وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 11
Differential Amplifiers

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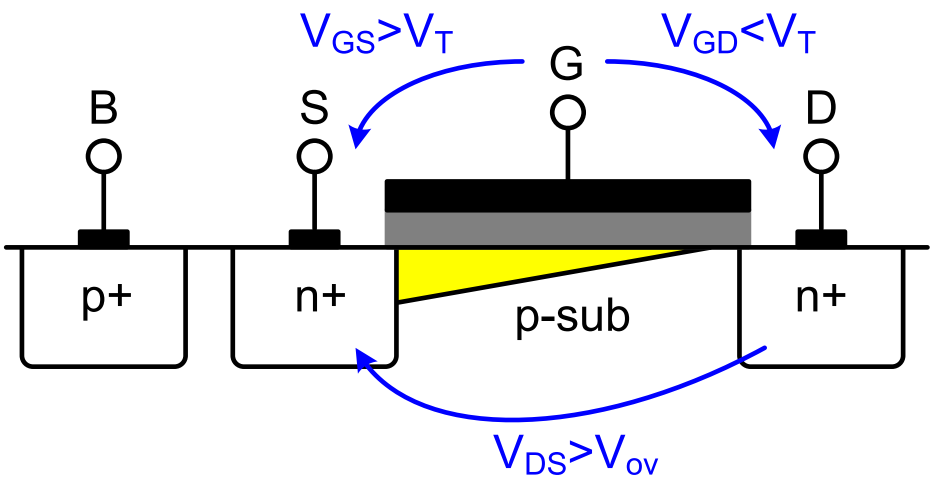
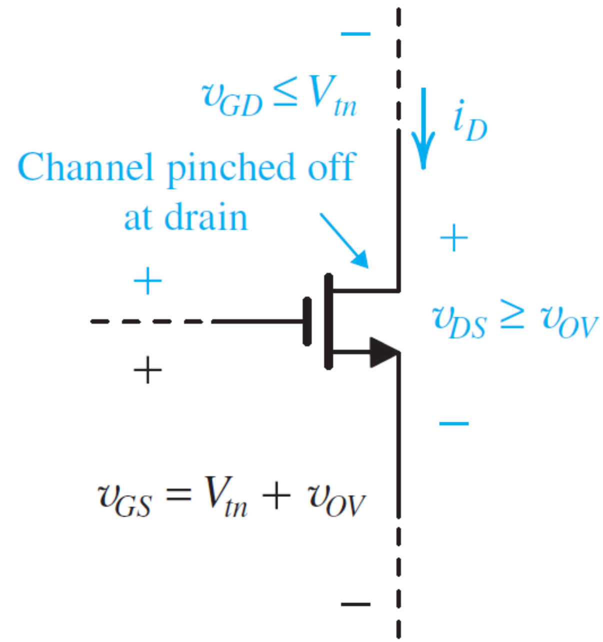
MOSFET in Saturation

- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L} \cdot V_{ov}^2(1 + \lambda V_{DS})$$





[Sedra/Smith, 2015]

Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_T$

ON

$V_{GS} > V_T$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_T$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_T$

$$I_D = \mu C_{ox} \frac{W}{L} \left(V_{ov} V_{DS} - \frac{V_{DS}^2}{2} \right)$$

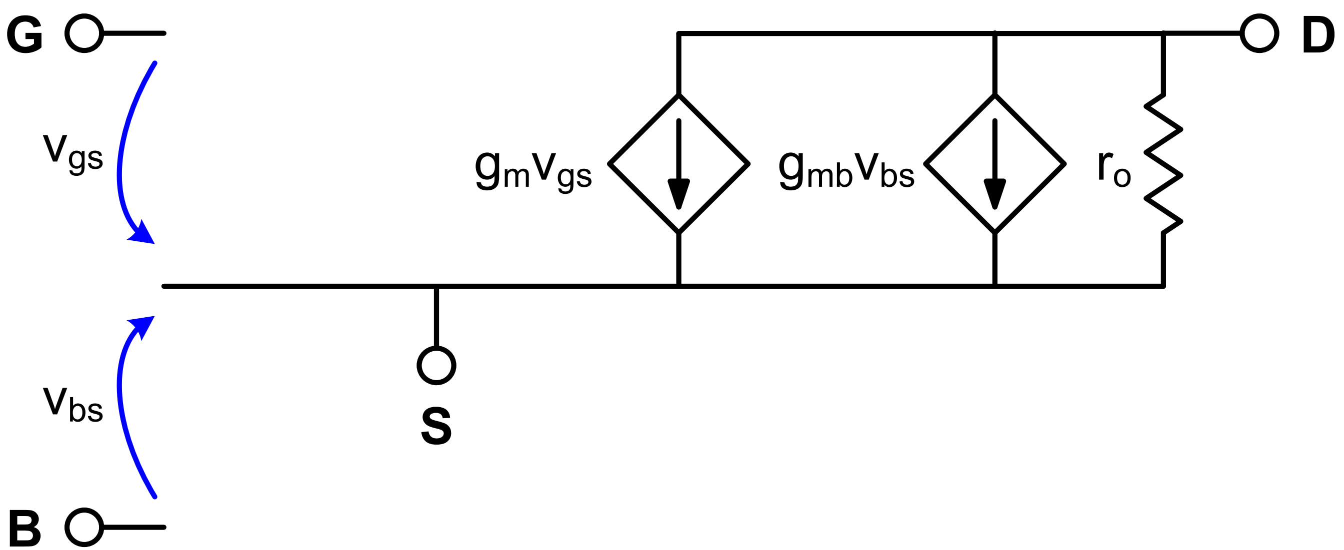
$$I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} V_{ov}^2 (1 + \lambda V_{DS})$$

Low-Frequency Small-Signal Model

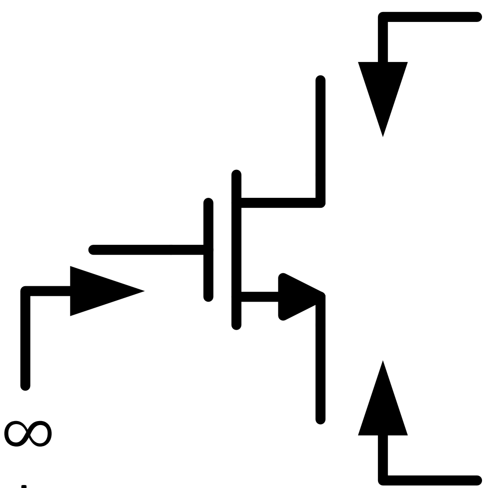
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} V_{ov} = \sqrt{\mu C_{ox} \frac{W}{L} \cdot 2 I_D} = \frac{2 I_D}{V_{ov}}$$

$$g_{mb} = \eta g_m, \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}} = \frac{1}{\lambda I_D}, \qquad \lambda \propto \frac{1}{L}$$



Rin/out Shortcuts Summary



$$r_o[1 + (g_m + g_{mb})R_S]$$

H.I.N.

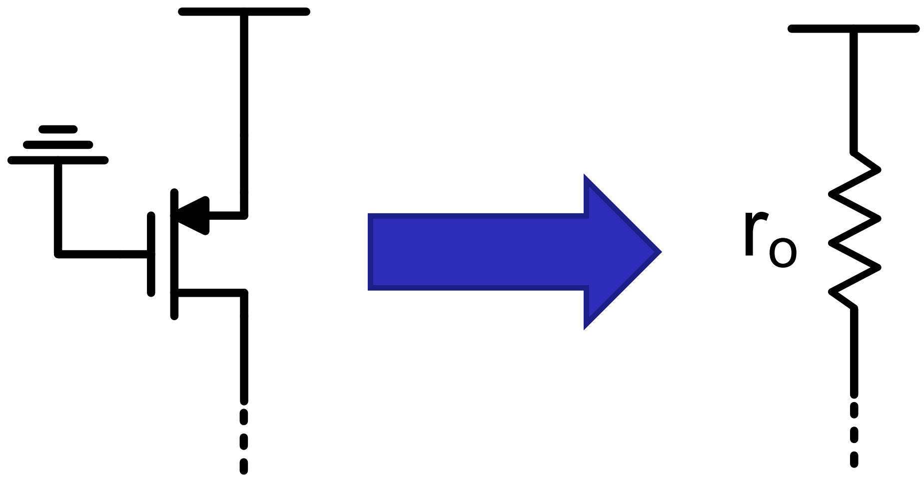
∞

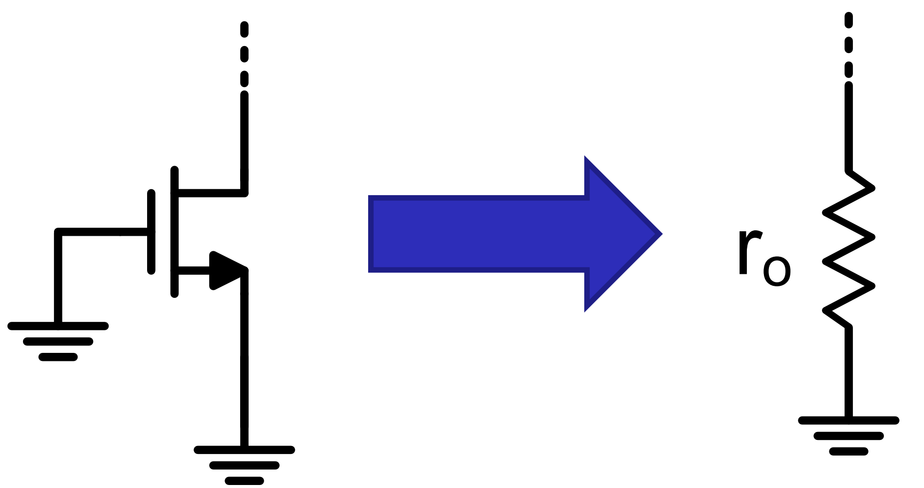
At low frequencies **ONLY**

$$\frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

L.I.N.

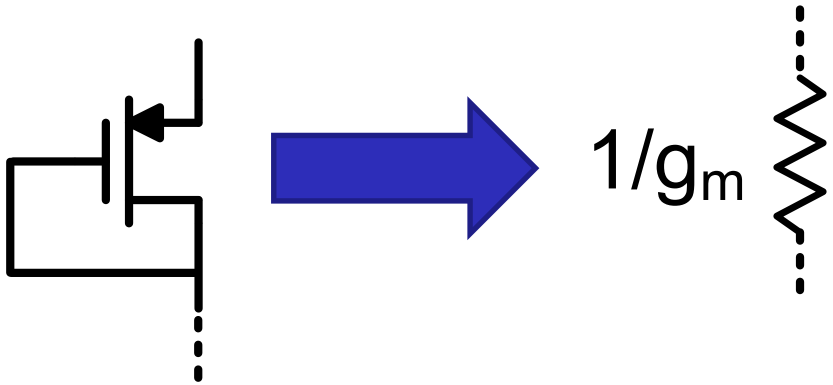
Active Load (Source OFF)

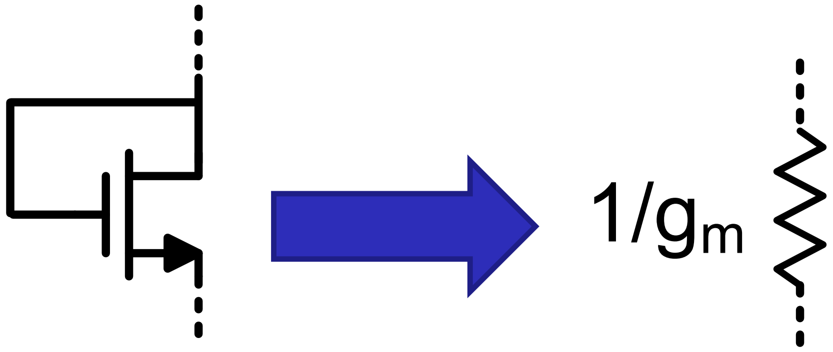




Diode Connected (Source Absorption)

- Always in saturation
- Bulk effect: $g_m \rightarrow g_m + g_{mb}$





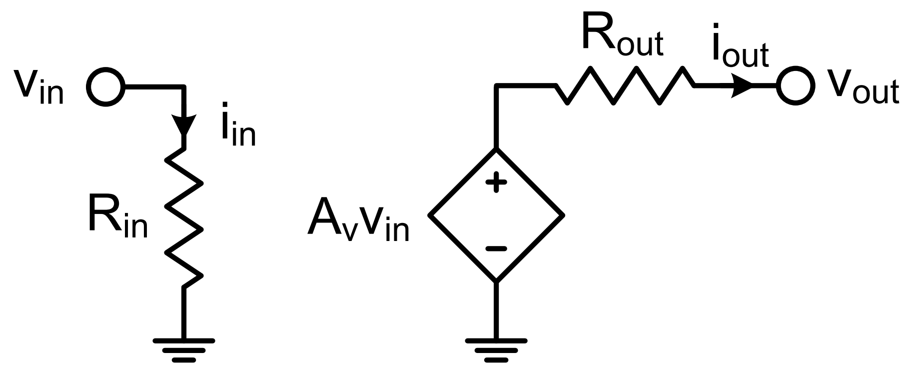
Why GmRout?

$$R_{out} = \frac{v_x}{i_x} @ v_{in} = 0$$

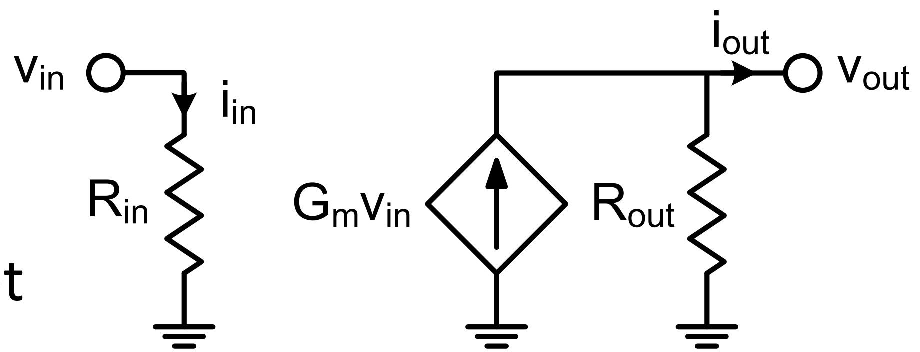
$$G_m = \frac{i_{out,sc}}{v_{in}}$$

$$A_v = G_m R_{out}$$

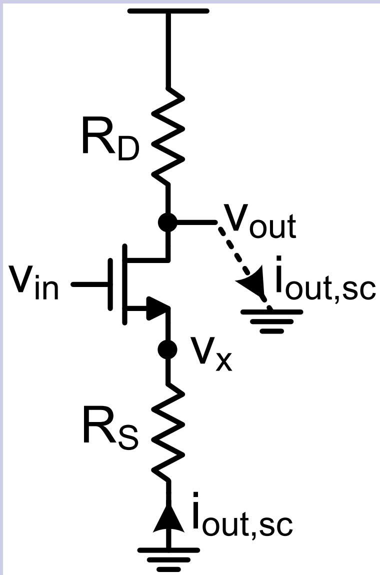
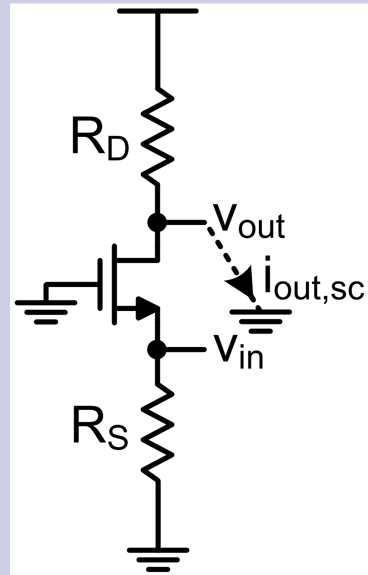
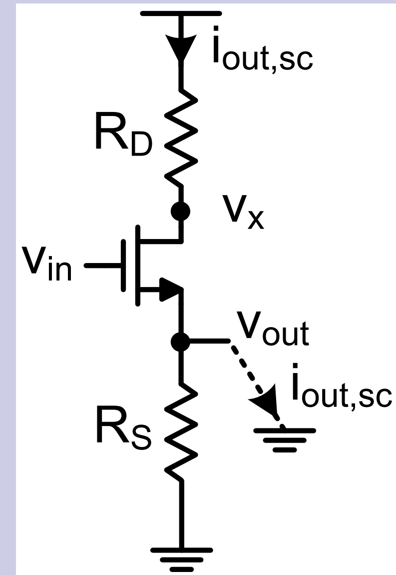
$$A_i = G_m R_{in}$$



