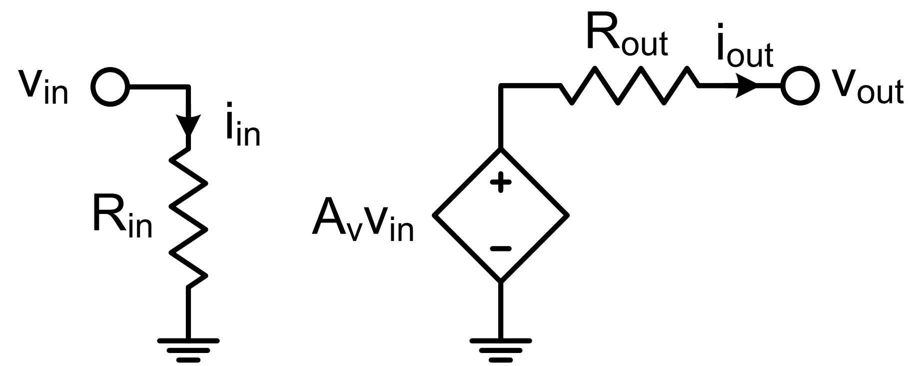
Why GmRout?

$$R_{out} = \frac{v_x}{i_x} \; @ \; v_{in} = 0$$

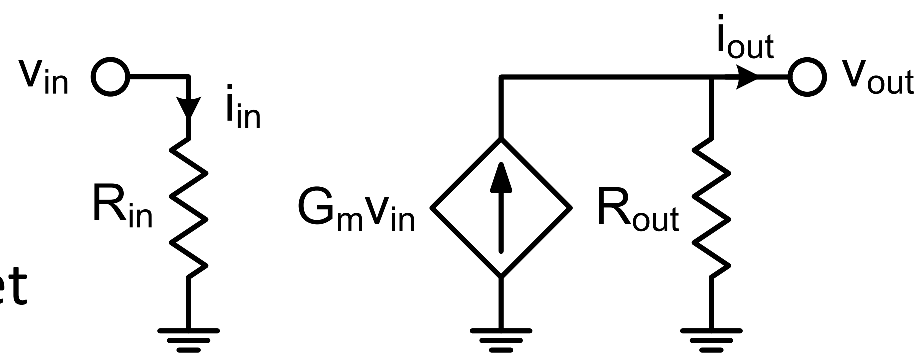
$$G_m = \frac{i_{out,sc}}{v_{in}}$$

$$A_v = G_m R_{out}$$

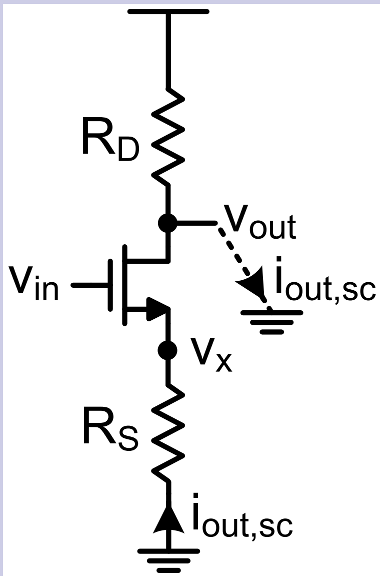
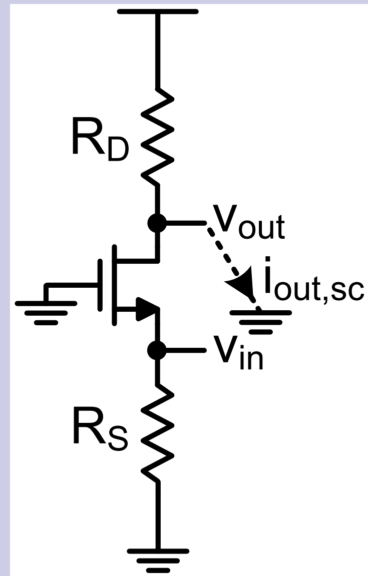
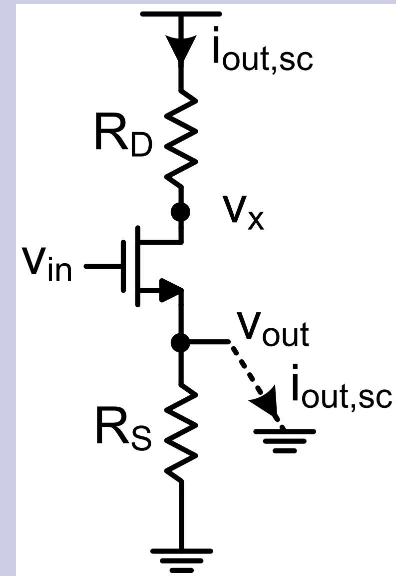
$$A_i = G_m R_{in}$$



- Divide and conquer
  - Rout simplified: vin=0
  - Gm simplified: vout=0
  - We already need Rin/out
  - We can quickly and easily get Rin/out from the shortcuts

