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Analog IC Design

Lecture 14
Negative Feedback

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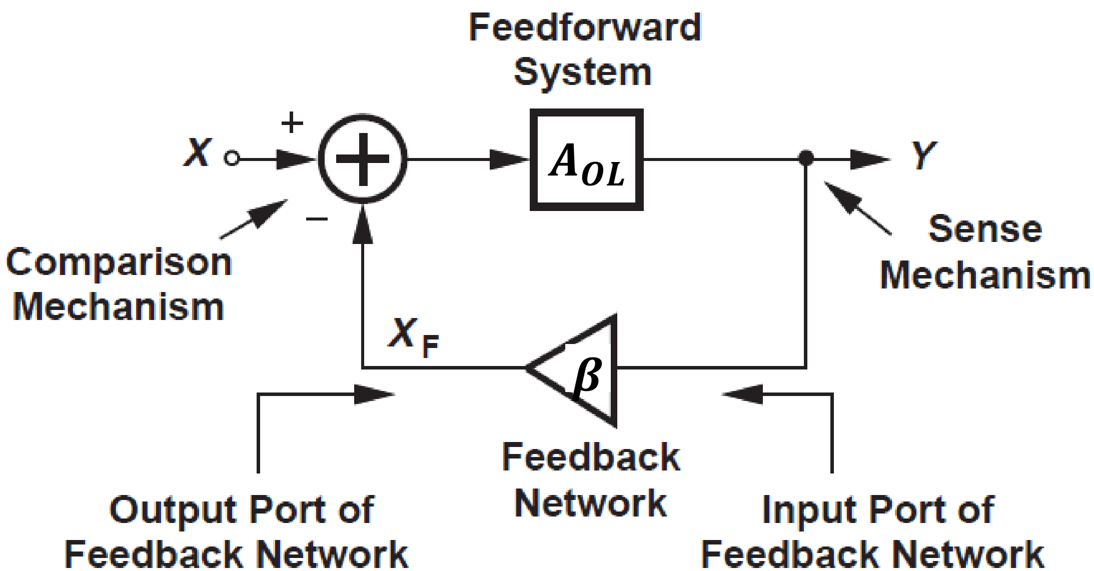
General Feedback System

- A_{OL} = Open loop (OL) gain $\gg 1$
- $A_{CL} = \frac{Y}{X}$ = Closed loop (CL) gain
- Error signal = $X - X_F$

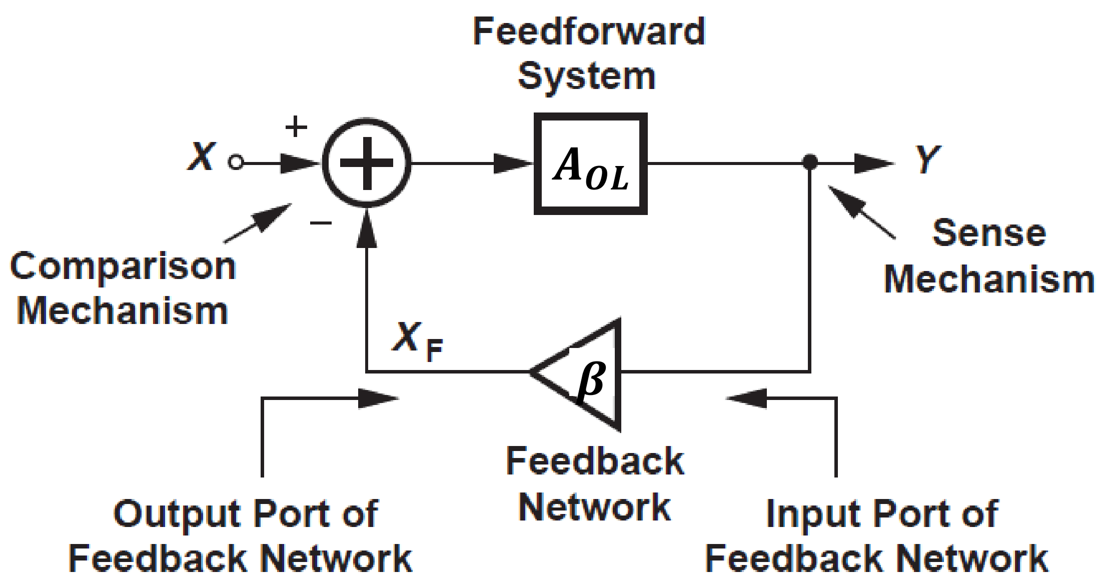


 [Razavi, 2014]

General Feedback System

- A_{OL} = Open loop (OL) gain $\gg 1$
- $A_{CL} = \frac{Y}{X}$ = Closed loop (CL) gain
- Error signal = $X - X_F$

$$Y = A_{OL}(X - X_F) = A_{OL}(X - \beta Y)$$
$$A_{CL} = \frac{Y}{X} = \frac{A_{OL}}{1 + \beta \cdot A_{OL}} \approx \frac{1}{\beta}$$

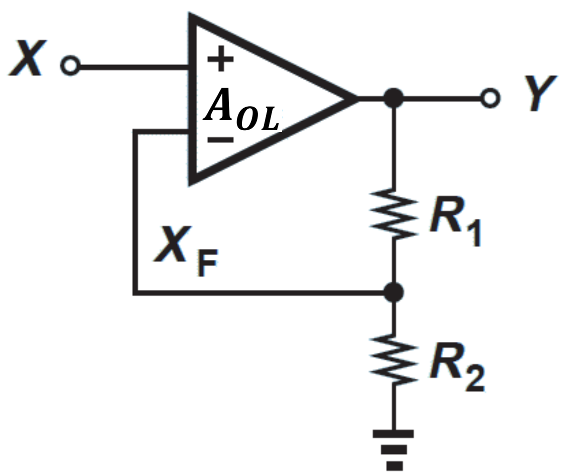


[Razavi, 2014]

Feedback Example

- Op-amp A_{OL} performs two functions: (1) subtraction of X and X_F and (2) amplification
- The network R_1 and R_2 performs two functions: (1) sensing the output voltage and (2) providing a feedback factor $\beta = \frac{R_2}{(R_1+R_2)}$

$$A_{CL} = \frac{Y}{X} = \frac{A_{OL}}{1+\beta \cdot A_{OL}} = \frac{A_{OL}}{1+\frac{R_2}{(R_1+R_2)} \cdot A_{OL}} \approx \frac{R_1+R_2}{R_2} = 1+\frac{R_1}{R_2}$$



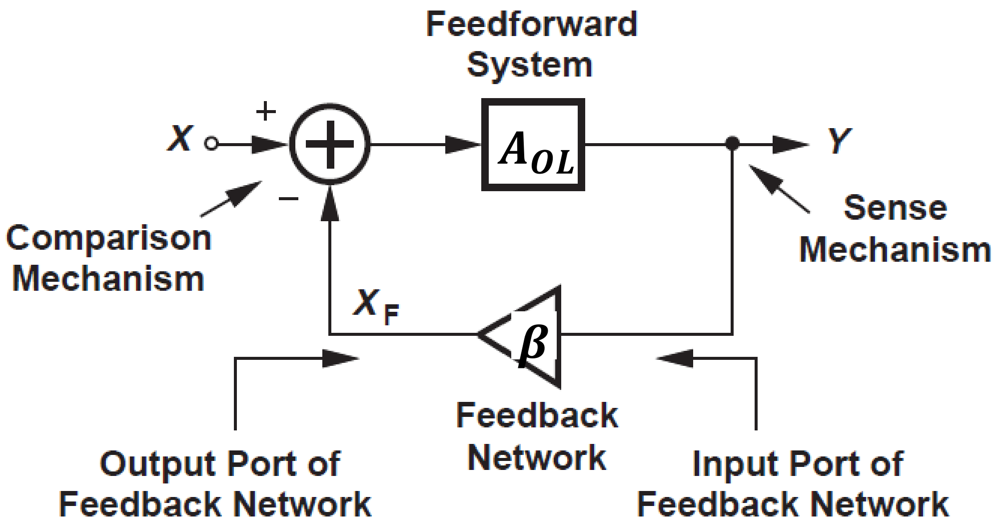
[Razavi, 2014]

