وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 08
Frequency Response (1)

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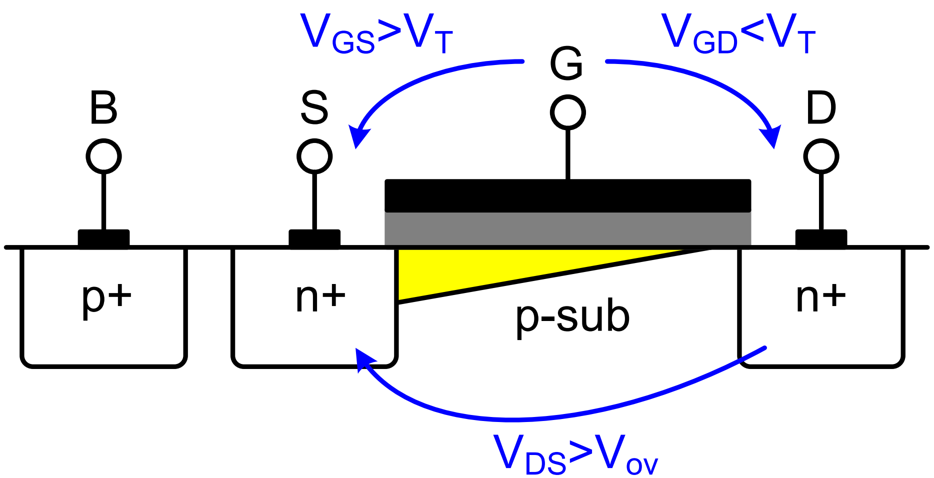
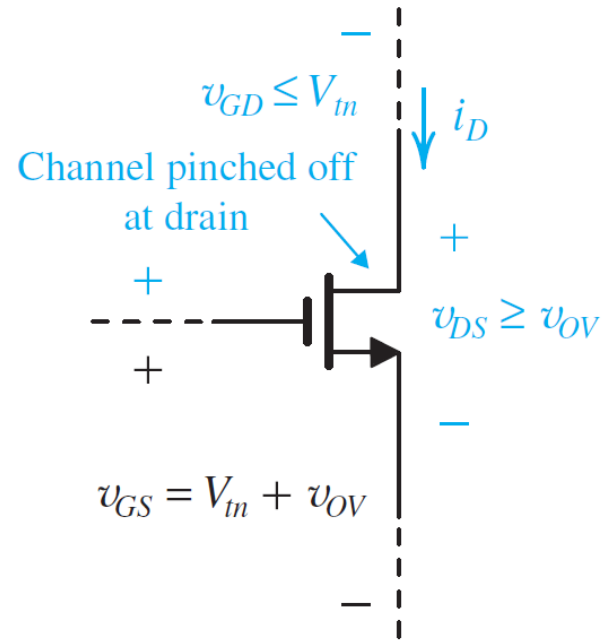
MOSFET in Saturation

- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2 (1 + \lambda V_{DS})$$





[Sedra/Smith, 2015]

Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_T$

ON

$V_{GS} > V_T$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_T$

$$I_D = \mu C_{ox} \frac{W}{L} \left(V_{ov} V_{DS} - \frac{V_{DS}^2}{2} \right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_T$

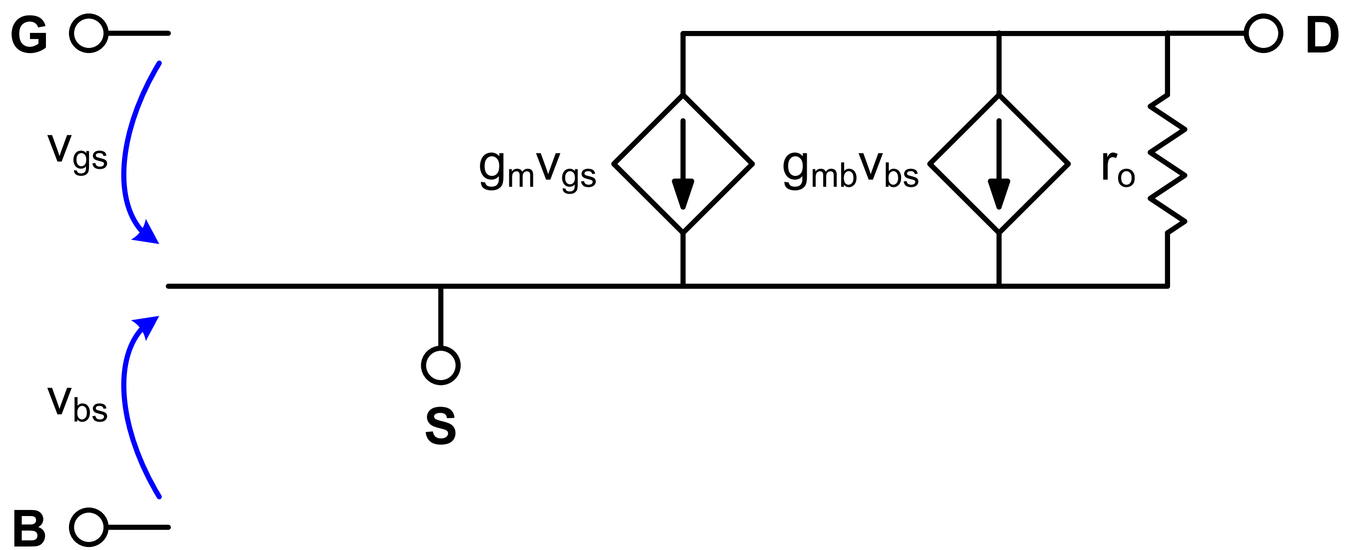
$$I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} V_{ov}^2 (1 + \lambda V_{DS})$$

Low-Frequency Small-Signal Model

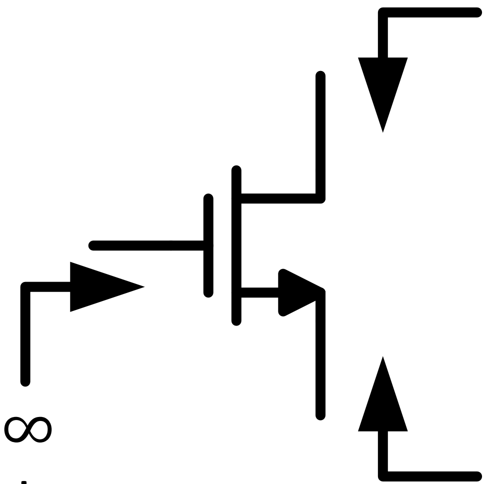
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox}\frac{W}{L}V_{ov} = \sqrt{\mu C_{ox}\frac{W}{L}\cdot 2I_D} = \frac{2I_D}{V_{ov}}$$

$$g_{mb} = \eta g_m, \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}} = \frac{1}{\lambda I_D}, \qquad \lambda \propto \frac{1}{L}$$



Rin/out Shortcuts Summary



$$r_o[1 + (g_m + g_{mb})R_S]$$

H.I.N.