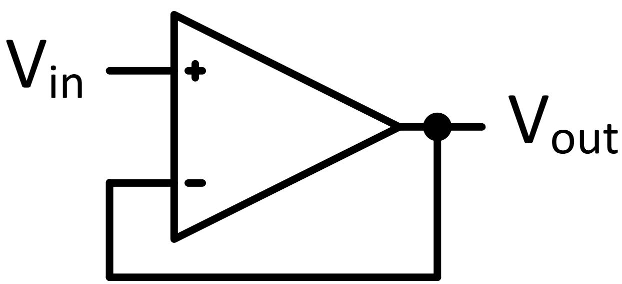


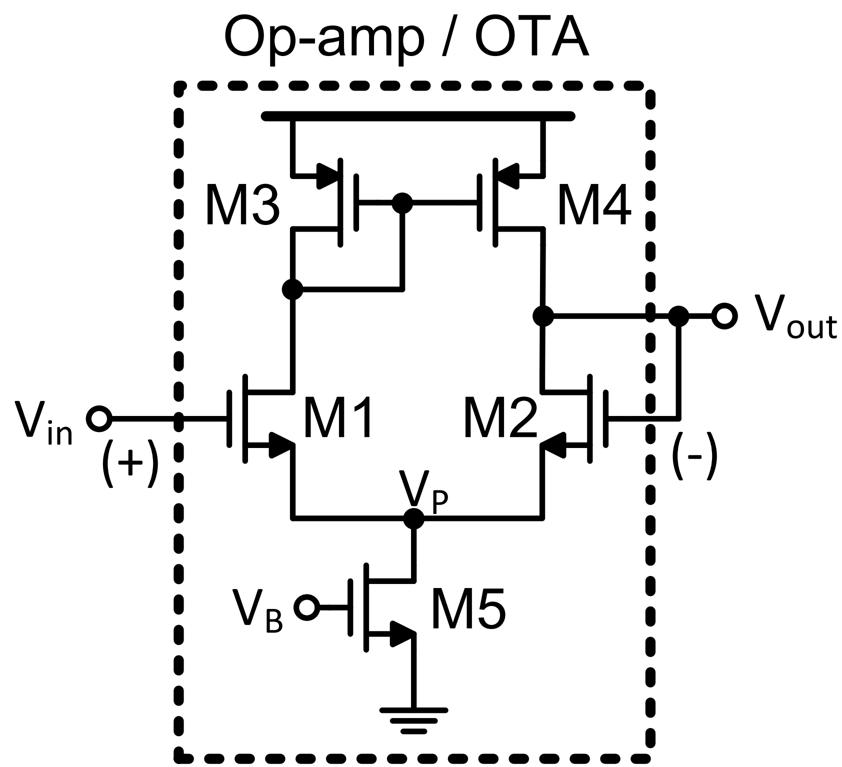
Summary of Basic Topologies

	CS	CG	CD (SF)
			
	Voltage & current amplifier	Current buffer	Voltage buffer
Rin	∞	$R_S // \dfrac{1}{g_m + g_{mb}}\left(1 + \dfrac{R_D}{r_o}\right)$	∞
Rout	$R_D // r_o[1 + (g_m + g_{mb})R_S]$	$R_D // r_o$	$R_S // \dfrac{1}{g_m + g_{mb}}\left(1 + \dfrac{R_D}{r_o}\right)$
Gm	$\dfrac{-g_m}{1 + (g_m + g_{mb})R_S}$	$g_m + g_{mb}$	$\dfrac{g_m}{1 + R_D/r_o}$

Have You Seen a Diff Amp Before?

- An op-amp is simply a high gain differential amplifier
- The gain can be increased by using cascodes and multi-stage amplifiers



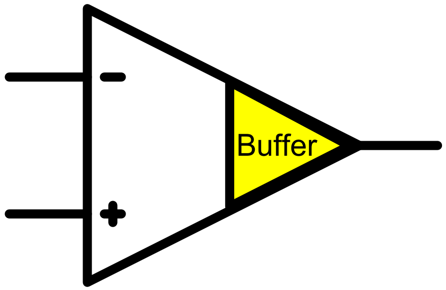


Differential Amplifier

	Pseudo Diff Amp	Diff Pair (w/ ideal CS)	Diff Pair (w/ R_{SS})
A_{vd}	$-g_m R_D$	$-g_m R_D$	$-g_m R_D$
A_{vCM}	$-g_m R_D$	0	$\frac{-g_m R_D}{1 + 2(g_m + g_{mb})R_{SS}}$
A_{vd}/A_{vCM}	1	∞	$2(g_m + g_{mb})R_{SS} \gg 1$

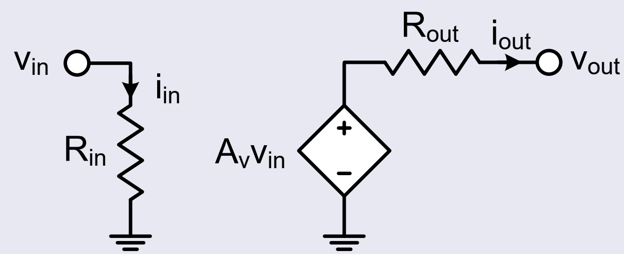
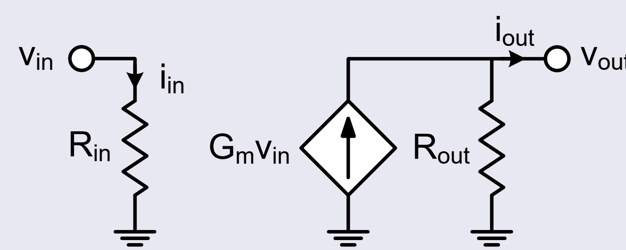
Op-Amp vs OTA

- Ideal op-amp has infinite R_{in}, infinite gain, and zero R_{out}
- Practical op-amp has HIGH R_{in}, HIGH gain, and LOW R_{out}
  - LOW R_{out} required to avoid loading when driving resistive loads
  - The op-amp is usually implemented as a multistage amplifier
  - The last stage (output stage) is a buffer to provide LOW R_{out}
- IC CMOS op-amps usually drive capacitive loads
  - There is no need for LOW R_{out}
  - The output stage (buffer) is not required
  - These modern op-amps are usually called Operational Transconductance Amplifiers (OTAs)



Op-Amp vs OTA

- In short, an OTA is an op-amp without an output stage (buffer)
- Some designers just use op-amp name and symbol for both