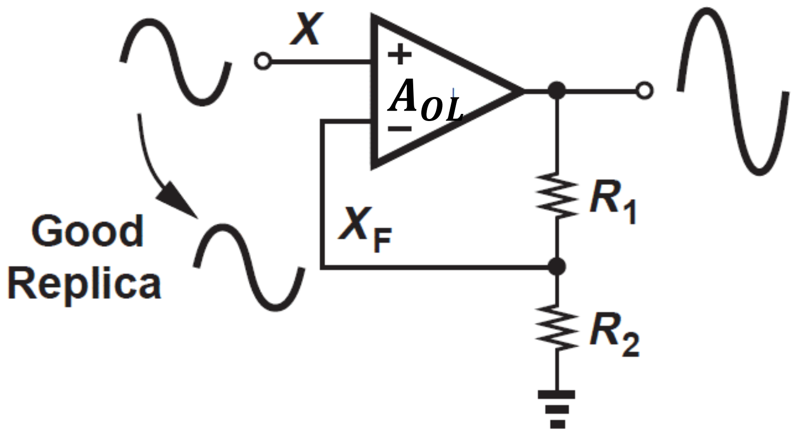
Feedback Example

- Negative feedback loop works to minimize the error signal

$$Error = E = X - X_F = X - \beta Y = X - \beta A_{OL} E$$

$$E = \frac{X}{1 + \beta A_{OL}} \rightarrow 0$$

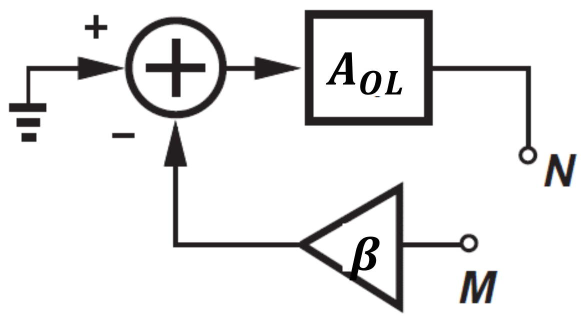


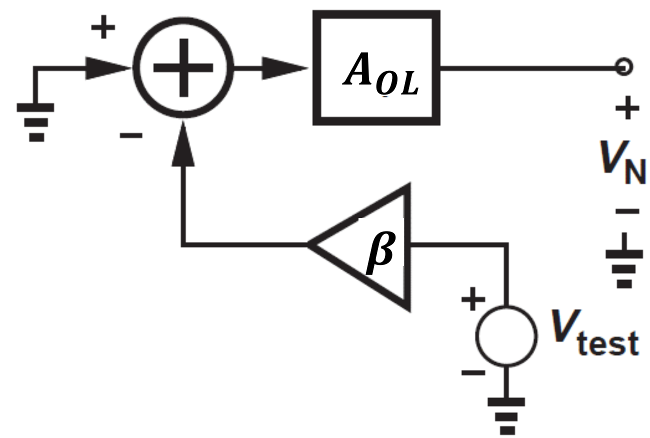


[Razavi, 2014]

Loop Gain

- Break the loop → Apply a test source → Calculate the gain around the loop
- Loop gain $= \beta \cdot A_{OL}$
- But the loading changes when we break the loop!
  - We may add a dummy load
  - STB simulation takes care of this ☺





[Razavi, 2014]

Why Negative Feedback?

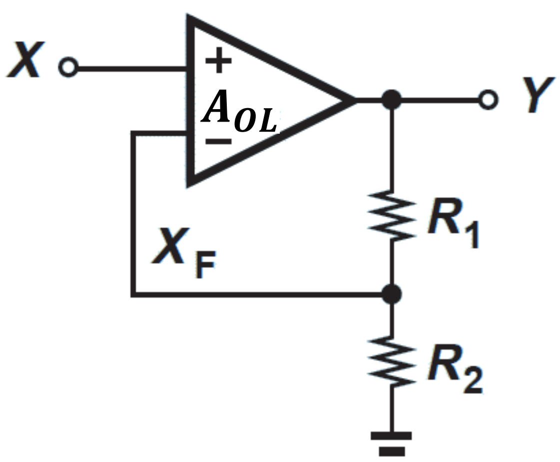
- We use a very high gain amplifier (A_{OL}), but end up with a much smaller gain $\frac{A_{OL}}{1+\beta \cdot A_{OL}} \approx \frac{1}{\beta}$
- We can design high gain amplifiers, but we really do not need all that gain
- High gain is the balance that we use to buy other useful properties
- Negative feedback properties
  - Gain Desensitization → Accurate, stable, and linear gain
  - Bandwidth Extension
  - Modification of I/O Impedances

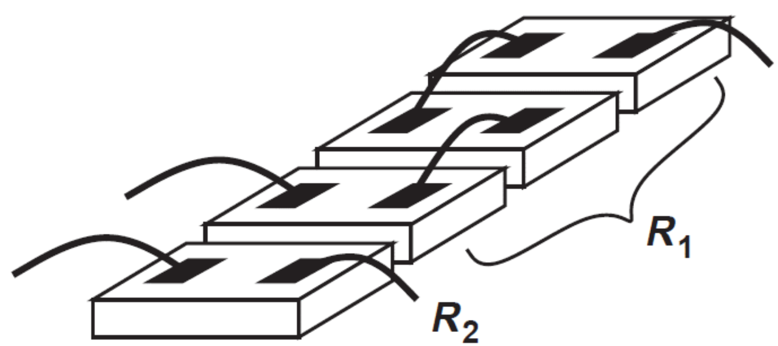
Gain Desensitization

- In IC design, we cannot control absolute values due to PVT, load, and input signal variations
- But we can precisely control ratios of MATCHED components

$$A_{CL} = \frac{Y}{X} = \frac{A_{OL}}{1 + \beta \cdot A_{OL}} = \frac{A_{OL}}{1 + \frac{R_2}{(R_1 + R_2)} \cdot A_{OL}} \approx \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

- $R_1 = 3R \ and \ R_2 = R \Rightarrow A_{CL} = 4 \Rightarrow$ Accurate, stable, and linear





[Razavi, 2014]

Bandwidth Extension

$$A_{CL}(s) = \frac{A_{OL}(s)}{1 + \beta \cdot A_{OL}(s)}$$

$$A_{OL}(s) = \frac{A_o}{1 + \frac{s}{\omega_{p,OL}}}$$

$$A_{CL}(s) = \frac{\frac{A_o}{(1 + \beta A_o)}}{1 + \frac{s}{(1 + \beta A_o)\omega_{p,OL}}}$$

$$\omega_{p,CL} = (1 + \beta A_o)\omega_{P,OL}$$

- CL DC gain reduced by $(1 + \beta A_o)$
- CL bandwidth extended by $(1 + \beta A_o)$
- GBW (and UGF) remains constant