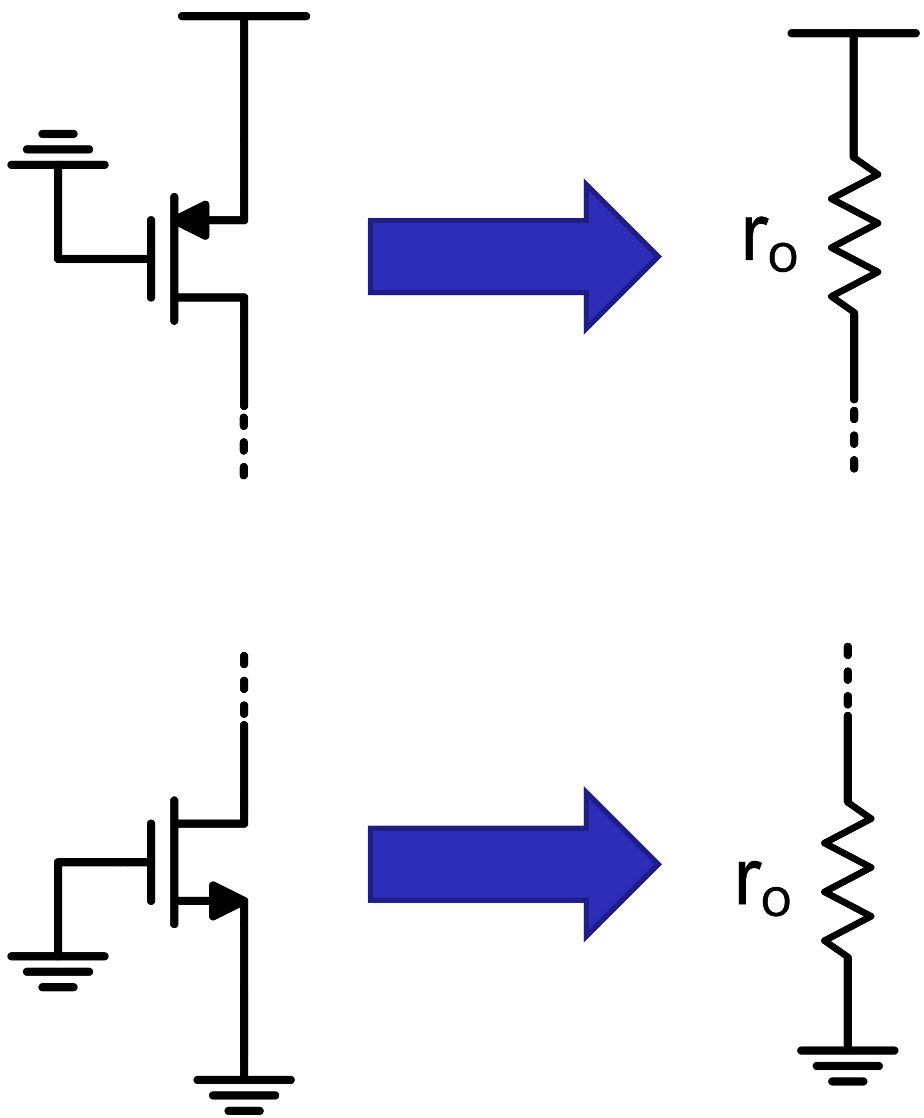
∞

At low frequencies **ONLY**

$$\frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

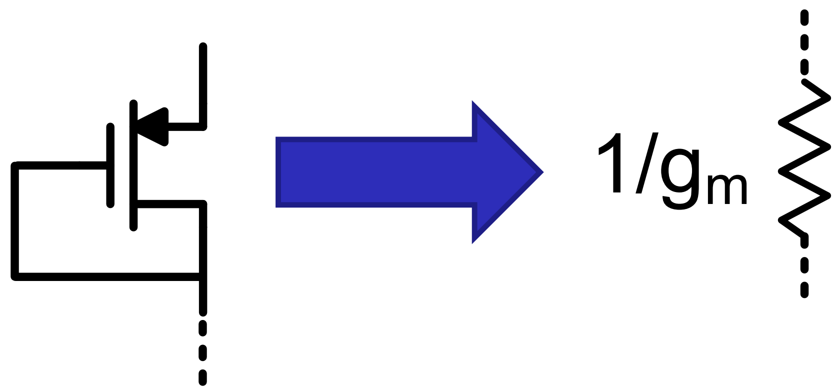
L.I.N.

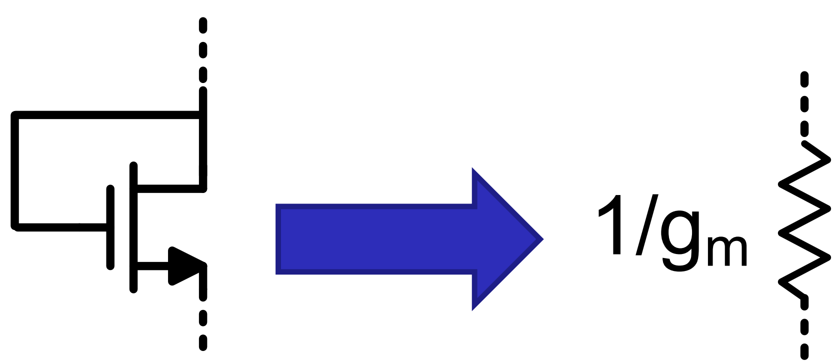
Active Load (Source OFF)



Diode Connected (Source Absorption)

- Always in saturation
- Bulk effect: $g_m \rightarrow g_m + g_{mb}$





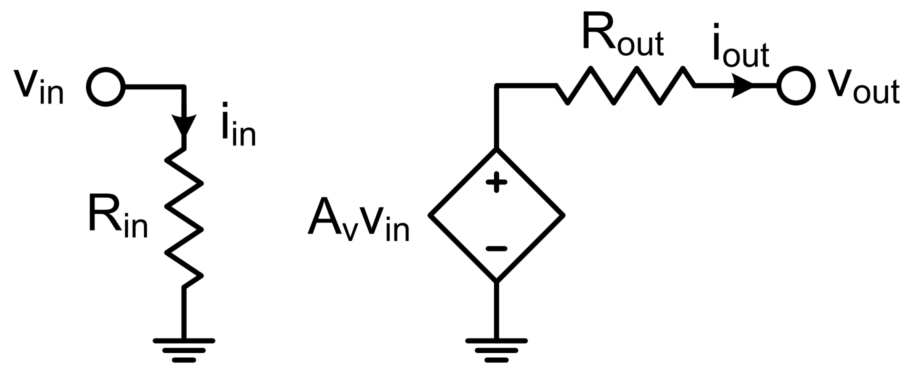
Why GmRout?