<table>
<tr><th></th><th>Op-amp</th><th>OTA</th></tr>
<tr><td>Rout</td><td>LOW</td><td>HIGH</td></tr>
<tr><td>Model</td><td>
</td><td>
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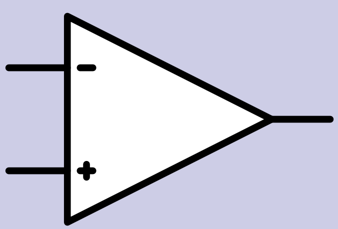
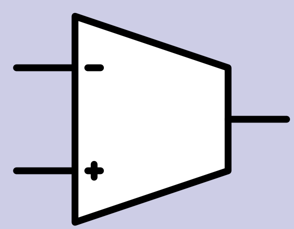
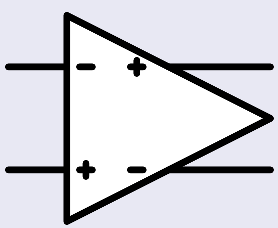
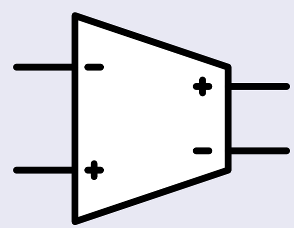
<tr><td>Diff input, SE output</td><td></td><td></td></tr>
<tr><td>Fully diff</td><td></td><td></td></tr>
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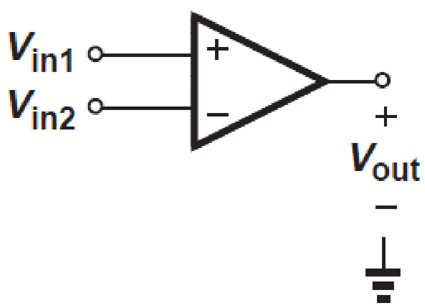
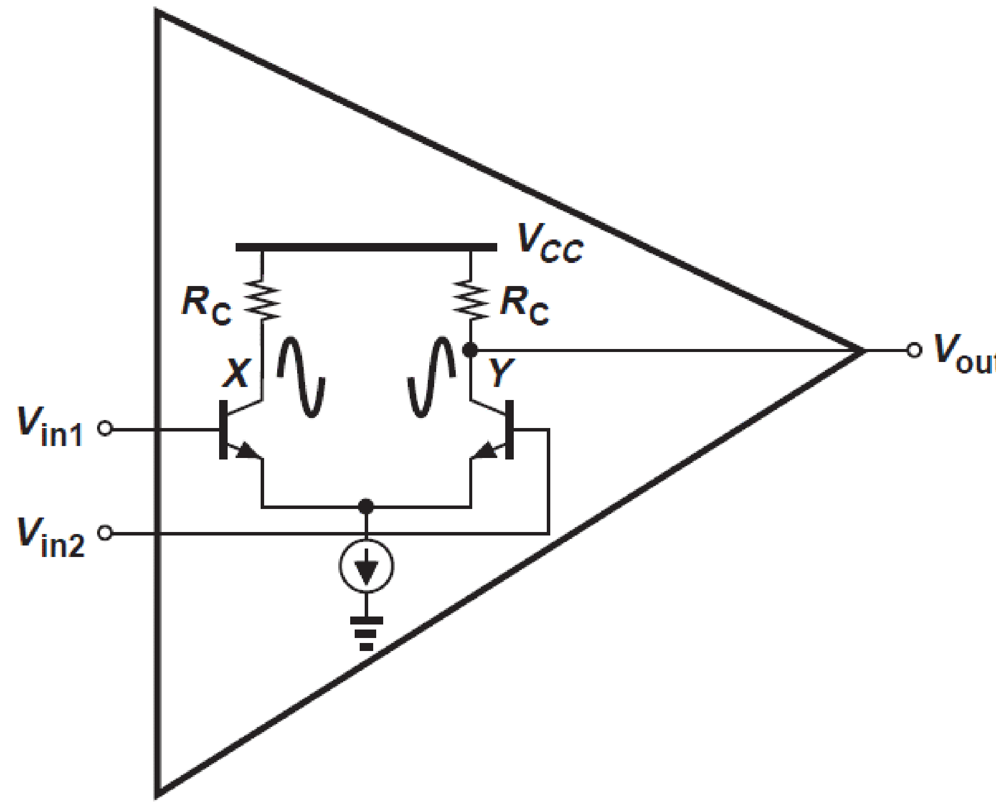
How to Get SE Output?

- Trivial solution: discard one output!
  - But the gain is halved (and the CMRR is poor)
- Better solution: use a diff to SE converter (but how?!)

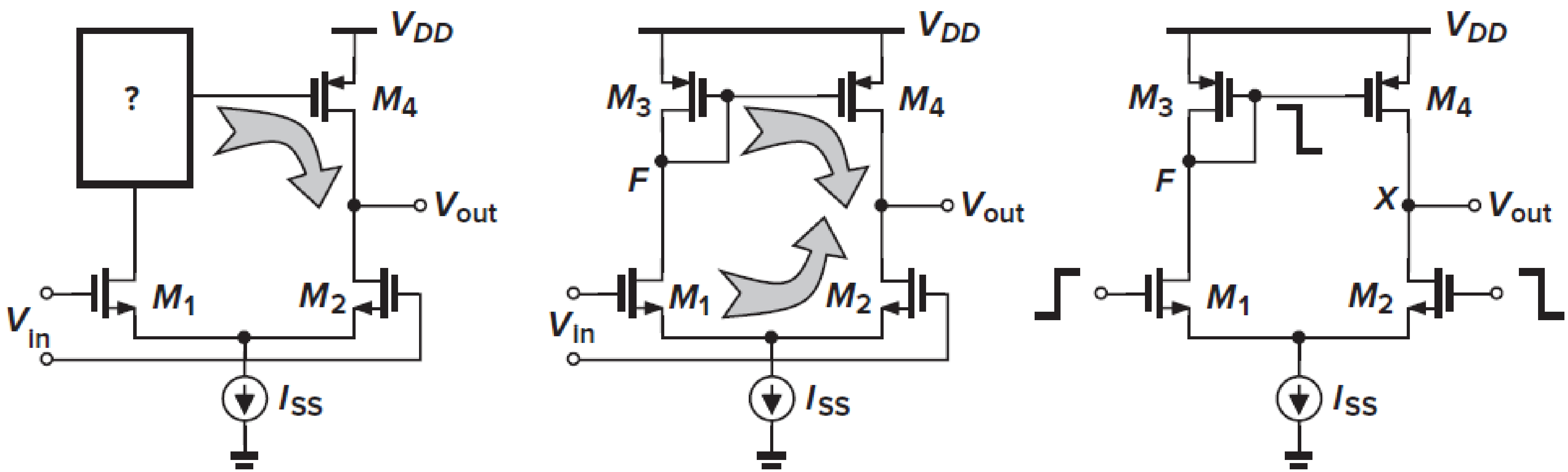




 [Razavi, 2014]

5T OTA

- A.k.a. diff pair with active load, diff pair with CM load, unbalanced diff pair
- Can be viewed in two ways
  1. M4 is a CS amplifier that transfers the small signal voltage from the left side to the right side
  2. We use a current mirror to transfer the small signal current from the left side to the right side