Bandwidth Extension

$$A_{CL}(s) = \frac{\frac{A_o}{(1+\beta A_o)}}{1 + \frac{s}{(1+\beta A_o)\omega_{p,OL}}}$$

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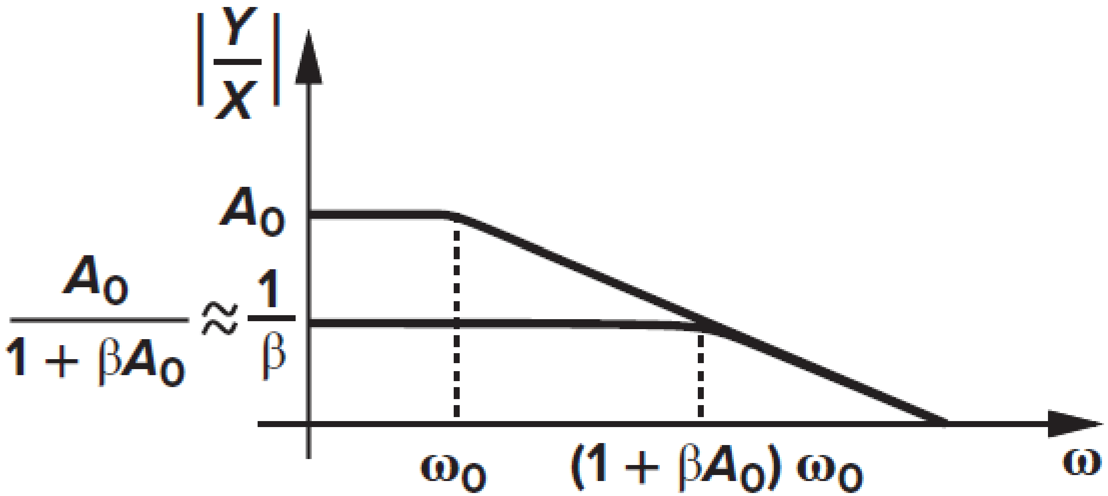
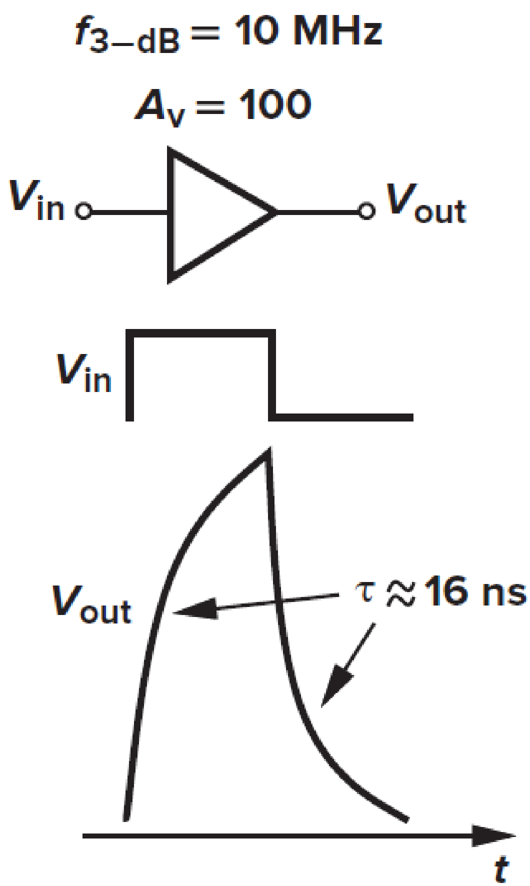
- CL bandwidth extended by $(1+\beta A_o)$
- GBW (and UGF) remains constant

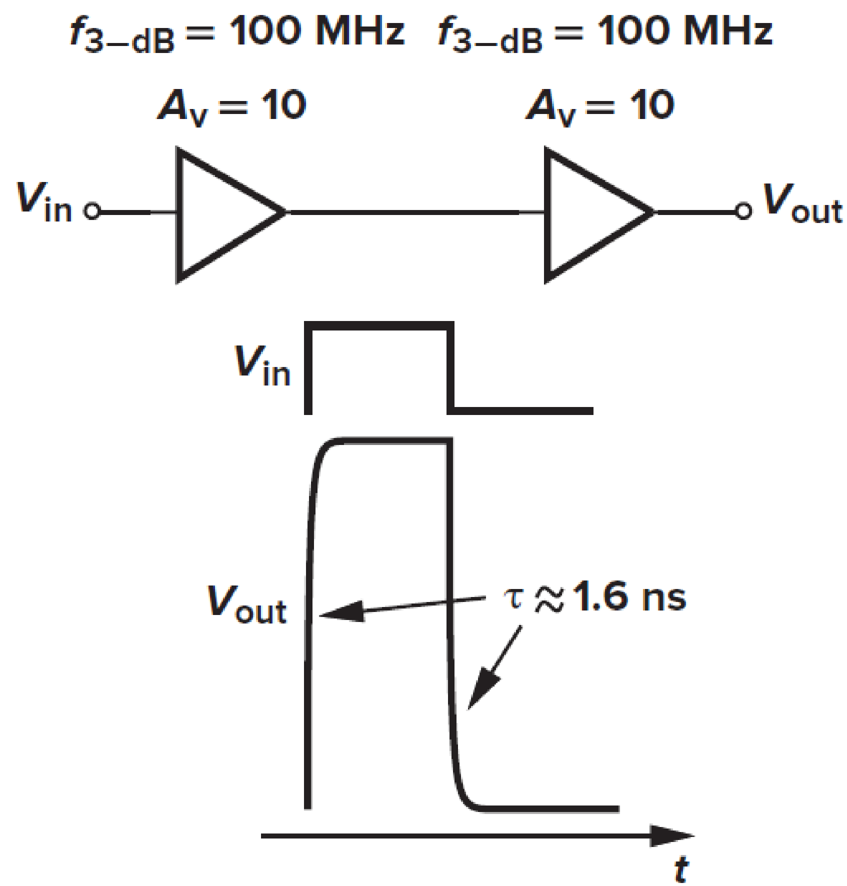


[Razavi, 2017]

Bandwidth Extension

- Cascade of feedback amplifiers provides the same gain and a much faster response → But power consumption is doubled





 [Razavi, 2017]

