وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 18
Noise

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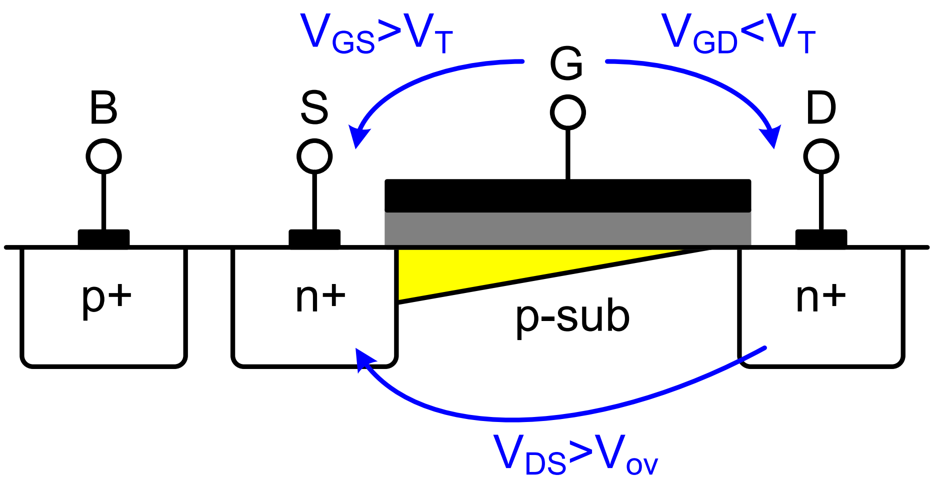
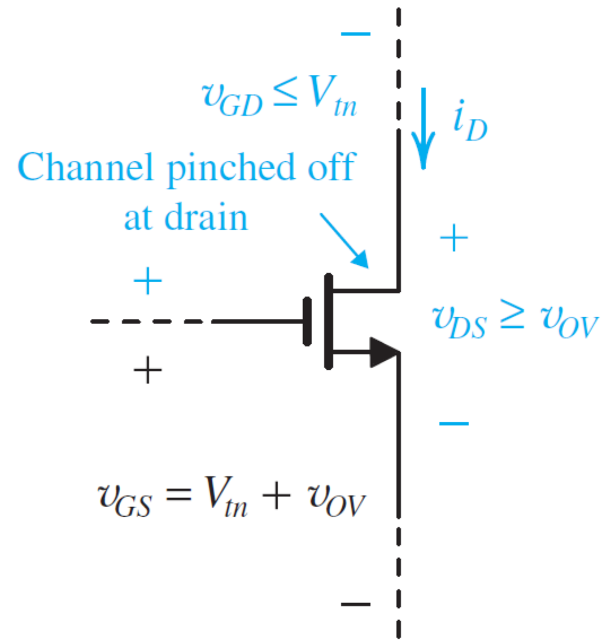
MOSFET in Saturation

- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L} \cdot V_{ov}^2(1 + \lambda V_{DS})$$





[Sedra/Smith, 2015]

Regions of Operation Summary

OFF (Subthreshold)

$$V_{GS} < V_T$$

ON

$$V_{GS} > V_T$$

Triode

$$V_{DS} < V_{ov}$$

Or

$$V_{GD} > V_T$$

$$I_D = \mu C_{ox} \frac{W}{L} \left(V_{ov} V_{DS} - \frac{V_{DS}^2}{2} \right)$$

Pinch-Off (Saturation)

$$V_{DS} \geq V_{ov}$$

Or

$$V_{GD} \leq V_T$$

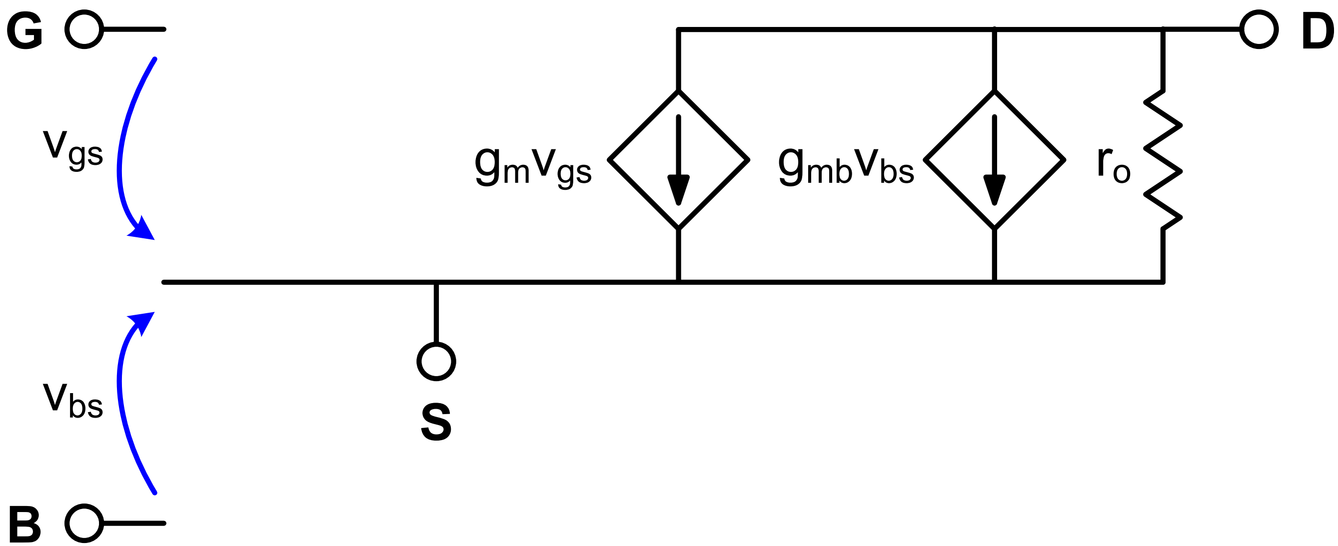
$$I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} V_{ov}^2 (1 + \lambda V_{DS})$$

Low-Frequency Small-Signal Model

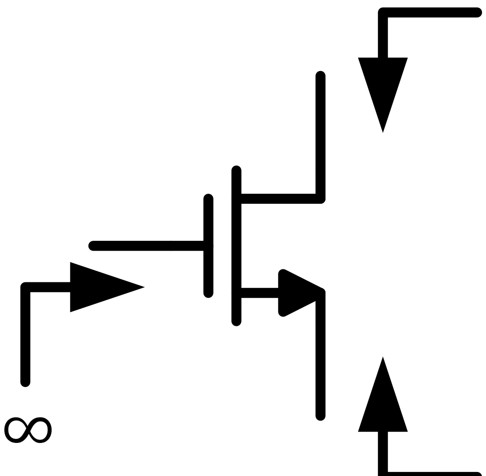
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox}\frac{W}{L}V_{ov} = \sqrt{\mu C_{ox}\frac{W}{L}\cdot 2I_D} = \frac{2I_D}{V_{ov}}$$

$$g_{mb} = \eta g_m, \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}} = \frac{1}{\lambda I_D}, \qquad \lambda \propto \frac{1}{L}$$



Rin/out Shortcuts Summary



$r_o[1 + (g_m + g_{mb})R_S]$

H.I.N.

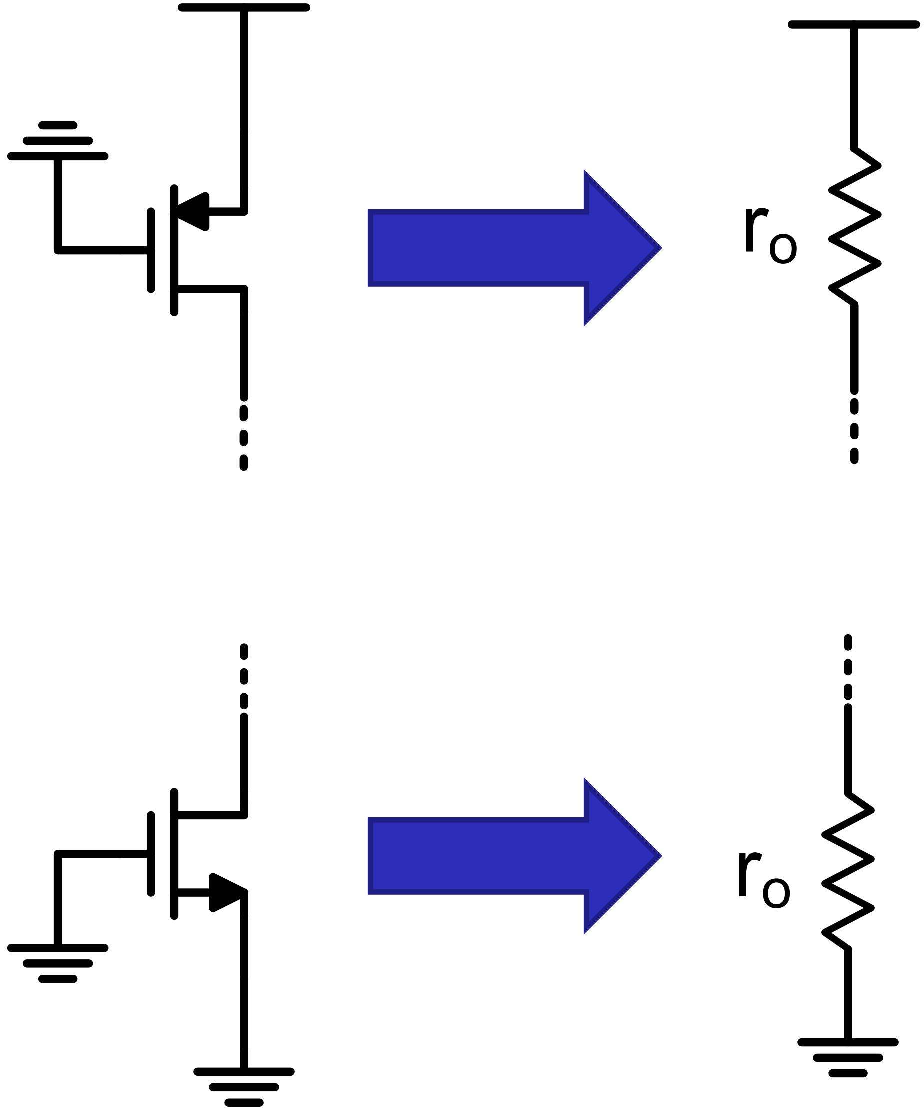
∞

At low frequencies **ONLY**

$$\frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

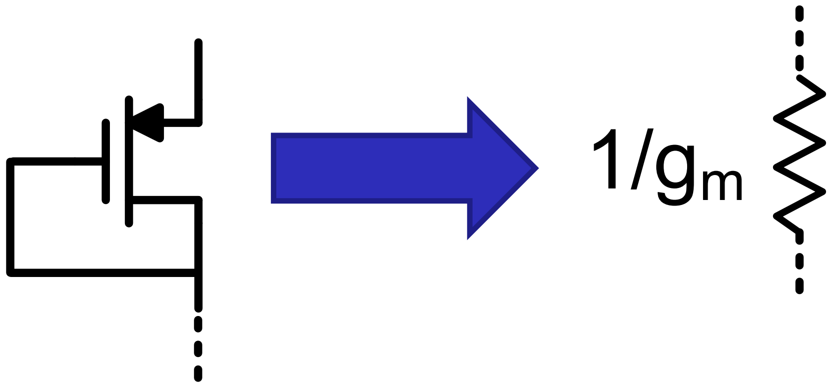
L.I.N.

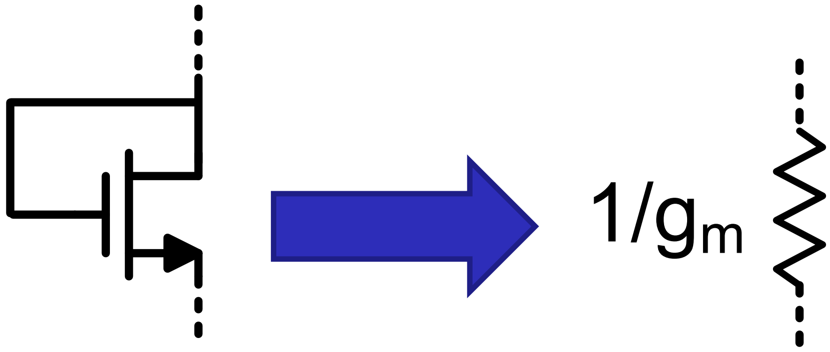
Active Load (Source OFF)



Diode Connected (Source Absorption)

- Always in saturation
- Bulk effect: $g_m \rightarrow g_m + g_{mb}$





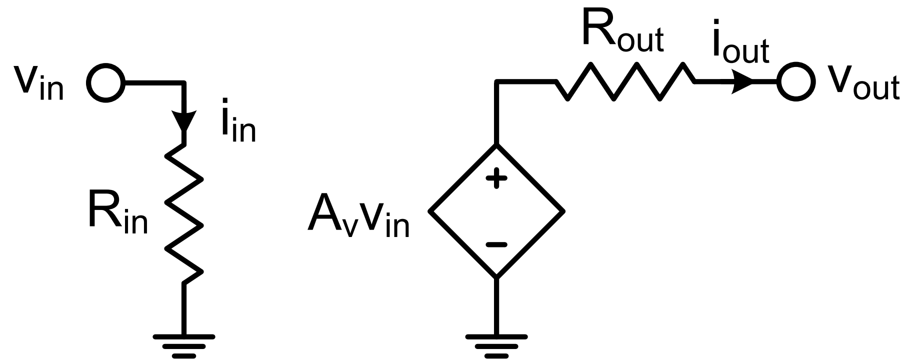
Why GmRout?

$$R_{out} = \frac{v_x}{i_x} @ v_{in} = 0$$

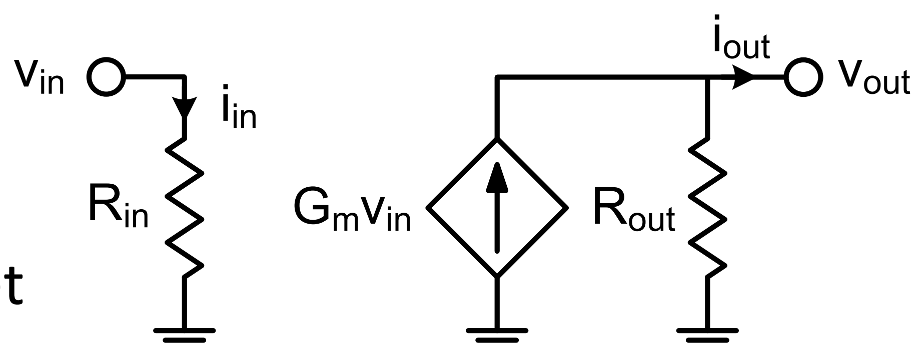
$$G_m = \frac{i_{out,sc}}{v_{in}}$$

$$A_v = G_m R_{out}$$

$$A_i = G_m R_{in}$$

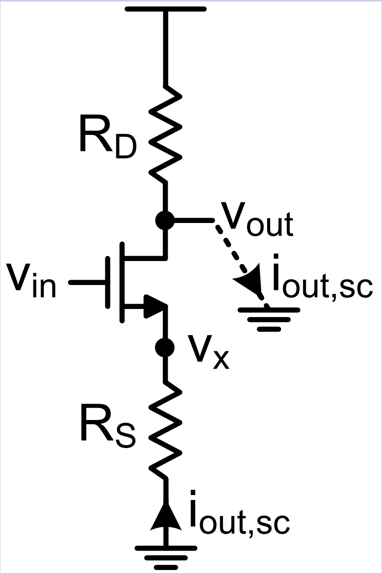


- Divide and conquer
  - Rout simplified: vin=0
  - Gm simplified: vout=0
  - We already need Rin/out
  - We can quickly and easily get Rin/out from the shortcuts

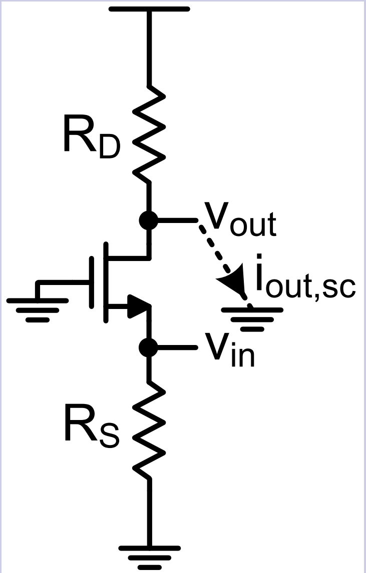


Summary of Basic Topologies

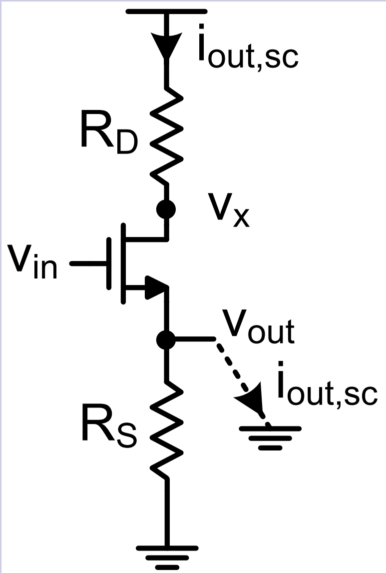
CS



CG



CD (SF)



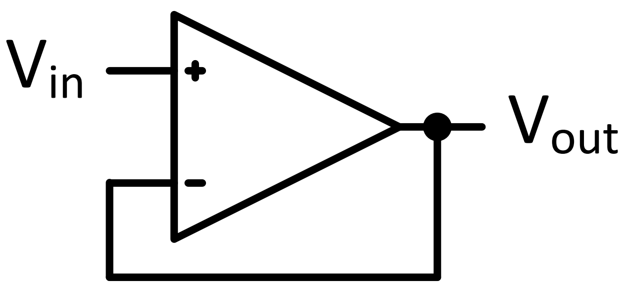
	CS	CG	CD (SF)
	Voltage & current amplifier	Current buffer	Voltage buffer
Rin	∞	$R_S // \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$	∞
Rout	$R_D // r_o[1 + (g_m + g_{mb})R_S]$	$R_D // r_o$	$R_S // \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$
Gm	$\boldsymbol{\frac{-g_m}{1 + (g_m + g_{mb})R_S}}$	$\boldsymbol{g_m + g_{mb}}$	$\boldsymbol{\frac{g_m}{1 + R_D/r_o}}$