- Divide and conquer
  - Rout simplified: vin=0
  - Gm simplified: vout=0
  - We already need Rin/out
  - We can quickly and easily get Rin/out from the shortcuts

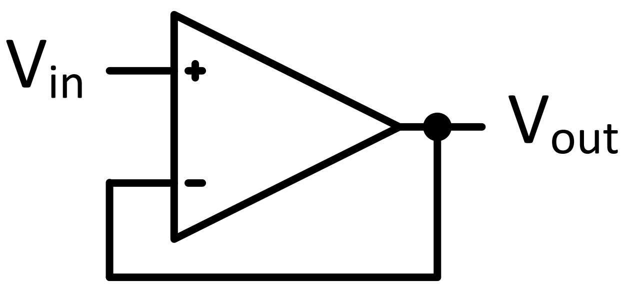


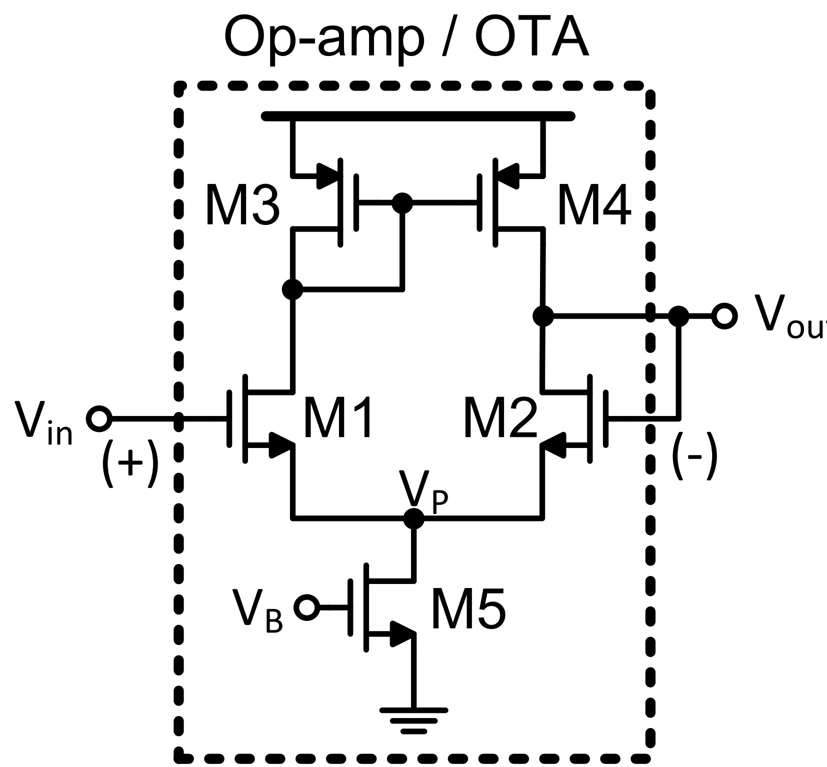
Summary of Basic Topologies

	CS	CG	CD (SF)
			
	Voltage & current amplifier	Current buffer	Voltage buffer
Rin	∞	$R_S // \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$	∞
Rout	$R_D // r_o[1 + (g_m + g_{mb})R_S]$	$R_D // r_o$	$R_S // \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$
Gm	$\frac{-g_m}{1 + (g_m + g_{mb})R_S}$	$g_m + g_{mb}$	$\frac{g_m}{1 + R_D / r_o}$

Have You Seen a Diff Amp Before?

- An op-amp is simply a high gain differential amplifier
- The gain can be increased by using cascodes and multi-stage amplifiers





Single-Ended (SE) vs Differential

- SE: measured with respect to a fixed potential (usually the ground)

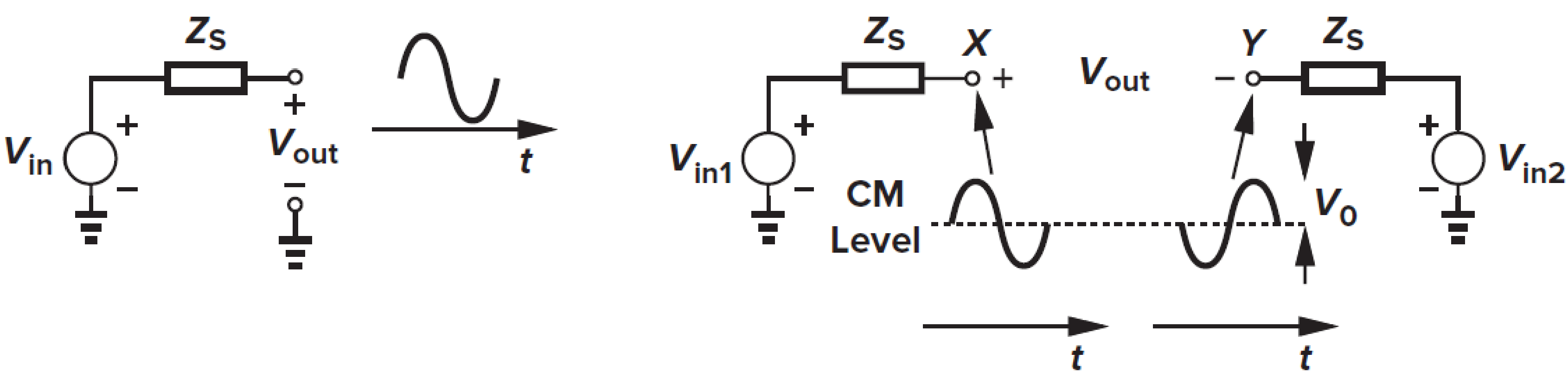
$$v_{out,SE} = V_o \sin\omega t + V_{CM}$$

  - Single-ended peak-to-peak swing is 2Vo

- Diff: measured between two nodes that have *equal* and *opposite* signal around a common-mode (CM) level

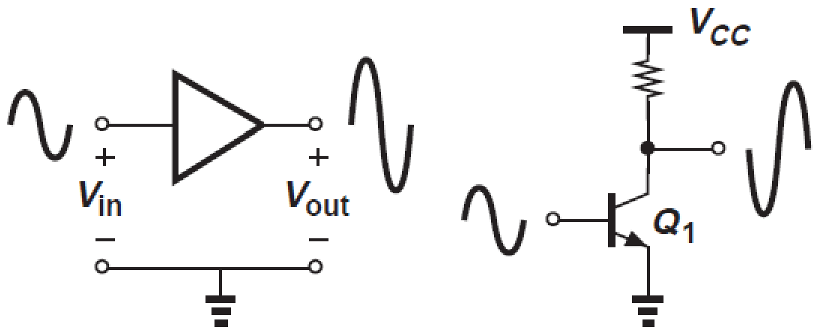
$$\begin{aligned} v_{out,diff} &= v_x - v_y \\ &= (V_o \sin\omega t + V_{CM}) - (-V_o \sin\omega t + V_{CM}) \end{aligned}$$

  - Differential peak-to-peak swing is 4Vo

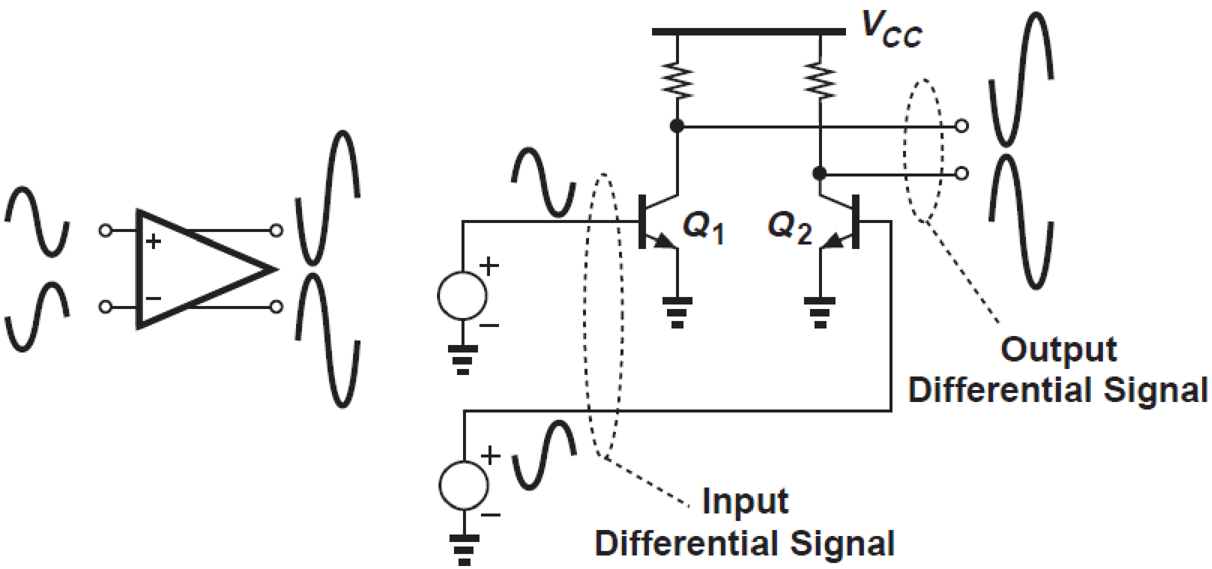


[Razavi, 2017]

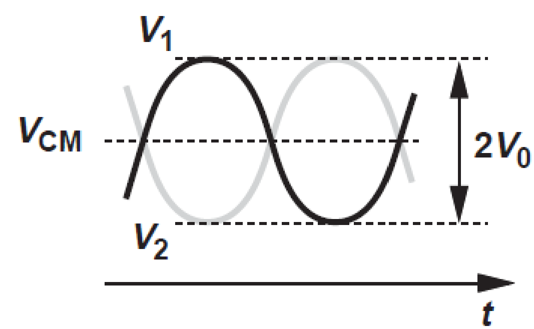
Single-Ended (SE) vs Differential



(a)



(b)



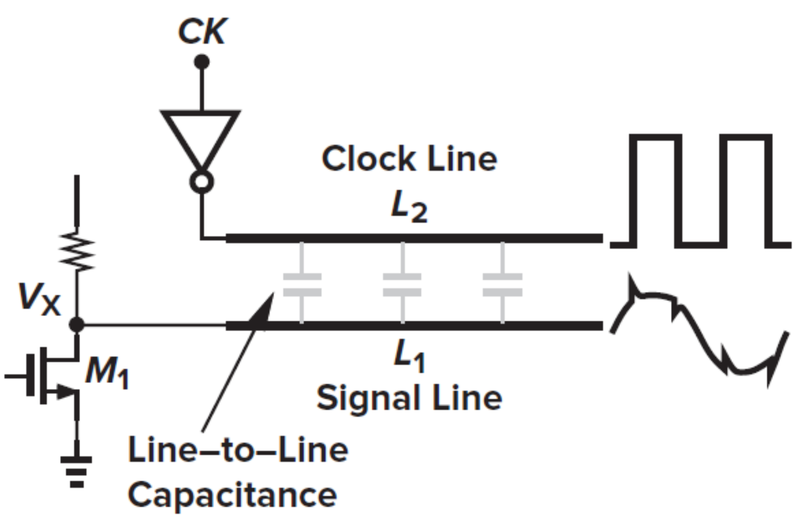
(c)

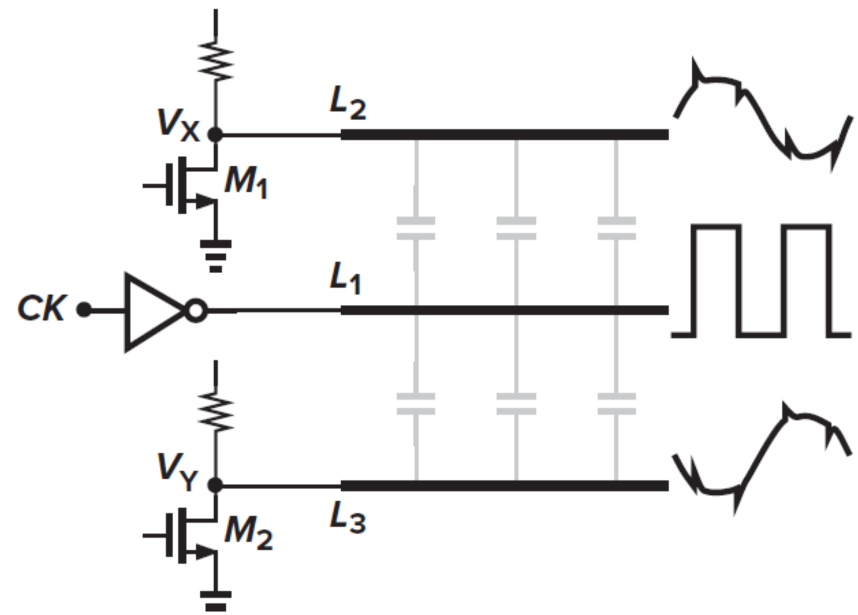
[Razavi, 2014]

Why Differential?

$$v_{out,SE} = V_o \sin \omega t + V_{CM} + V_{CMnoise}$$

$$\begin{aligned} v_{out,diff} &= v_x - v_y \\ &= (V_o \sin \omega t + V_{CM} + V_{CMnoise}) - (-V_o \sin \omega t + V_{CM} + V_{CMnoise}) \\ &= 2V_o \sin \omega t \end{aligned}$$

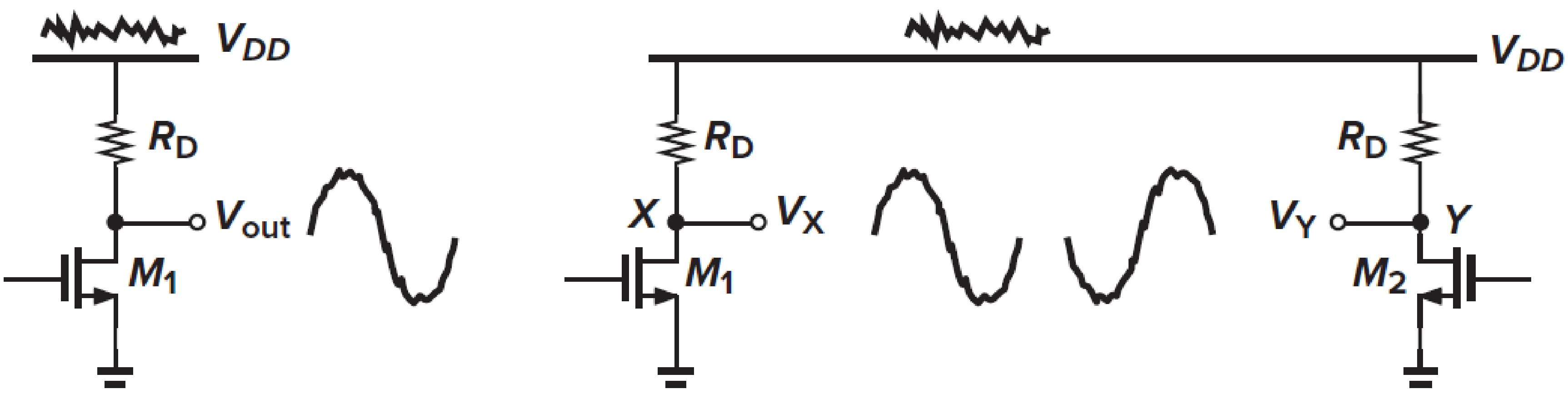




Why Differential?