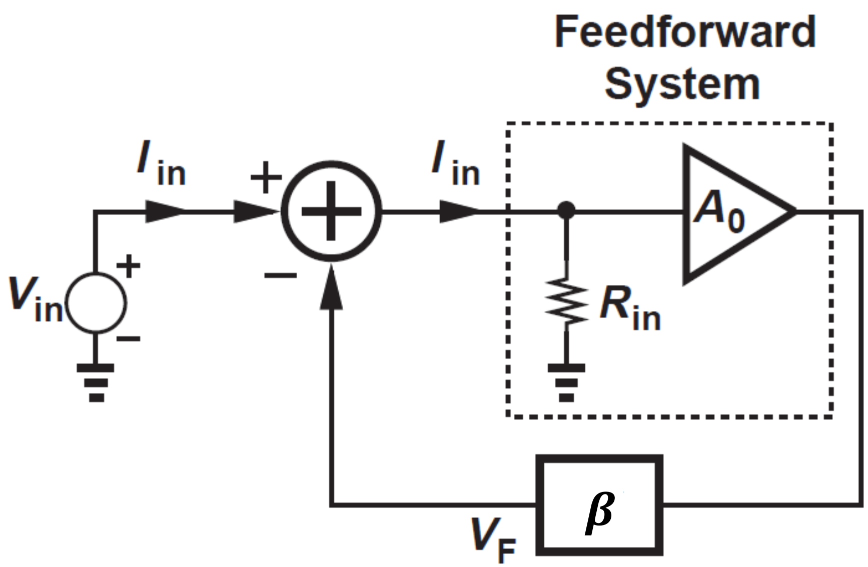
Modification of I/O Impedances

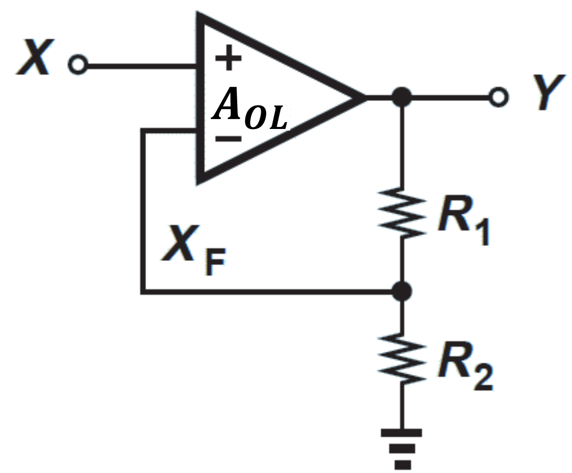
- Example: voltage sensing – voltage mixing feedback

$$I_{in}R_{in} = V_{in} - V_F$$

$$= V_{in} - (I_{in}R_{in})A_0\boldsymbol{\beta}$$

$$\frac{V_{in}}{I_{in}} = R_{in}(1 + \boldsymbol{\beta}A_0)$$





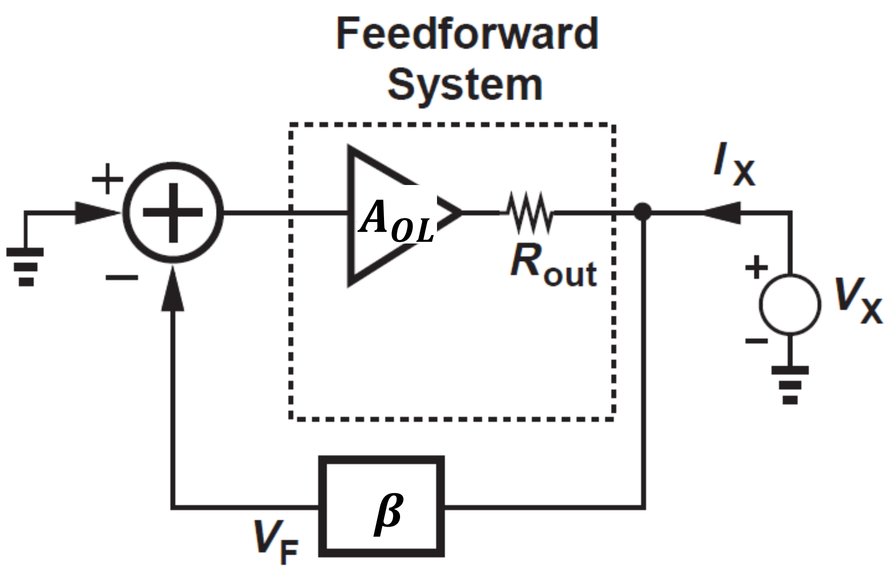
[Razavi, 2014]

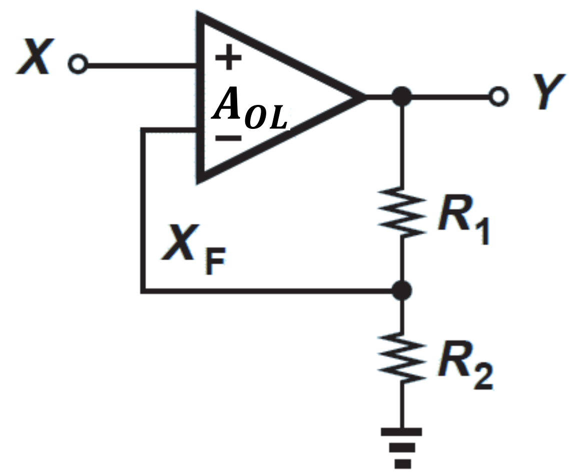
Modification of I/O Impedances

- Example: voltage sensing – voltage mixing feedback

$$I_X = \frac{V_X - (-\beta A_0 V_X)}{R_{out}}$$

$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + \beta A_0}$$

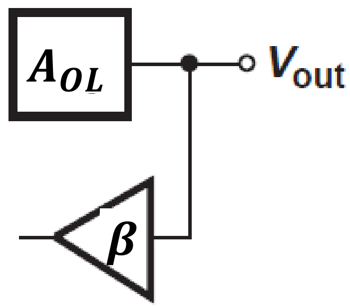




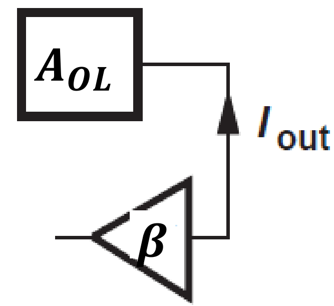
[Razavi, 2014]

Modification of I/O Impedances

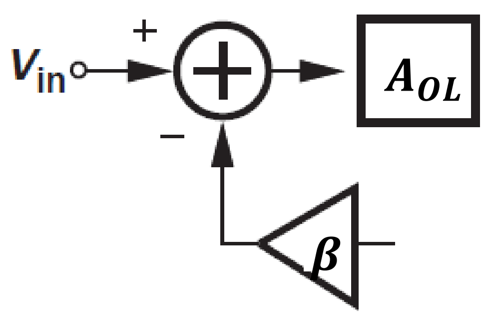
- Shunt sensing/mixing → R decreases
- Series sensing/mixing → R increases



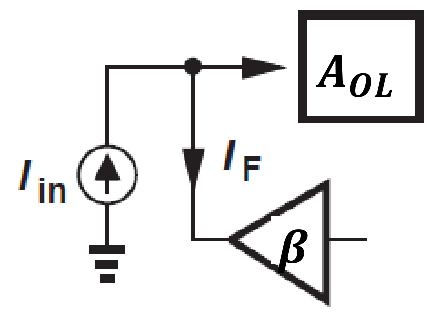
Output impedance falls by 1+ loop gain.



Output impedance rises by 1+ loop gain.



Input impedance rises by 1+ loop gain.

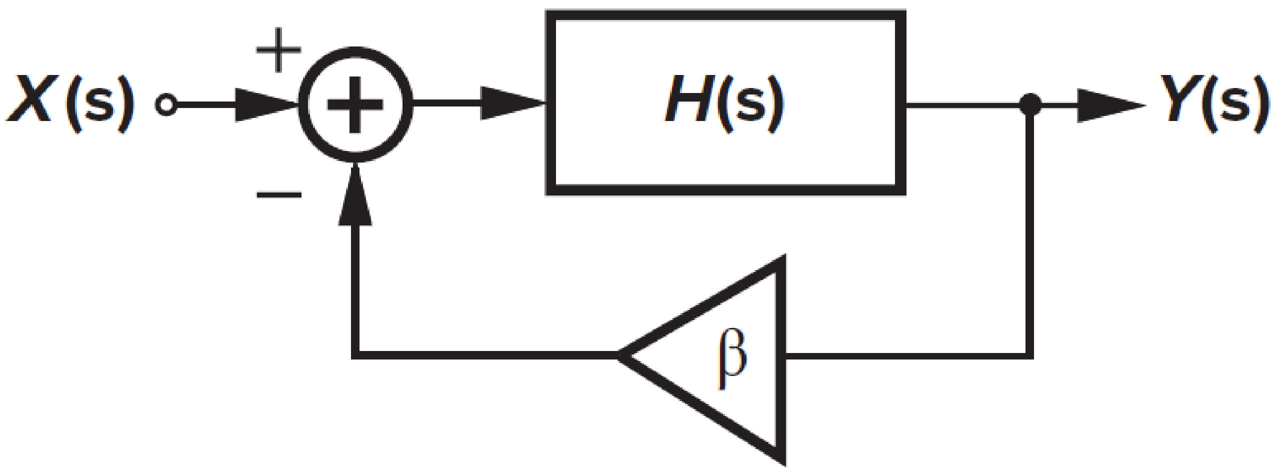


Input impedance falls by 1+ loop gain.