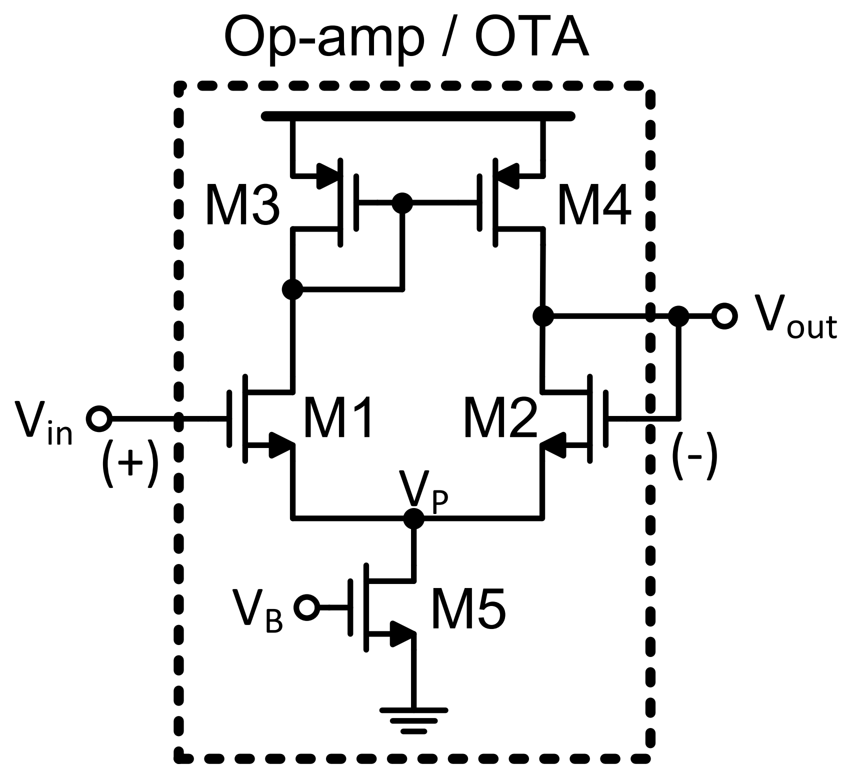
Differential Amplifier

	Pseudo Diff Amp	Diff Pair (w/ ideal CS)	Diff Pair (w/ R_{SS})
A_{vd}	$-g_m R_D$	$-g_m R_D$	$-g_m R_D$
A_{vCM}	$-g_m R_D$	0	$\dfrac{-g_m R_D}{1+2(g_m+g_{mb})R_{SS}}$
A_{vd}/A_{vCM}	1	∞	$2(g_m+g_{mb})R_{SS} \gg 1$

What is an OTA / Op-Amp?

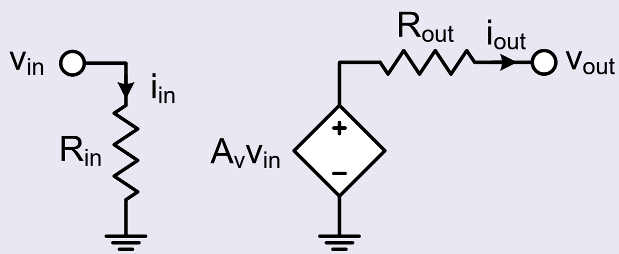
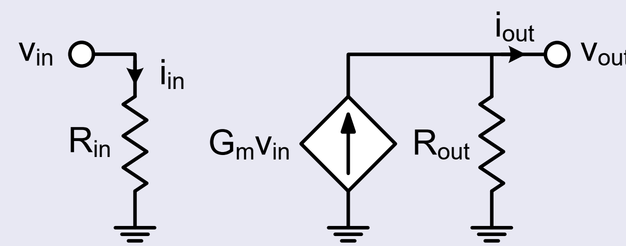
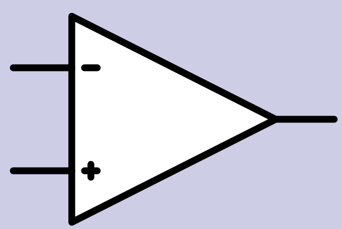
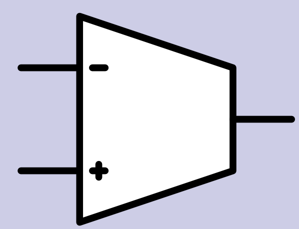
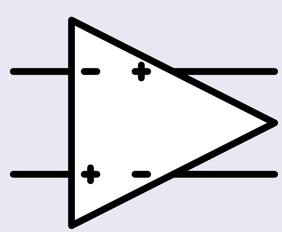
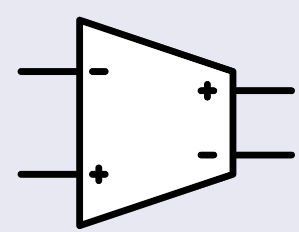
- An op-amp is simply a high gain differential amplifier
- The gain can be increased by using cascodes and multi-stage amplifiers





Op-Amp vs OTA

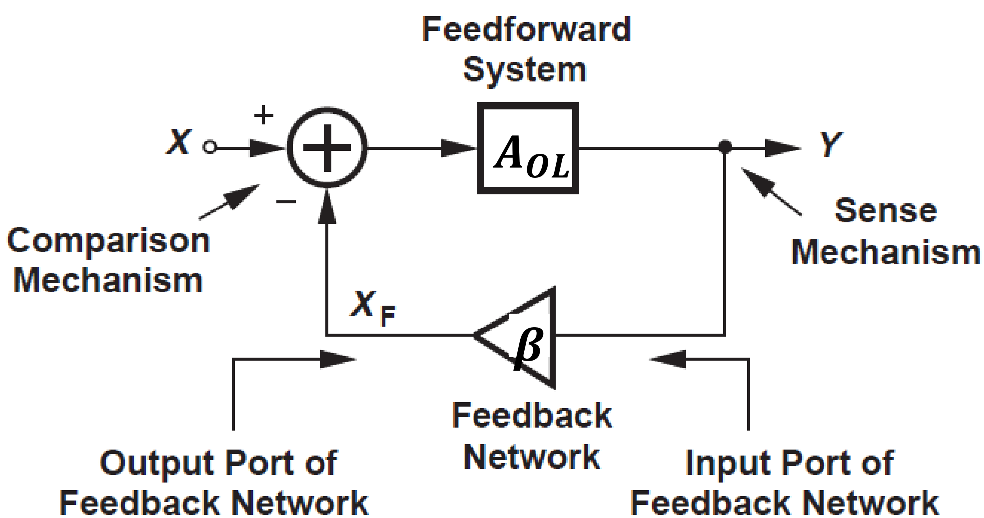
- An OTA is an op-amp without an output stage (buffer)
- Some designers just use op-amp name and symbol for both

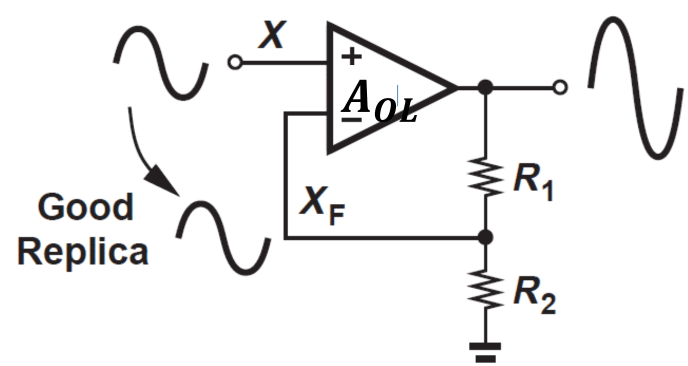
	Op-amp	OTA
Rout	**LOW**	**HIGH**
Model		
Diff input, SE output		
Fully diff		

Negative Feedback

- A_{OL} = Open loop (OL) gain $\gg 1$
- $A_{CL} = \frac{Y}{X}$ = Closed loop (CL) gain
- Error signal = $X - X_F$

$$Y = A_{OL}(X - X_F) = A_{OL}(X - \beta Y)$$
$$A_{CL} = \frac{Y}{X} = \frac{A_{OL}}{1 + \beta \cdot A_{OL}} \approx \frac{1}{\beta}$$

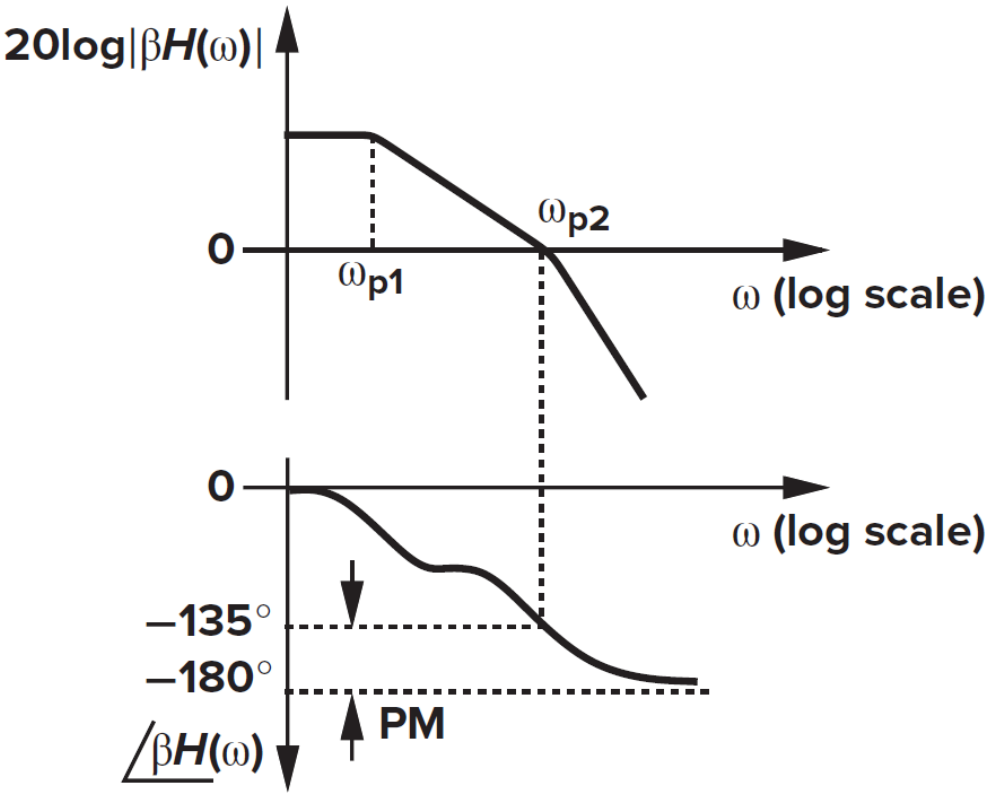


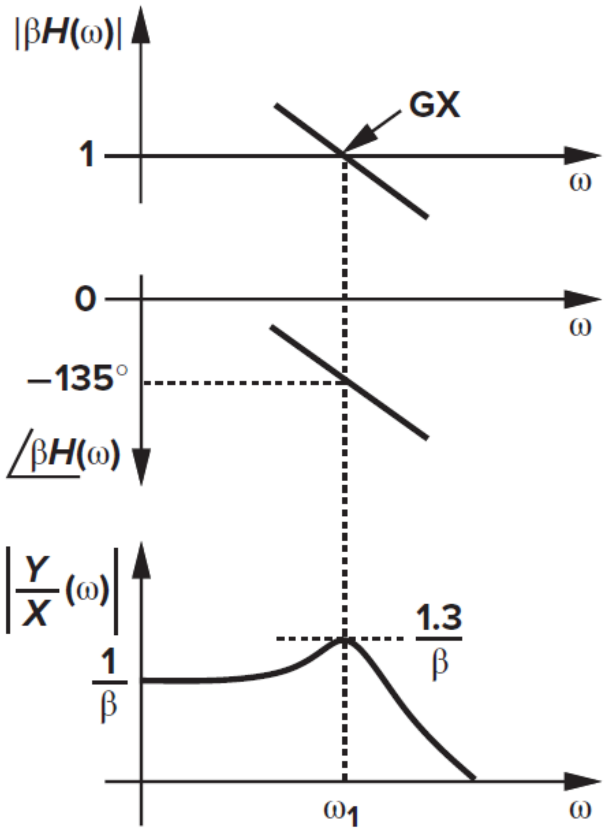


[Razavi, 2014]

Stability: Phase Margin

- If $\omega_{p2} = \omega_u$: PM = 45° → typically inadequate (peaking/ringing)
- The ultimate ω_u cannot exceed ω_{p2} → $\omega_{p1} < \omega_u < \omega_{p2}$
  - For $\omega < \omega_u$ the Bode plot is similar to a 1st order system





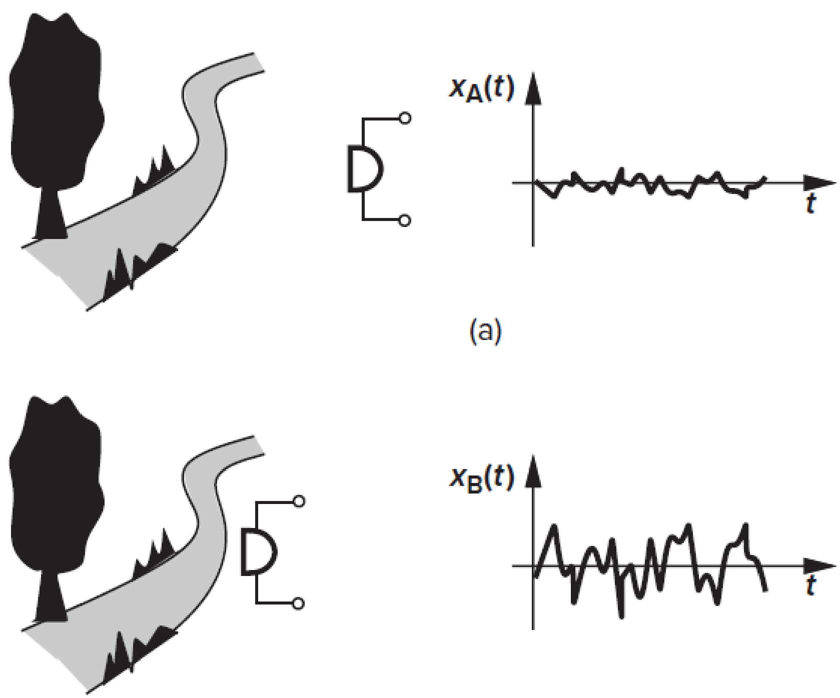
[Razavi, 2017]

In This Lecture: Part 1

- Noise representation
- Noise power
- Signal-to-noise ratio (SNR)
- Resistor thermal noise
- MOSFET thermal and flicker noise
- Input-referred noise

Noise in Time Domain

- Noise is a statistical random process
- We cannot predict its instantaneous value in advance
- But we can predict its average power
  - Noise of water flow is louder as we get closer to the river



(a)

(b)

[Razavi, 2017]

Noise Power

- Average power of a periodic signal (in Watts)

$$P_{av} = \frac{1}{T} \int_{-T/2}^{+T/2} \frac{v^2(t)}{R_L} dt$$

- Average power of a noise signal (in Watts)

$$P_{av} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{+T/2} \frac{x^2(t)}{R_L} dt$$

- Drop R_L from the definition (P_{avg} now in V^2)

$$P_{av} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x^2(t) dt$$

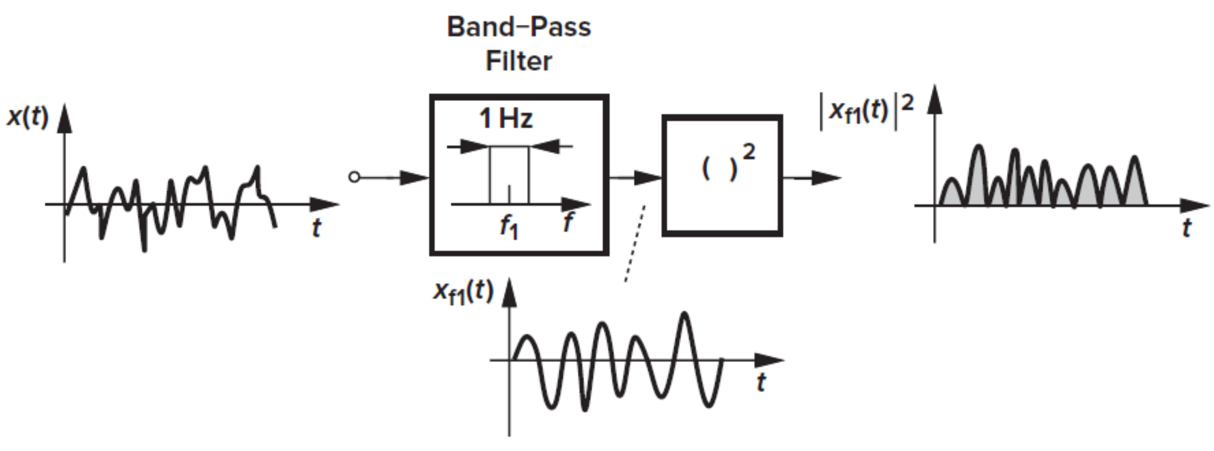
- RMS (root-mean-square) noise voltage

$$V_{nrms} = \sqrt{P_{avg}}$$

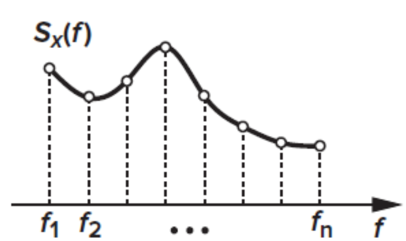
[Razavi, 2017]

Noise in Frequency Domain

- PSD: Power spectral density, $S_x(f)$, of a noise waveform x(t) is the average power carried by $x(t)$ in a one-hertz bandwidth around f
  - Measured in $Watts/Hz$ or V^2/Hz
- Voltage noise density: $V_n(f) = \sqrt{S_x(f)} \rightarrow V/\sqrt{Hz}$