$$v_{out,SE} = V_o \sin \omega t + V_{CM} + V_{CMnoise}$$

$$\begin{aligned} v_{out,diff} &= v_x - v_y \\ &= (V_o \sin \omega t + V_{CM} + V_{CMnoise}) - (-V_o \sin \omega t + V_{CM} + V_{CMnoise}) \\ &= 2V_o \sin \omega t \end{aligned}$$

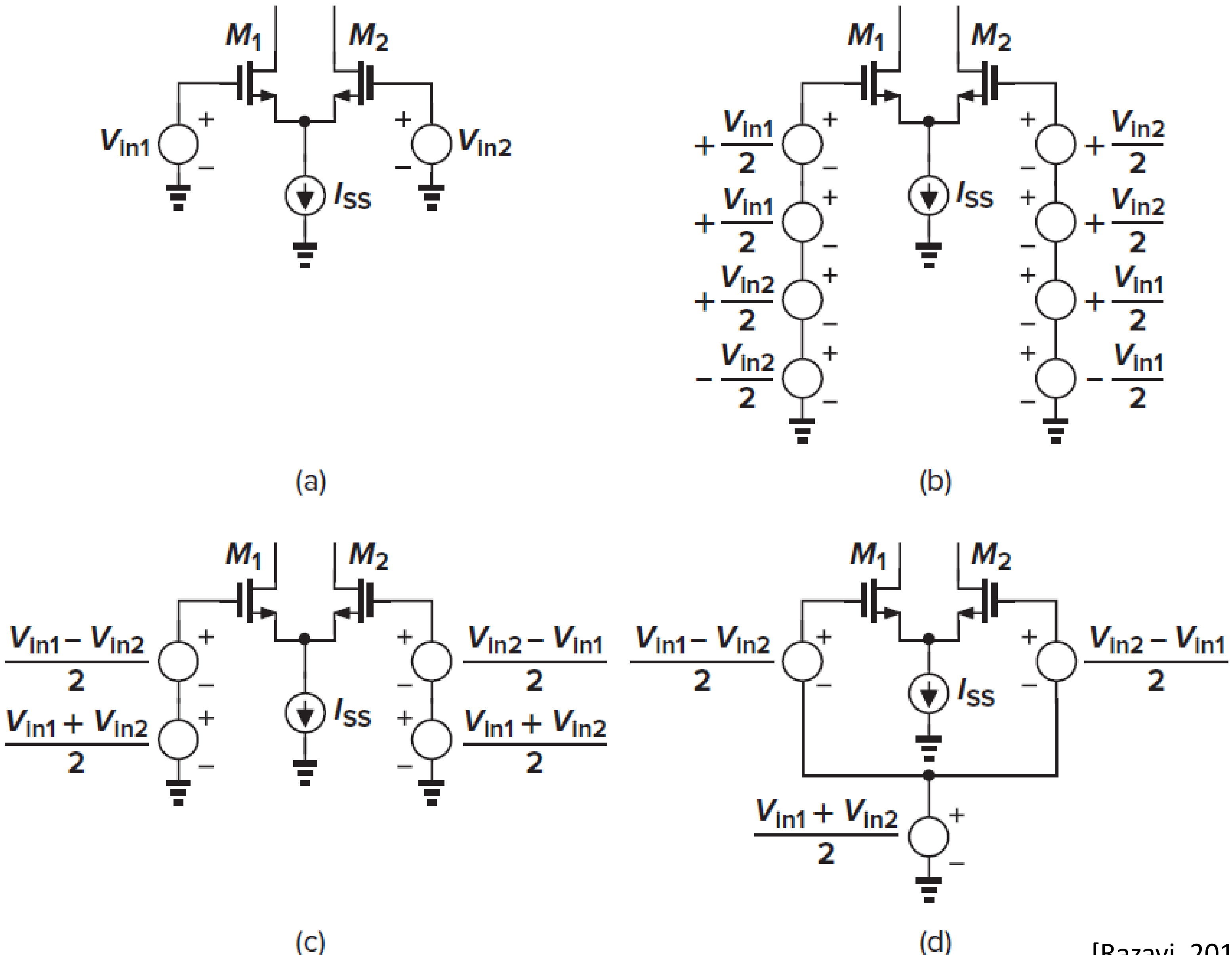


[Razavi, 2017]

Why Differential?

- Pros
  - **Common-mode (CM) noise rejection**
  - Larger maximum signal swing
  - Simpler biasing (no need for bypass or coupling capacitors)
  - Higher linearity (H.W.: Read Section 14.1.2 in Razavi)
- Cons
  - Doubling the area
  - Doubling the power consumption
- The advantages of differential operation by far outweigh the disadvantages
- Differential operation has become the de facto choice in today's high-performance analog and mixed-signal circuits

Diff Amp with Arbitrary Inputs



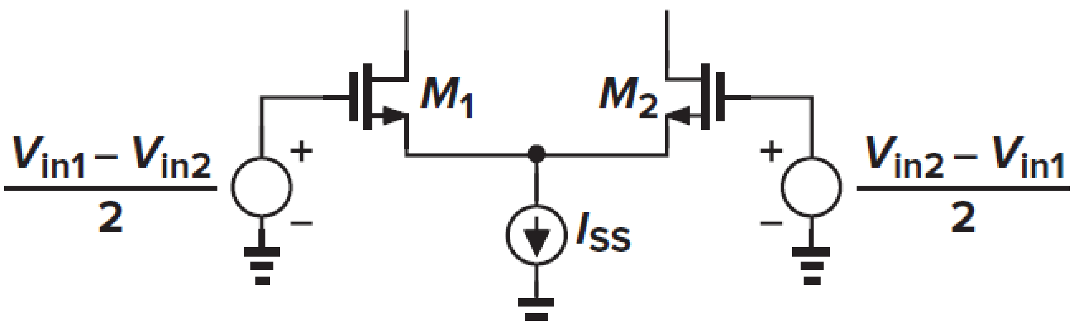
[Razavi, 2017]

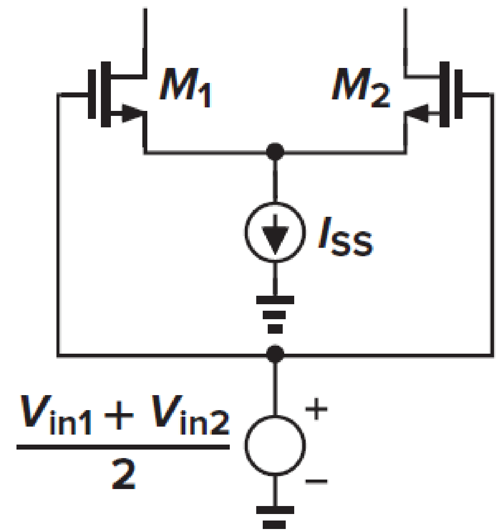
Separate CM and Diff by Superposition

$$v_{id} = v_{in1} - v_{in2}$$

$$v_{id1} = \frac{v_{id}}{2} \quad \text{and} \quad v_{id2} = -\frac{v_{id}}{2}$$

$$v_{iCM} = \frac{v_{in1} + v_{in2}}{2}$$

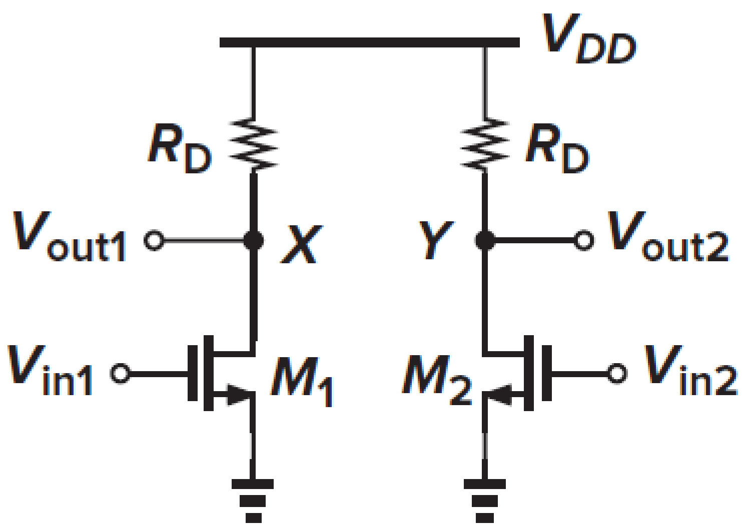




 [Razavi, 2017]

"Pseudo" Diff Amp Analysis

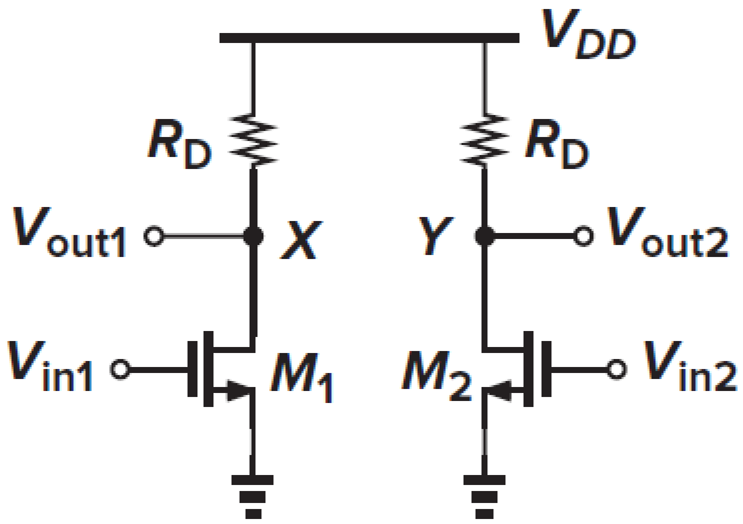
A. Small signal analysis
   1. Diff small signal analysis
   2. CM small signal analysis
B. Large signal analysis
   1. Diff large signal analysis
   2. CM large signal analysis



[Razavi, 2017]

A1. Diff Small Signal Analysis

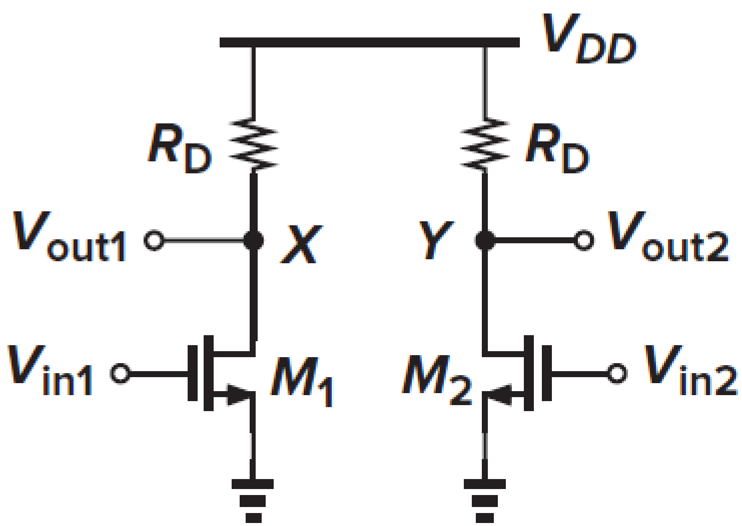
- $v_{in1} = \frac{v_{id}}{2}$ and $v_{in2} = -\frac{v_{id}}{2}$
- $v_{out1} = -g_m R_D \left(\frac{v_{id}}{2}\right)$ and $v_{out2} = -g_m R_D \left(-\frac{v_{id}}{2}\right)$
- $v_{od} = v_{out1} - v_{out2} = -g_m R_D (v_{id})$
- $A_{vd} = \frac{v_{od}}{v_{id}} = -g_m R_D = \frac{v_{out1}}{v_{in1}} = \frac{v_{out2}}{v_{in2}} = A_{v,SE}$



[Razavi, 2017]

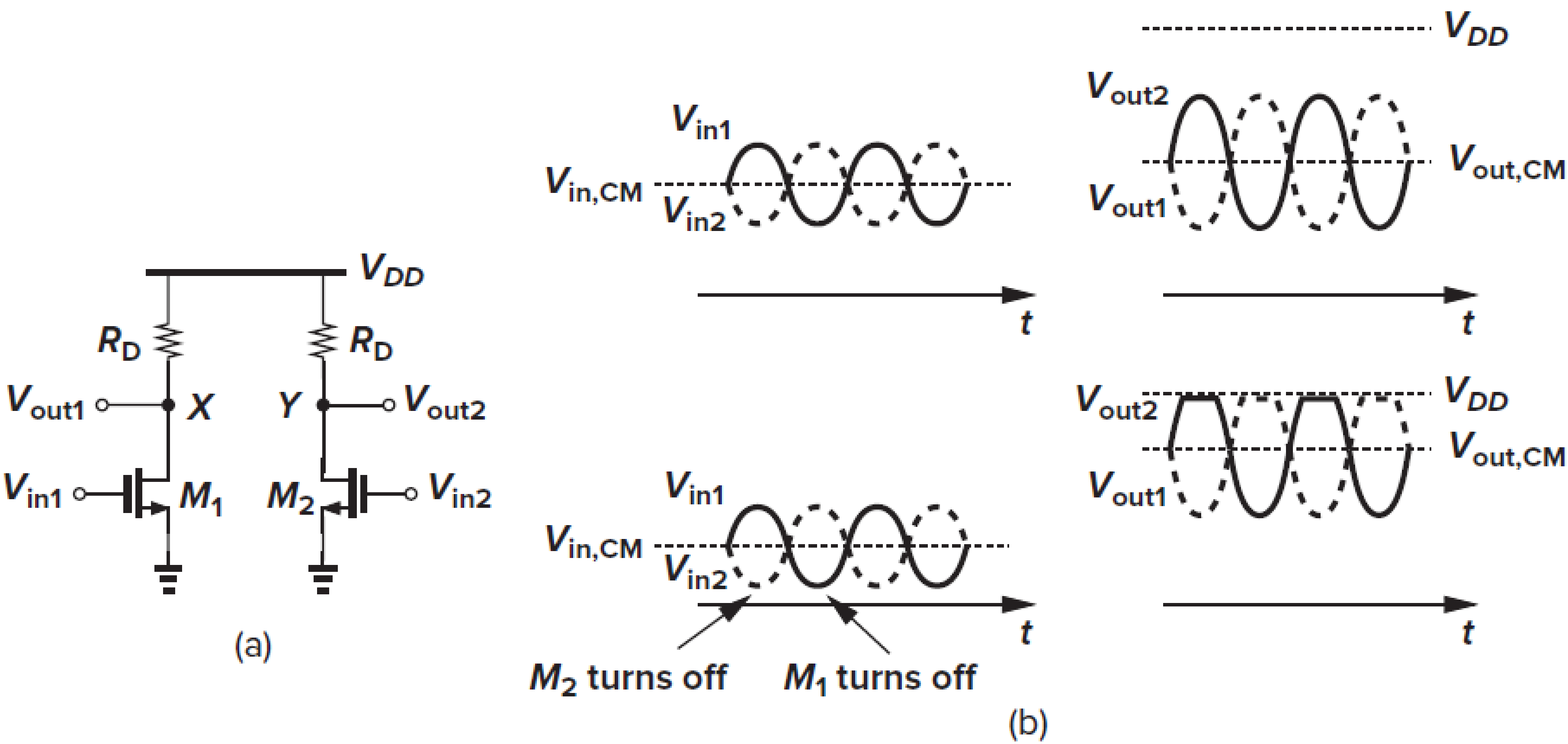
A2. CM Small Signal Analysis

- $v_{in1} = v_{iCM}$ and $v_{in2} = v_{iCM}$
- $v_{out1} = -g_m R_D(v_{iCM})$ and $v_{out2} = -g_m R_D(v_{iCM})$
- $v_{oCM} = \frac{v_{out1}+v_{out2}}{2} = -g_m R_D(v_{iCM})$
- $A_{vCM} = \frac{v_{oCM}}{v_{iCM}} = -g_m R_D = A_{vd} \rightarrow A_{vd}/A_{vCM} = 1$
- The output CM level is sensitive to the input CM level
- CM input is not “completely” rejected