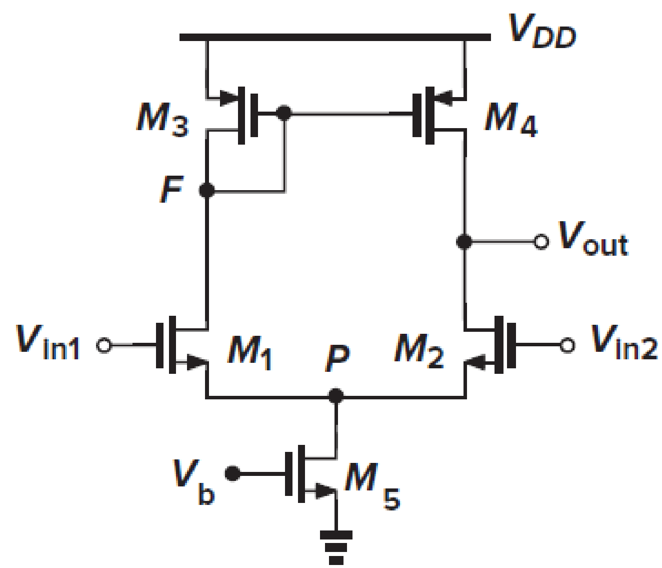


[Razavi, 2017]

5T OTA Analysis

- **Half-circuit principle cannot be used due to asymmetry**

- Four types of analysis
  A. Small signal analysis
     1. Diff small signal analysis
     2. CM small signal analysis
  B. Large signal analysis
     1. Diff large signal analysis
     2. CM large signal analysis



[Razavi, 2017]

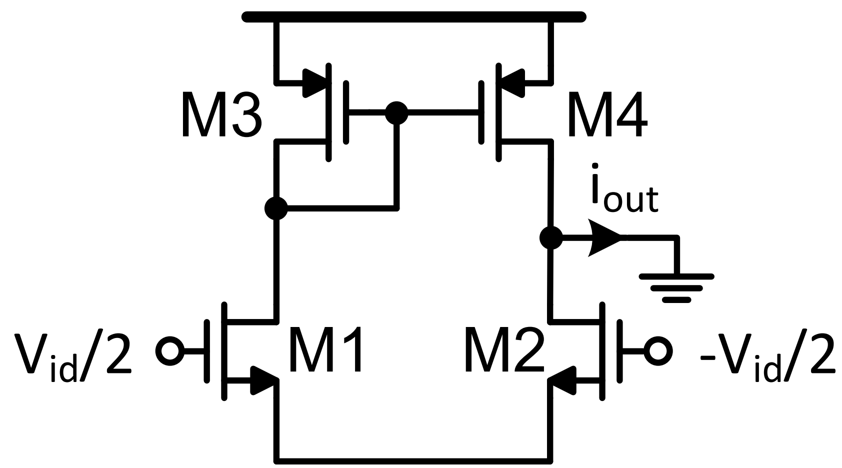
A1. Diff Small Signal Analysis

- Solve it as a single stage using GmRout (apply superposition)

$$i_{out} = i'_{out} + i''_{out}$$

$$i_{out} = \frac{g_{m1}}{1 + g_{m1}(1/g_{m2})} \cdot 2 \cdot \frac{v_{id}}{2} + \frac{g_{m2}}{1 + g_{m2}(1/g_{m1})} \cdot 2 \cdot \frac{v_{id}}{2}$$

$$G_m = \frac{i_{out}}{v_{id}} \approx g_{m1,2}$$

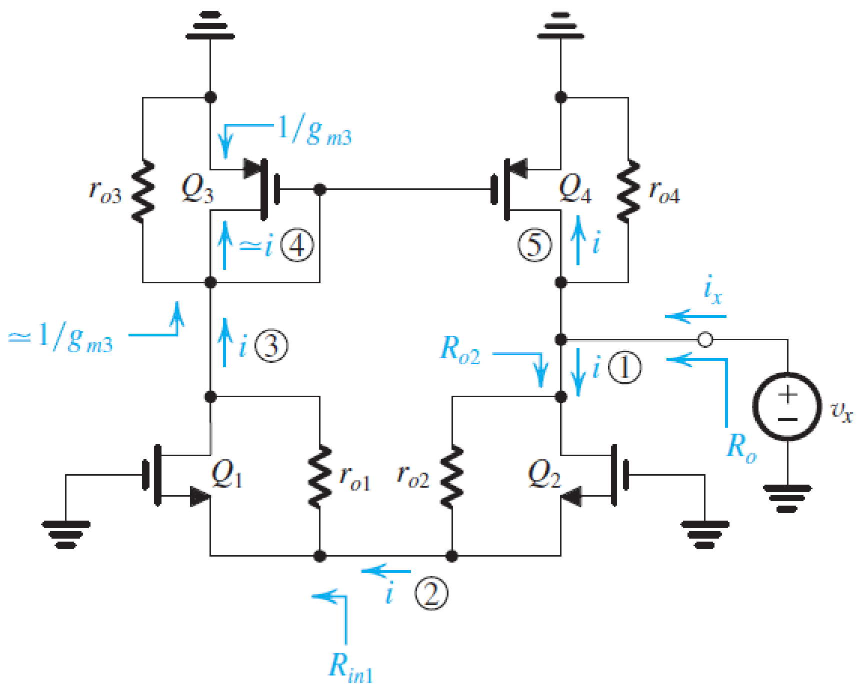


A1. Diff Small Signal Analysis

$$R_{out} = \frac{v_x}{i_x} = \frac{v_x}{2i} // r_{o4}$$

$$\frac{v_x}{i} = R_{o2} \approx r_{o2}\left(1 + g_{m2}\frac{1}{g_{m1}}\right) \approx 2r_{o2}$$

$$R_{out} \approx r_{o2} // r_{o4}$$

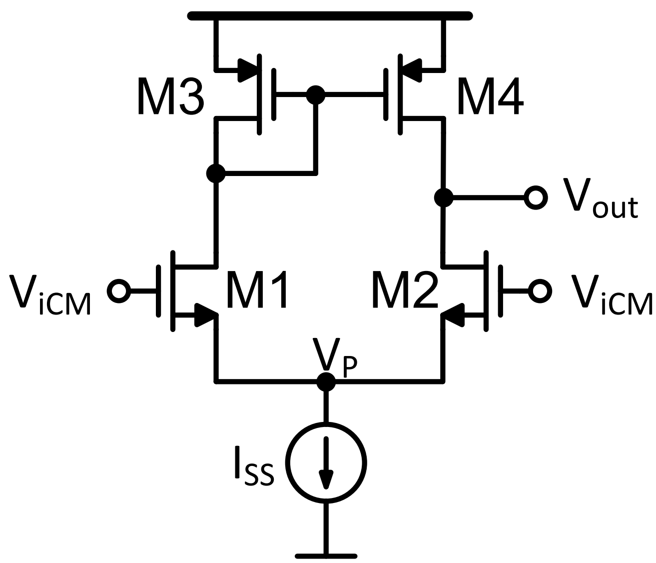


[Sedra/Smith, 2015]