(a)

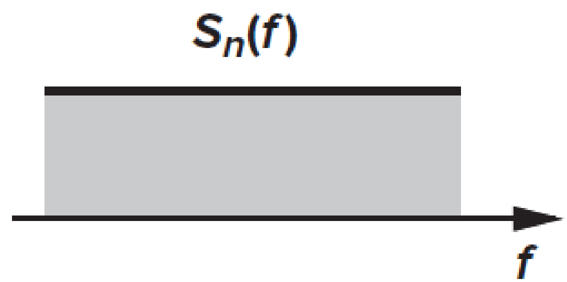


(b)

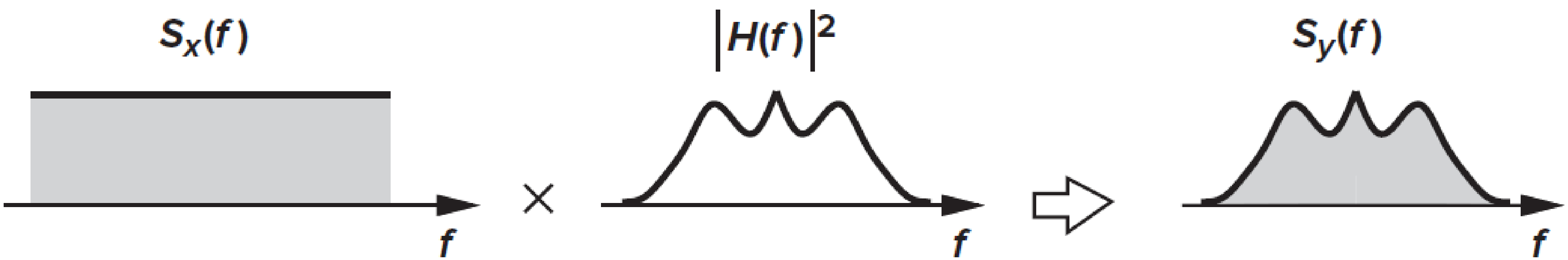
[Razavi, 2017]

White Noise and Noise Shaping

- White noise: PSD displays the same value at all frequencies (similar to white light)



- The noise spectrum is shaped by the system transfer function



- Average noise power is the area under the curve

$$P_{noise} = \int_{-\infty}^{+\infty} S_{noise}(f)df$$

[Razavi, 2017]

Example

- $S_{noise}(f) = 5 \times 10^{-16} V^2/Hz\ and\ BW = 1MHz$

$$P_{noise} = \int_0^{1\ \text{MHz}} S_{noise}(f)df$$

$$= 5 \times 10^{-10}\ \text{V}^2$$

$$V_{nrms} = \sqrt{P_{noise}} = 22.4\mu Vrms$$

[Razavi, 2017]

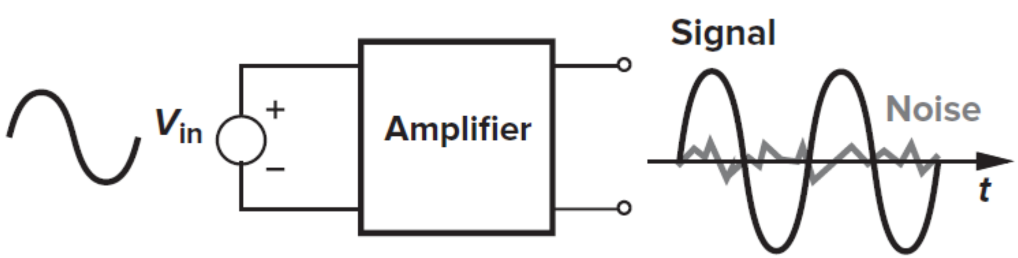
Signal-to-Noise Ratio (SNR)

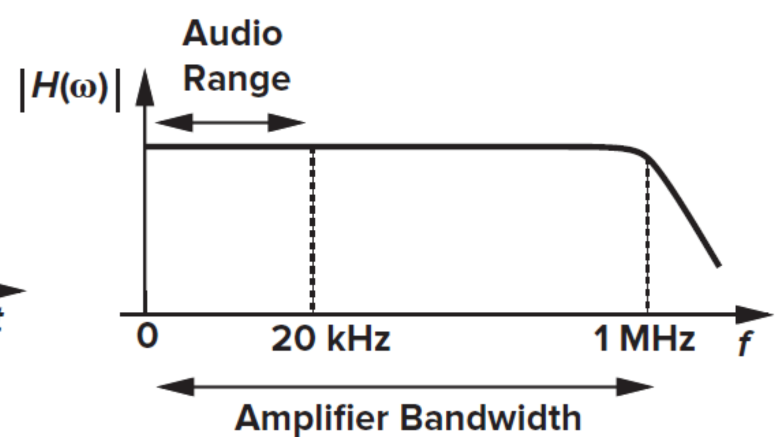
$$SNR = \frac{P_{signal}}{P_{noise}} = \frac{V_{sigrms}^2}{V_{nrms}^2}$$

$$P_{noise} = \int_{-\infty}^{+\infty} S_{noise}(f)df$$

$$V_{nrms} = \sqrt{P_{noise}}$$

- **Wide BW is not good BW → The BW should just fit the signal**



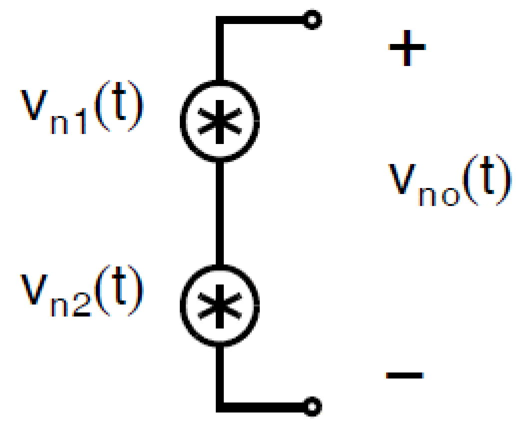


[Razavi, 2017]

Multiple Noise Sources

- Noise adds in time domain

$$v_{no}(t) = v_{n1}(t) + v_{n2}(t)$$



- But RMS values do not simply add

$$V^2_{no(rms)} = \frac{1}{T}\int_0^T [v_{n1}(t) + v_{n2}(t)]^2\, dt$$

$$V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)} + \frac{2}{T}\int_0^T v_{n1}(t)v_{n2}(t)\, dt$$

- Correlation coefficient $(-1 \le C \le 1)$

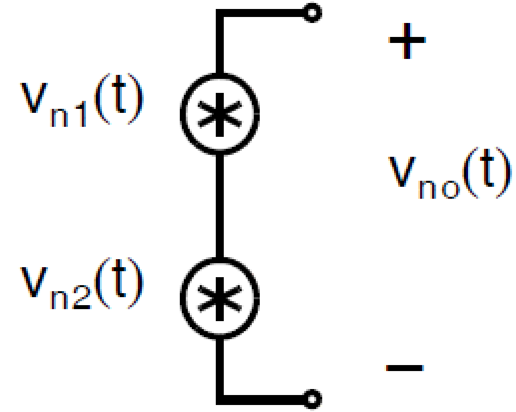
$$C \equiv \frac{\frac{1}{T}\int_0^T v_{n1}(t)v_{n2}(t)\, dt}{V_{n1(rms)}V_{n2(rms)}}$$

[Johns and Martin, 2015]

Multiple Noise Sources

- Correlation coefficient $(-1 \le C \le 1)$

$$C \equiv \frac{\frac{1}{T}\int_0^T v_{n1}(t)v_{n2}(t)\,dt}{V_{n1(rms)}V_{n2(rms)}}$$



- Total RMS noise voltage

$$V_{no(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2 + 2CV_{n1(rms)}V_{n2(rms)}$$

- Usually we have uncorrelated noise sources (C = 0)

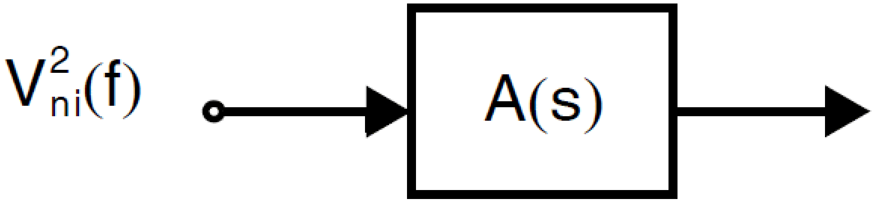
$$V_{no(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2$$

- The largest noise contributor dominates: $3^2 + 1^2 \approx 3^2$
- Note that for the other extreme case of fully correlated signals

$$V_{no(rms)}^2 = [V_{n1(rms)} \pm V_{n2(rms)}]^2$$

[Johns and Martin, 2015]

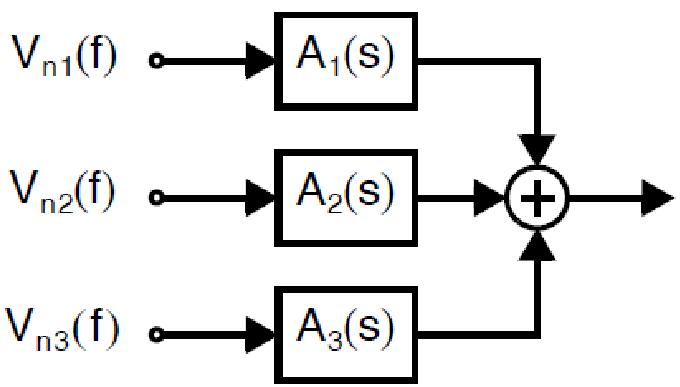
Multiple Noise Sources



$$V_{no}^2(f) = |A(j2\pi f)|^2 V_{ni}^2(f)$$
$$V_{no}(f) = |A(j2\pi f)| V_{ni}(f)$$

- ❑ Uncorrelated noise signals remain uncorrelated, even when filtered by a circuit’s magnitude response

Uncorrelated noise sources



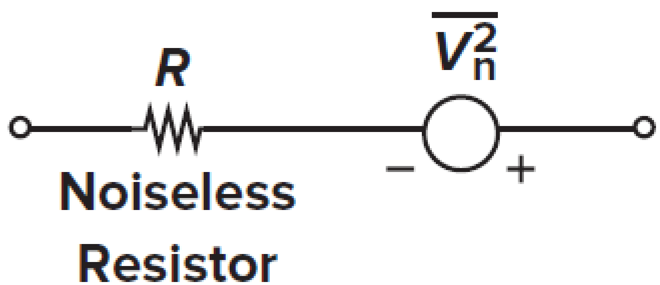
$$V_{no}(f) = \left(\sum_{i=1,2,3} |A_i(j2\pi f)|^2 V_{ni}^2(f) \right)^{1/2}$$

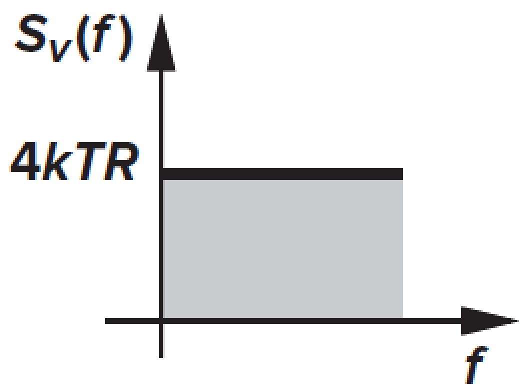
[Johns and Martin, 2015]

Resistor Thermal Noise

- The random motion of electrons in a conductor introduces fluctuations in the voltage measured across the conductor, even if the average current is zero

$$V_n^2(f) = S_v(f) = 4kTR$$





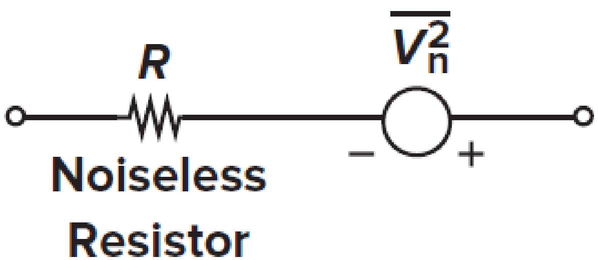
- A number to remember
  - For $R = 1k\Omega$ → $V_n(f) \approx 4nV/\sqrt{Hz}$

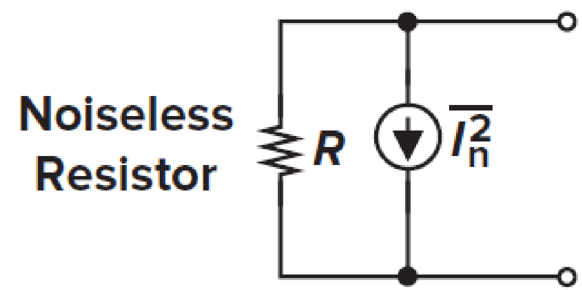
[Razavi, 2017]

Resistor Thermal Noise

- The random motion of electrons in a conductor introduces fluctuations in the voltage measured across the conductor, even if the average current is zero

$$I_n^2(f) = \frac{V_n^2(f)}{R^2} = \frac{4kT}{R}$$



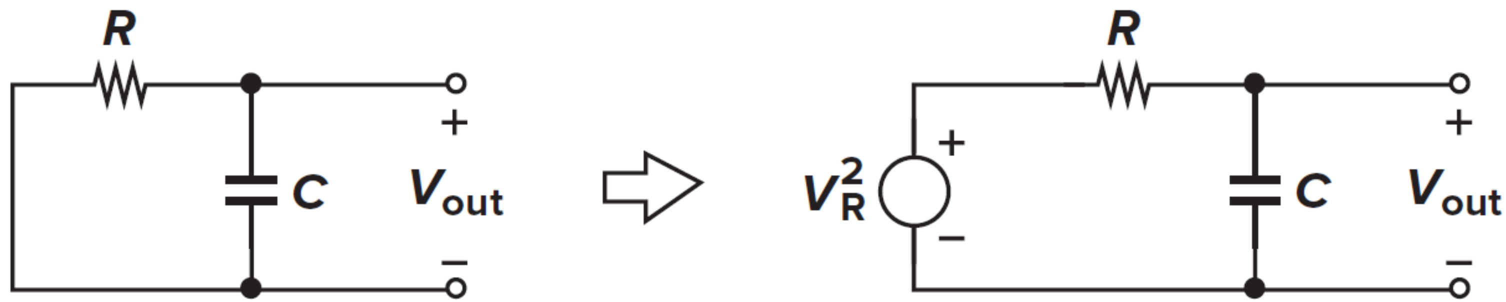


- A number to remember
  - For $R = 1k\Omega$ → $I_n(f) \approx 4pA/\sqrt{Hz}$

[Razavi, 2017]

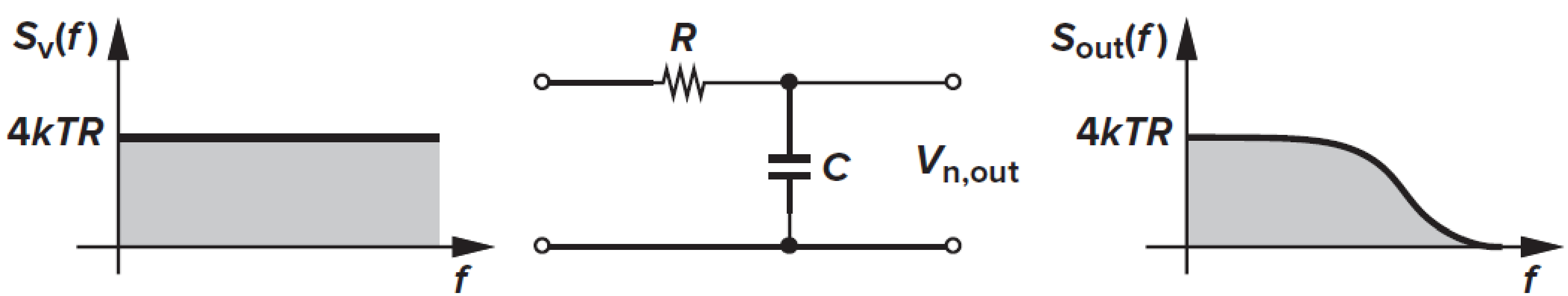
Noise in RC Circuit

- Resistors never exist alone → The BW is always limited by a cap



$$S_{out}(f) = S_v(f)\left|\frac{V_{out}(\omega)}{V_R(\omega)}\right|^2$$

$$V_{nrms}^2 = P_{nout} = \int_{-\infty}^{\infty} S_{out}(f)\,df = \frac{kT}{C}$$



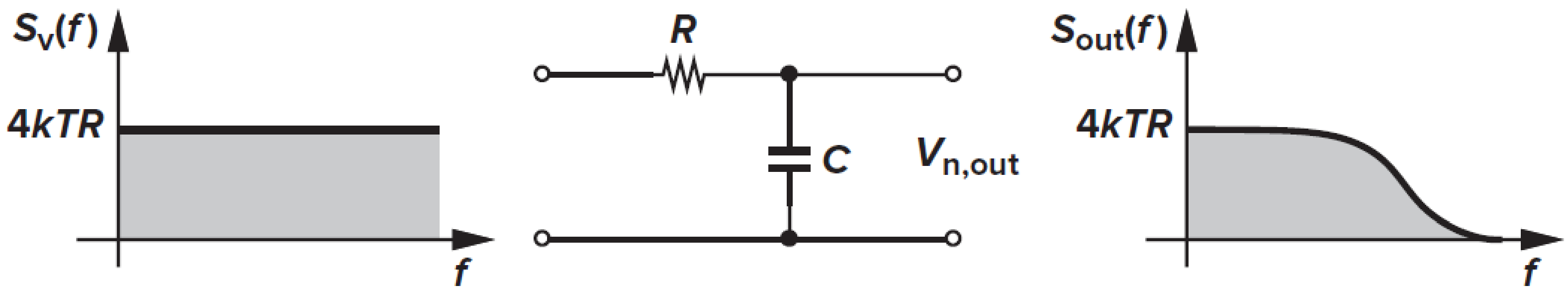
[Razavi, 2017]