B2. CM Large Signal Analysis

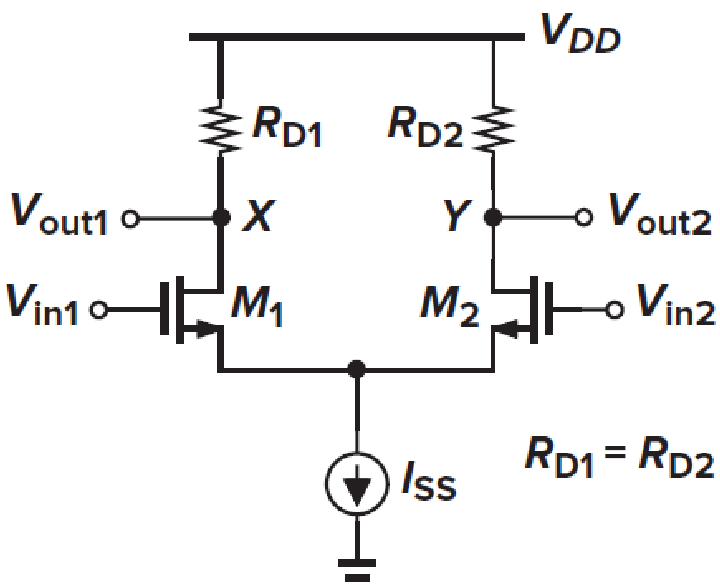
- The transistors are biased by the input CM level
- The OP point is sensitive to the input CM level



[Razavi, 2017]

"True" Diff Amp (Diff Pair) Analysis

A. Small signal analysis
   1. Diff small signal analysis
   2. CM small signal analysis
B. Large signal analysis
   1. Diff large signal analysis
   2. CM large signal analysis

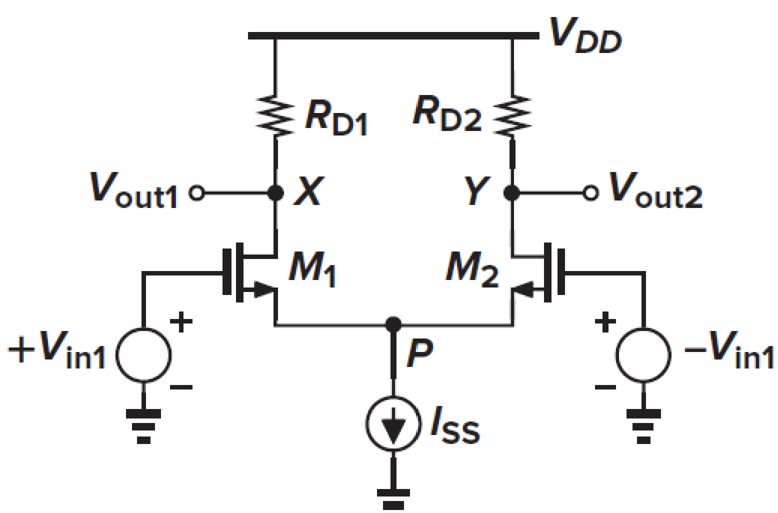


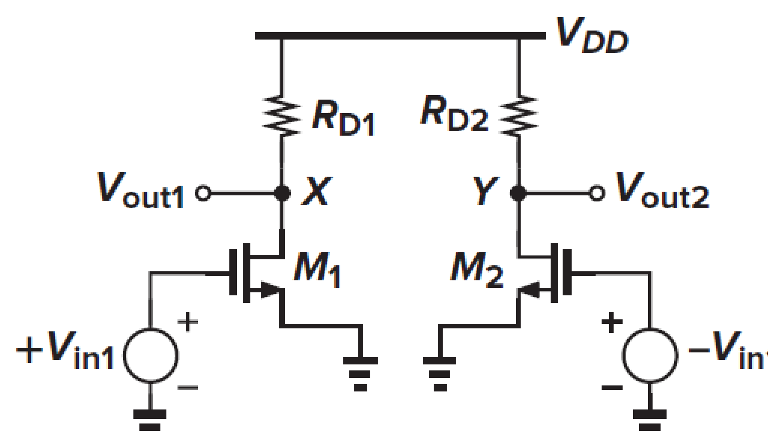
$R_{D1} = R_{D2} = R_D$

[Razavi, 2017]

A1. Diff Small Signal Analysis

- METHOD #1: **Half-Circuit Principle (exploit symmetry)**
- $v_{out1} = -g_m R_D \left(\frac{v_{id}}{2}\right)$ and $v_{out2} = -g_m R_D \left(-\frac{v_{id}}{2}\right)$
- $v_{od} = v_{out1} - v_{out2} = -g_m R_D (v_{id})$
- $A_{vd} = \frac{v_{od}}{v_{id}} = -g_m R_D = \frac{v_{out1}}{v_{in1}} = \frac{v_{out2}}{v_{in2}} = A_{v,half-circuit} = A_{v,SE}$

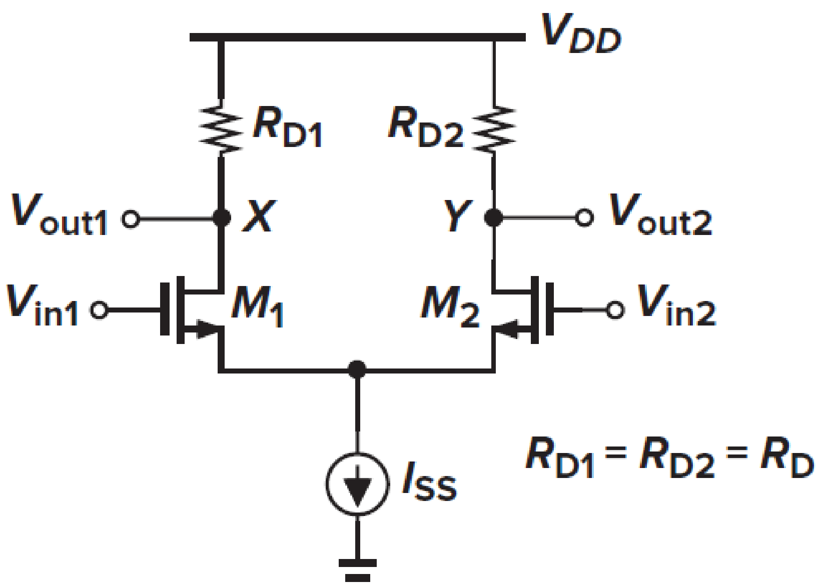




[Razavi, 2017]

A1. Diff Small Signal Analysis

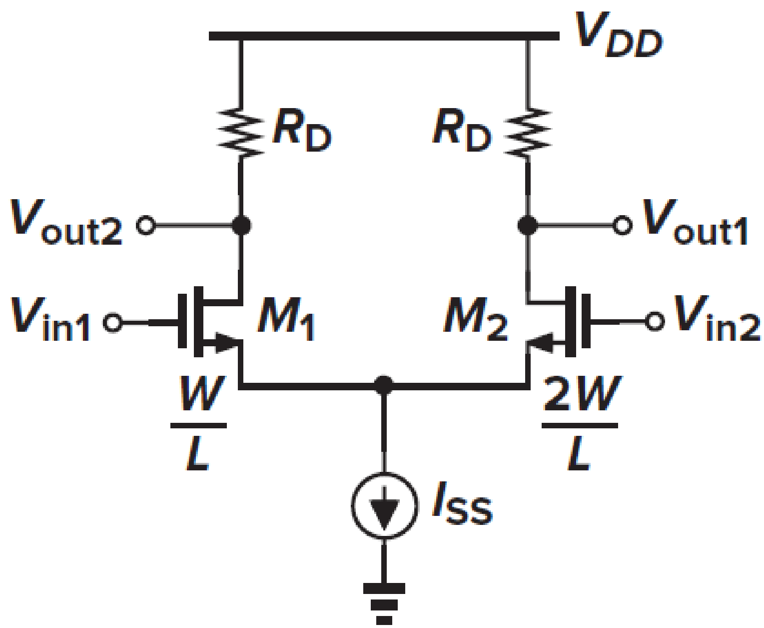
- METHOD #2: **Super-position (H.W.)**
- For v_{in1} to v_{out1}: CS (M1) degenerated by M2
- For v_{in1} to v_{out2}: CD (M1) + CG (M2)
- Similarly for v_{in2}
- Same result as half-circuit principle (H.W.)
- Lengthy analysis! (but may be necessary if not symmetric)



[Razavi, 2017]

A1. Diff Small Signal Analysis

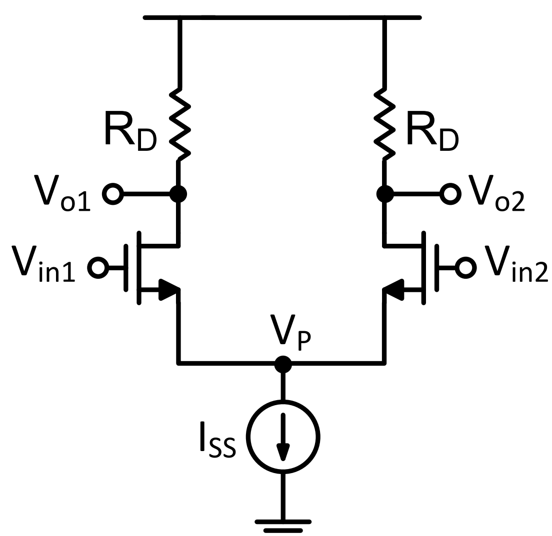
- Half-circuit principle does not work in this case

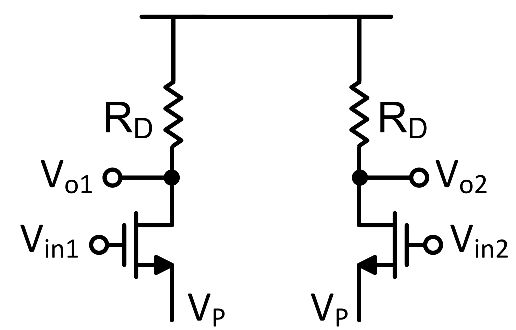


 [Razavi, 2017]

A2. CM Small Signal Analysis

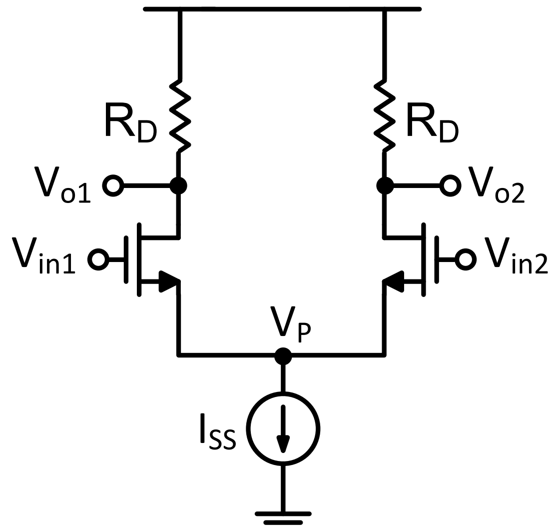
- METHOD #1: **Half-Circuit Principle (exploit symmetry)**
- $v_{out1} = 0$ and $v_{out2} = 0$
- $v_{oCM} = \frac{v_{out1}+v_{out2}}{2} = 0$
- $A_{vCM} = \frac{v_{oCM}}{v_{iCM}} = 0 = A_{v,half-circuit}$ → $A_{vd}/A_{vCM} \rightarrow \infty$
- The output CM level is NOT sensitive to the input CM level
- CM input is “completely” rejected (compare with pseudo diff amp)





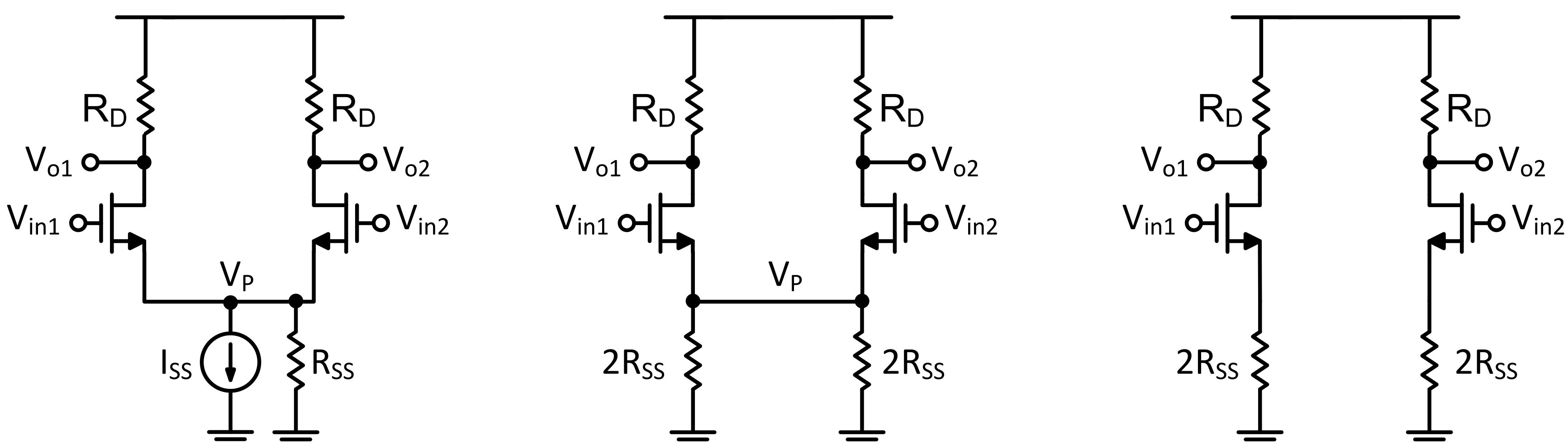
A2. CM Small Signal Analysis

- METHOD #2: **Super-position (H.W.)**
- For v_{in1} to v_{out1}: CS (M1) degenerated by M2
- For v_{in1} to v_{out2}: CD (M1) + CG (M2)
- Similarly for v_{in2}
- Same result as half-circuit principle (H.W.)
- Lengthy analysis! (but may be necessary if not symmetric)