A2. CM Small Signal Analysis

- If the tail CS is ideal
  - The two sides will generate current in the same direction
  - Thus both currents must be zero

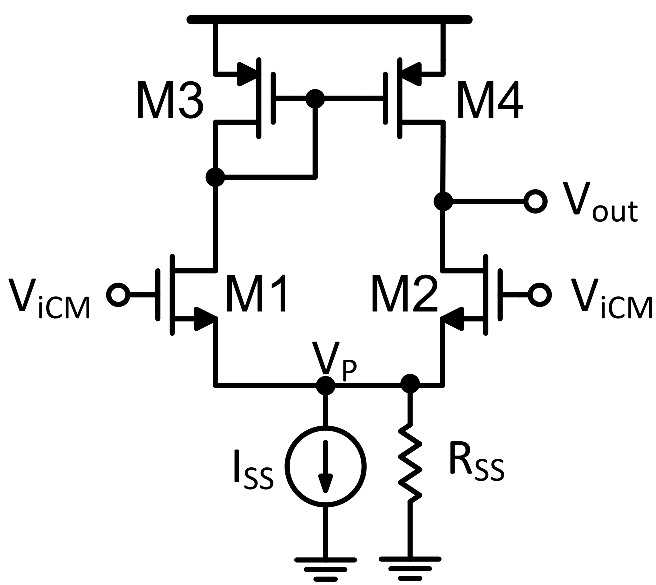
$$V_{out} = 0 \;\Rightarrow\; A_{vCM} = \frac{V_{out}}{V_{iCM}} = 0 \;\Rightarrow\; CMRR = \frac{A_{vd}}{A_{vCM}} \rightarrow \infty$$



A2. CM Small Signal Analysis

- For non-ideal tail CS: $CMRR \neq \infty$
- **Method #1:** By intuition: M1 and M2 generate the same current (Δi_d) → ∴ M3 and M4 have EXACTLY the same current → ∴ Not just their VGS is equal, but their VDS must be equal → $\therefore V_{out} = V_F$
  - As if we have s.c. between V_{out} and V_F (two halves in parallel)

$$A_{vCM} = \frac{V_{out}}{V_{iCM}} = G_m R_{out} \approx \frac{-2g_{m1,2}}{1 + 2g_{m1,2}R_{SS}} \frac{1}{2g_{m3,4}} \approx -\frac{1}{2g_{m3,4}R_{SS}}$$



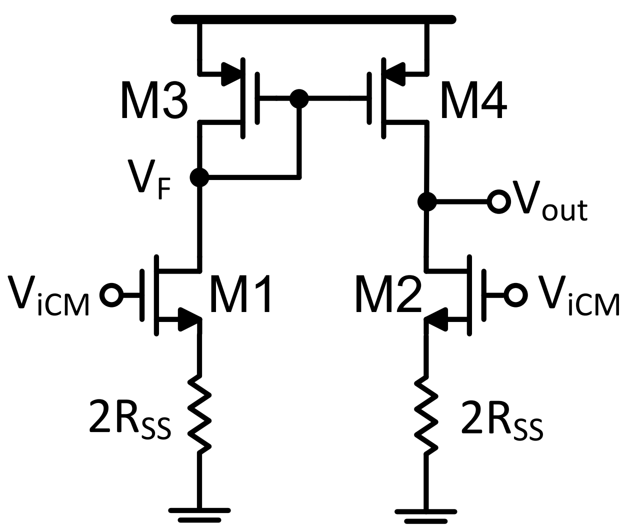
A2. CM Small Signal Analysis

- **Method #2:** By analysis (Do NOT neglect r_{o3}, otherwise $A_{vCM} \to 0$)

$$G_m = \frac{i_{o,sc}}{v_{iCM}} = \frac{g_{m1}}{1+2g_{m1}R_{SS}} \cdot \left(\frac{1}{g_{m3}} // r_{o3}\right) \cdot g_{m4} - \frac{g_{m2}}{1+2g_{m2}R_{SS}}$$

$$= \frac{g_{m1,2}}{1+2g_{m1,2}R_{SS}} \left(\frac{g_{m3,4}r_{o3,4}}{1+g_{m3,4}r_{o3,4}} - 1\right) \approx \frac{-1}{2g_{m3,4}r_{o3,4}R_{SS}}$$

$$A_{vCM} = G_m R_{out} \approx \frac{-1}{2g_{m3,4}r_{o3,4}R_{SS}} \cdot r_{o4} = -\frac{1}{2g_{m3,4}R_{SS}}$$



CMRR

$$A_{vd} = \frac{V_{out}}{V_{id}} \approx g_{m1,2}(r_{o2}//r_{o4})$$

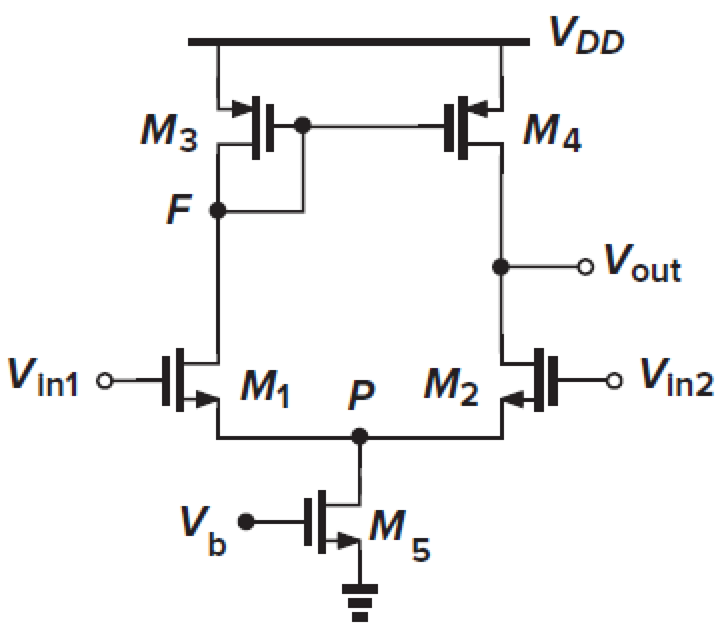
$$A_{vCM} = \frac{V_{out}}{V_{iCM}} \approx -\frac{1}{2g_{m3,4}R_{SS}}$$