Noise in RC Circuit

- Resistors never exist alone → The BW is always limited by a cap

$$V_{nrms}^2 = \frac{kT}{C}$$

- RMS noise is independent of R!!!
- A number to remember
  - For $C = 1pF$ → $V_{nrms} \approx 64\mu Vrms$

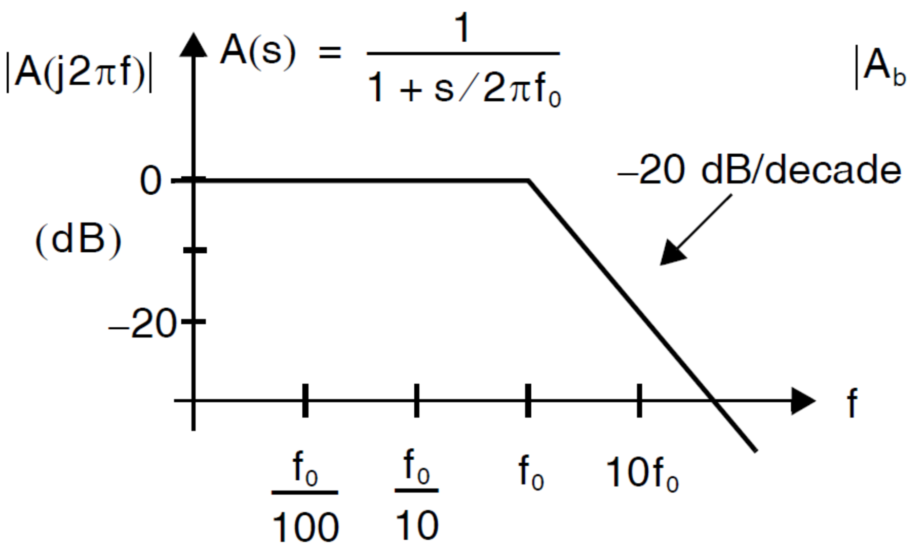


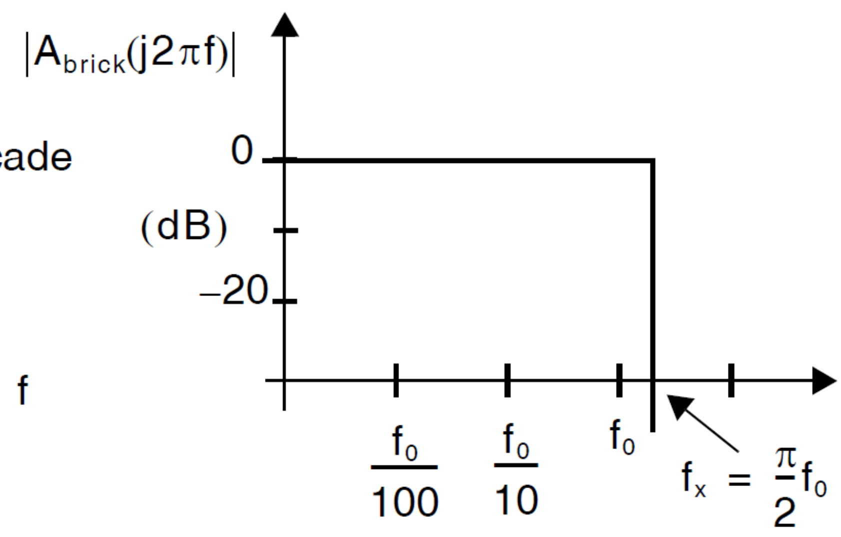
[Razavi, 2017]

Equivalent Noise Bandwidth

$$V_{nrms}^2 = P_{nout} = \int_{-\infty}^{\infty} S_{out}(f)\,df = S_{out}(f) \times B_N$$

$$S_{out}(f) \times B_N = 4kTR \times B_N = \frac{kT}{C} \quad \rightarrow \quad B_N = \frac{1}{4RC} = \frac{\pi}{2} f_p$$





[Johns and Martin, 2015]

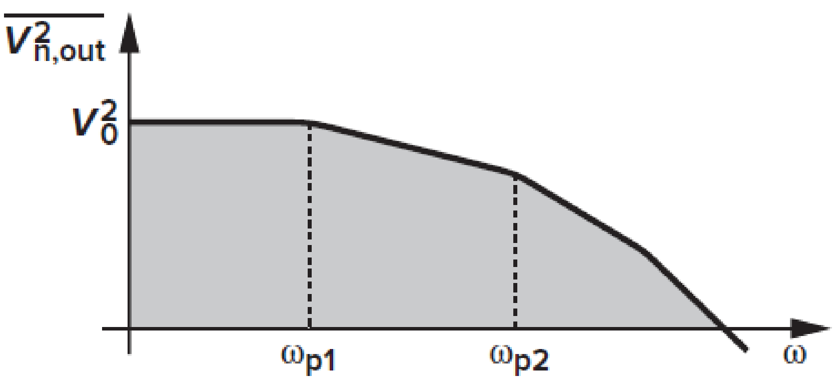
Equivalent Noise Bandwidth

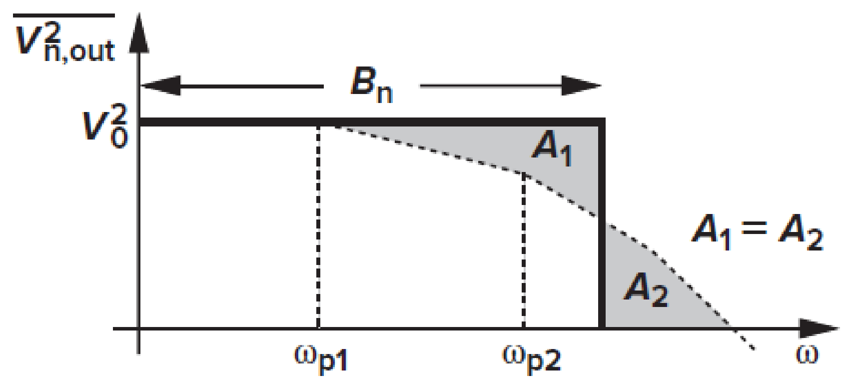
$$V_{nrms}^2 = P_{nout} = \int_{-\infty}^{\infty} S_{out}(f)\, df = S_{out}(f) \times B_N$$

$$S_{out}(f) \times B_N = 4kTR \times B_N = \frac{kT}{C} \quad \rightarrow \quad B_N = \frac{1}{4RC} = \frac{\pi}{2} f_p$$

- In general

$$B_N = \frac{\int_{-\infty}^{\infty} S_{out}(f)\, df}{S_{out}(f)}$$





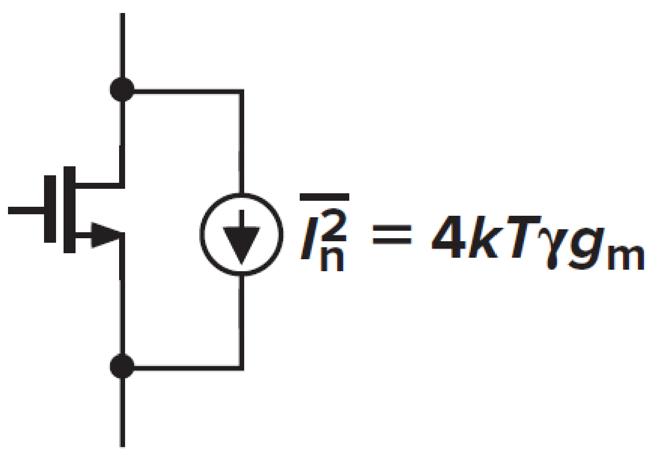
[Razavi, 2017]

MOSFET Channel Thermal Noise

- MOS transistors also exhibit thermal noise
- The most significant source is the noise generated in the channel
- For long-channel MOS devices

$$I_n^2(f) = 4kT\gamma g_m$$

  - Similar to a resistor with $R = \dfrac{1}{\gamma g_m}$
- $\gamma = \frac{2}{3}$ for long channel MOS, but close to 1 for short channel MOS

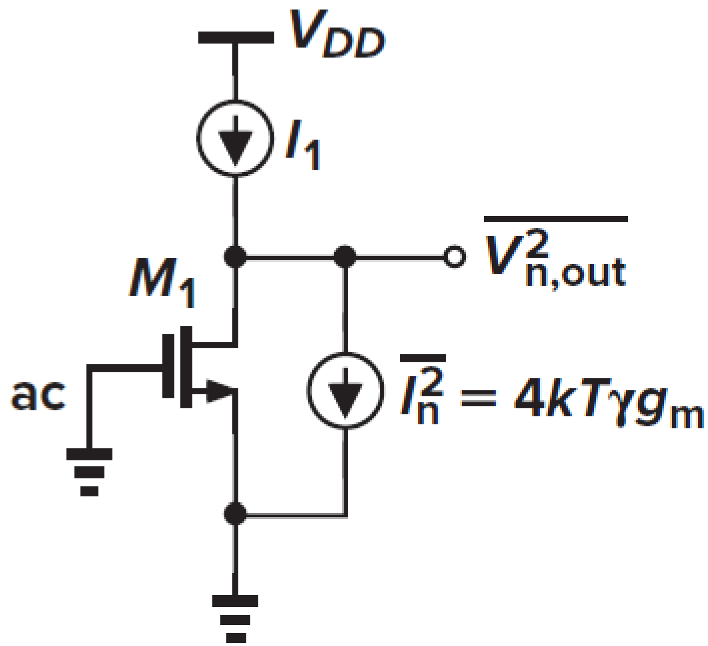


[Razavi, 2017]

MOSFET Channel Max Voltage Noise

$$\overline{V_n^2} = \overline{I_n^2} r_O^2$$

$$= (4kT\gamma g_m) r_O^2$$



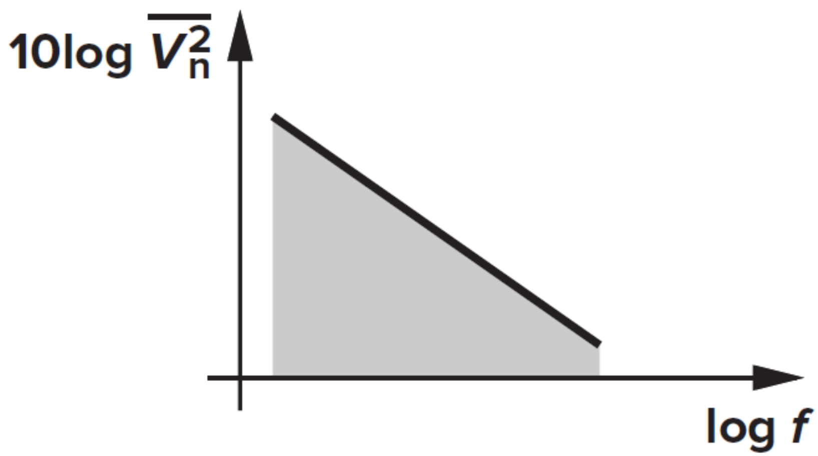
[Razavi, 2017]

MOSFET Flicker Noise

- Mainly due to dangling bonds at the interface between the gate oxide and the silicon substrate

$$V_n^2(f) = \frac{K}{C_{ox}WL}\frac{1}{f}$$

- K: Flicker noise coefficient
- A.k.a. $\mathbf{1/f}$ Noise
- Can be reduced by increasing device area



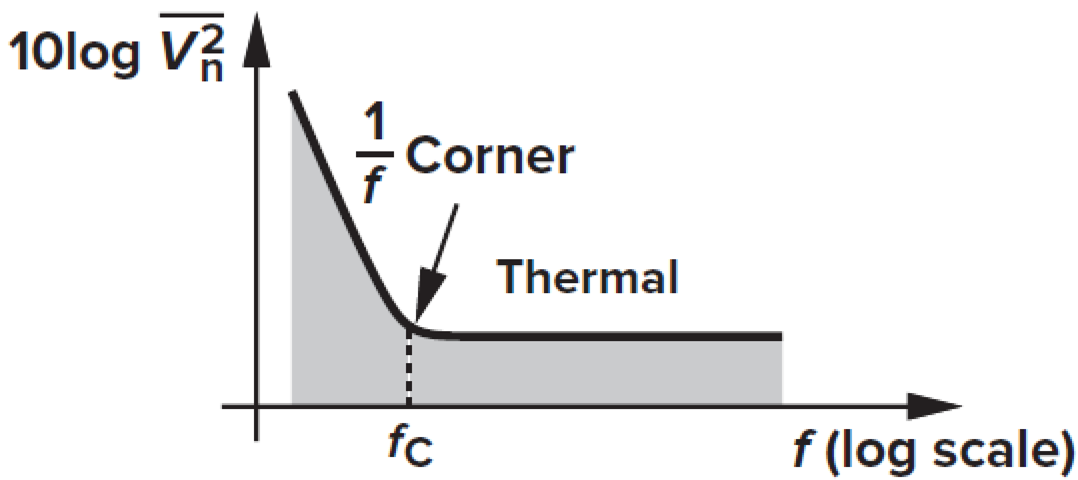
[Razavi, 2017]

Flicker Noise Corner

- Serves as a measure of what part of the band is mostly corrupted by flicker noise

$$4kT\gamma g_m = \frac{K}{C_{ox}WL} \cdot \frac{1}{f_C} \cdot g_m^2$$

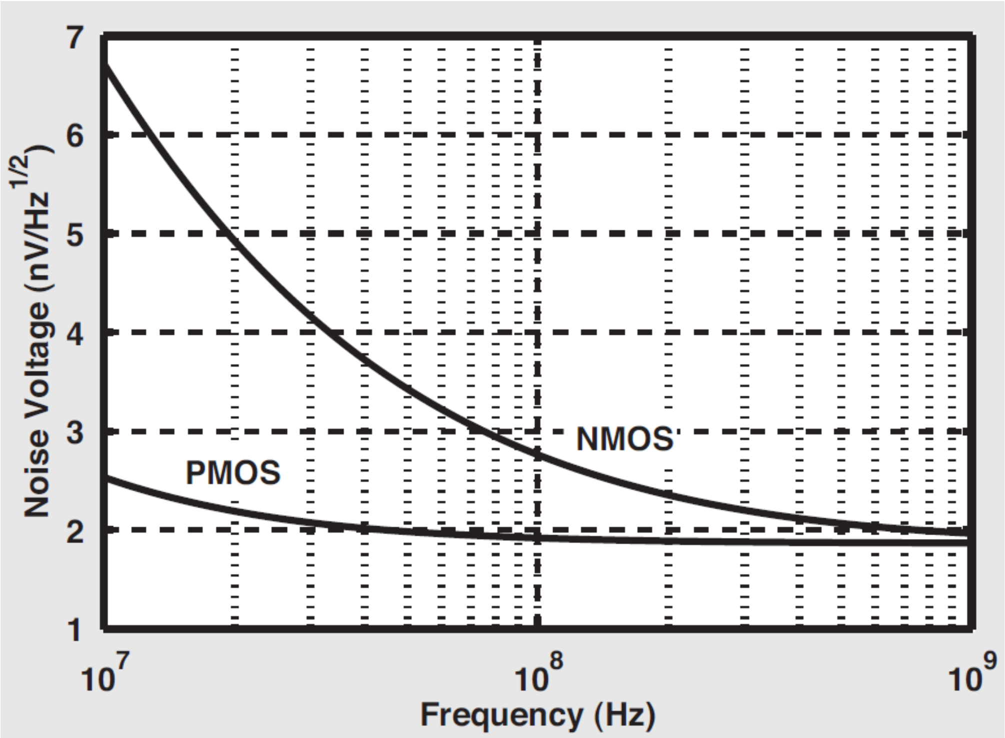
$$f_C = \frac{K}{\gamma C_{ox}WL} g_m \frac{1}{4kT}$$



[Razavi, 2017]

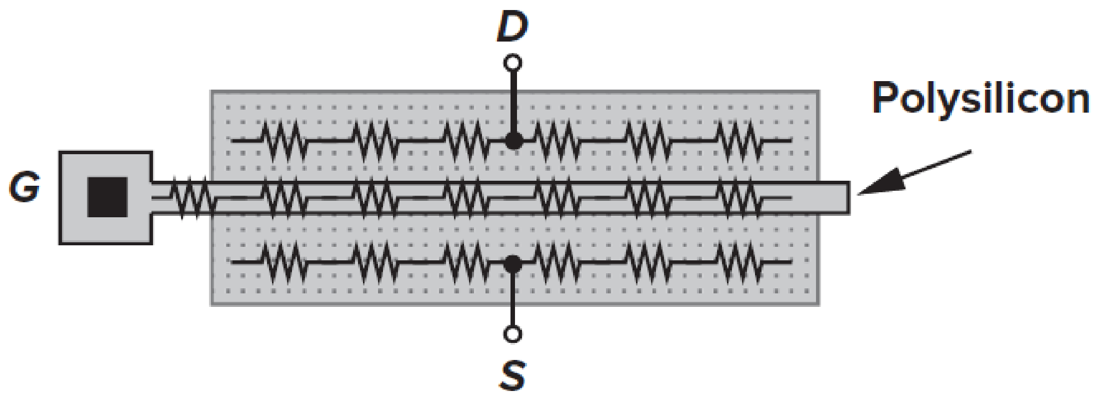
Short Channel MOS Flicker Noise

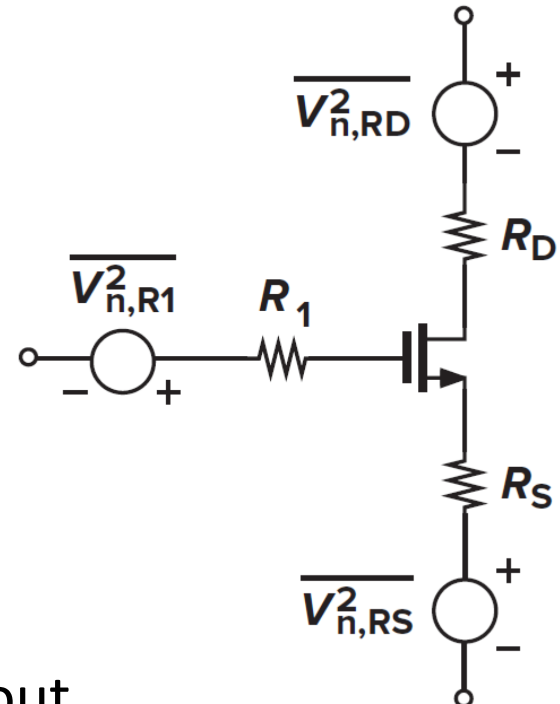
- PMOS and NMOS devices with W/L = 5 µm/40 nm and ID = 250 µA
- PMOS devices exhibit significantly less noise
- NMOS flicker noise corner is as high as several hundred megahertz
- For low flicker noise, gate area must be increased substantially