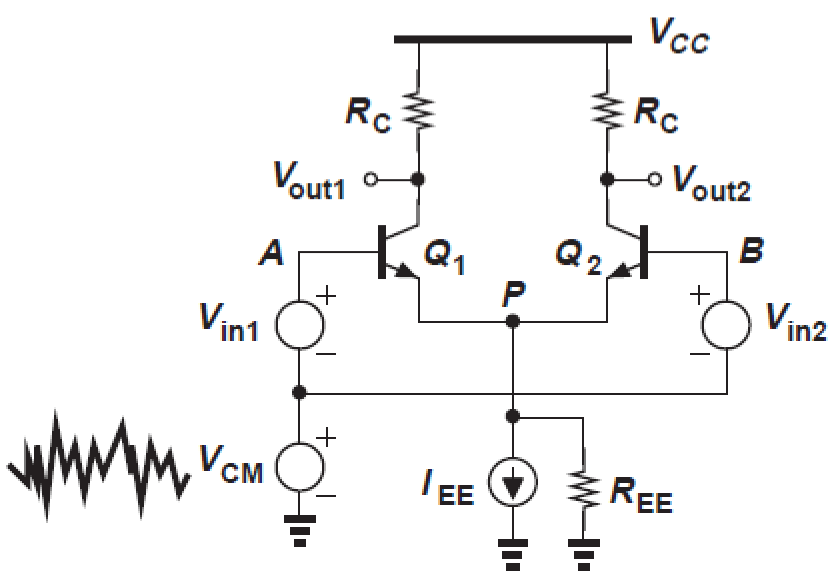


A2. CM Small Signal Analysis

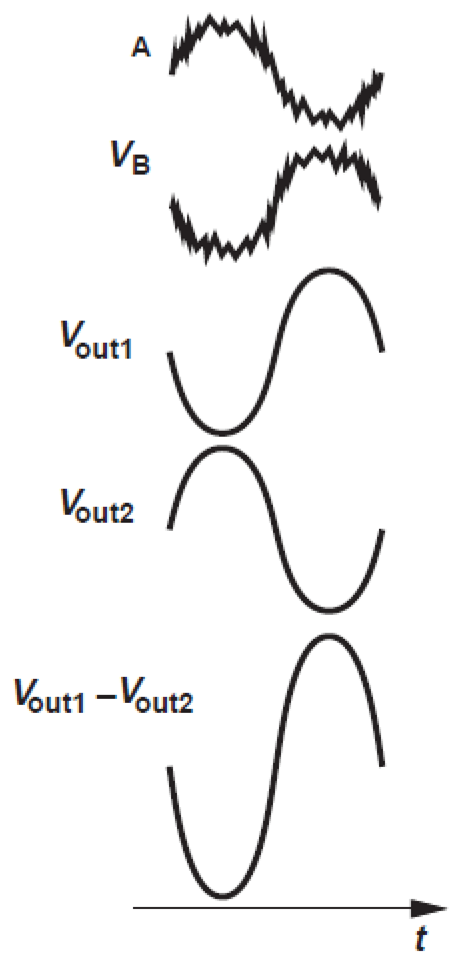
- METHOD #1: **Half-Circuit Principle (exploit symmetry)**
- $A_{vCM} = \frac{v_{oCM}}{v_{iCM}} = \frac{-g_m R_D}{1+2(g_m+g_{mb})R_{SS}} = A_{v,half-circuit}$
- $A_{vd}/A_{vCM} \approx 2(g_m + g_{mb})R_{SS} \gg 1$
- CM input is "partially" rejected (compare with pseudo diff amp)



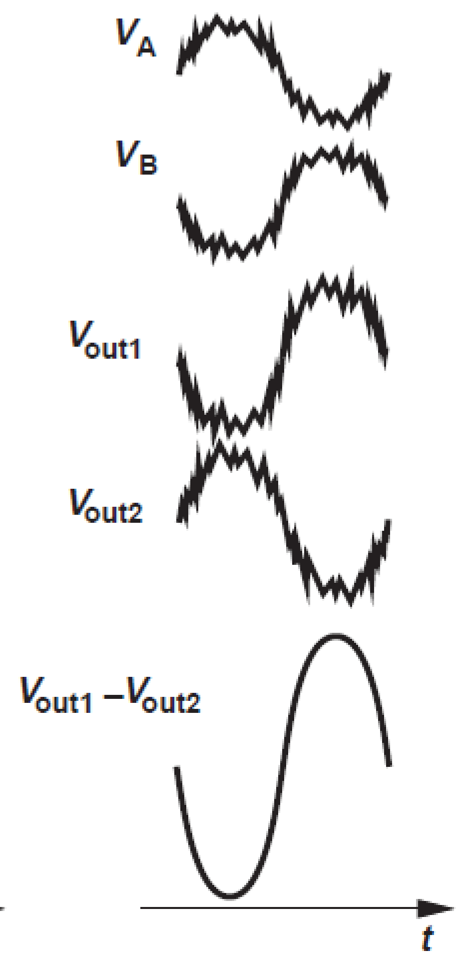
A2. CM Small Signal Analysis



$R_{EE} \rightarrow \infty$



Finite R_{EE}



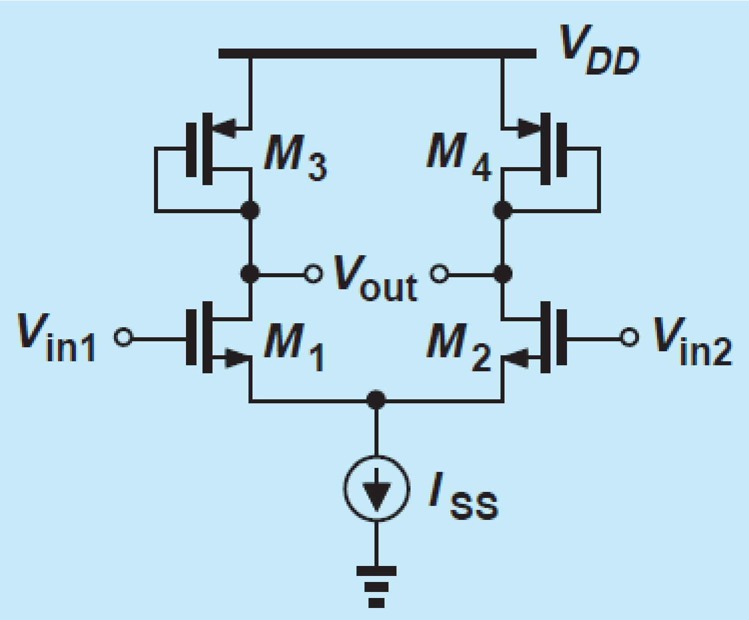
[Razavi, 2014]

Recapping Small Signal Analysis

	Pseudo Diff Amp	Diff Pair (w/ ideal CS)	Diff Pair (w/ R_{SS})
A_{vd}	$-g_m R_D$	$-g_m R_D$	$-g_m R_D$
A_{vCM}	$-g_m R_D$	0	$\dfrac{-g_m R_D}{1+2(g_m+g_{mb})R_{SS}}$
A_{vd}/A_{vCM}	1	∞	$2(g_m+g_{mb})R_{SS} \gg 1$

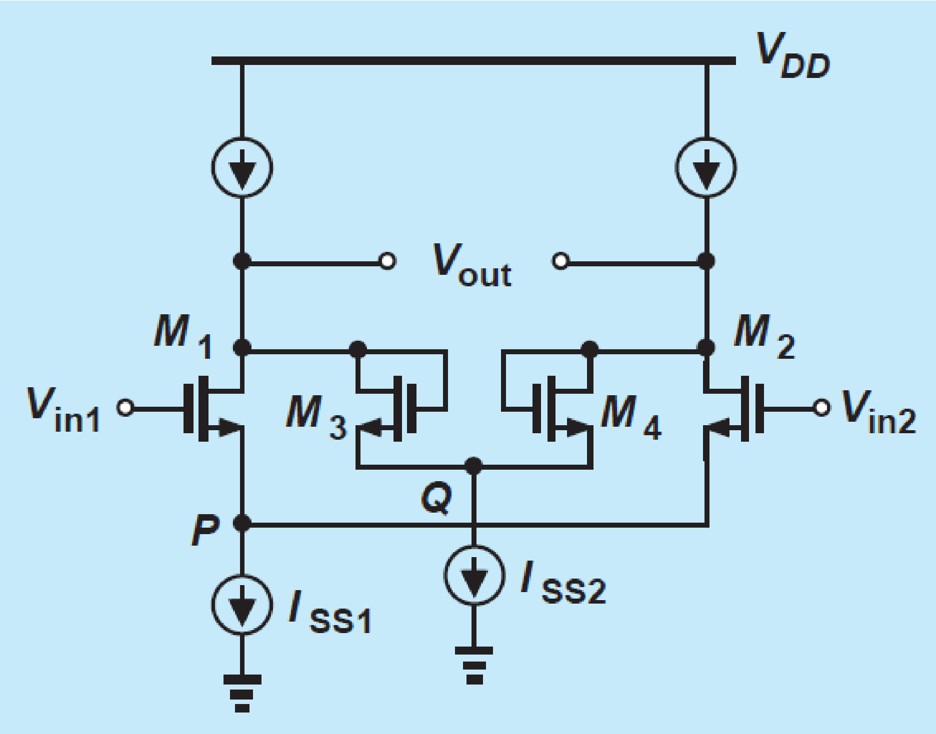
Bonus Question

- Assume symmetry and neglect body effect. Calculate A_{vd}.



Bonus Question

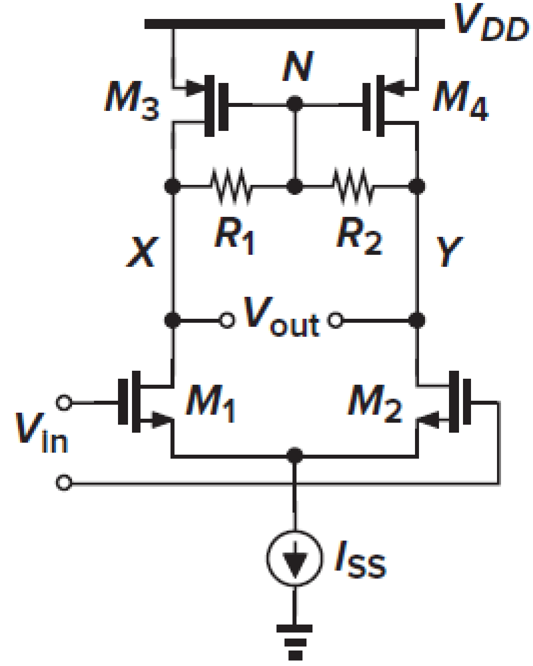
- Assume symmetry and neglect body effect. Calculate A_{vd}.



[Razavi, 2014]

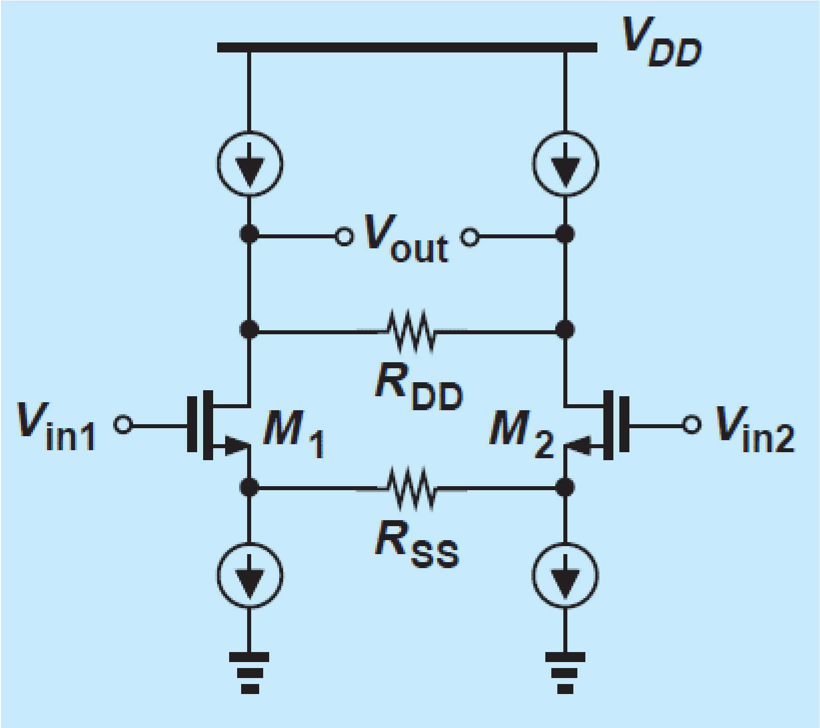
Bonus Question

- Assume symmetry and neglect Channel Length Modulation (CLM) (ro). Calculate A_{vd}.



Bonus Question

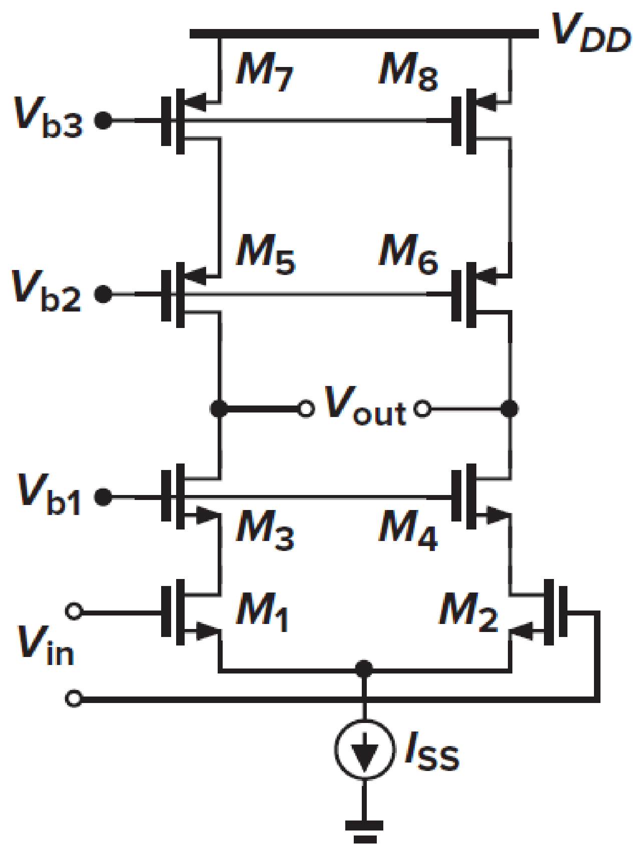
- Assume symmetry, assume R_{SS} is large, and neglect CLM (ro). Calculate A_{vd}.



[Razavi, 2014]

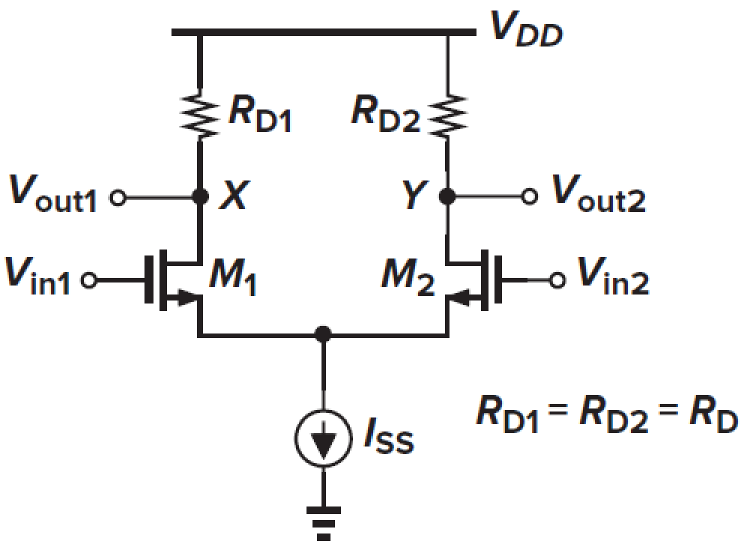
Bonus Question

- Assume symmetry, and assume all transistors have the same g_m and r_o. Calculate A_{vd}.