[Razavi, 2014]

Stability of Feedback System

$$H_{CL}(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + \beta H(s)}$$

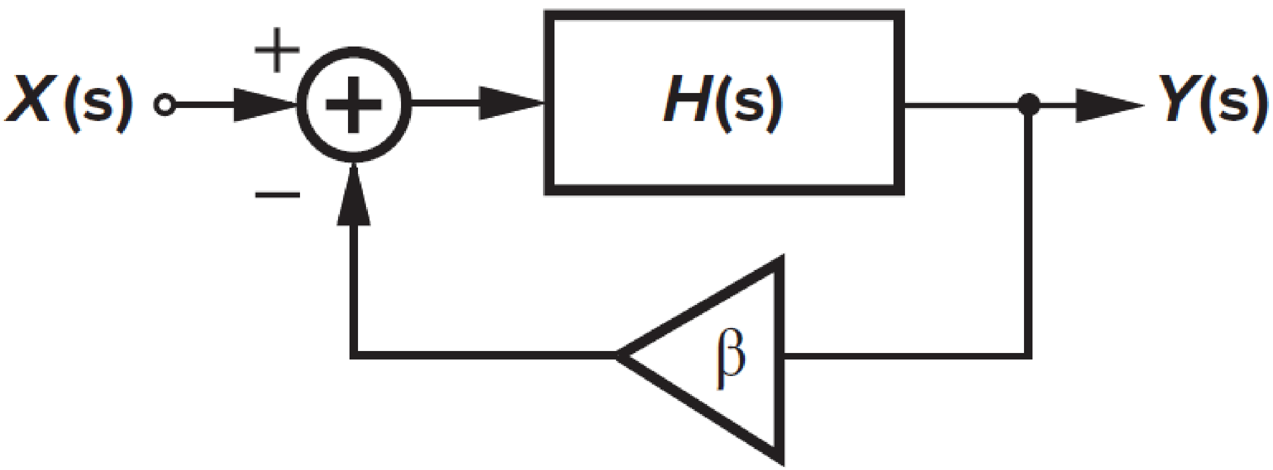


[Razavi, 2017]

Barkhausen's Oscillation Criteria

$$H_{CL}(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + \beta H(s)}$$

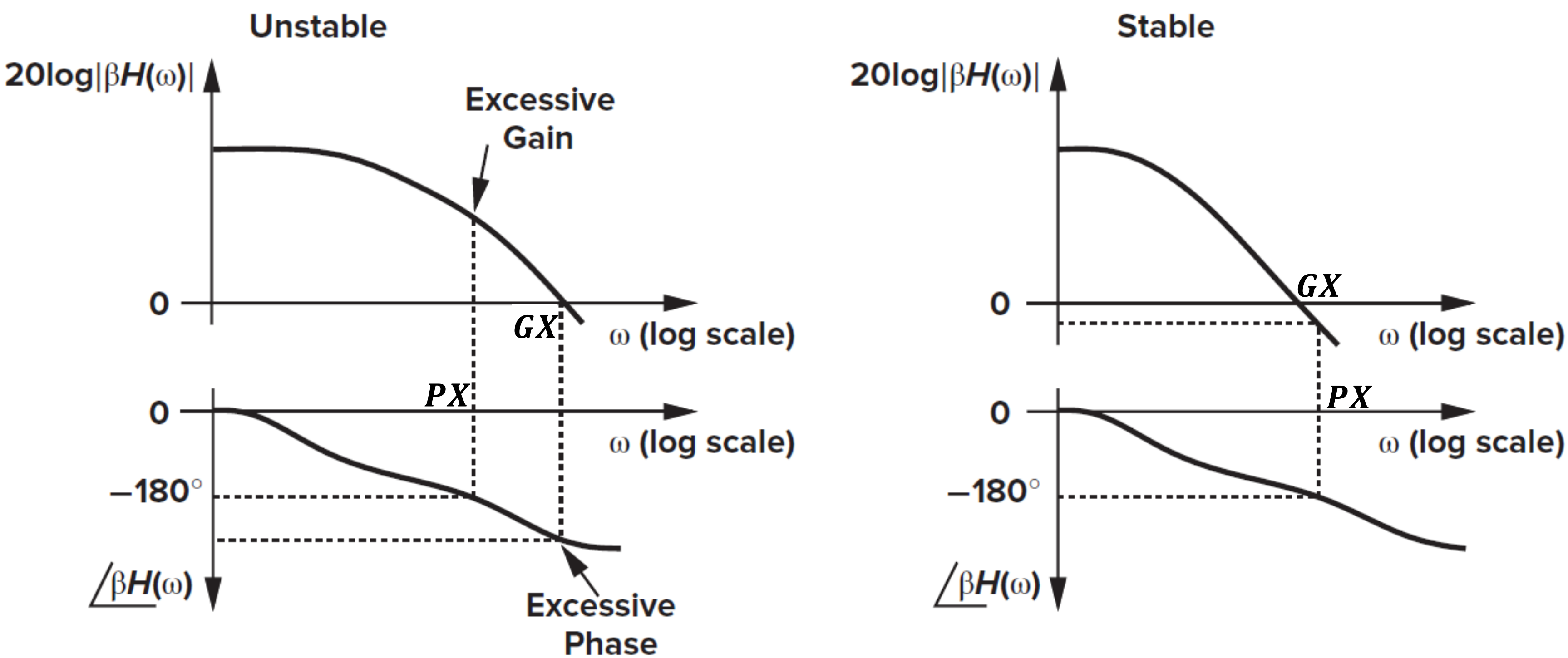
$$|\beta H(s)| = 1$$
$$\angle \beta H(s) = -180$$



[Razavi, 2017]