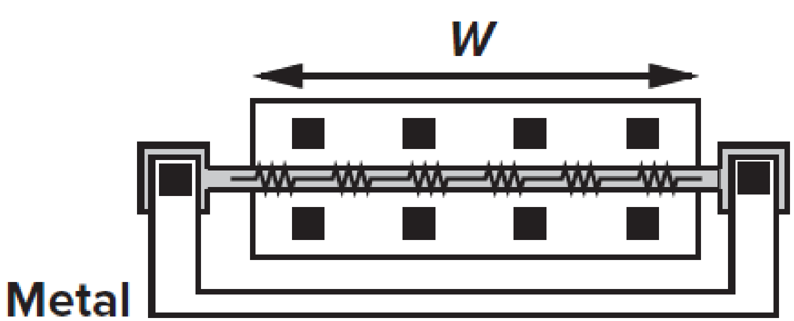
[Razavi, 2017]

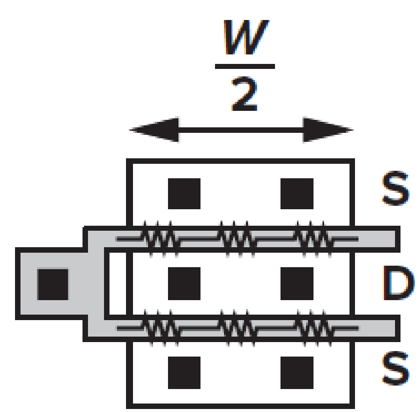
MOSFET Terminal Resistances Noise





- ❑ Terminal resistances reduced by proper layout



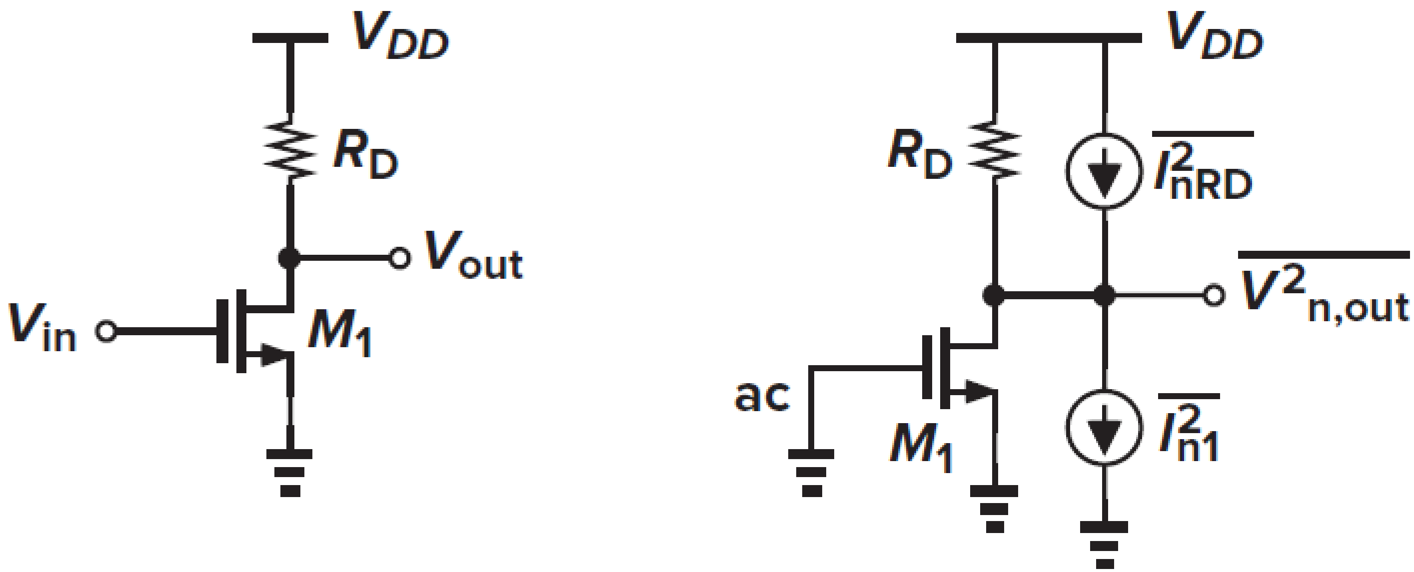


[Razavi, 2017]

Noise Analysis Procedure

- Identify the sources of noise (e.g., resistors and transistors) and write down the noise density of each
- Set the input signal to zero
- Find the transfer function from each noise source to the output (as if the source were a deterministic signal)
- Utilize the theorem $S_Y(f) = S_X(f)|H(f)|^2$ to calculate the output noise spectrum contributed by each noise source
- Add all of the output spectra, paying attention to correlated and uncorrelated sources
- This procedure gives the output noise spectrum, which must then be integrated to yield the total output noise

Ex: Output Noise in CS Amplifier

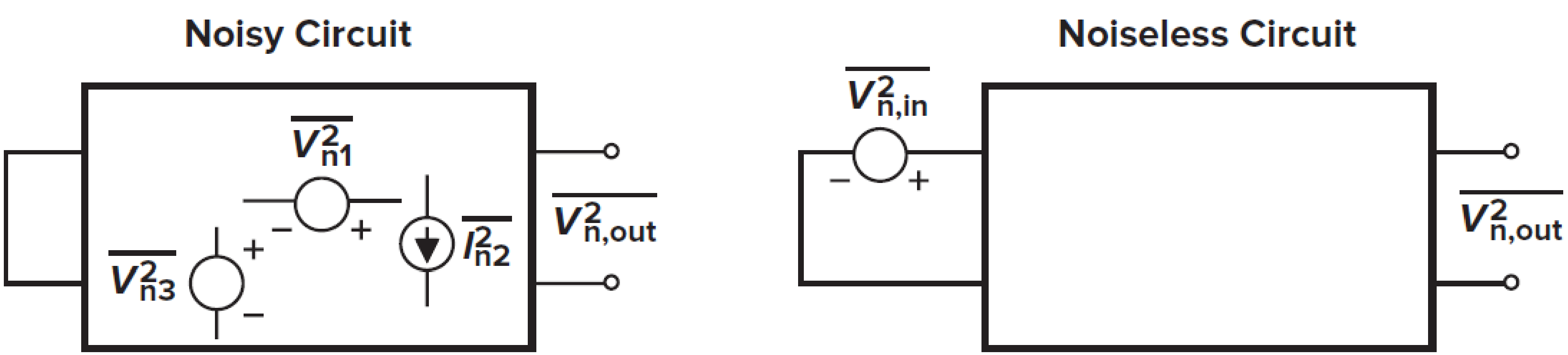


$$\overline{V^2_{n,out}} = \left(4kT\gamma g_m + \frac{K}{C_{ox}WL} \cdot \frac{1}{f} \cdot g_m^2 + \frac{4kT}{R_D}\right) R_D^2$$

[Razavi, 2017]

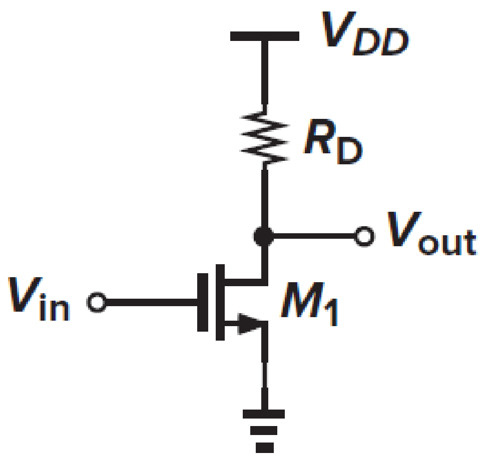
Input-Referred Noise

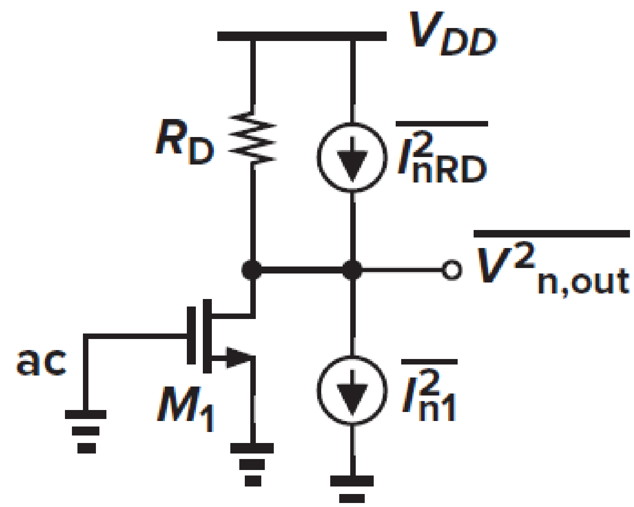
- The output-referred noise does not allow a fair comparison of the performance of different circuits because it depends on the gain
  - The signal is multiplied by the gain as well!
  - The noise should be referred to the input



[Razavi, 2017]

Ex: Input-Referred Noise in CS Amplifier





$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{A_v^2}$$

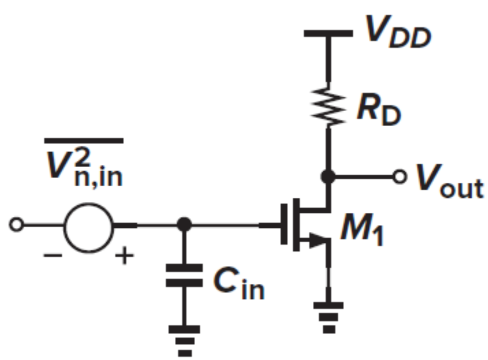
$$= \left(4kT\gamma g_m + \frac{K}{C_{ox}WL}\cdot\frac{1}{f}\cdot g_m^2 + \frac{4kT}{R_D}\right)R_D^2\frac{1}{g_m^2R_D^2}$$

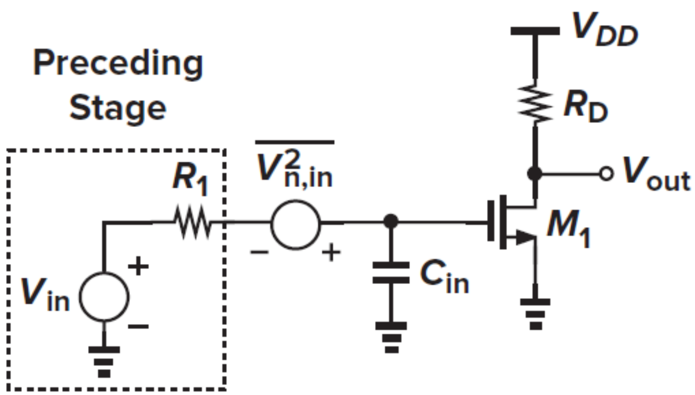
$$= 4kT\frac{\gamma}{g_m} + \frac{K}{C_{ox}WL}\cdot\frac{1}{f} + \frac{4kT}{g_m^2R_D}$$

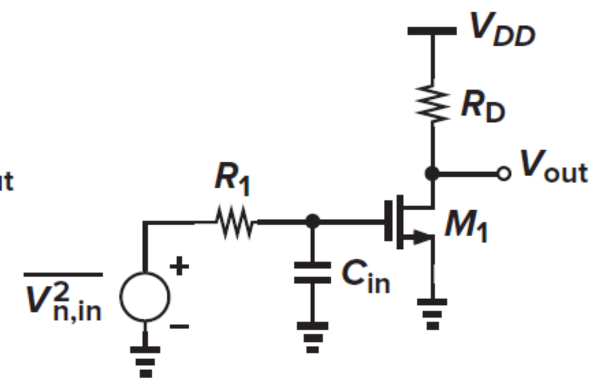
[Razavi, 2017]

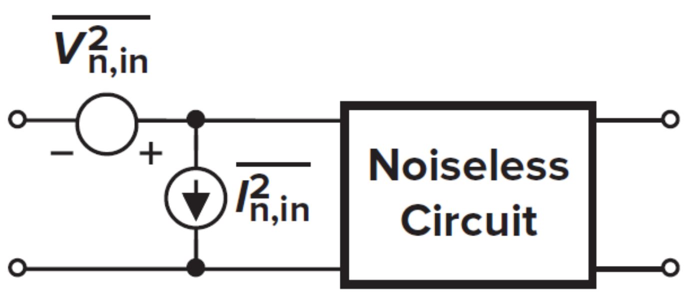
Input-Referred Noise

- The noise due to M1 channel should be independent of the source impedance R_1, but if $R_1 \rightarrow \infty$ output noise is zero!!!
- Modeling the noise by $V^2_{n,in}$ alone is not sufficient
- A complete model can be provided by $V^2_{n,in}$ and $I^2_{n,in}$





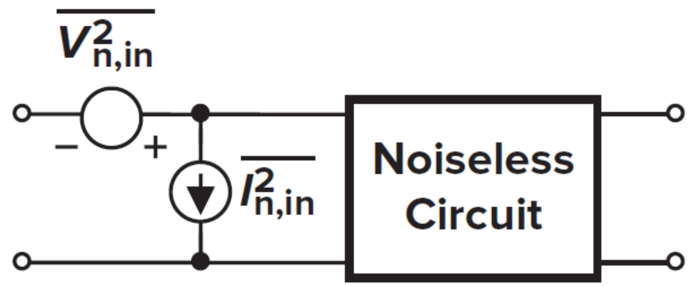


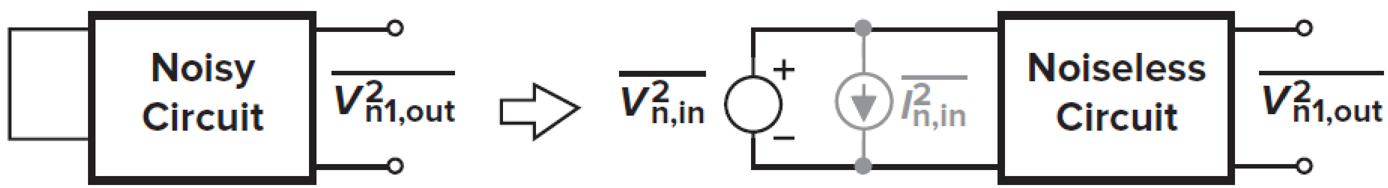


[Razavi, 2017]

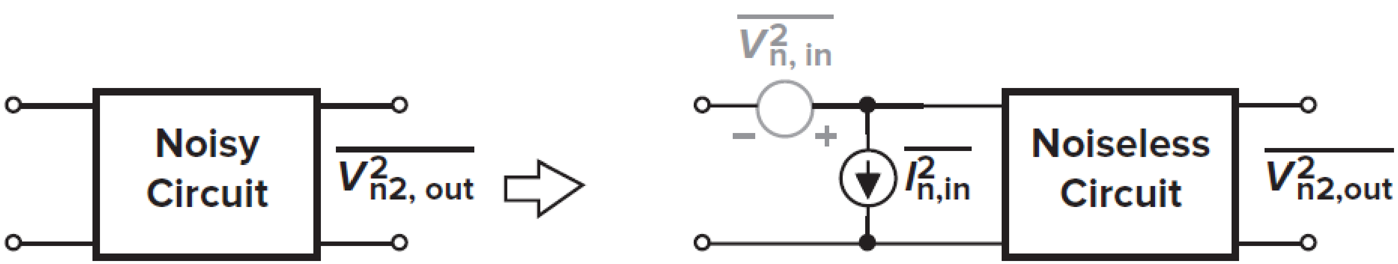
Input-Referred Noise

- Zero source impedance → Calculate $V_{n,in}^2$
- Infinite source impedance → Calculate $I_{n,in}^2$





(a)



(b)

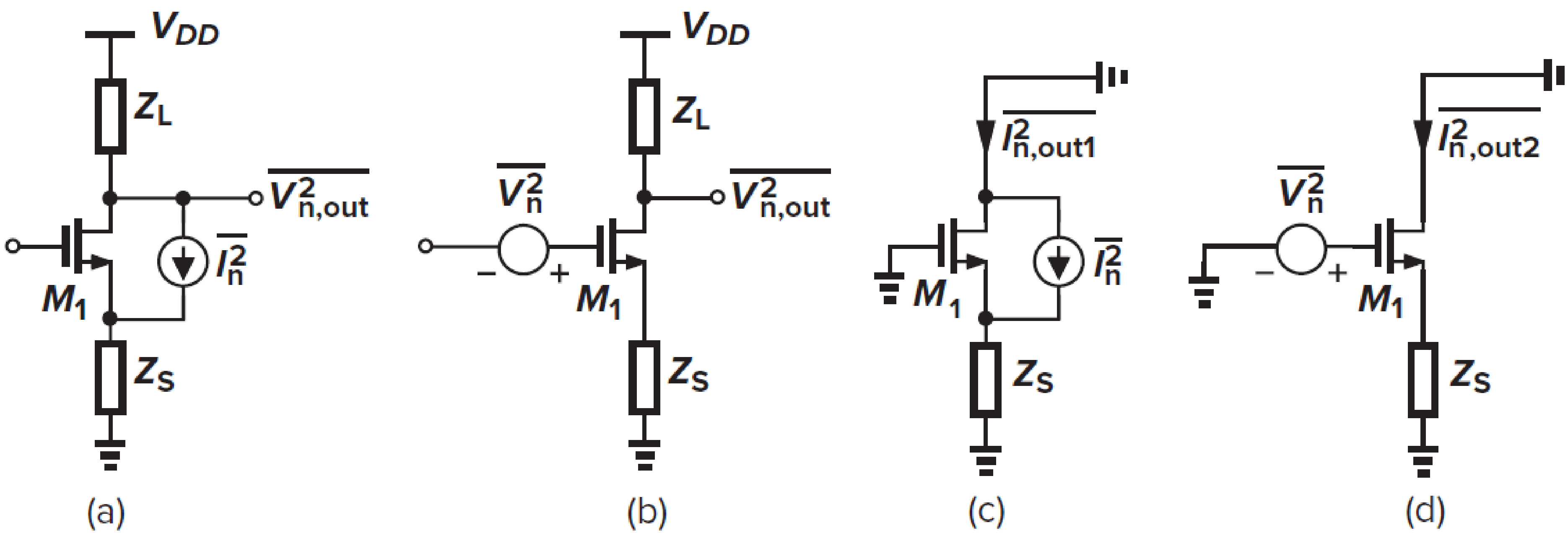
[Razavi, 2017]