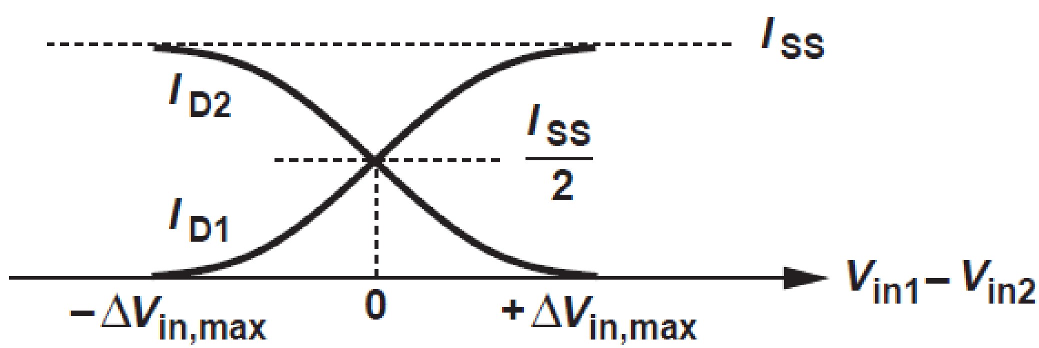


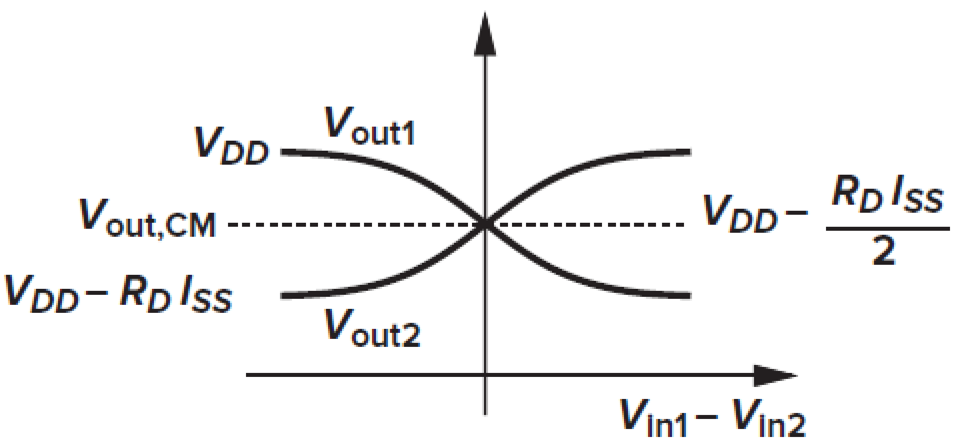
 [Razavi, 2017]

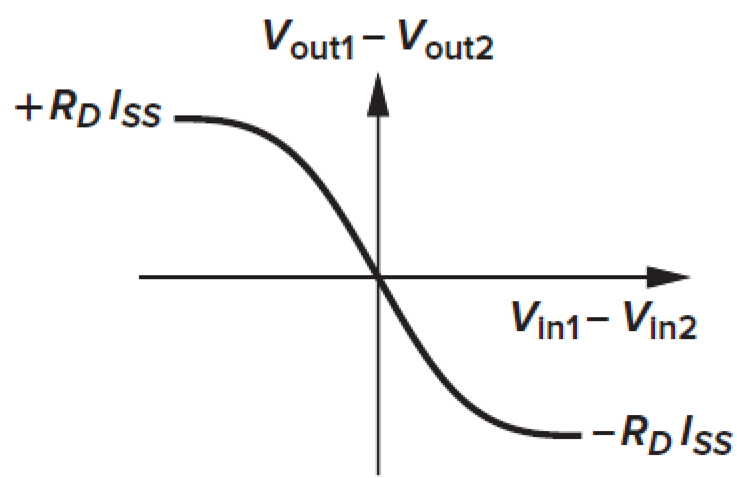
B1. Diff Large Signal Analysis



$$\Delta V_{in,max} = \sqrt{2}V_{ov,eq}$$







[Razavi, 2014]

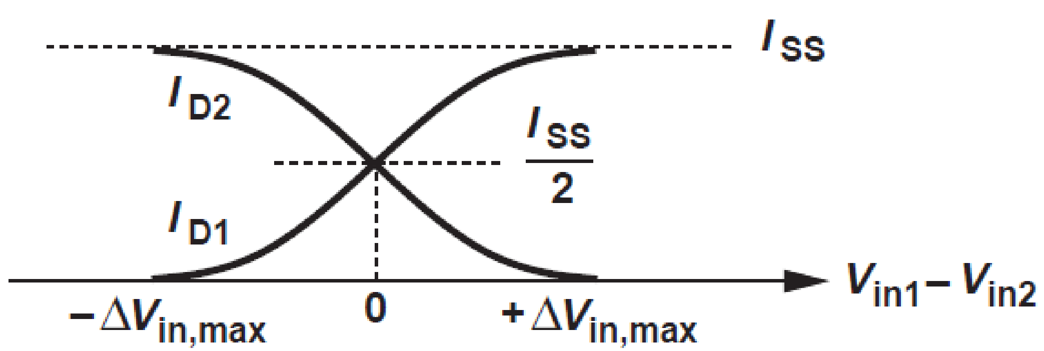
B1. Diff Large Signal Analysis

- Analysis using square law (H.W., see Razavi Section 4.2.2)

$$I_{D1} = \frac{I_{SS}}{2} + \frac{V_{id}}{4}\sqrt{\mu C_{ox}\frac{W}{L}\left(4I_{SS} - \mu C_{ox}\frac{W}{L}V_{id}^2\right)}$$

$$I_{D2} = \frac{I_{SS}}{2} - \frac{V_{id}}{4}\sqrt{\mu C_{ox}\frac{W}{L}\left(4I_{SS} - \mu C_{ox}\frac{W}{L}V_{id}^2\right)}$$

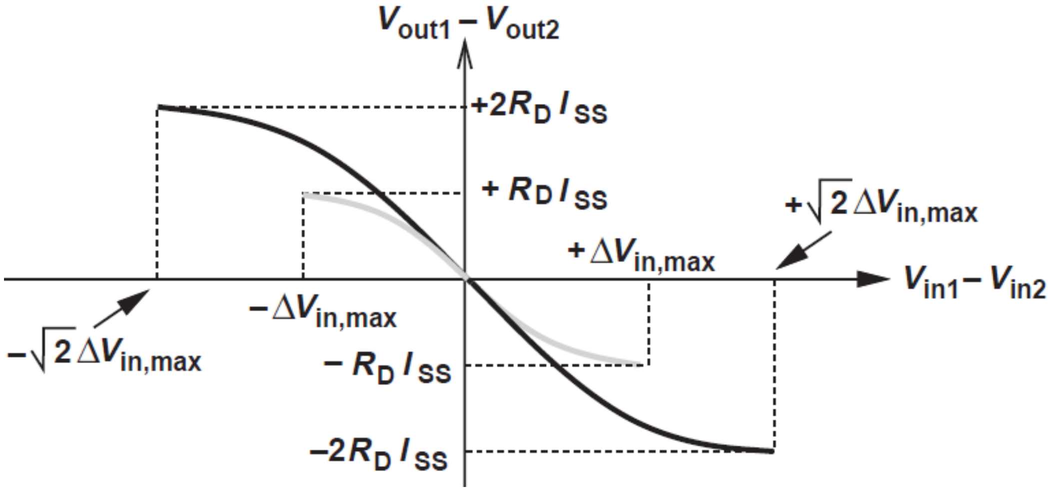
$$\Delta V_{in,max} = \sqrt{2}V_{ov,eq}$$



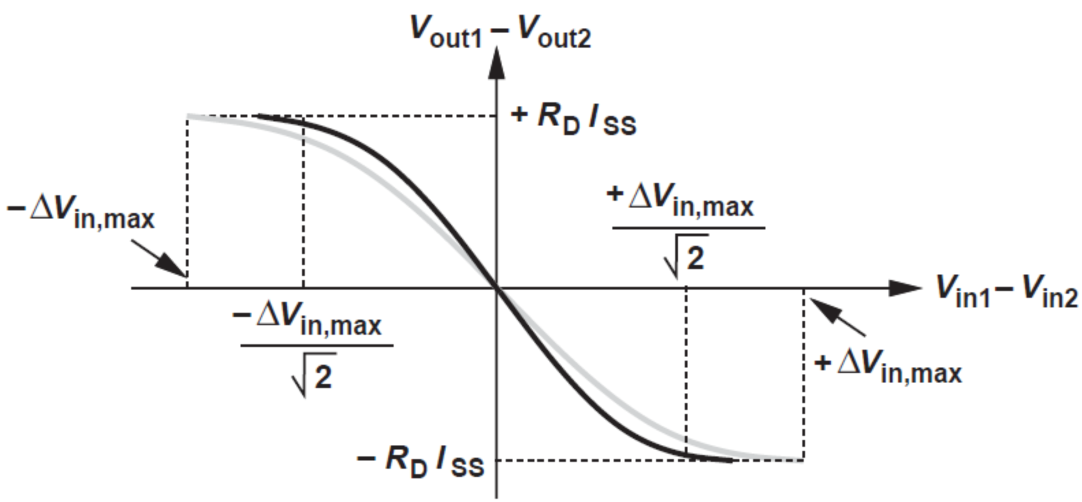
[Razavi, 2014]

B1. Diff Large Signal Analysis

- If tail current is doubled



- If aspect ratio is doubled

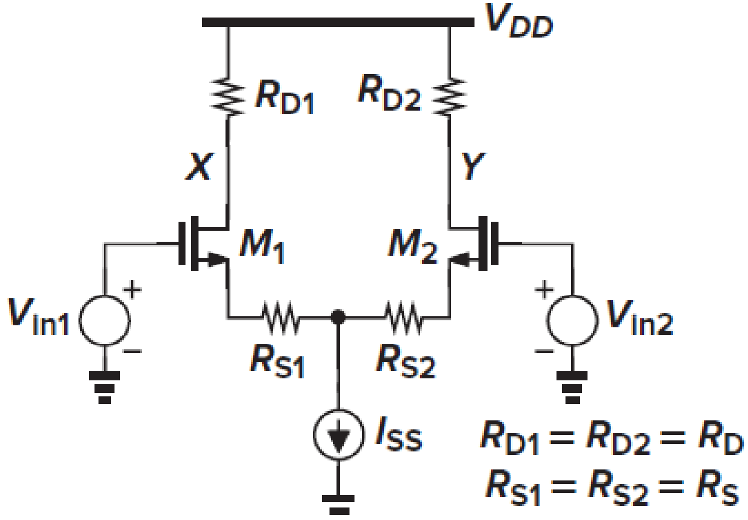


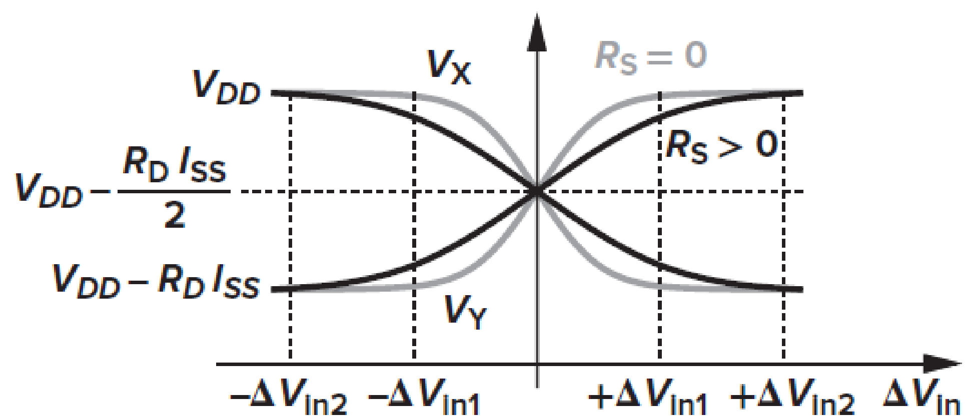
B1. Diff Large Signal Analysis

- Degeneration extends linear range

$$\Delta V_{in2} = \Delta V_{in1} + I_{SS} R_S$$

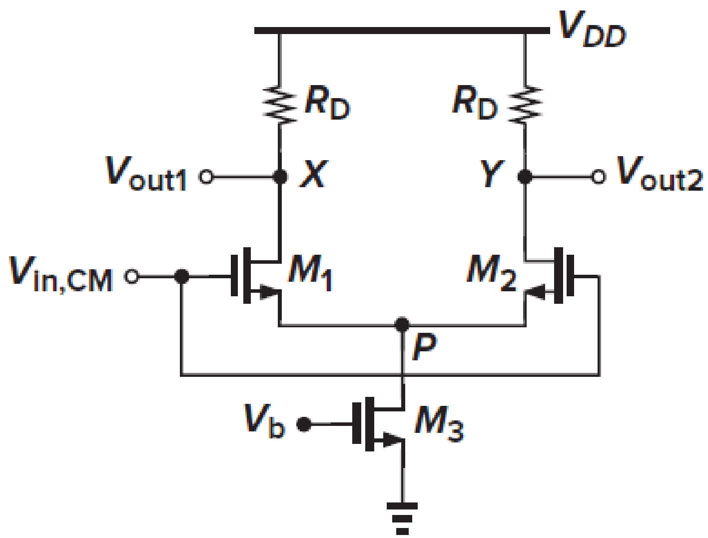
- But headroom is reduced





B2. CM Large Signal Analysis

- The tail current source suppresses the effect of input CM level variations on OP point of M1 and M2 and output CM level
  - But all transistors must be in saturation



- M3 in sat

$$V_{inCM} \geq V_{TH} + V_{ov1} + V_{ov3}$$

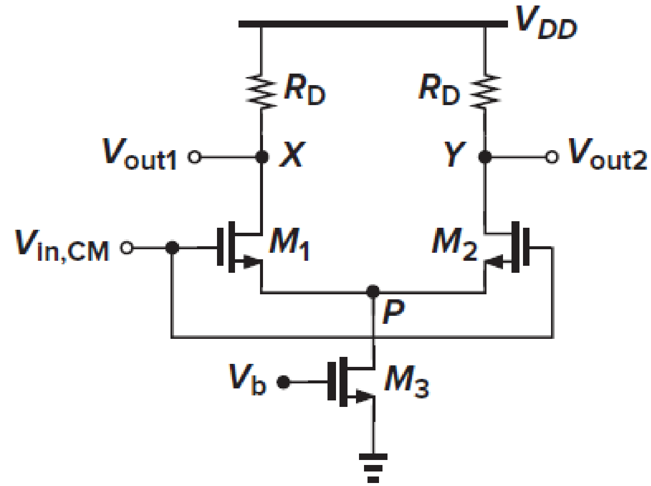
- M1,2 in sat

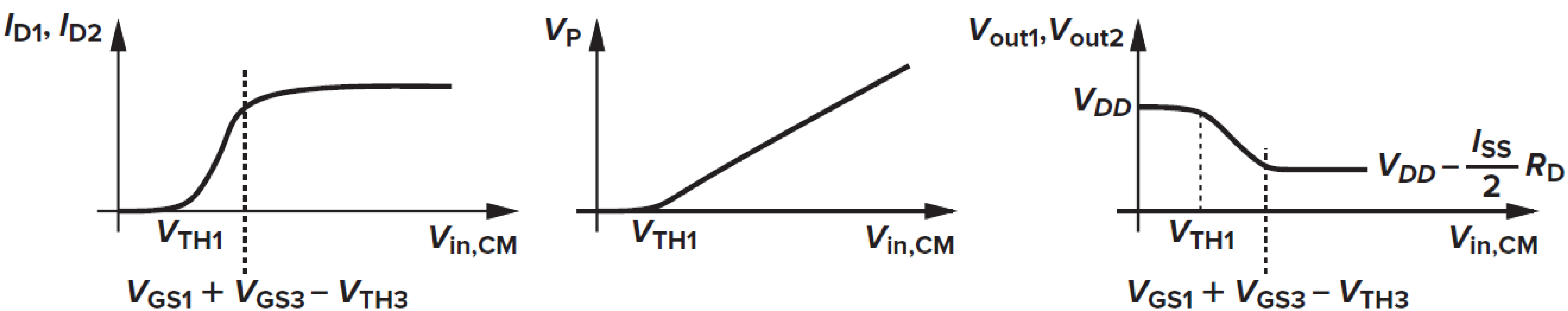
$$V_{inCM} \leq V_{DD} - \frac{I_{SS}}{2} R_D + V_{TH}$$