$$R_{out} = \frac{v_x}{i_x} @ v_{in} = 0$$

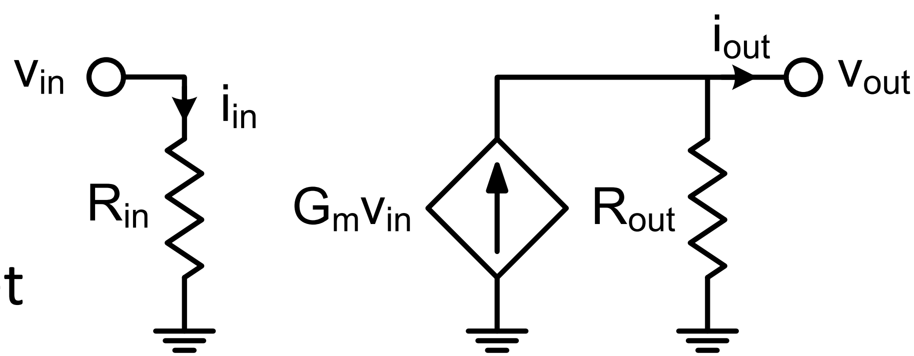
$$G_m = \frac{i_{out,sc}}{v_{in}}$$

$$A_v = G_m R_{out}$$

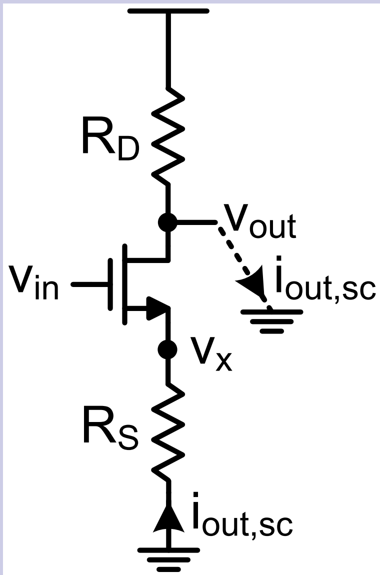
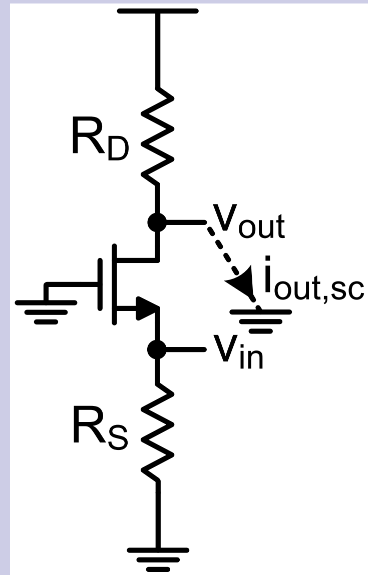
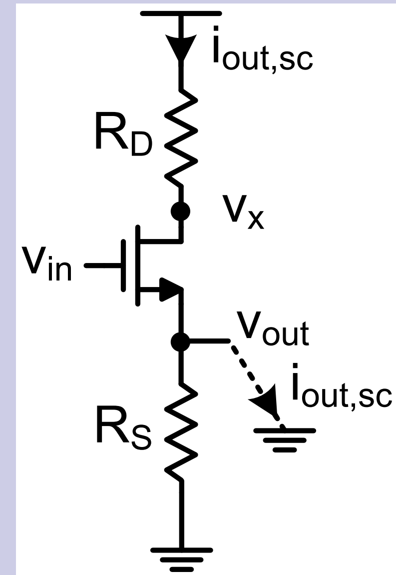
$$A_i = G_m R_{in}$$



- Divide and conquer
  - Rout simplified: vin=0
  - Gm simplified: vout=0
  - We already need Rin/out
  - We can quickly and easily get Rin/out from the shortcuts

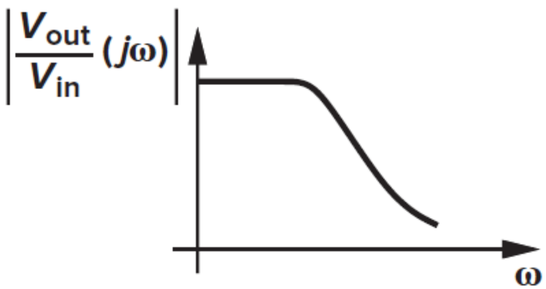


Summary of Basic Topologies

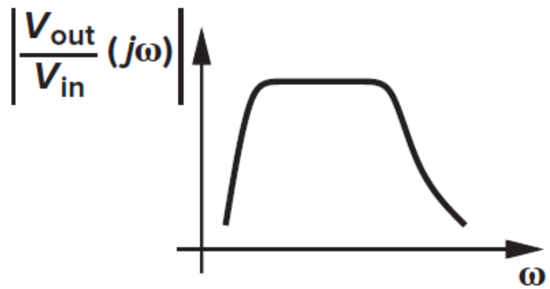
	CS	CG	CD (SF)
			
	Voltage & current amplifier	Current buffer	Voltage buffer
Rin	∞	$R_S // \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$	∞
Rout	$R_D // r_o[1 + (g_m + g_{mb})R_S]$	$R_D // r_o$	$R_S // \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$
Gm	$\boldsymbol{\frac{-g_m}{1 + (g_m + g_{mb})R_S}}$	$\boldsymbol{g_m + g_{mb}}$	$\boldsymbol{\frac{g_m}{1 + R_D/r_o}}$