In This Lecture: Part 2

- Noise in Amplifiers
  - Common source amplifier
  - Common gate amplifier
  - Common drain amplifier
  - Cascode amplifier
  - Differential amplifier
  - Common OTA topologies

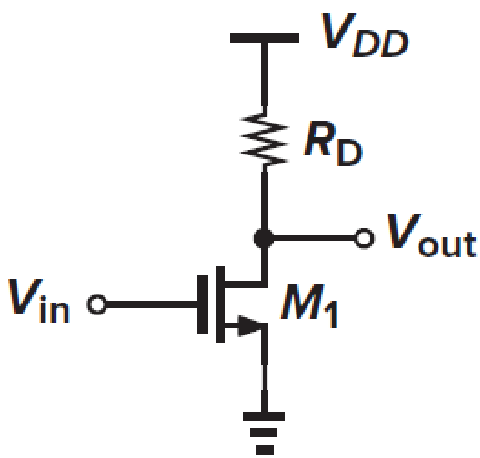
Useful Lemma

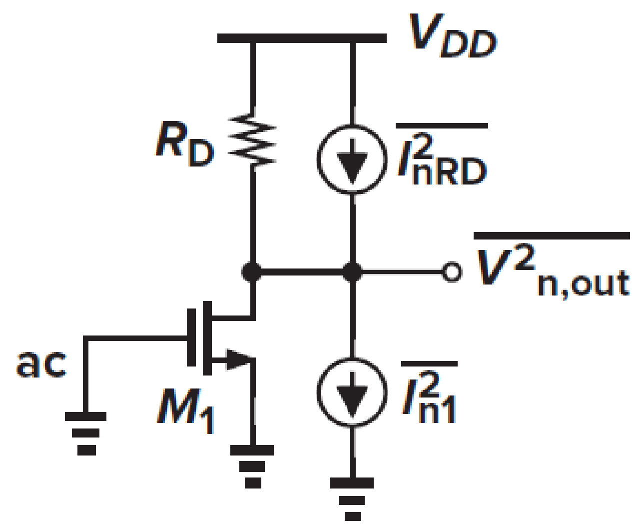
- You can show that (a) and (b) are equivalent if $I_n^2 = g_m^2 V_n^2$. Neglect body effect and CLM (the result is valid even if we consider them).
  - Hint: Show that the short circuit output current is the same in (c) and (d).



[Razavi, 2017]

Common Source Amplifier





$$\overline{V^2_{n,in}} = 4kT\left(\frac{\gamma}{g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{K}{C_{ox}WL}\frac{1}{f}$$

- If we focus on thermal noise and substitute for g_m

$$\overline{V^2_{n,in}} = 4kT\left[\frac{\gamma(V_{GS}-V_{TH})}{2I_D} + \frac{(V_{GS}-V_{TH})^2}{4I_D \cdot I_D R_D}\right]$$

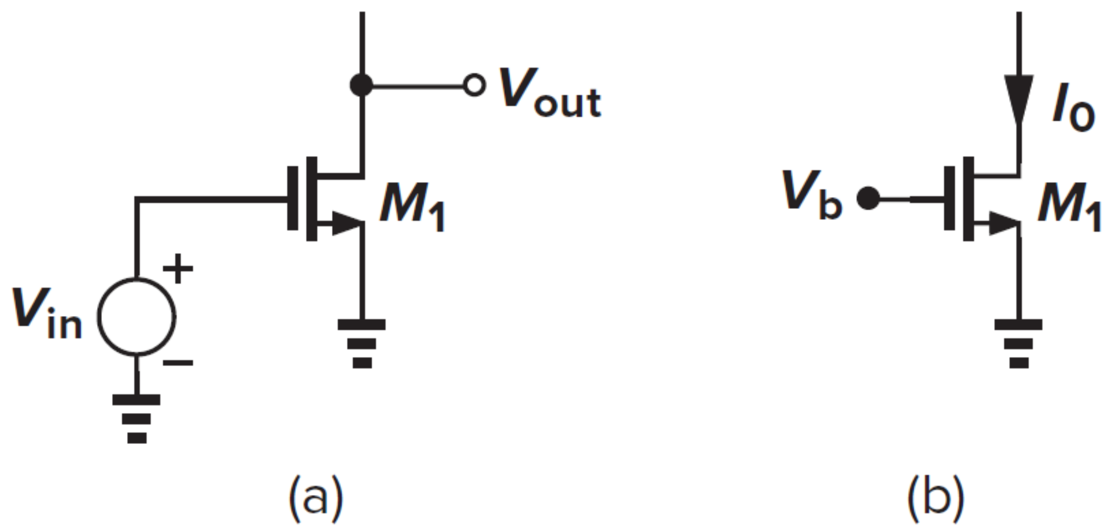
- **Tradeoff between noise and power consumption**

[Razavi, 2017]

Common Source Amplifier

- From noise perspective, do we want small or large g_m?
  - Amplifier/transconductor (a) → Maximize g_m
  - Constant current source (b) → Minimize g_m

$$\overline{V_{n,in}^2} = 4kT\left(\frac{\gamma}{g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{K}{C_{ox}WL}\frac{1}{f}$$



[Razavi, 2017]

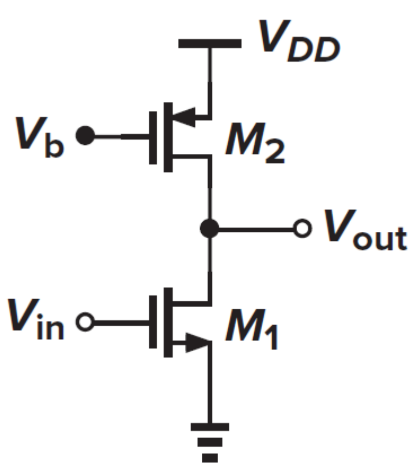
Common Source Amplifier

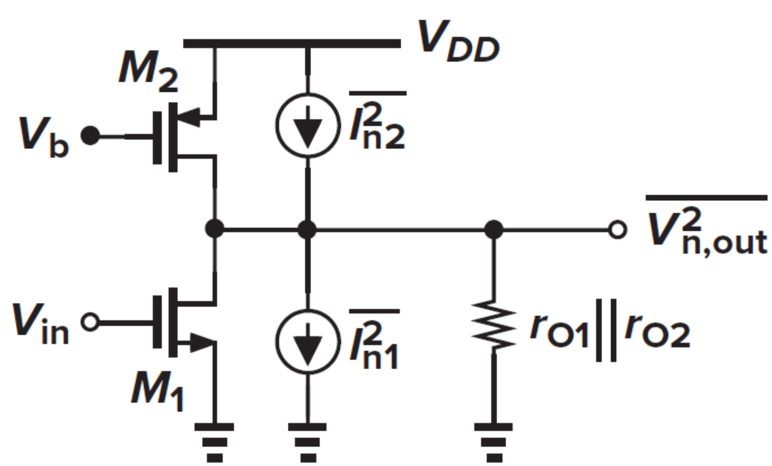
- Maximize g_{m1} (transconductor) and minimize g_{m2} (current source)

$$\overline{V_{n,out}^2} = 4kT(\gamma g_{m1} + \gamma g_{m2})(r_{O1} \| r_{O2})^2$$

$$\overline{V_{n,in}^2} = 4kT(\gamma g_{m1} + \gamma g_{m2})\frac{1}{g_{m1}^2}$$

$$= 4kT\gamma\left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right)$$



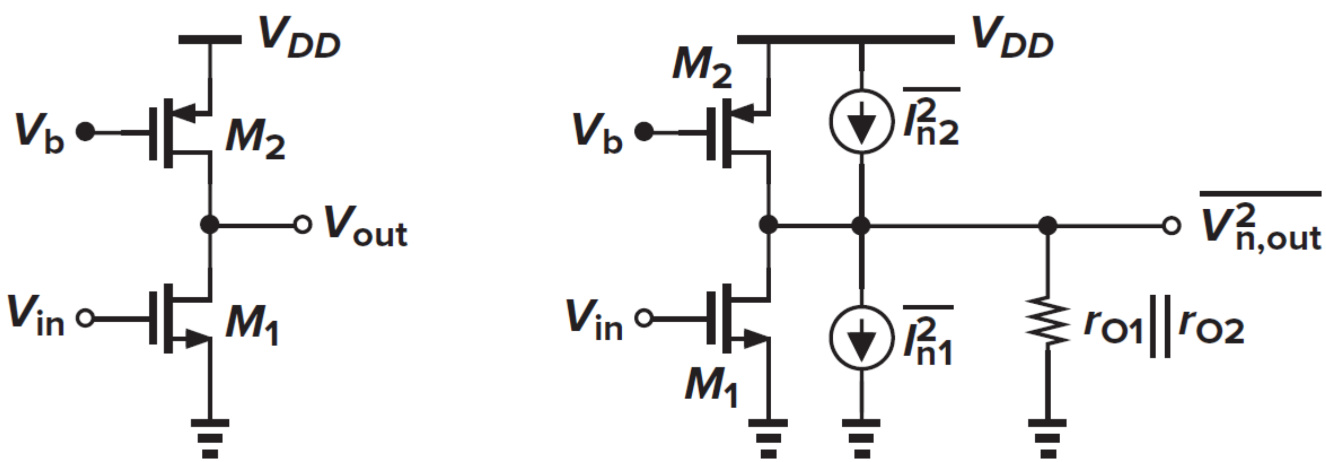


[Razavi, 2017]

Common Source Amplifier

- Assume a load capacitance C_L

$$
\begin{aligned}
V_{n,out,rms}^2 &= V_{n,out}^2 \times B_N \\
&= 4kT\gamma\left(\frac{1}{g_{m1}}+\frac{g_{m2}}{g_{m1}^2}\right)\times g_{m1}^2(r_{o1}//r_{o2})^2\times\frac{1}{4(r_{o1}//r_{o2})C_L} \\
&= (g_{m1}+g_{m2})(r_{o1}//r_{o2})\frac{kT\gamma}{C_L}
\end{aligned}
$$



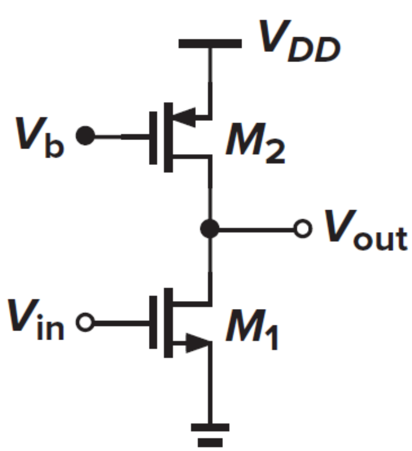
[Razavi, 2017]

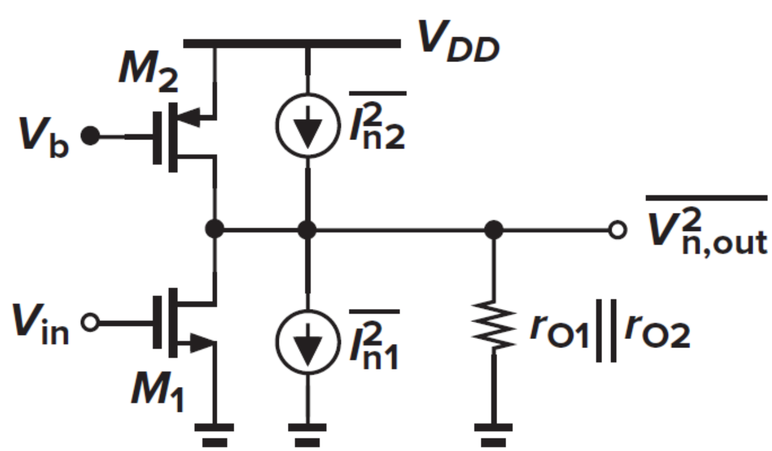
Common Source Amplifier

- Assume a sinusoidal input with amplitude $= V_m$

$$\mathrm{SNR}_{out} = \left[\frac{g_{m1}(r_{O1}\|r_{O2})V_m}{\sqrt{2}}\right]^2 \cdot \frac{1}{\gamma(g_{m1}+g_{m2})(r_{O1}\|r_{O2})(kT/C_L)}$$

$$= \frac{C_L}{2\gamma kT} \cdot \frac{g_{m1}^2(r_{O1}\|r_{O2})}{g_{m1}+g_{m2}} V_m^2$$





[Razavi, 2017]

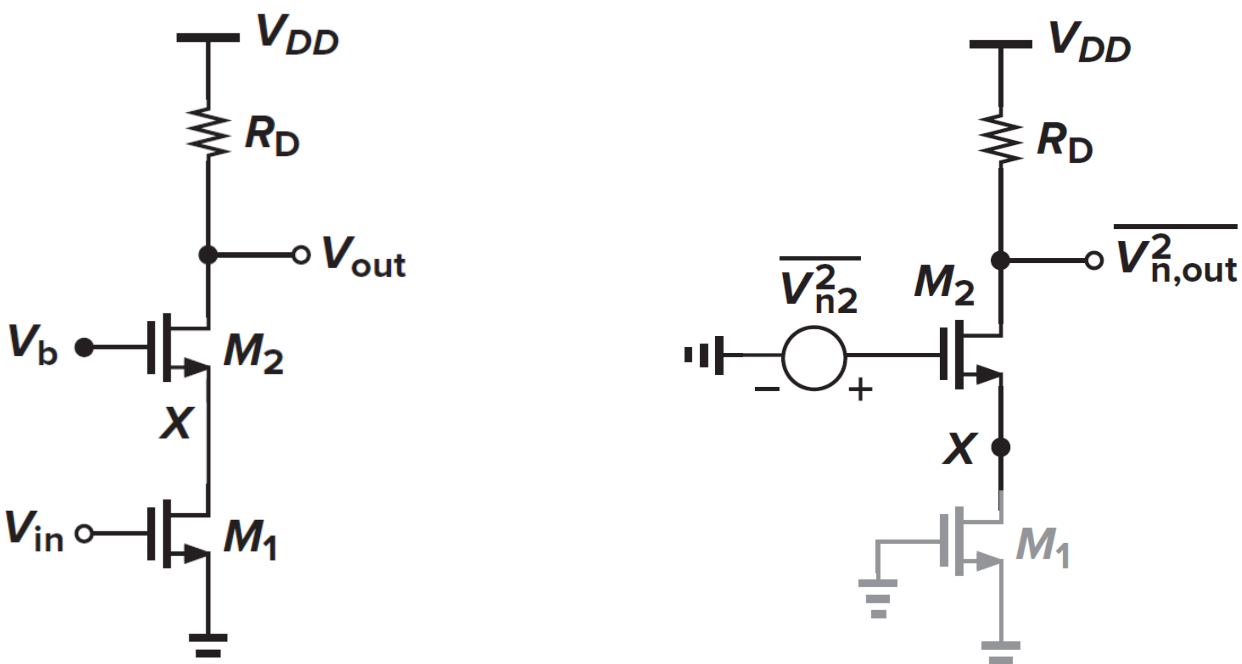
Common Gate and Common Drain

- Read [Razavi, 2017] Section 7.4.2 and 7.4.3 (FYI)

Cascode Amplifier

- Noise of M2 is negligible because V_{out}/V_{n2} is much smaller than V_{out}/V_{n1} (dominant noise contributions from high gain paths)
  - But M2 noise contribution may be large at high frequencies

$$\overline{V_{n,in}^2}|_{M1,RD} = 4kT\left(\frac{\gamma}{g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right)$$

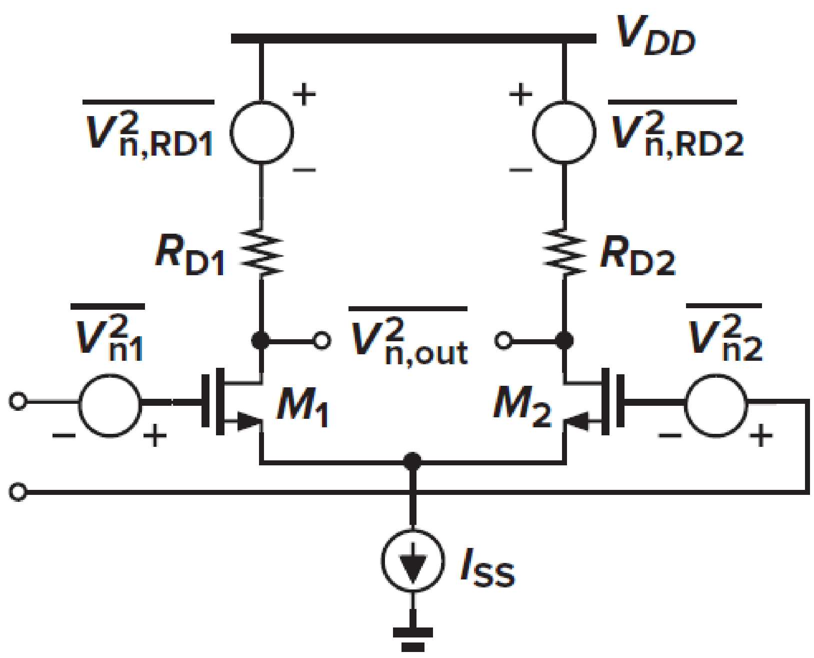


[Razavi, 2017]