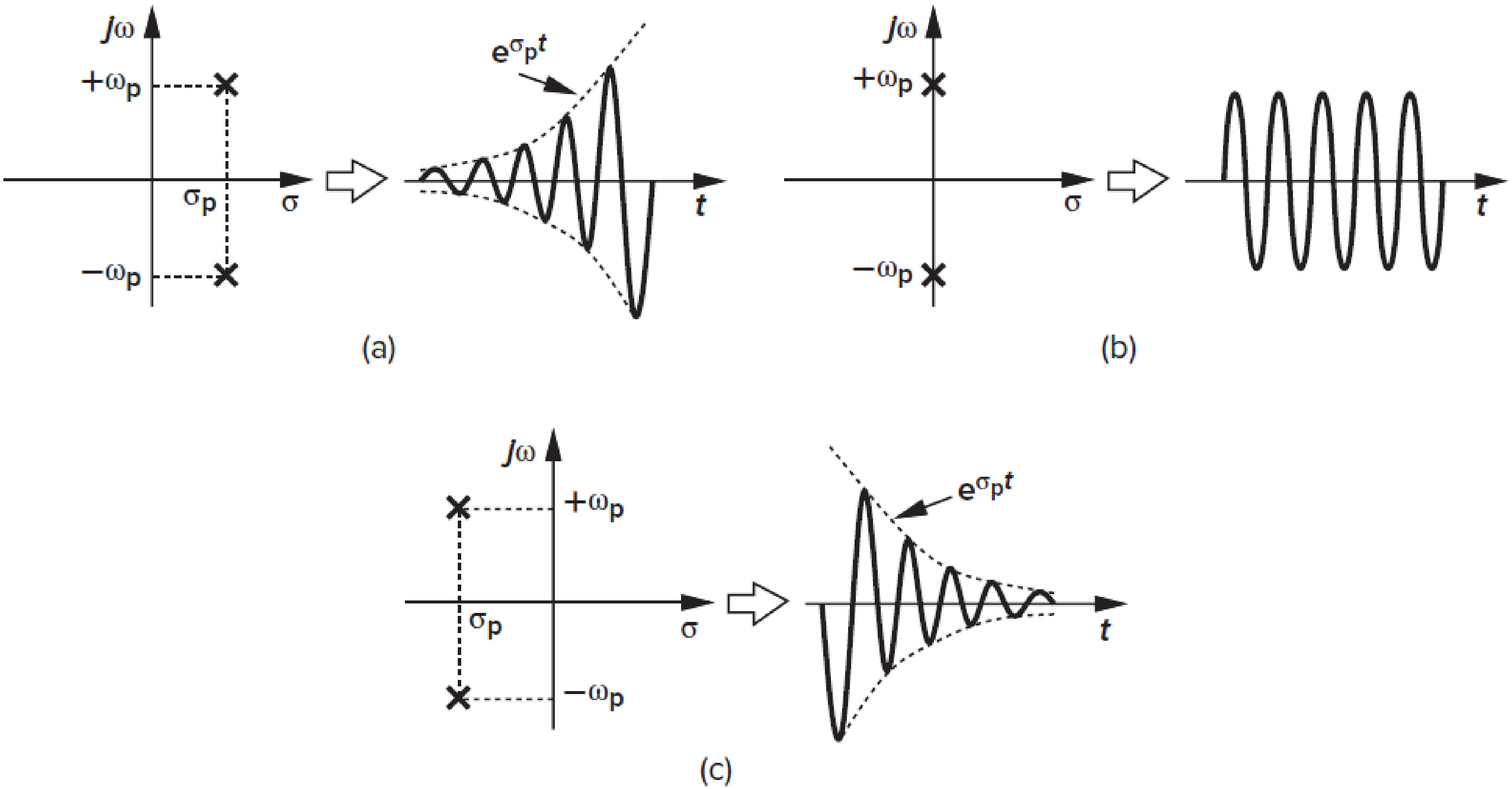
Stable vs Unstable System: Bode Plot

- Gain crossover frequency (GX): @ $|\beta H(s)| = 1$
- Phase crossover frequency (PX): @ $\angle\beta H(s) = -180$
- For a stable system: GX < PX



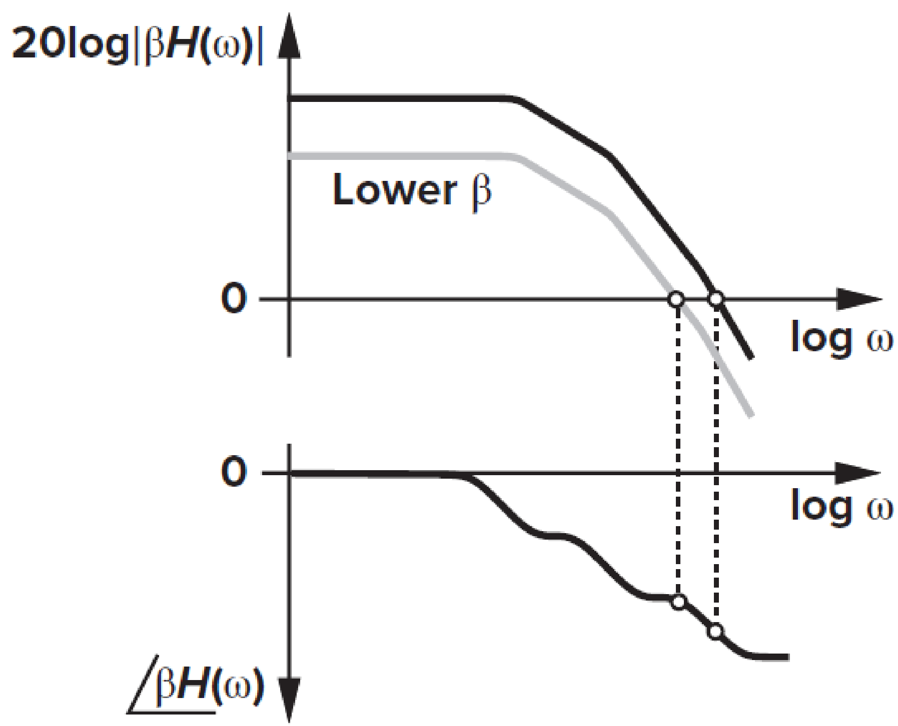
[Razavi, 2017]

Stable vs Unstable System: Pole-Zero Plot



Effect of Feedback Factor ($\boldsymbol{\beta}$)

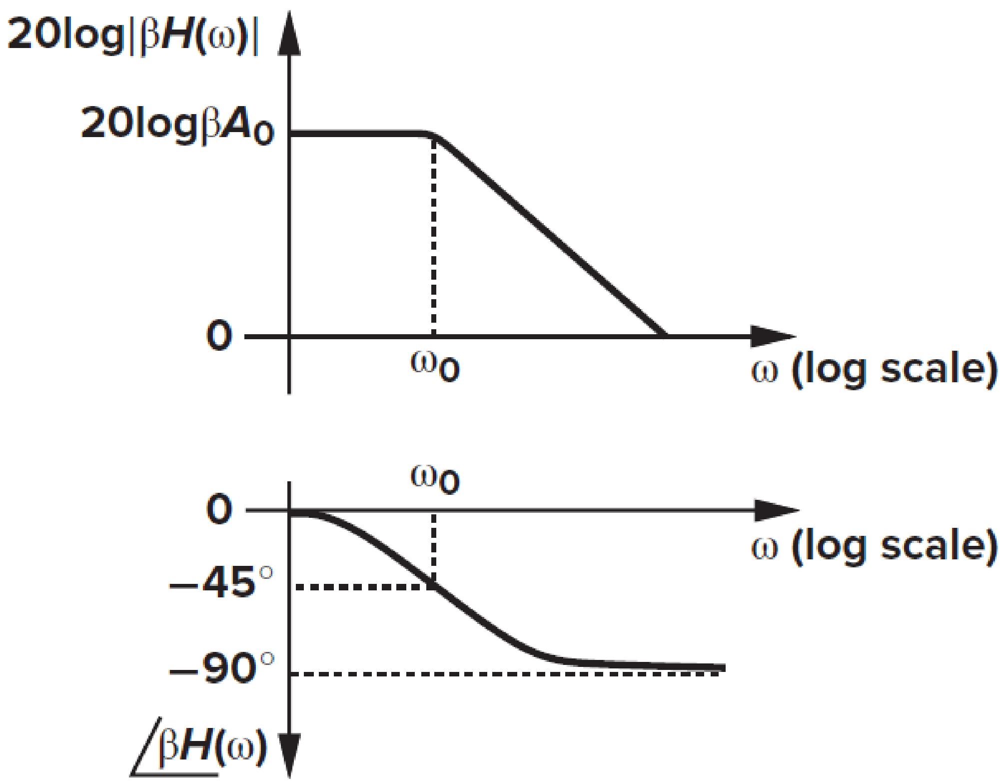
- We assume β is independent of frequency: $\angle\beta H$ independent of β
- If we apply no feedback ($\beta = 0$), the circuit will never oscillate
- Worst-case stability corresponds to $\beta = 1$ → $\beta H = H$ → OL gain
  - Worst case for unity-gain feedback → buffer → smallest CL gain