[Razavi, 2017]

B2. CM Large Signal Analysis

- The tail current source suppresses the effect of input CM level variations on OP point of M1 and M2 and output CM level
  - But all transistors must be in saturation

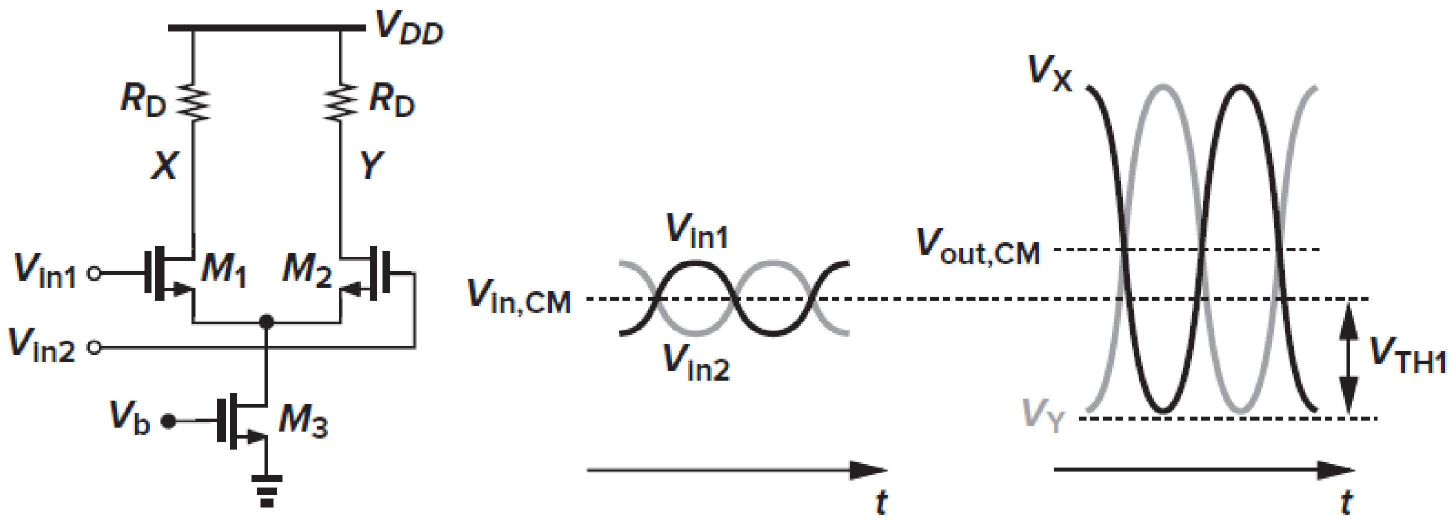




[Razavi, 2017]

Max Allowable Signal Swing

- Max output is VDD, min output is set by keeping M1,2 in sat
$$max\ pk2pk\ swing = 2 \times \left(V_{DD} - (V_{inCM} - V_{TH})\right)$$
- If V_{inCM} is set to its min value: $V_{inCM} = V_{TH} + V_{ov1} + V_{ov3}$
$$max\ pk2pk\ swing = 2 \times (V_{DD} - V_{ov1} - V_{ov3})$$
  - Can be deduced intuitively by noting that M1 and M3 are vertically stacked
- For SE amp: $max\ pk2pk\ swing = (V_{DD} - V_{ov1})$



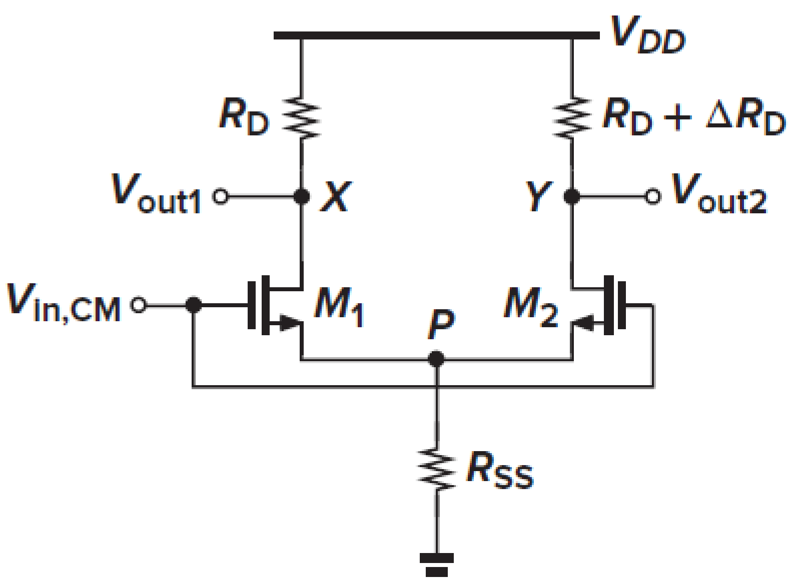
[Razavi, 2017]

Effect of Mismatch (in Load)

- Most dangerous effect: CM to diff conversion
- Example #1: Mismatch in load resistance

$$A_{vCM2d} = \frac{v_{od}}{v_{iCM}} = \frac{v_{out1} - v_{out2}}{v_{iCM}} = -\left(\frac{g_m R_D}{1 + 2g_m R_{SS}} - \frac{g_m (R_D + \Delta R_D)}{1 + 2g_m R_{SS}}\right)$$

$$= \frac{g_m \Delta R_D}{1 + 2g_m R_{SS}} \approx \frac{\Delta R_D}{2R_{SS}}$$



[Razavi, 2017]

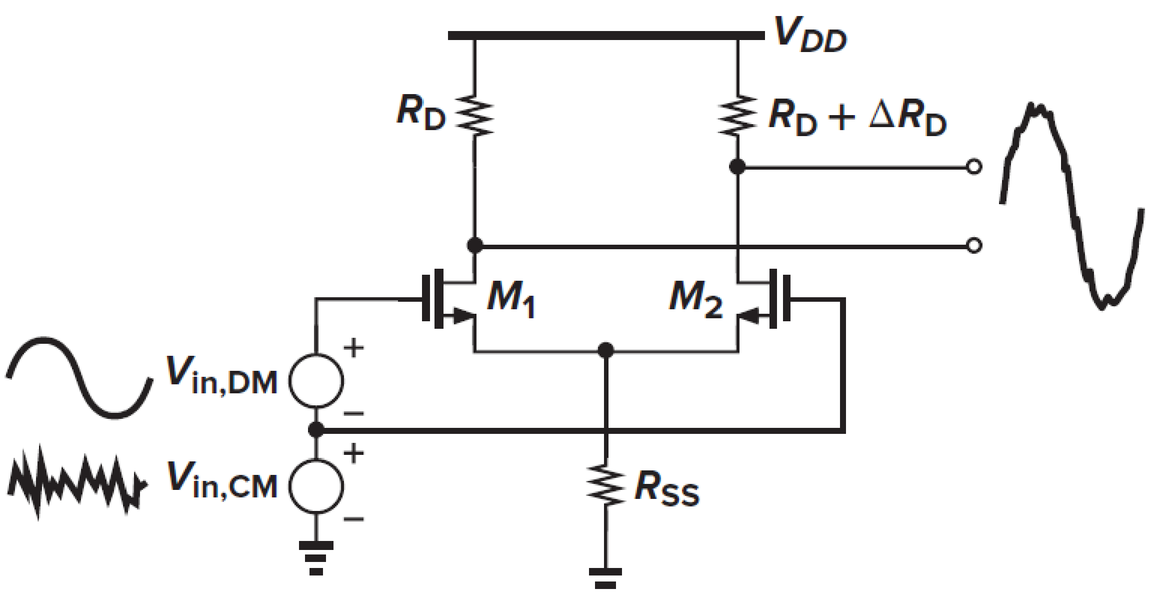
Effect of Mismatch (in Load)

- Example #1: Mismatch in load resistance

$$A_{vCM2d} \approx \frac{\Delta R_D}{2R_{SS}}$$

- Common-mode rejection ration (CMRR) (@low frequency!)

$$CMRR = \frac{A_{vd}}{A_{vCM2d}} \approx 2g_m R_{SS} \frac{R_D}{\Delta R_D}$$



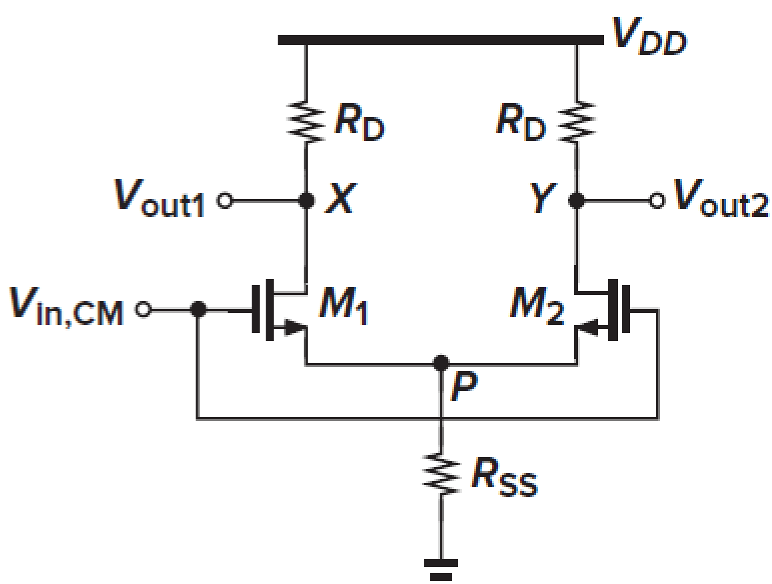
[Razavi, 2017]

Effect of Mismatch (in Input Pair)

- Most dangerous effect: CM to diff conversion
- Example #2: Mismatch in input pair (M1 and M2)

$$A_{vCM2d} = \frac{v_{od}}{v_{iCM}} = \frac{v_{out1} - v_{out2}}{v_{iCM}} = -\frac{g_{m1}R_D}{1 + g_{m1}\left(\frac{1}{g_{m2}}||R_{SS}\right)} + \frac{g_{m2}R_D}{1 + g_{m2}\left(\frac{1}{g_{m1}}||R_{SS}\right)}$$

$$= -\frac{\Delta g_m R_D}{1 + (g_{m1} + g_{m2})R_{SS}} \approx -\frac{\Delta g_m R_D}{(g_{m1} + g_{m2})R_{SS}}$$



[Razavi, 2017]

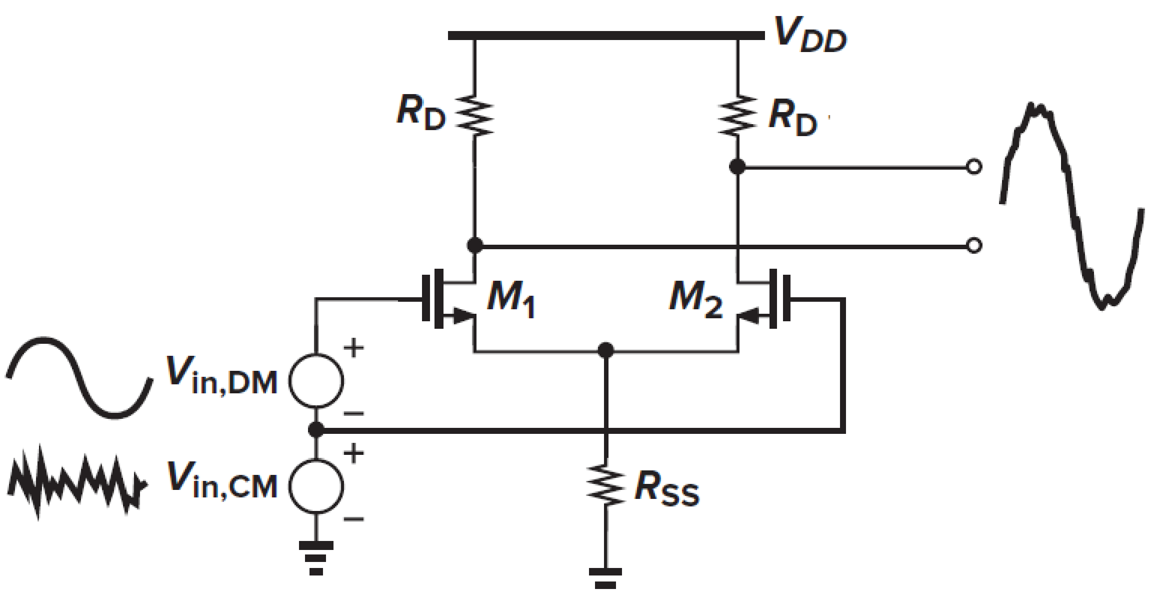
Effect of Mismatch (in Input Pair)

- Example #2: Mismatch in input pair (M1 and M2)

$$A_{vCM2d} \approx \frac{\Delta g_m R_D}{(g_{m1} + g_{m2})R_{SS}}$$

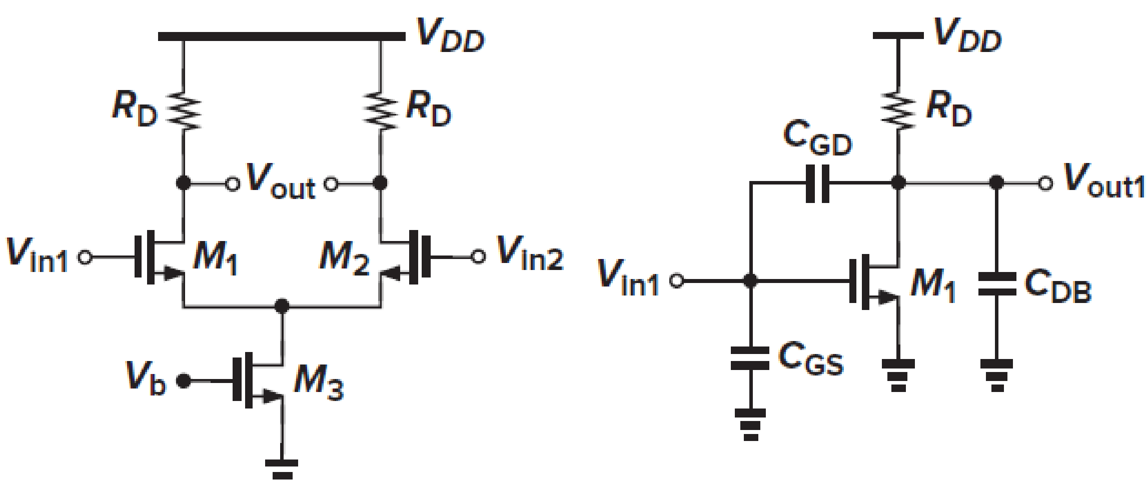
- Common-mode rejection ration (CMRR) (@low frequency!)

$$CMRR = \frac{A_{vd}}{A_{vCM2d}} \approx 2g_m R_{SS} \frac{g_m}{\Delta g_m}$$



Diff Frequency Response

- The freq response of the diff amp is itself the freq response of the half-circuit
- Note that the number of poles/zeros in the diff amp is the same as the number of poles/zeros in the half-circuit
  - The two halves are added, not multiplied
  - Ex: $A(s) = \frac{A_o}{1+\frac{s}{\omega_p}} + \frac{A_o}{1+\frac{s}{\omega_p}} = \frac{2A_o}{1+\frac{s}{\omega_p}}$ (what if there is mismatch?)