Frequency Response

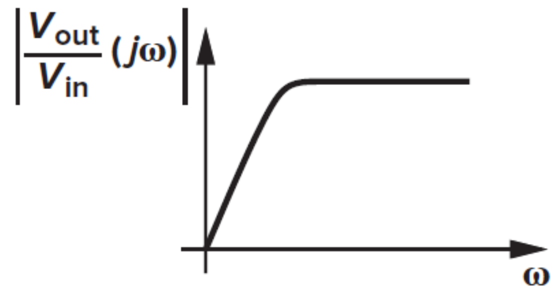
LPF



BPF



HPF



Poles and Zeros

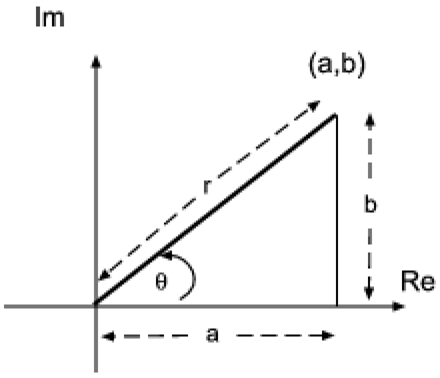
- Transfer function

$$H(s) = \frac{N(s)}{D(s)}$$

- Zeros: roots of numerator => $N(s)$
- Poles: roots of denominator => $D(s)$
- Frequency response: $s \Rightarrow j\omega$

$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = |H(j\omega)|e^{j\phi}$$

- Magnitude$(a + jb) = \sqrt{a^2 + b^2}$
- Phase$(a + jb) = \tan^{-1}\frac{b}{a}$

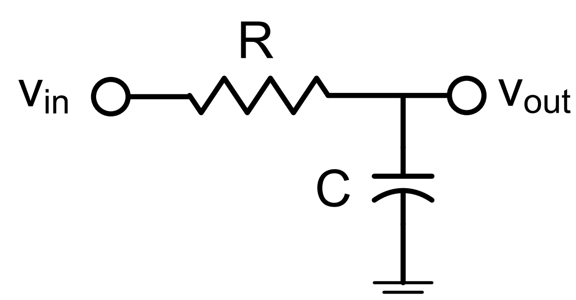


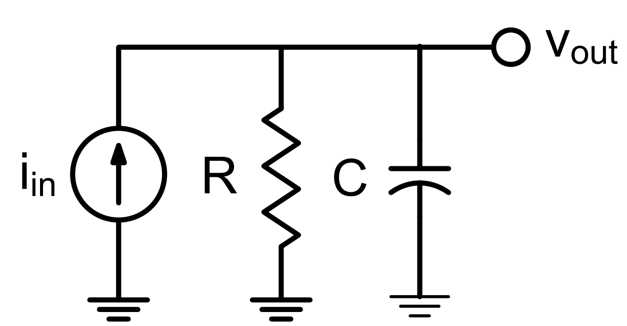
1^{st} Order LPF

$$H(s) = \frac{v_{out}}{v_{in}} = \frac{1/sC}{R + 1/sC} = \frac{1}{1 + sRC} = \frac{1}{1 + s\tau}$$

$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + \frac{j\omega}{\omega_c}}$$

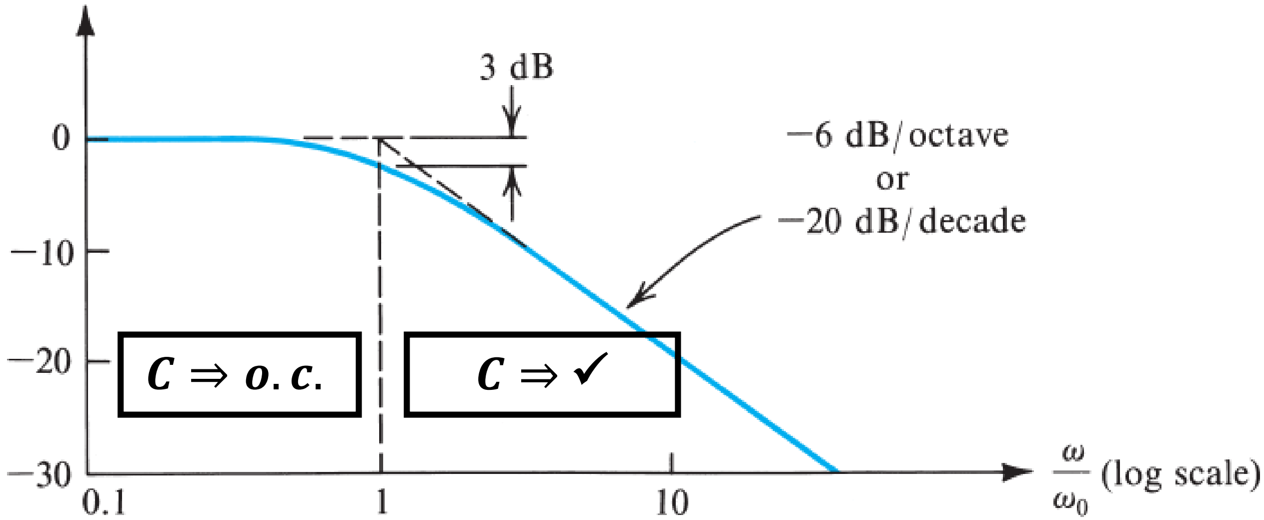
- ❑ $\tau = RC$: time constant
- ❑ $\omega_c = \frac{1}{\tau} = \frac{1}{RC}$: cutoff/corner frequency
- ❑ Poles: $s_p = -\frac{1}{\tau} = -\omega_c$
- ❑ Zeros: ?
- ❑ $|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$
- ❑ $P\big(H(j\omega)\big) = -\tan^{-1}\frac{\omega}{\omega_c}$