[Razavi, 2017]

Single-Pole System: Bode Plot

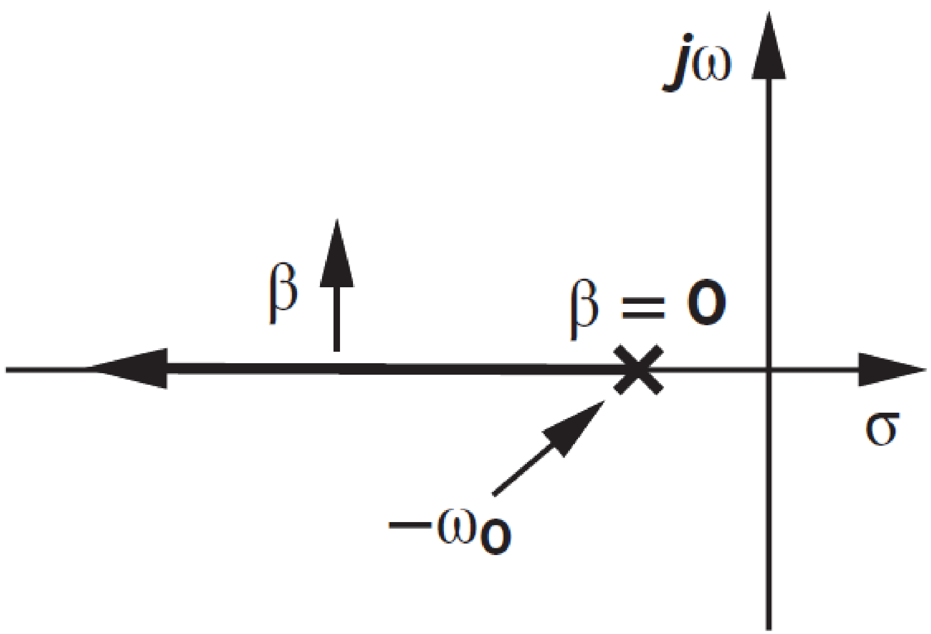
$$\frac{Y}{X}(s) = \frac{\dfrac{A_0}{1+\beta A_0}}{1+\dfrac{s}{\omega_0(1+\beta A_0)}}$$



[Razavi, 2017]

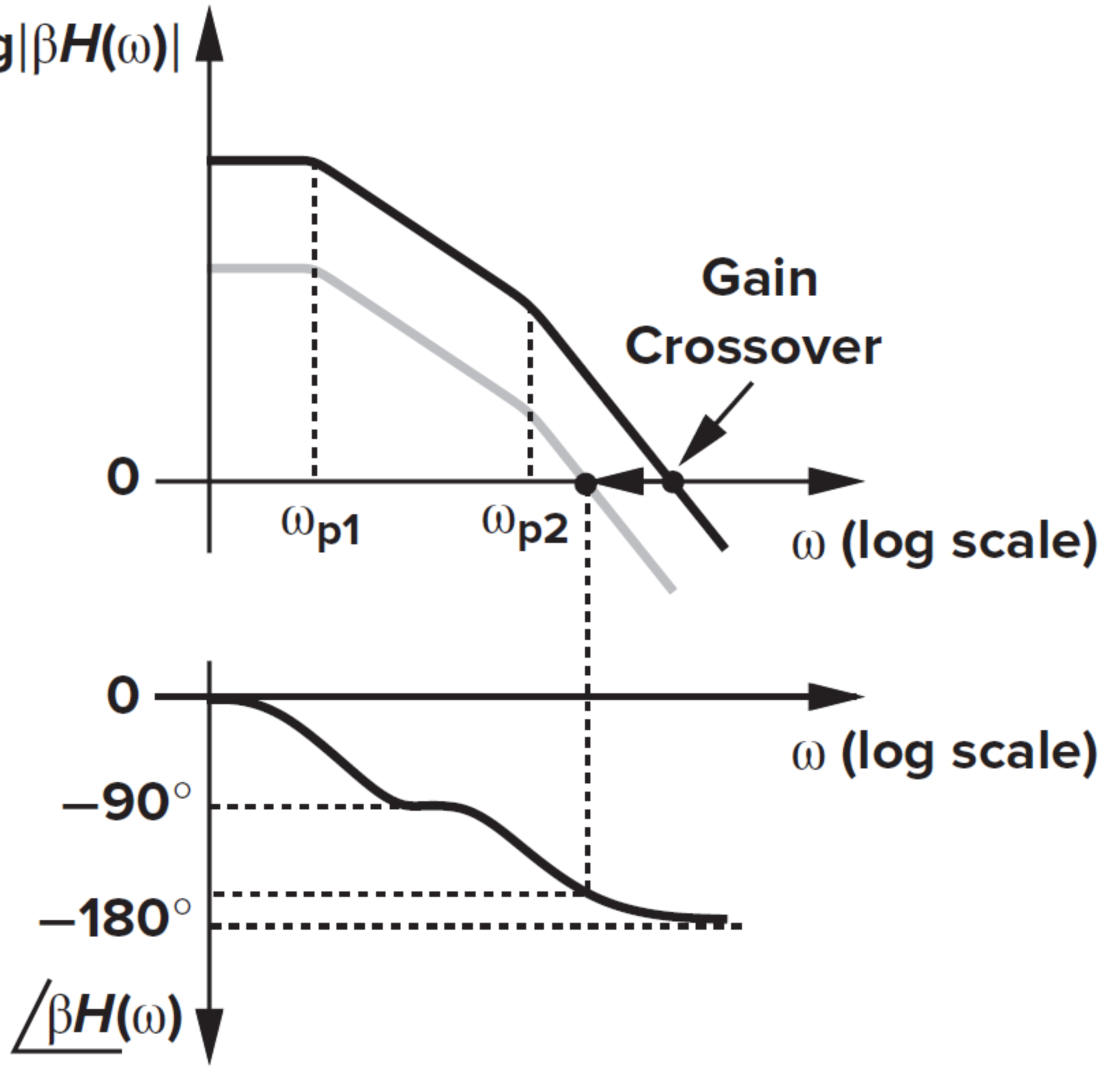
Single-Pole System: Root Locus

- The locus exists on real axis to the left of an odd number of poles and zeros.
- The locus starts at the open-loop poles and end at the open-loop zeros or at infinity.