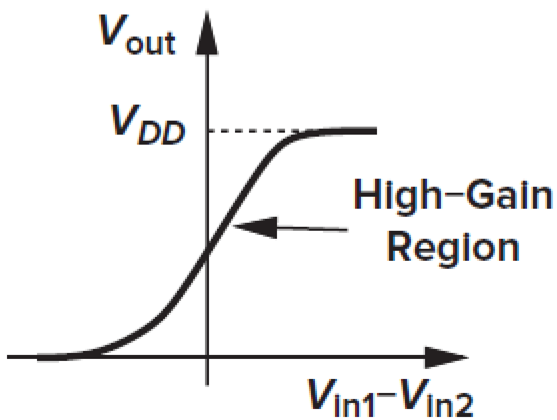
$$CMRR \approx g_{m1,2}(r_{o2}//r_{o4}) \cdot 2g_{m3,4}R_{SS} \sim (g_m r_o)^2$$

- Note that the CMRR is much better than the trivial case of getting SE output by dropping one differential output
  - For the trivial case $CMRR \approx g_m R_{SS} \sim g_m r_o$
- Note that we have been assuming perfect symmetry!
  - CM noise will affect the SE output even in the case of perfect symmetry → A clear disadvantage compared to fully diff OTA
- Note that effective R_{SS} degrades at high frequency

B1. Diff Large Signal Analysis

- $V_{id} = (V_{in1} - V_{in2}) \ll 0$
  - M1 and M3 OFF → M4 OFF → large resistance
  - M2 and M5 ON (triode) → small resistance
  - $V_{out} \approx 0$

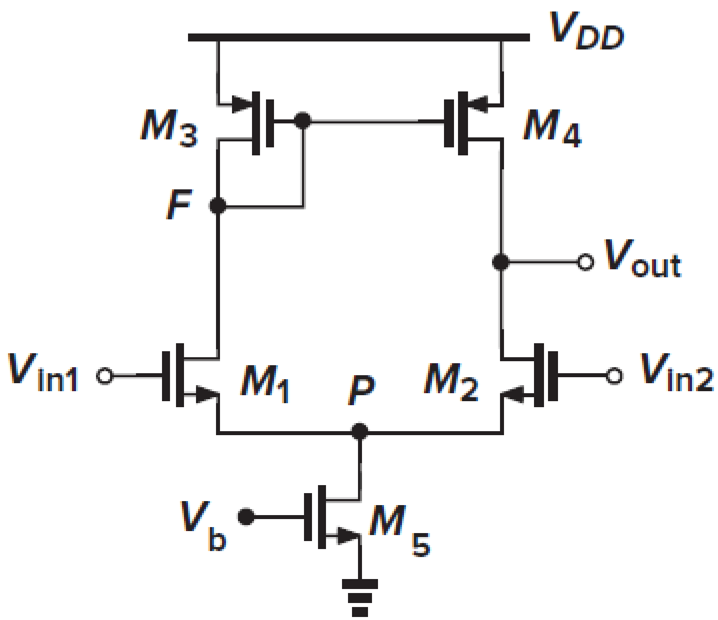


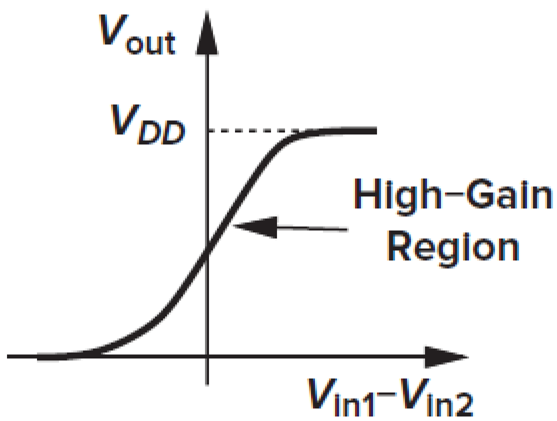


[Razavi, 2017]

B1. Diff Large Signal Analysis

- $V_{id} = (V_{in1} - V_{in2}) \gg 0$
  - M1 and M3 ON → M4 ON (triode) → small resistance
  - M2 and M5 OFF → large resistance
  - $V_{out} \approx V_{DD}$

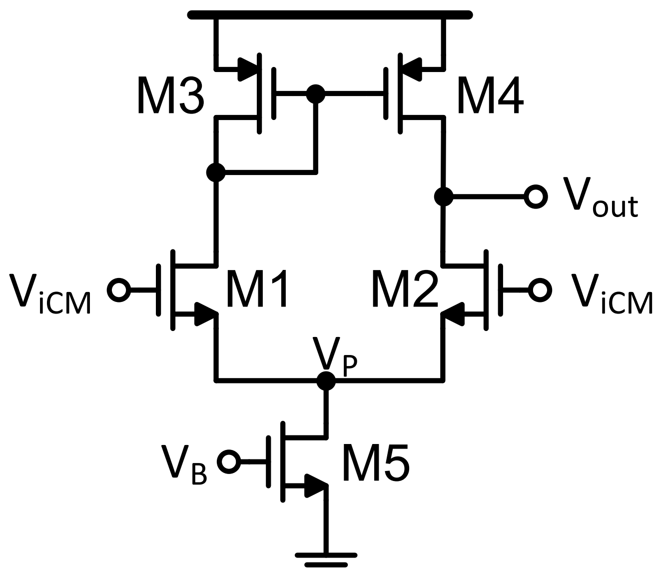




[Razavi, 2017]