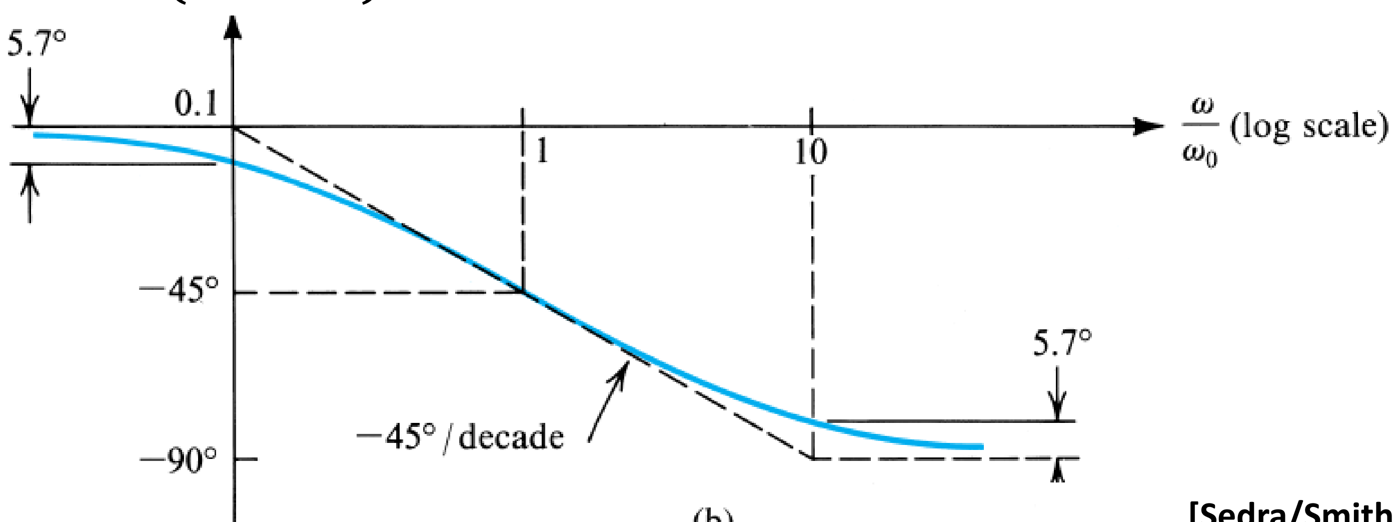
1st Order LPF

$20\log|H(j\omega)|\ (dB)$



(a)

$P(H(j\omega))$



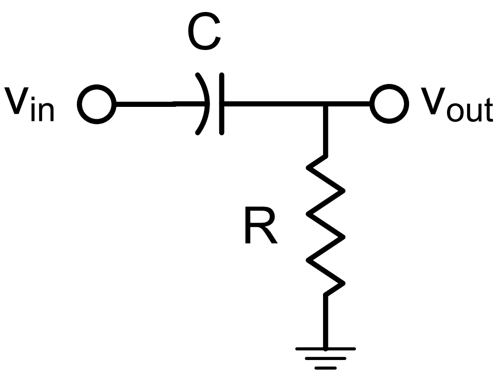
(b)

1^{st} Order HPF

$$H(s) = \frac{v_{out}}{v_{in}} = \frac{R}{R + 1/sC} = \frac{sRC}{1 + sRC} = \frac{s\tau}{1 + s\tau}$$

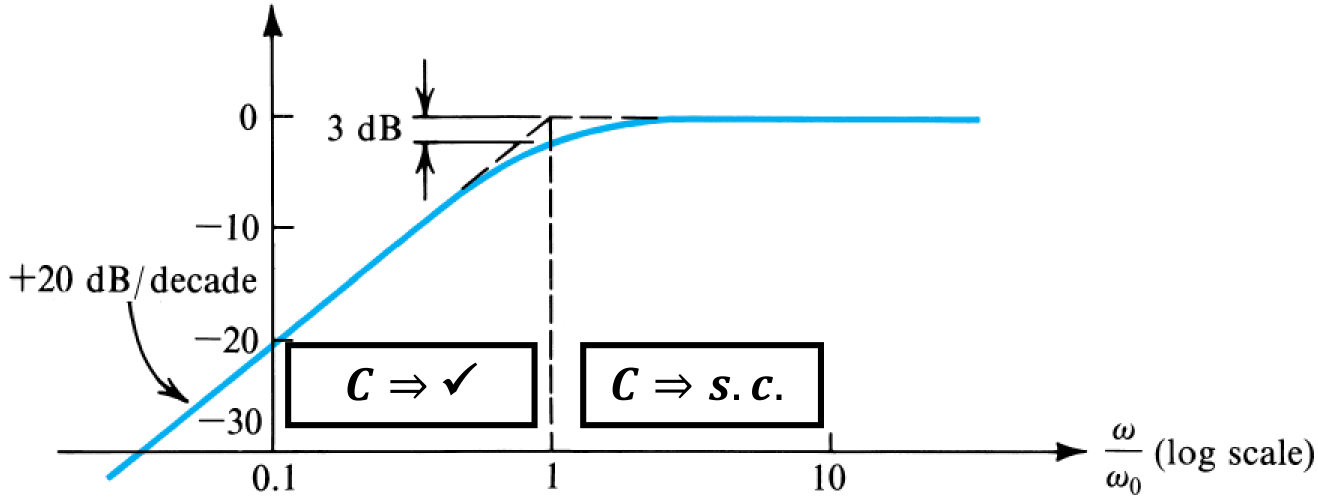
$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC} = \frac{\frac{j\omega}{\omega_c}}{1 + \frac{j\omega}{\omega_c}}$$

- Poles: $s_p = -\frac{1}{\tau} = -\omega_c$
- Zeros: $s_z = 0$
- $|H(j\omega)| = \frac{\frac{\omega}{\omega_c}}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$
- $P\left(H(j\omega)\right) = 90^o - \tan^{-1}\frac{\omega}{\omega_c}$



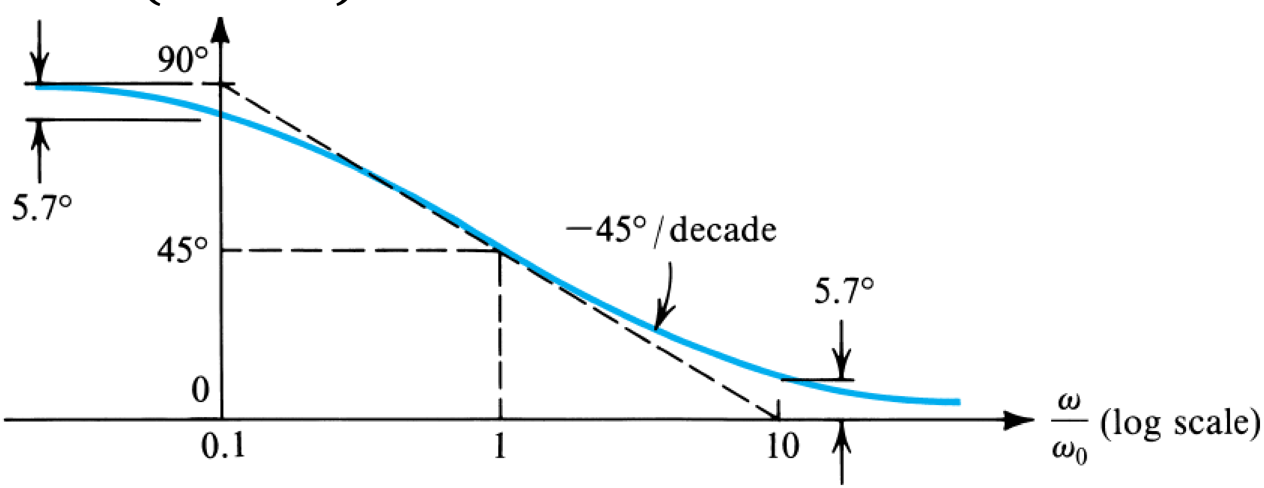
1st Order HPF

$20\log|H(j\omega)|\ (dB)$



(a)