Differential Pair with Resistive Load

- Twice the noise density of a CS amplifier (and twice the power consumption)

$$\overline{V_{n,in,tot}^2} = 8kT\left(\frac{\gamma}{g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{2K}{C_{ox}WL}\frac{1}{f}$$



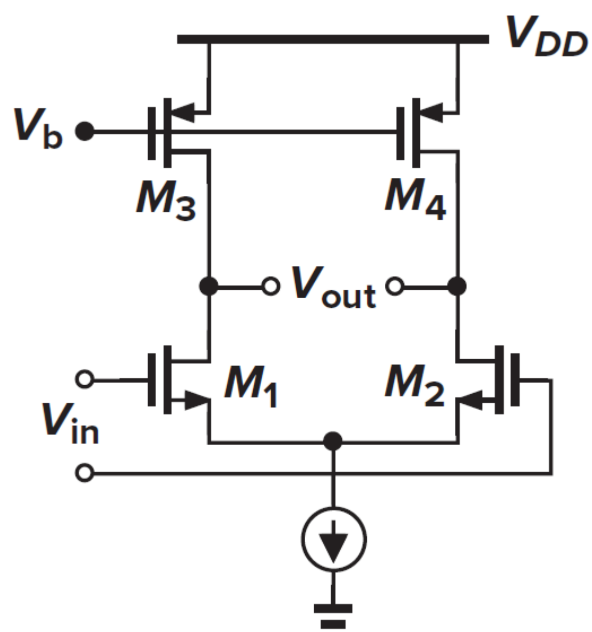
[Razavi, 2017]

Differential Pair with Active Load

- Noise sources of M1 and M2 are already input referred
- Noise of M3 and M4 can be viewed as noise of PMOS input pair

$$\overline{V_{n,in}^2} = 2\overline{V_{n1}^2} + 2\frac{g_{m3}^2}{g_{m1}^2}\overline{V_{n3}^2}$$

$$\overline{V_{n,in}^2} = 8kT\gamma\left(\frac{1}{g_{m1}} + \frac{g_{m3}}{g_{m1}^2}\right) + \frac{2K_N}{C_{ox}(WL)_1 f} + \frac{2K_P}{C_{ox}(WL)_3 f}\frac{g_{m3}^2}{g_{m1}^2}$$

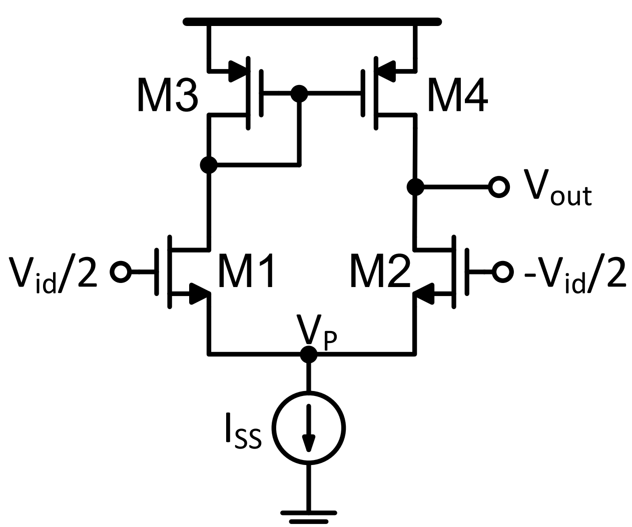


[Razavi, 2017]

Differential Pair with CM Load (5T OTA)

- Noise sources of M1 and M2 are already input referred
- M3 is diode connected: V_{n3} in series with $1/g_{m3}$
  - Drain of M1 is H.I.N.: $V_{gs4} \approx V_{n3}$
- Same as last slide: $\overline{V_{n,in}^2} = 2\overline{V_{n1}^2} + 2\frac{g_{m3}^2}{g_{m1}^2}\overline{V_{n3}^2}$

$$\overline{V_{n,in}^2} = 8kT\gamma\left(\frac{1}{g_{m1}} + \frac{g_{m3}}{g_{m1}^2}\right) + \frac{2K_N}{C_{ox}(WL)_1 f} + \frac{2K_P}{C_{ox}(WL)_3 f}\frac{g_{m3}^2}{g_{m1}^2}$$

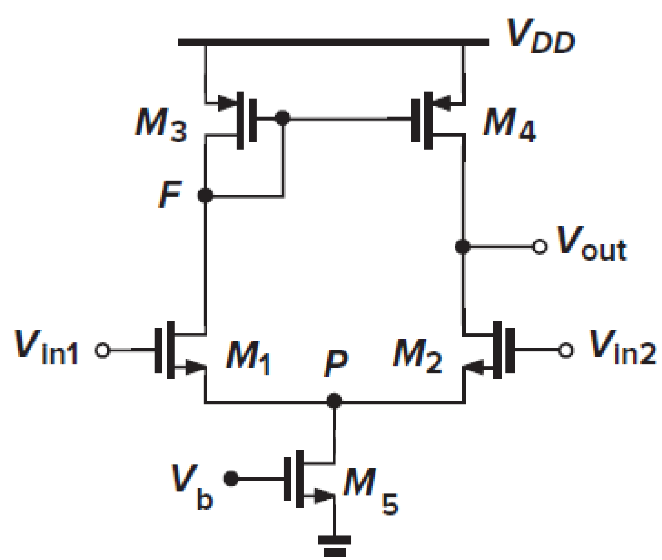


Differential Pair with CM Load (5T OTA)

- Noise of tail current source appears at output, even if we assume perfect matching
  - V_{out} and V_X respond similarly to CM input → $R_{out} = 1/g_{m3}$
- Noise of tail current source split equally between M1 and M2

$$V_{n,in,M5}^2 = \left(\frac{V_{n5}}{2}\right)^2 \cdot g_{m5}^2 \cdot \frac{1}{g_{m3}^2} \cdot \frac{1}{g_{m1}^2 (r_{o2}//r_{o4})^2}$$

- This noise component is relatively negligible



[Razavi, 2017]

Differential Pair: Design for Low Noise

$$\overline{V_{n,in}^2} = 8kT\gamma\left(\frac{1}{g_{m1}} + \frac{g_{m3}}{g_{m1}^2}\right) + \frac{2K_N}{C_{ox}(WL)_1 f} + \frac{2K_P}{C_{ox}(WL)_3 f}\frac{g_{m3}^2}{g_{m1}^2}$$

- Thermal noise: $V_{n,in}^2 = \frac{8kT\gamma}{g_{m1}}\left(1 + \frac{g_{m3}}{g_{m1}}\right)$
  - Maximize g_{m1} → I_D ↑ → power consumption ↑
  - Minimize g_{m3} → V_{ov3} ↑ → headroom ↓
- Flicker noise: $V_{n,in}^2 = \frac{2K_N}{C_{ox}W_1L_1}\frac{1}{f} + \frac{2K_PL_1}{C_{ox}W_1L_3^2}\frac{\mu_p}{\mu_n}\frac{1}{f}$

$$V_{n,in,rms}^2 = \frac{2}{C_{ox}W_1}\left(\frac{K_N}{L_1} + \frac{K_PL_1}{L_3^2}\frac{\mu_p}{\mu_n}\right)\ln\frac{f_{max}}{f_{min}}$$

  - If $L_1 = L_3$: NMOS input pair dominates since $K_N > K_P\ and\ \mu_n > \mu_p$
  - $W_{3,4}$ has no effect
  - Increase W_1 → I_D ↑ (if V_{ov} is constant) → power consumption ↑
  - Increase L_3 → V_{ov3} ↑ → headroom ↓ → also area and parasitics ↑

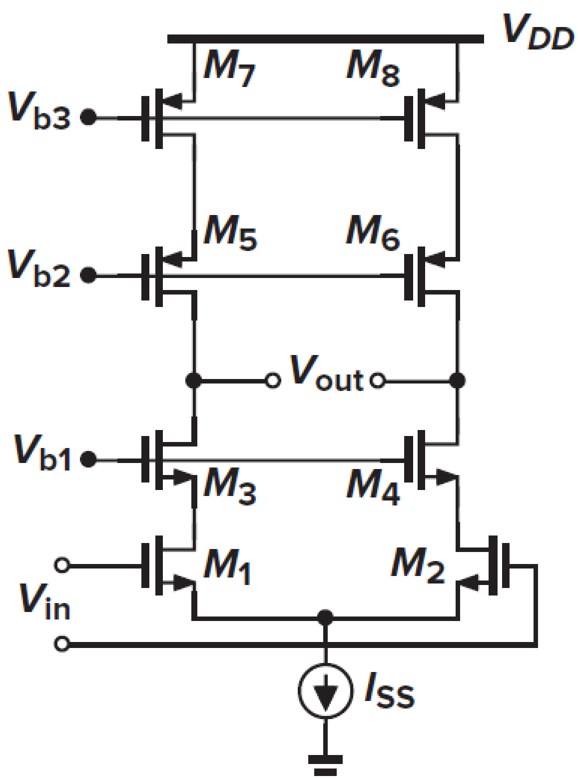
Noise in OTAs

- In all OTA topologies, at least four devices contribute to the input noise: two input transistors and two "load" transistors
- Quick check to determine if a transistor dominant noise contributors:
  - Estimate the gain from the gate of the transistor to the output: is it comparable to the gain from the input to the output?

Telescopic Cascode

- The noise of the cascode devices is negligible at low frequencies
- M1–M2 and M7–M8 are the primary noise sources

$$\overline{V_n^2} = 4kT\left(2\frac{\gamma}{g_{m1,2}} + 2\frac{\gamma g_{m7,8}}{g_{m1,2}^2}\right) + 2\frac{K_N}{(WL)_{1,2}C_{ox}f} + 2\frac{K_P}{(WL)_{7,8}C_{ox}f}\frac{g_{m7,8}^2}{g_{m1,2}^2}$$

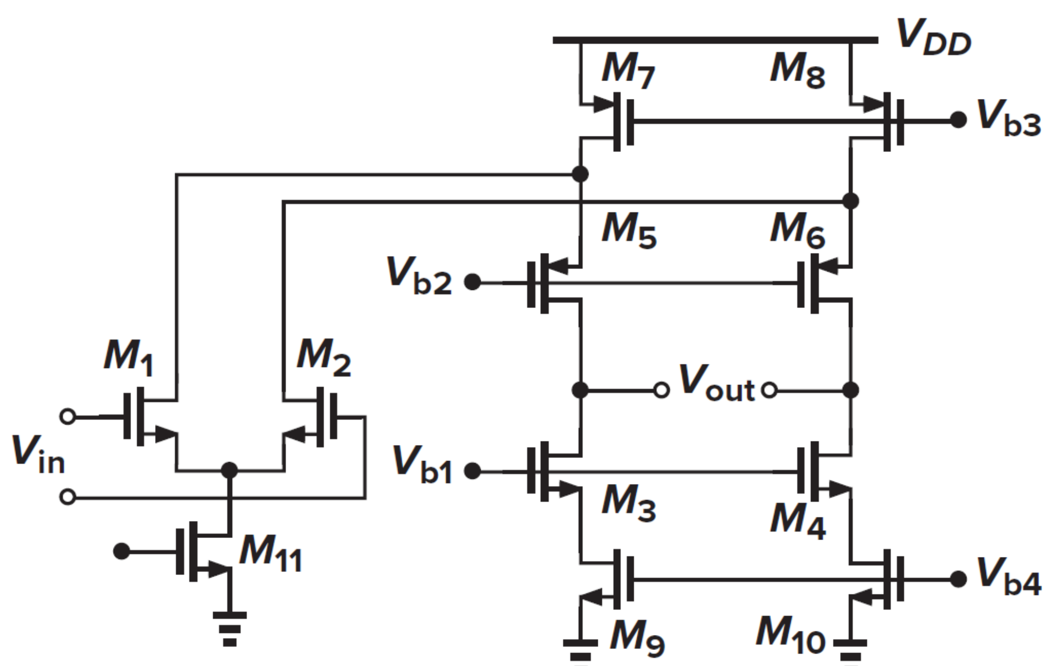


[Razavi, 2017]

Folded Cascode

- The noise of the cascode devices is negligible at low frequencies
- M1–M2, M7–M8, and M9–M10 are the primary noise sources

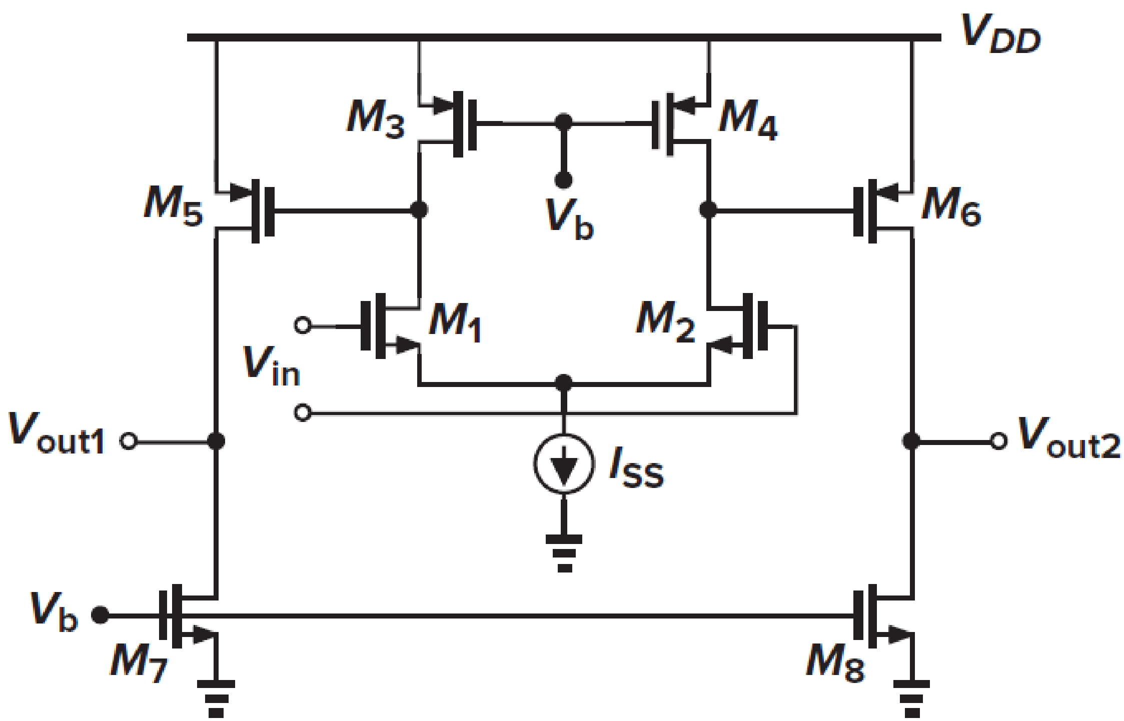
$$\overline{V_{n,int}^2} = 8kT\left(\frac{\gamma}{g_{m1,2}} + \gamma\frac{g_{m7,8}}{g_{m1,2}^2} + \gamma\frac{g_{m9,10}}{g_{m1,2}^2}\right)$$



[Razavi, 2017]

Two-Stage OTA

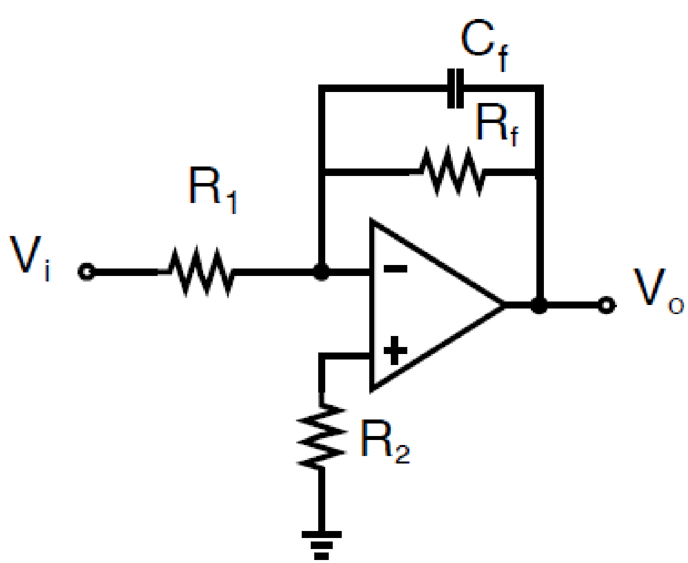
- The noise resulting from the second stage is usually negligible because it is divided by the gain of the first stage when referred to the main input

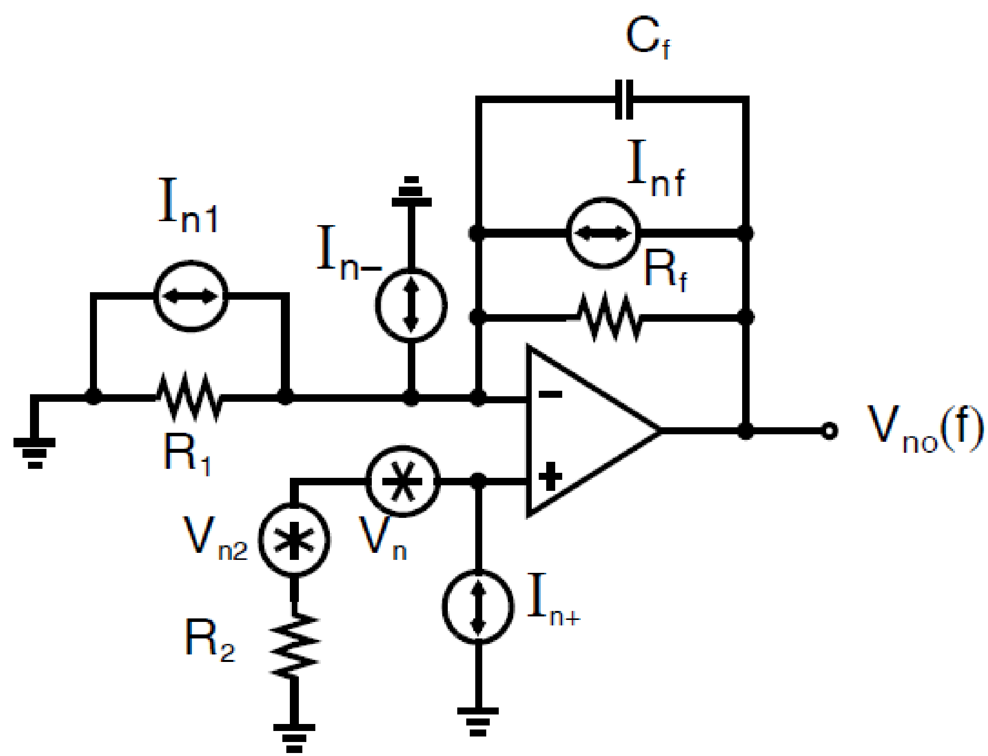


[Razavi, 2017]