B2. CM Large Signal Analysis

- For high gain, all transistors must be in saturation



- Tail CS in sat

$$V_{iCM} \geq V_{THN} + V_{ov1} + V_{ov5}$$

- Input pair in sat

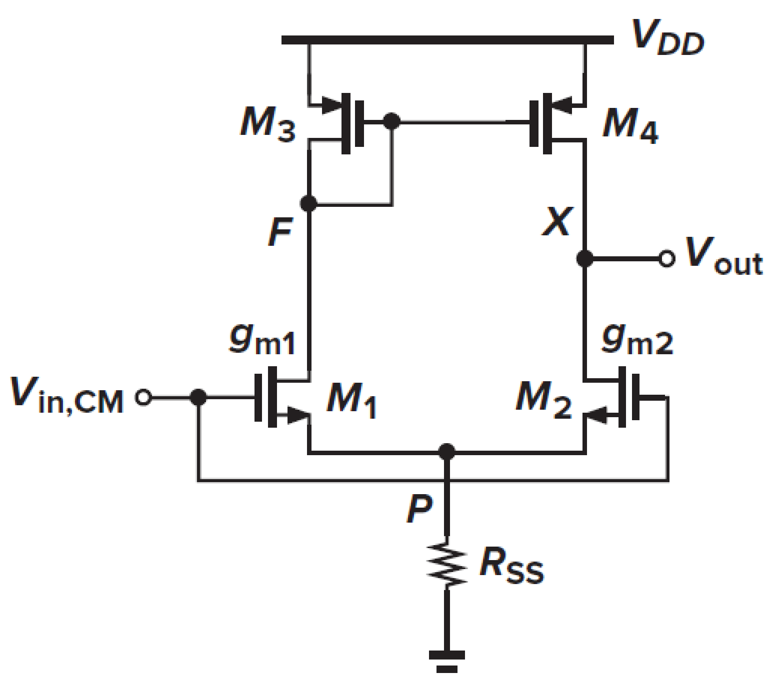
$$V_{iCM} \leq V_{DD} - |\boldsymbol{V_{THP}}| - V_{ov3} + V_{THN}$$

Effect of Mismatch (in Input Pair)

- Use superposition (left + right)

$$A_{vCM} = \frac{v'_{out}}{v_{iCM}} + \frac{v''_{out}}{v_{iCM}} = \frac{2g_{m1}R_{out}}{1 + g_{m1}\left(\frac{1}{g_{m2}}||R_{SS}\right)} - \frac{2g_{m2}R_{out}}{1 + g_{m2}\left(\frac{1}{g_{m1}}||R_{SS}\right)}$$

$$= \frac{2\Delta g_m R_{out}}{1 + (g_{m1} + g_{m2})R_{SS}} \approx \frac{\Delta g_m R_{out}}{g_{m1,2}R_{SS}}$$



[Razavi, 2017]

CMRR with Mismatch

- Overall CM small signal response (matched + mismatch)

$$A_{vCM} \approx \frac{-\frac{g_{m1,2}}{g_{m3,4}}}{1 + 2g_{m1,2}R_{SS}} + \frac{2\Delta g_m R_{out}}{1 + 2g_{m1,2}R_{SS}} \approx \frac{-\frac{g_{m1,2}}{g_{m3,4}} + 2\Delta g_m R_{out}}{2g_{m1,2}R_{SS}}$$

- Common-mode rejection ration (CMRR) (@low frequency!)

$$A_{vd} = g_{m1,2}R_{out}$$

$$CMRR = \frac{A_{vd}}{A_{vCM}} \approx \frac{2g_{m1,2}R_{SS}}{-\frac{1}{g_{m3,4}R_{out}} + 2\frac{\Delta g_m}{g_{m1,2}}}$$

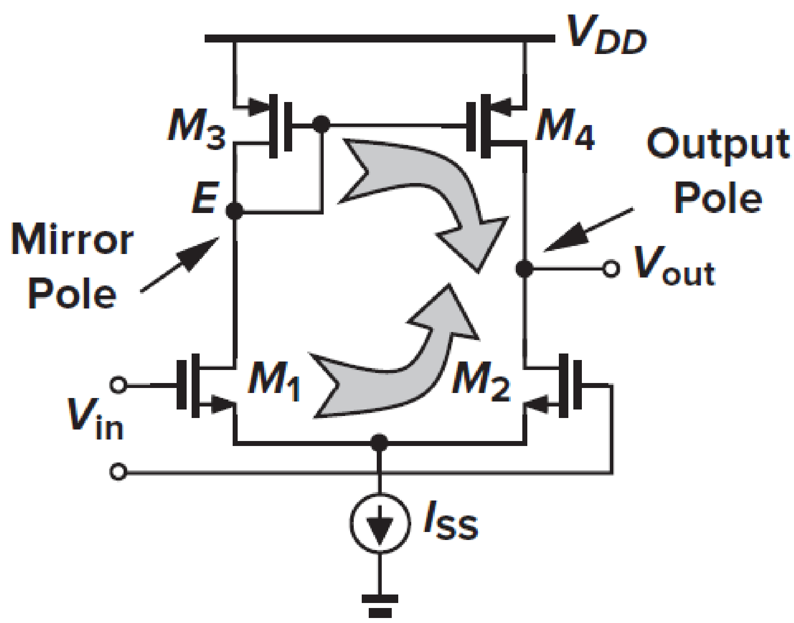
Frequency Response: Poles

$$\omega_{p1} \approx \frac{1}{R_{out}C_{out}}$$

$$C_{out} \approx C_{db4} + C_{db2} + f\left(C_{gd4}\right) + C_{gd2} + C_L$$

$$\omega_{p2} \approx \frac{g_{m3}}{C_E}$$

$$C_E \approx C_{gs3} + C_{gs4} + C_{db3} + C_{db1} + C_{gd1} + f\left(C_{gd4}\right)$$



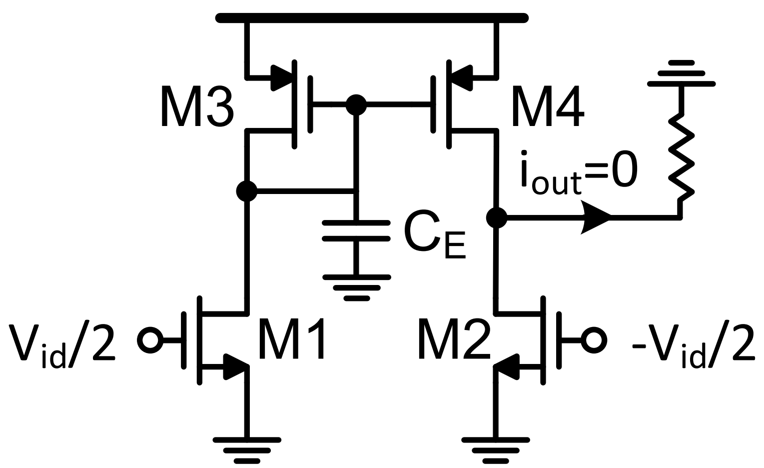
[Razavi, 2017]

Frequency Response: LHP Zero

$$g_{m1} \cdot \frac{1/g_{m3}}{1 + \frac{s_z C_E}{g_{m3}}} \cdot g_{m4} + g_{m2} = 0$$

$$s_z = -\frac{2g_{m3,4}}{C_E}$$

$$\omega_z = 2\omega_{p2}$$



Thank you!