 [Razavi, 2017]

Diff Frequency Response

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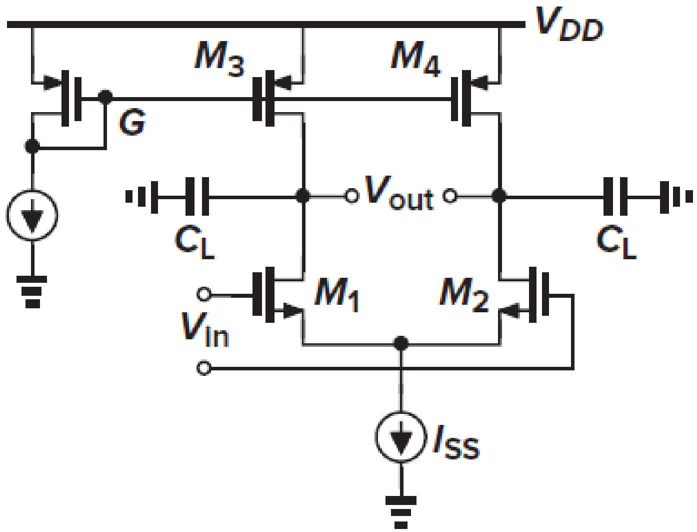
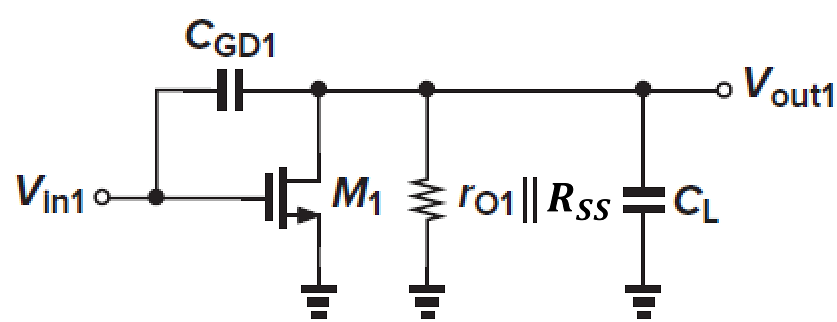
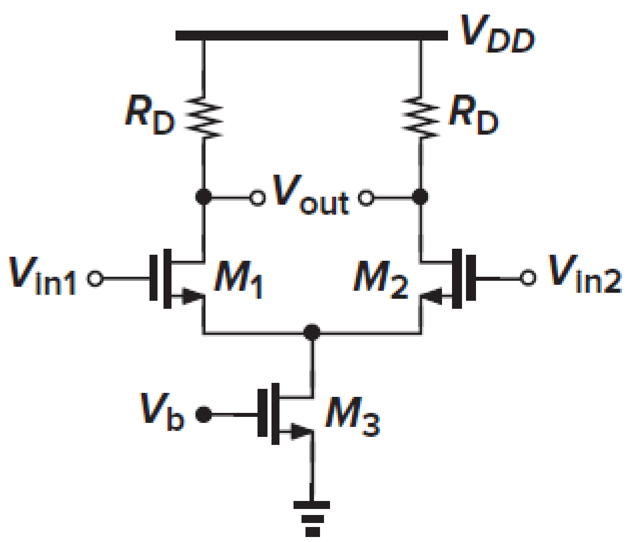
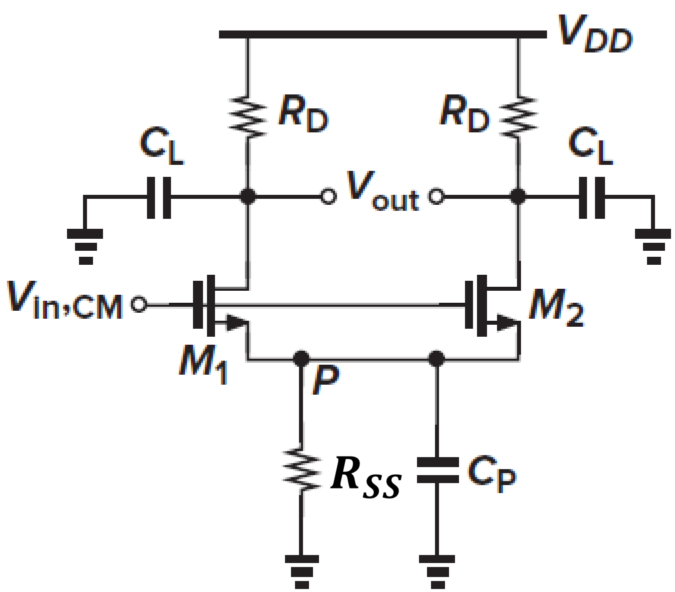




CM Frequency Response

- C_P degrades tail CS impedance at high frequency
- $C_P \approx C_{db3} + C_{gd3} + C_{sb1} + C_{sb2}$
- Trade-off between headroom and CMRR
  - M1-M3 are made wide to decrease V_{ov} and increase headroom
  - But C_P increases, and degrades CMRR





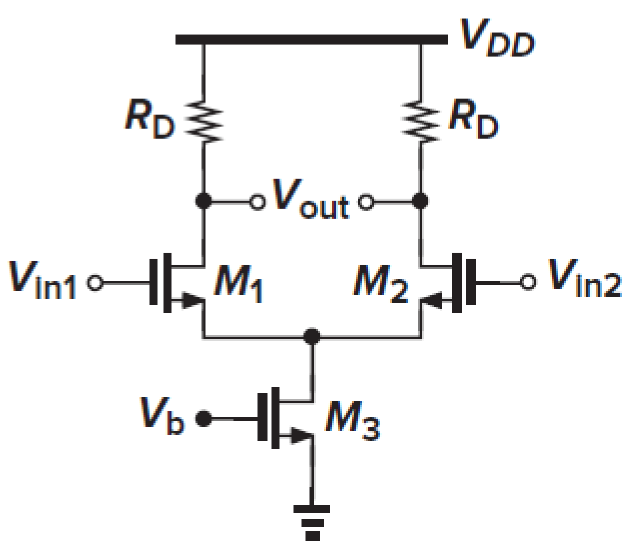
[Razavi, 2017]

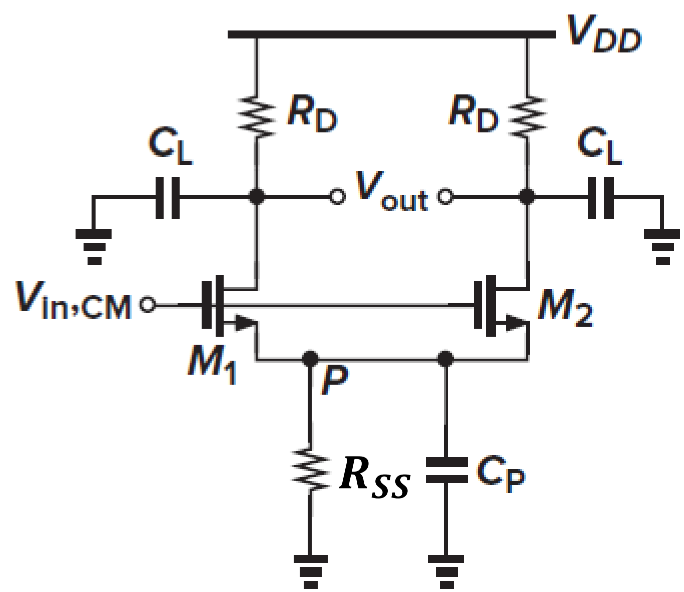
CM Frequency Response

- Mismatch in input pair (M1 and M2)

@Low frequency: $A_{vCM2d} \approx \dfrac{\Delta g_m R_D}{1+(g_{m1}+g_{m2})R_{SS}}$

@High frequency: $A_{vCM2d} \approx \dfrac{\Delta g_m \left(R_D // \frac{1}{sC_L}\right)}{1+(g_{m1}+g_{m2})\left(R_{SS} // \frac{1}{sC_P}\right)}$





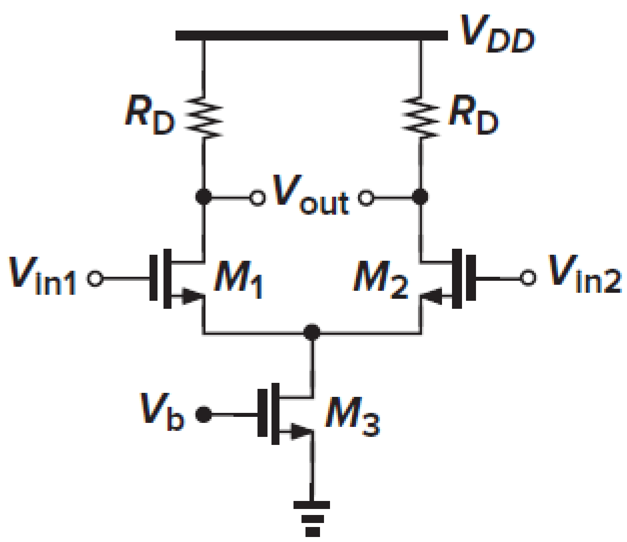
[Razavi, 2017]

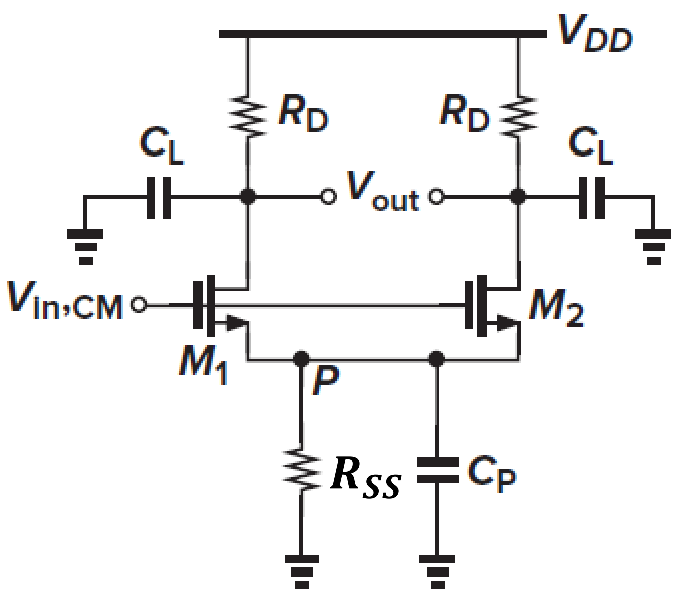
CM Frequency Response

- Mismatch in input pair (M1 and M2)

@Low frequency: $CMRR = \frac{A_{vd}}{A_{vCM2d}} \approx (1 + 2g_m R_{SS}) \frac{g_m}{\Delta g_m}$

@High frequency: $CMRR = \frac{A_{vd}(s)}{A_{vCM2d}(s)} \approx \left[1 + 2g_m \left(R_{SS} // \frac{1}{sC_P}\right)\right] \frac{g_m}{\Delta g_m}$





[Razavi, 2017]

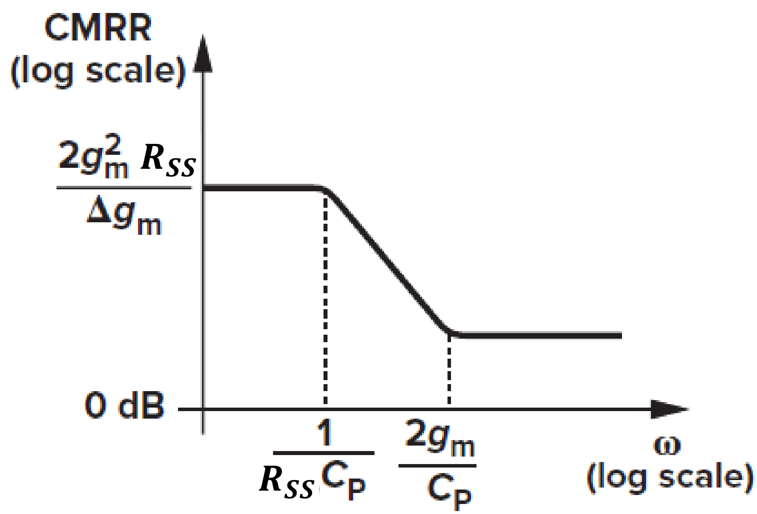
CM Frequency Response

- Mismatch in input pair (M1 and M2)

$$CMRR = \frac{A_{vd}(s)}{A_{vCM2d}(s)} \approx \left[1 + 2g_m\left(R_{SS}//\frac{1}{sC_P}\right)\right]\frac{g_m}{\Delta g_m}$$

$$\approx \frac{1 + s\frac{C_P}{2g_m}}{1 + sR_{SS}C_P} \cdot 2g_m R_{SS}\frac{g_m}{\Delta g_m}$$

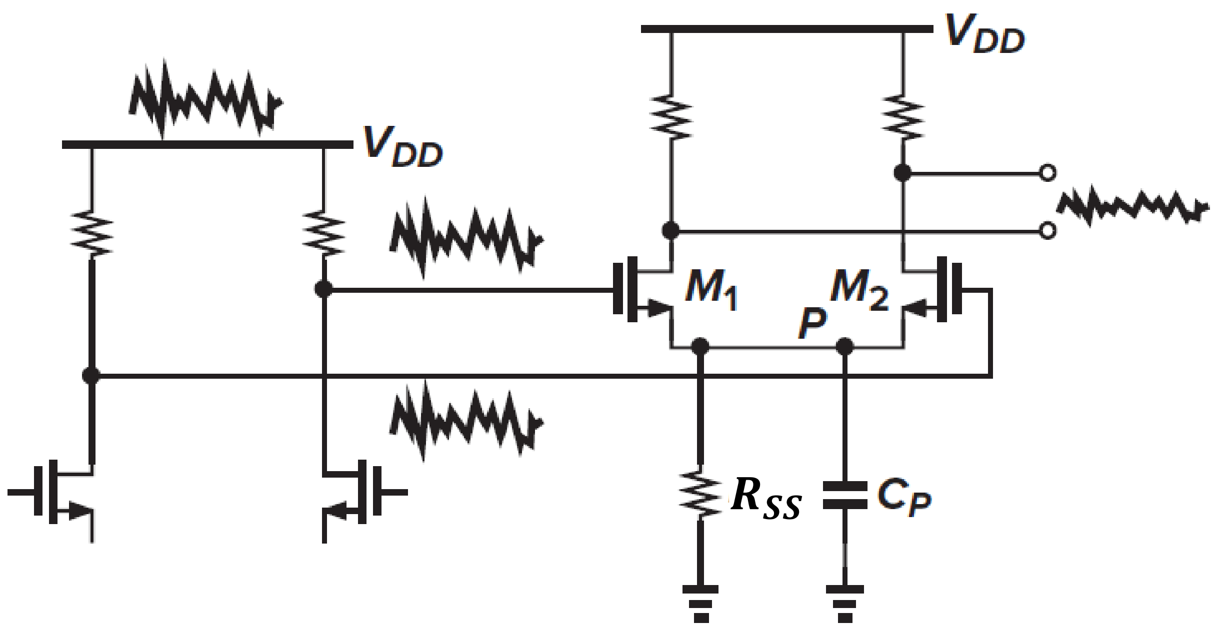
- $@\omega \uparrow\uparrow: CMRR \approx \frac{g_m}{\Delta g_m}$



[Razavi, 2017]

CM Frequency Response

- High frequency supply noise is a very serious issue
- Again: There is a trade-off between headroom and CMRR
  - More serious for low supply voltage



 [Razavi, 2017]

Thank you!