Noise in Closed Loop OTA / Op-Amp Circuits

- Example: Inverting amplifier (Rf and R1) and LPF (Rf and Cf)
- $V_n, I_{n-}, and\ I_{n+}$ model the op-amp equivalent input noise
- HW: Read Section 9.4.1 and solve Example 9.10 in Johns and Martin



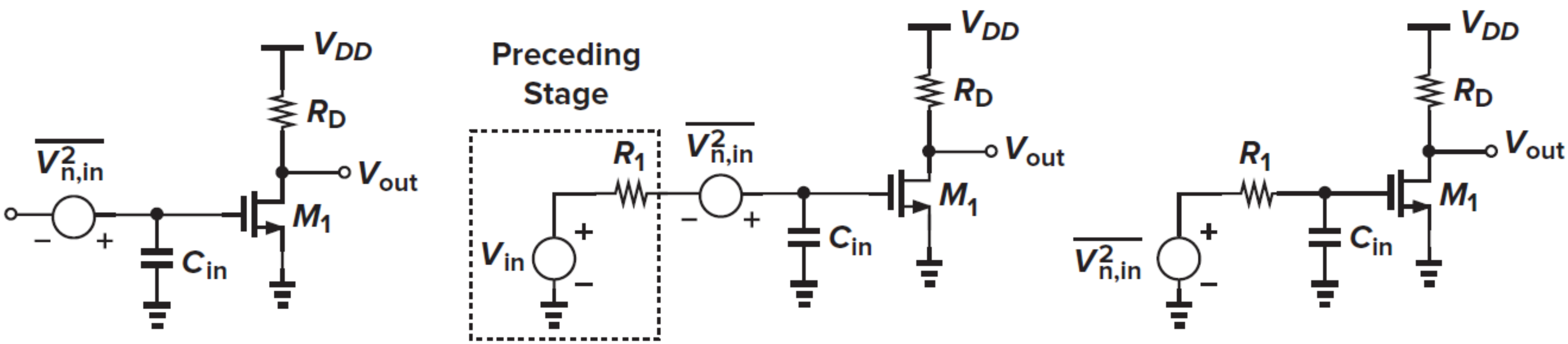


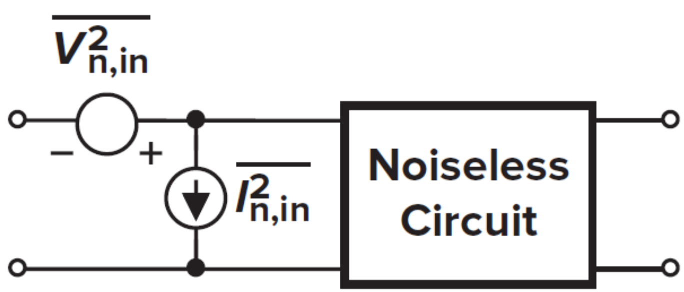
[Johns and Martin, 2015]

Thank you!

Input-Referred Noise

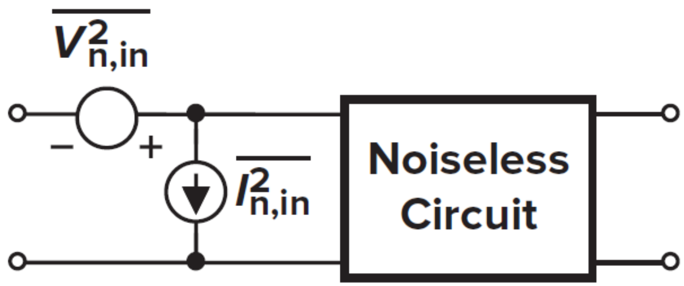
- The noise due to M1 channel should be independent of the source impedance R_1 → If $R_1 \rightarrow \infty$ output noise is zero!!!
- Modeling the noise by $V_{n,in}^2$ alone is not sufficient
- A complete model can be provided by $V_{n,in}^2$ and $I_{n,in}^2$

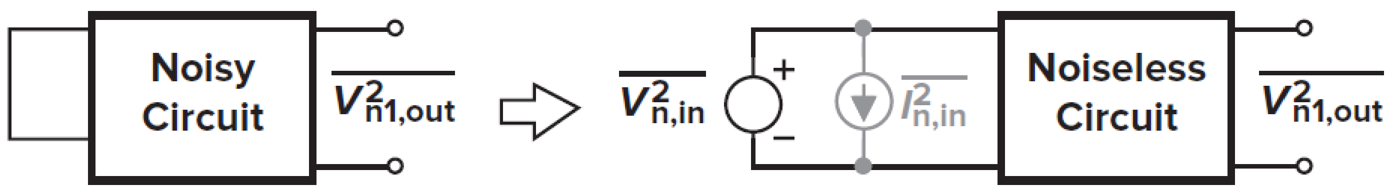




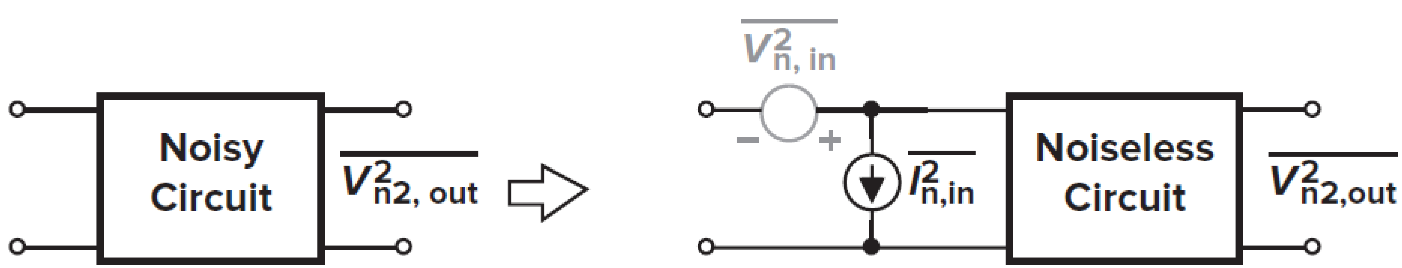
Input-Referred Noise

- Zero source impedance → Calculate $V_{n,in}^2$
- Infinite source impedance → Calculate $I_{n,in}^2$





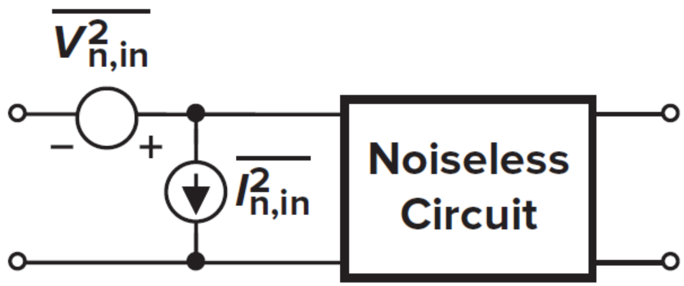
(a)



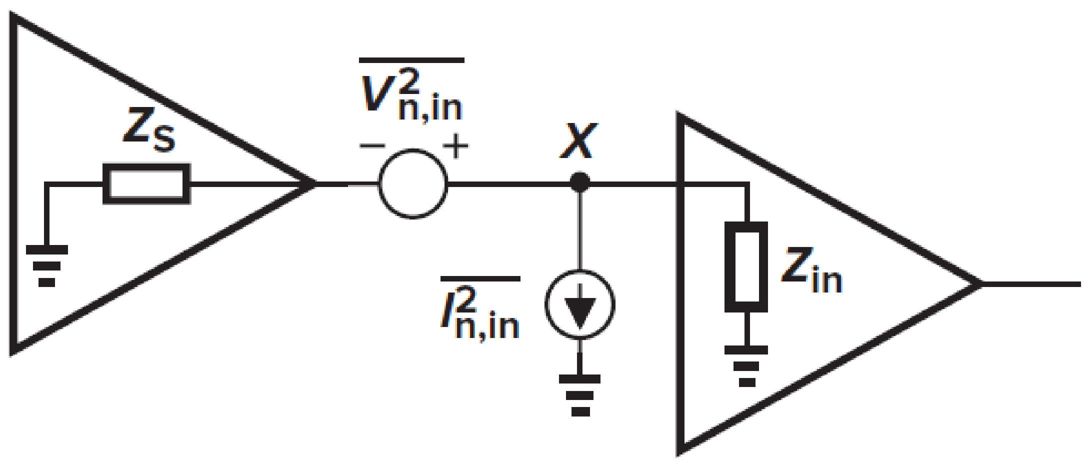
(b)

Input-Referred Noise

- Zero source impedance → Calculate $V^2_{n,in}$
- Infinite source impedance → Calculate $I^2_{n,in}$



$$V_{n,X} = \frac{Z_{in}}{Z_{in} + Z_S} V_{n,in} + \frac{Z_{in} Z_S}{Z_{in} + Z_S} I_{n,in}$$

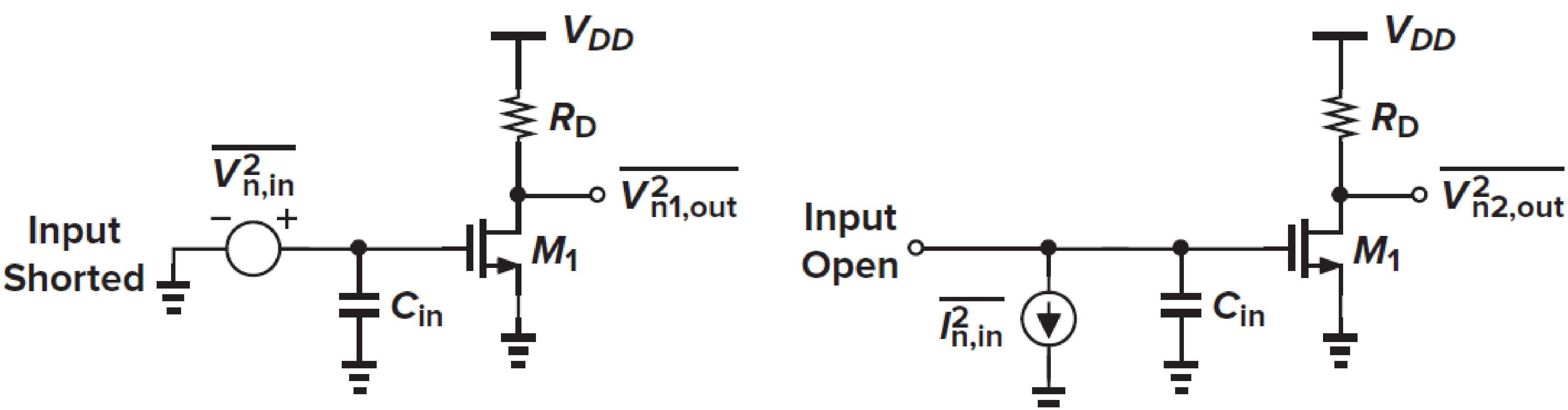


Ex: Input-Referred Noise in CS Amplifier

$$\overline{V_{n,in}^2} = 4kT\frac{\gamma}{g_m} + \frac{4kT}{g_m^2 R_D}$$

$$\overline{V_{n2,out}^2} = \overline{I_{n,in}^2}\left(\frac{1}{C_{in}\omega}\right)^2 g_m^2 R_D^2 = \left(4kT\gamma g_m + \frac{4kT}{R_D}\right) R_D^2$$

$$\overline{I_{n,in}^2} = (C_{in}\omega)^2 \frac{4kT}{g_m^2}\left(\gamma g_m + \frac{1}{R_D}\right)$$



(a) (b)

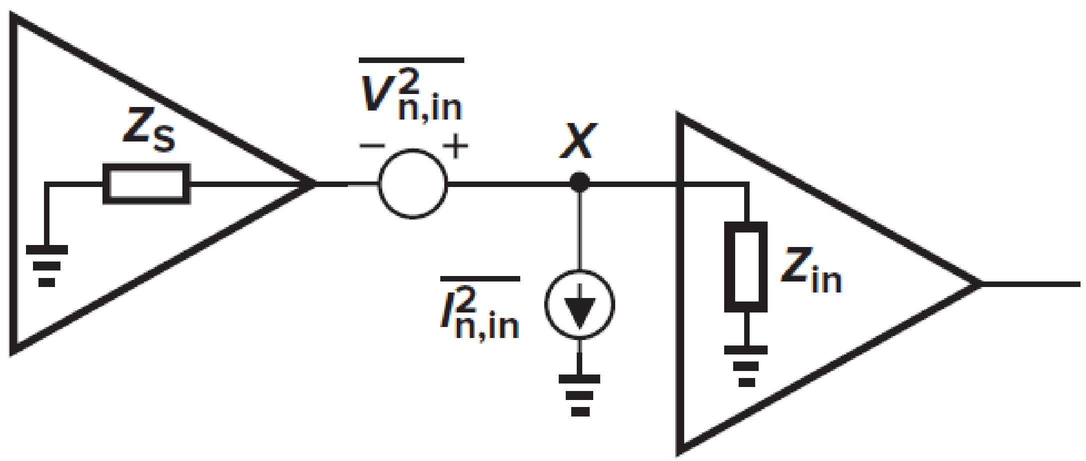
Input-Referred Noise

$$V_{n,X} = \frac{Z_{in}}{Z_{in} + Z_S} V_{n,in} + \frac{Z_{in} Z_S}{Z_{in} + Z_S} I_{n,in}$$

- $I_{n,in}^2$ can be neglected if

$$\overline{I_{n,in}^2} |Z_S|^2 \ll \overline{V_{n,in}^2} \quad \rightarrow \quad |Z_S|^2 \ll \frac{\overline{V_{n,in}^2}}{\overline{I_{n,in}^2}}$$

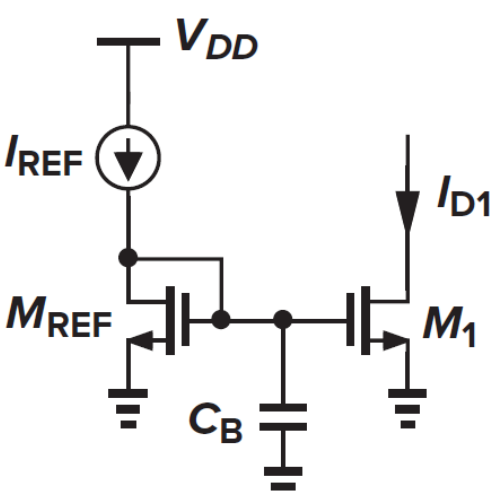
- It is all about the output impedance of the preceding stage

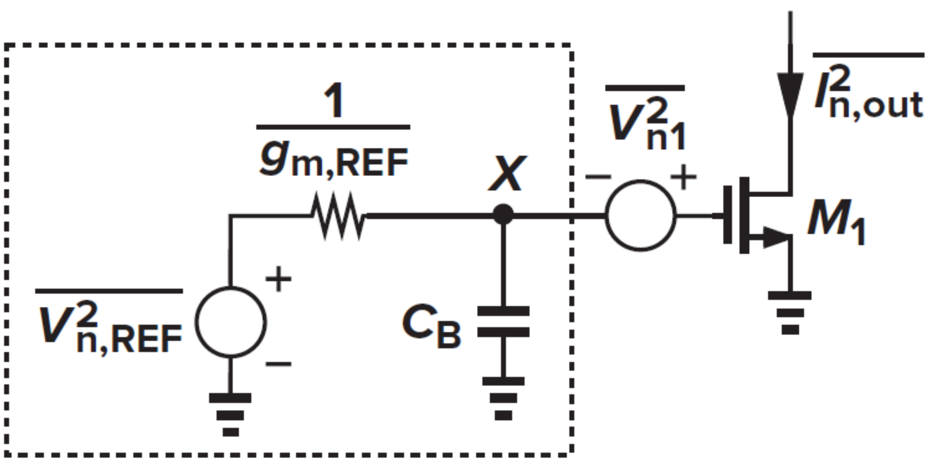


Current Mirror

- The diode-connected device may contribute substantial flicker noise unless an extremely large bypass capacitor is used
- Let $(W/L)_{REF} = \frac{1}{N}(W/L)_1 \rightarrow V_{n,REF}^2 = NV_{n1}^2$ (flicker noise $\propto \frac{1}{WL}$)

$$\overline{I_{n,out}^2} = \left(\frac{g_{m,REF}^2}{C_B^2\omega^2 + g_{m,REF}^2} \overline{V_{n,REF}^2} + \overline{V_{n1}^2} \right) g_{m1}^2 = \left(\frac{N g_{m,REF}^2}{C_B^2\omega^2 + g_{m,REF}^2} + 1 \right) g_{m1}^2 \overline{V_{n1}^2}$$





Current Mirror

$$\overline{I_{n,out}^2} = \left(\frac{g_{m,REF}^2}{C_B^2\omega^2 + g_{m,REF}^2} \overline{V_{n,REF}^2} + \overline{V_{n1}^2} \right) g_{m1}^2 = \left(\frac{N g_{m,REF}^2}{C_B^2\omega^2 + g_{m,REF}^2} + 1 \right) g_{m1}^2 \overline{V_{n1}^2}$$

- For M_{REF} contribution to be negligible: $(N-1)g_{m,REF}^2 \ll C_B^2\omega^2$
  - Let N = 5, $g_m = 5mS$, and $\omega = 1MHz$ → $C_B \gg 500pF$!!
- A resistance R_B can be used to lower the filter cutoff frequency
  - R_B can be implemented as a MOSFET in triode