$P\left(H(j\omega)\right)$

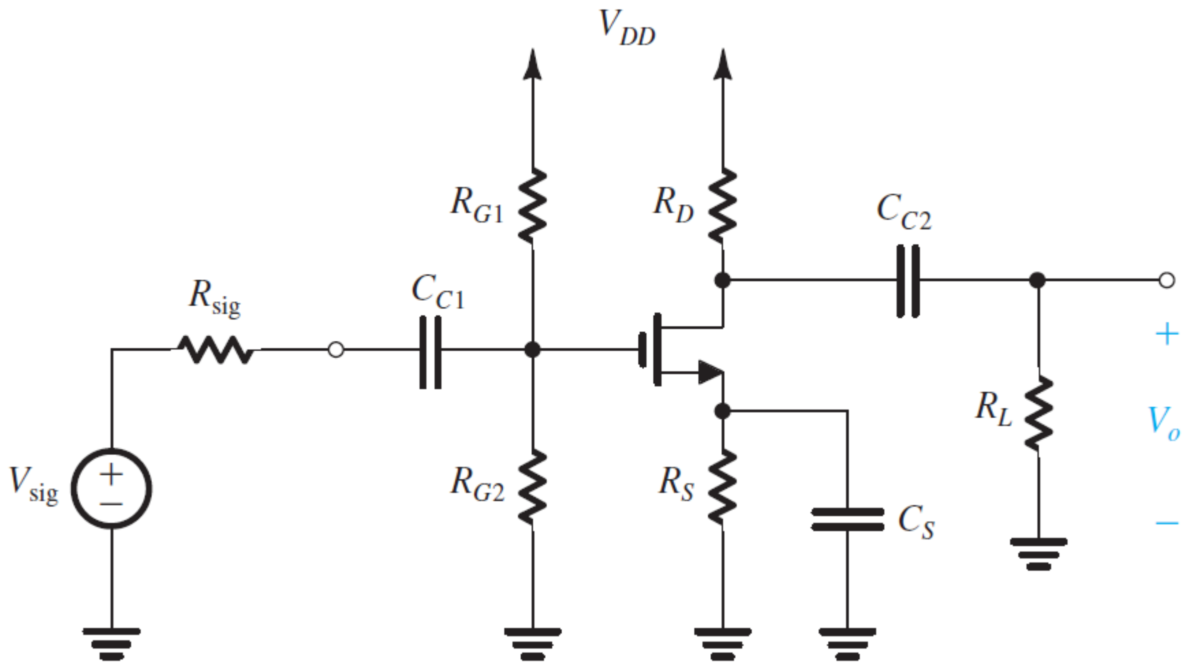


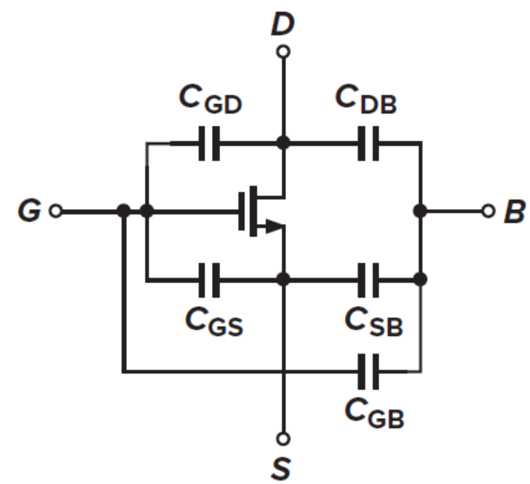
(b)

Bode Plot Rules

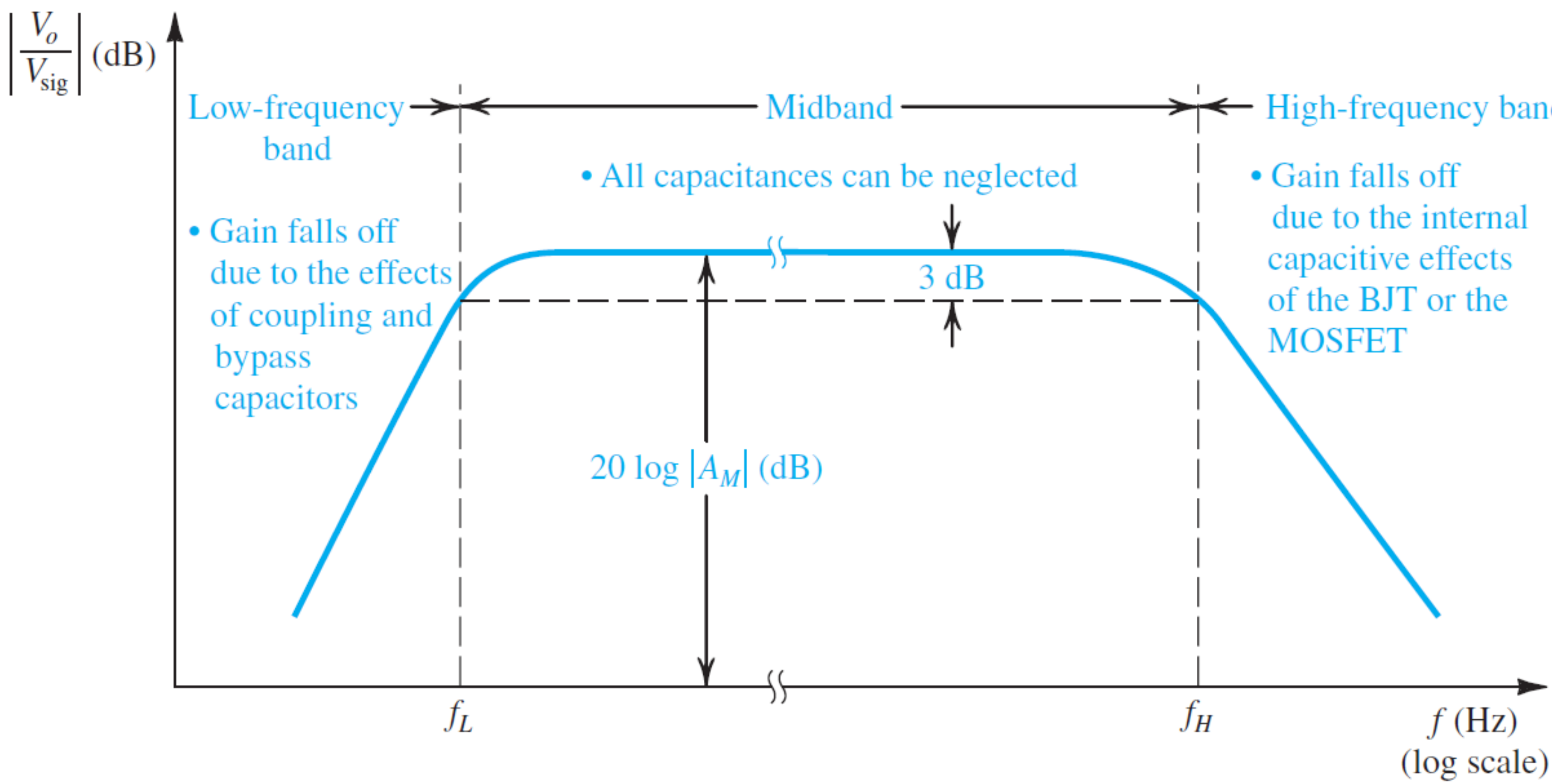
<table>
<tr><th></th><th>Pole</th><th>Zero</th></tr>
<tr><td>Magnitude</td><td>-20 dB/decade
Actual Mag @ pole: -3 dB</td><td>+20 dB/decade
Actual Mag @ zero: +3 dB</td></tr>
<tr><td rowspan="2">Phase</td><td rowspan="2">-90^{o}
Actual Phase @ pole: -45^{o}</td><td>LHP zero:
$+90^{o}$
Actual Phase @ zero: $+45^{o}$</td></tr>
<tr><td>RHP zero:
-90^{o}
Actual Phase @ zero: -45^{o}</td></tr>
</table>