5T OTA as a Buffer

- Tail CS in sat

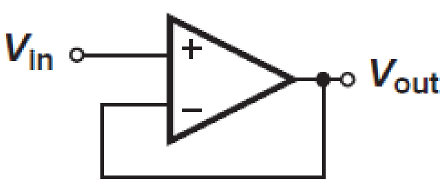
$$V_{iCM} \geq V_{THN} + V_{ov1} + V_{ov5}$$

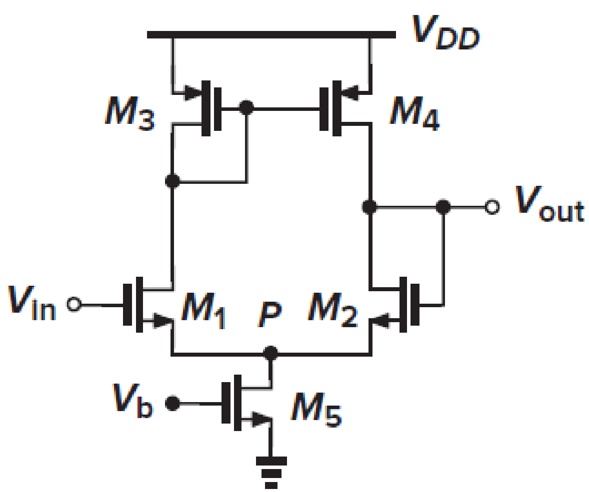
- Input pair in sat

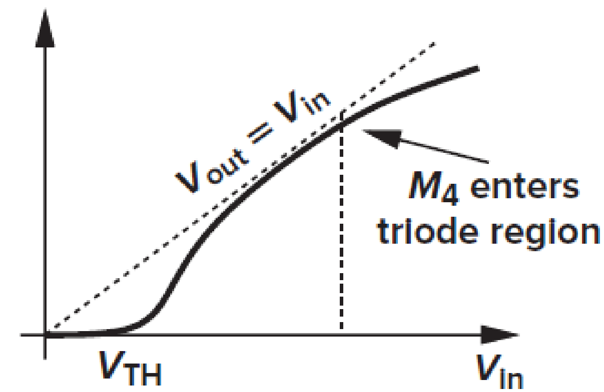
$$V_{iCM} \leq V_{DD} - |\boldsymbol{V_{THP}}| - V_{ov3} + V_{THN}$$

- Or M4 in sat

$$V_{iCM} \leq V_{DD} - V_{ov4}$$



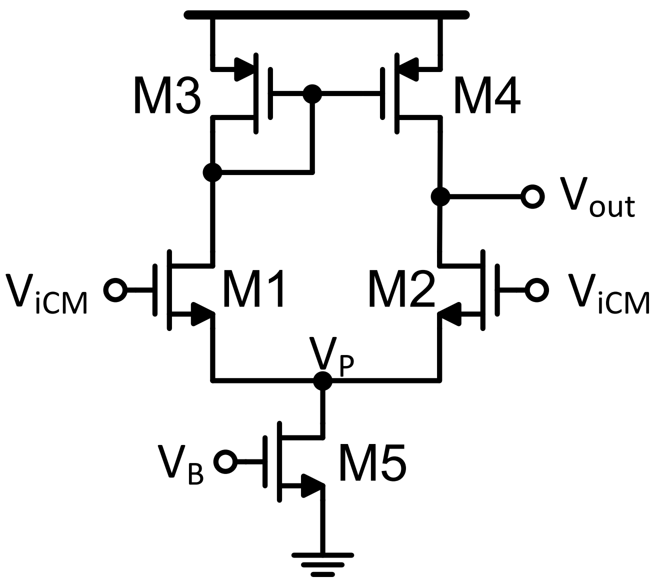


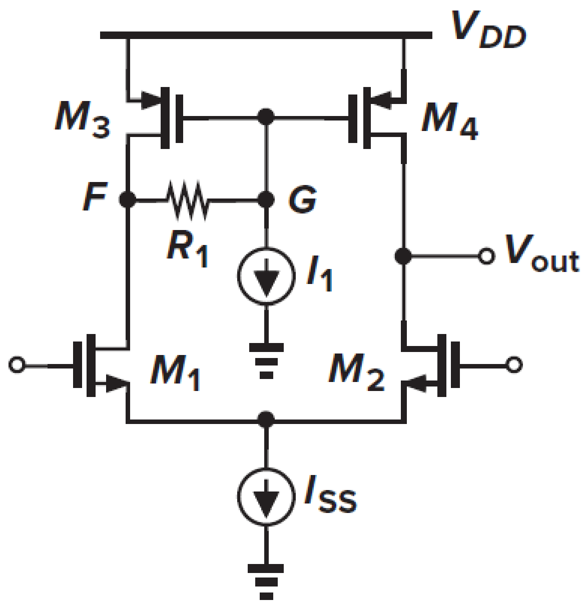


[Razavi, 2017]

Improving Headroom

- Max V_{iCM} can be improved by reducing the drop on M3





- Left (M3 consumes substantial headroom)

$$V_{iCM} \le V_{DD} - |\boldsymbol{V_{THP}}| - V_{ov3} + V_{THN}$$

- Right (set $I_1 R_1 \approx V_{THP}$, but $I_1 \ll I_{SS}/2$ to avoid disturbing symmetry)

$$V_{iCM} \le V_{DD} - V_{ov3} + V_{THN}$$