Where are the Capacitors?





Frequency Response



Coupling and bypass	✓	s.c.	s.c.
Intrinsic and load	o.c.	o.c.	✓

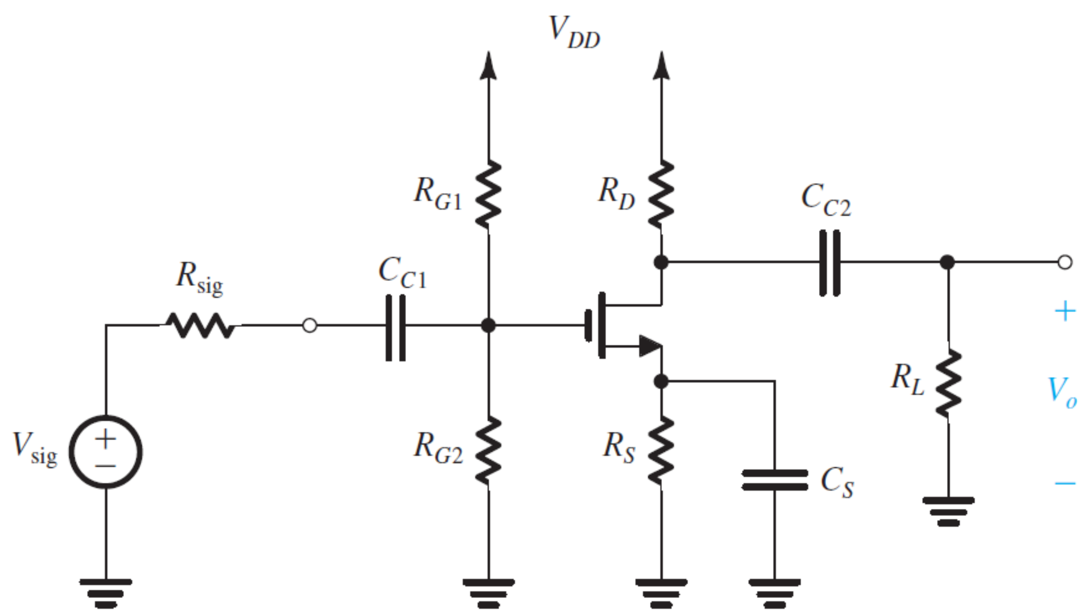
[Sedra/Smith, 2015]

SCTC and OCTC Techniques

- Low-frequency range (LFR) => Not common in IC design
  - Only consider one cap at a time
  - Assume other caps are s.c.
  - s.c. time constant (SCTC) technique
  - $\omega_L \approx \omega_{L1} + \omega_{L2} + \cdots$
  - Highest pole dominates (L.I.N. dominates)
- **High-frequency range (HFR) => More important in IC design**
  - **Only consider one cap at a time**
  - **Assume other caps are o.c.**
  - **o.c. time constant (OCTC) technique**
  - $\boldsymbol{\omega_H \approx \omega_{H1} // \omega_{H2} // \cdots}$
  - **Lowest pole dominates (H.I.N. dominates)**
- Both provide good approx if one pole is dominant (and poles are real)

Effect of Bypass Capacitor

- Does C_S act as a LPF or a HPF?

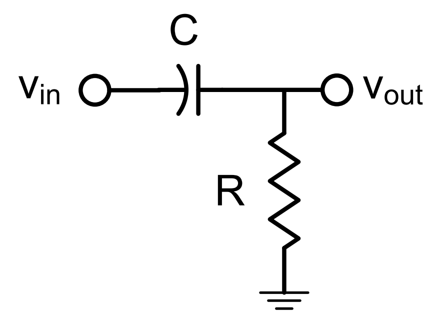


[Sedra/Smith, 2015]

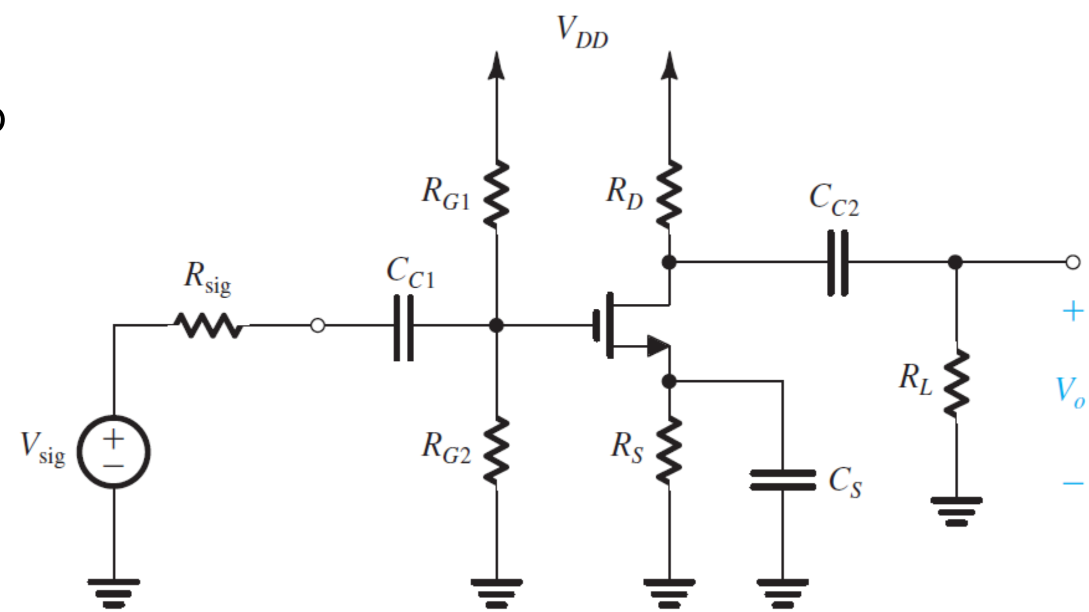
Calculating Zeros by Inspection

1. Find the value $s = s_z$ that makes $H(s) = 0 \Rightarrow v_{out} = 0$

- ❑ Ex1: C_{c1}: $v_o = 0\ if\ Z_{C_1} = \infty$
  - $Z_{C_1} = \frac{1}{sC_1}$
  - $\Rightarrow s_{z1} = 0$

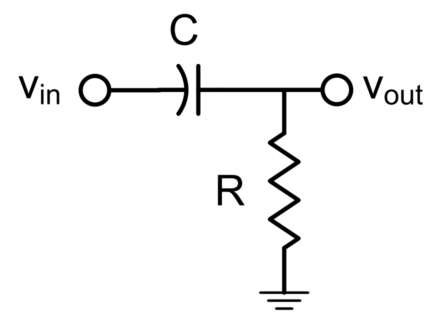


- ❑ Ex2: C_S: $v_o = 0\ if\ Z_S = \infty$
  - $Z_S = \frac{R_S}{1+sR_SC_S}$
  - $\Rightarrow s_{z2} = -\frac{1}{R_SC_S}$

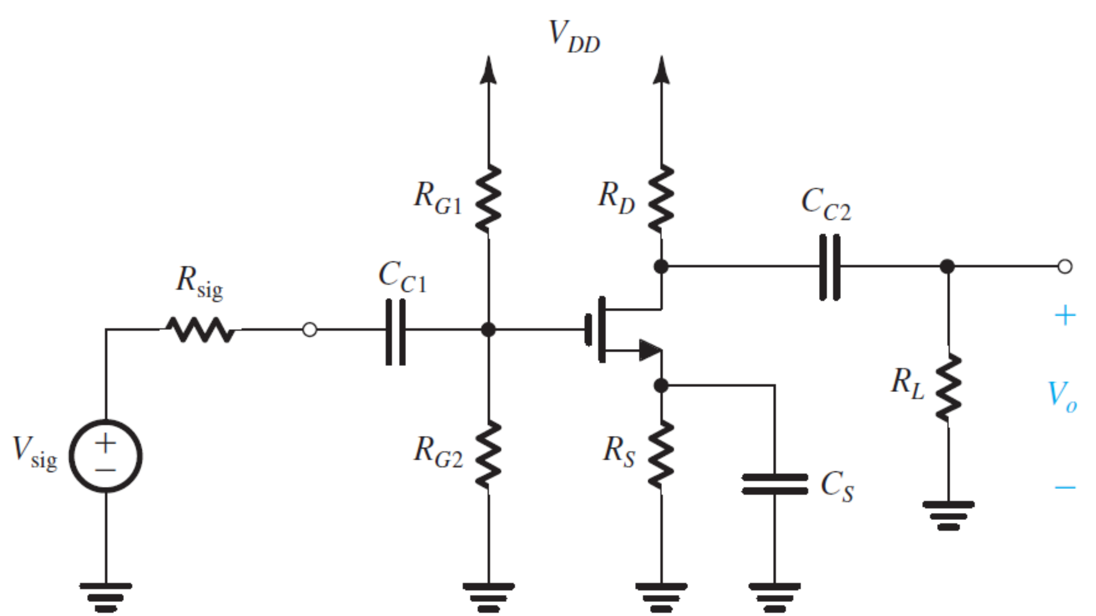


Calculating Poles by Inspection

1. Set $v_{in} = 0$
2. Calculate thevenin resistance $\left(R_{th,i}\right)$ seen by each cap $\left(C_i\right)$
3. $s_{p,i} = -\frac{1}{R_{th,i}C_i}$



- ❑ Ex1: C_{c1}: $R_{th} = R_{sig} + R_G$
  - $\Rightarrow s_{p1} = -\frac{1}{(R_{sig}+R_G)C_{c1}}$
- ❑ Ex2: C_S: $R_{th} \approx R_S // \frac{1}{g_m}$
  - $\Rightarrow s_{p2} = -\frac{1}{\left(R_S//\frac{1}{g_m}\right)C_S}$



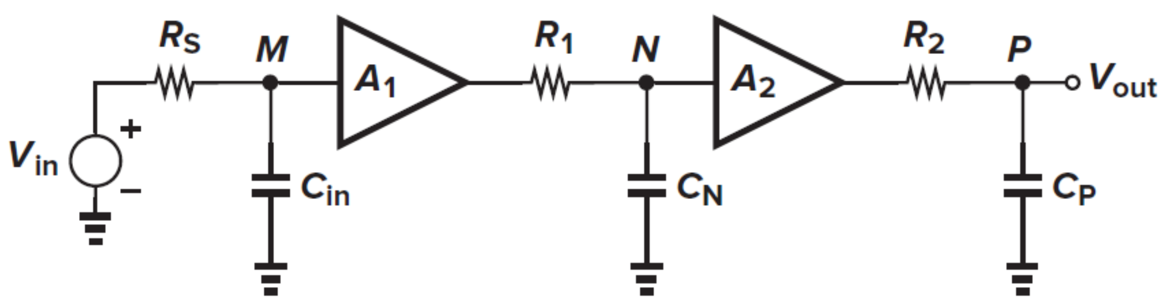
[Sedra/Smith, 2015]

Calculating Poles by Inspection

1. Set $v_{in} = 0$
2. Calculate thevenin resistance $\left(R_{th,i}\right)$ seen by each cap (C_i)
3. $s_{p,i} = -\dfrac{1}{R_{th,i}C_i}$

- Examples:
  - Each node is associated with a pole
  - H.I.N. dominates

$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + R_S C_{in} s} \cdot \frac{A_2}{1 + R_1 C_N s} \cdot \frac{1}{1 + R_2 C_P s}$$



Thank you!