[Razavi, 2017]

Two-Pole System: Bode Plot



[Razavi, 2017]

Two-Pole System: Root Locus

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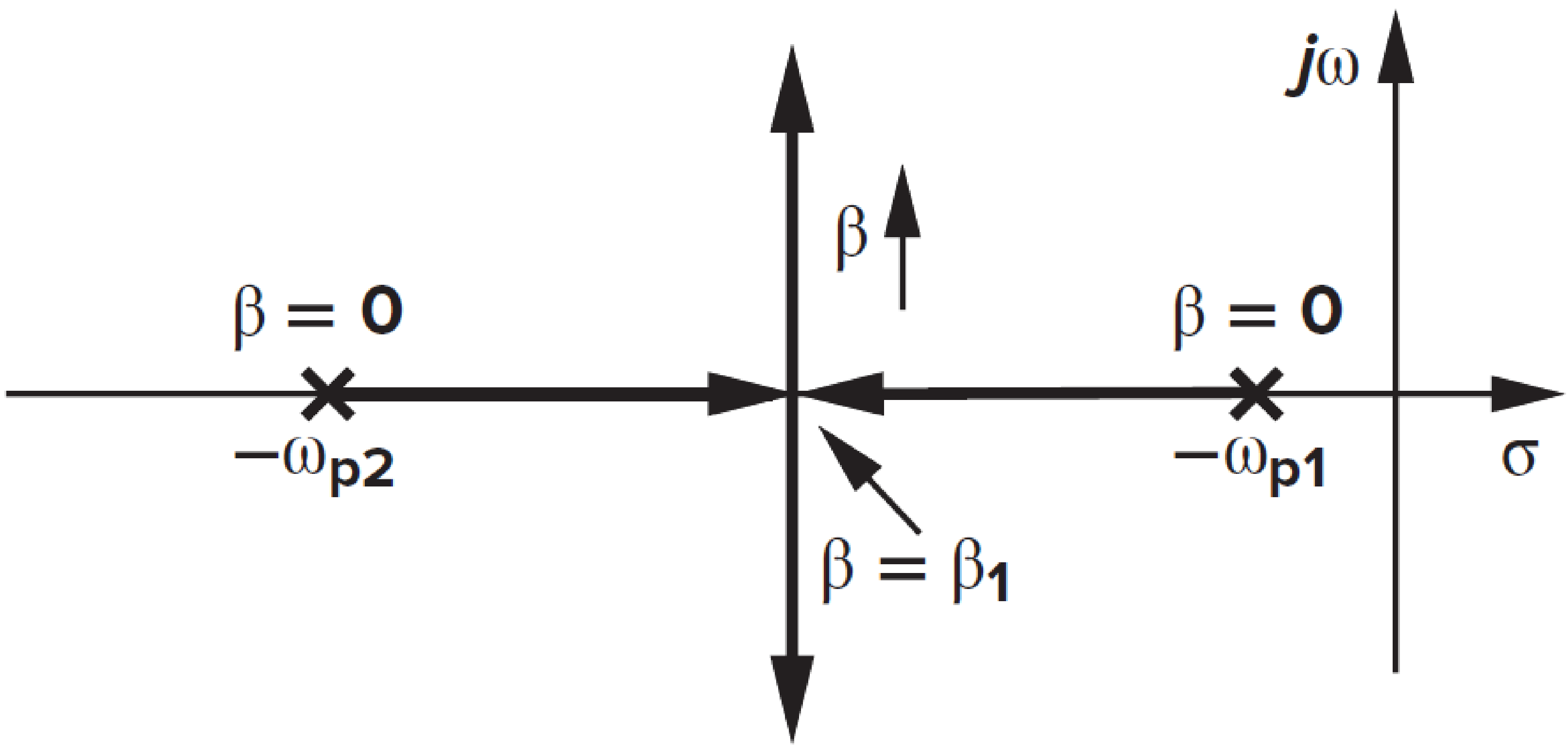
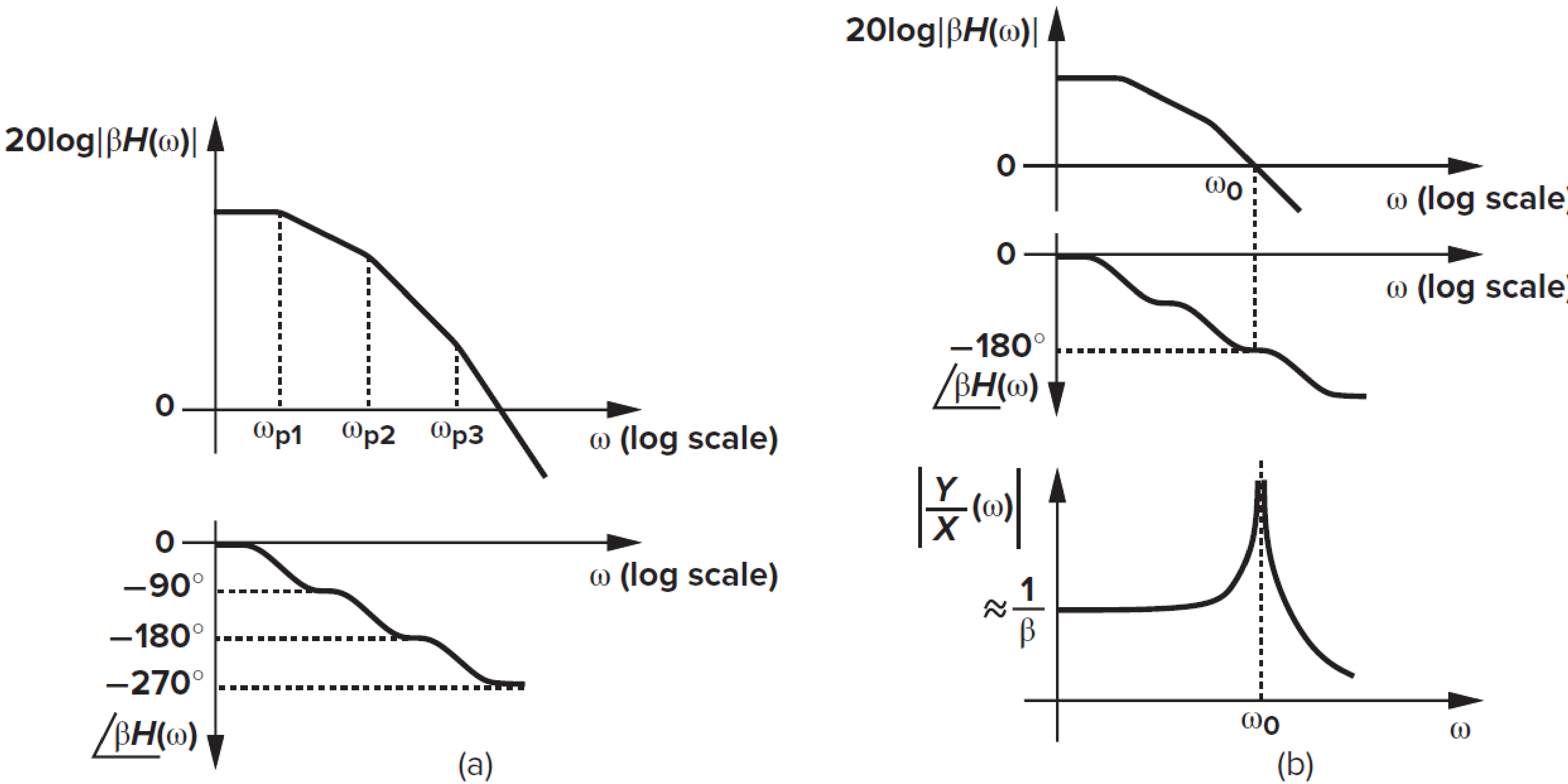


[Razavi, 2017]

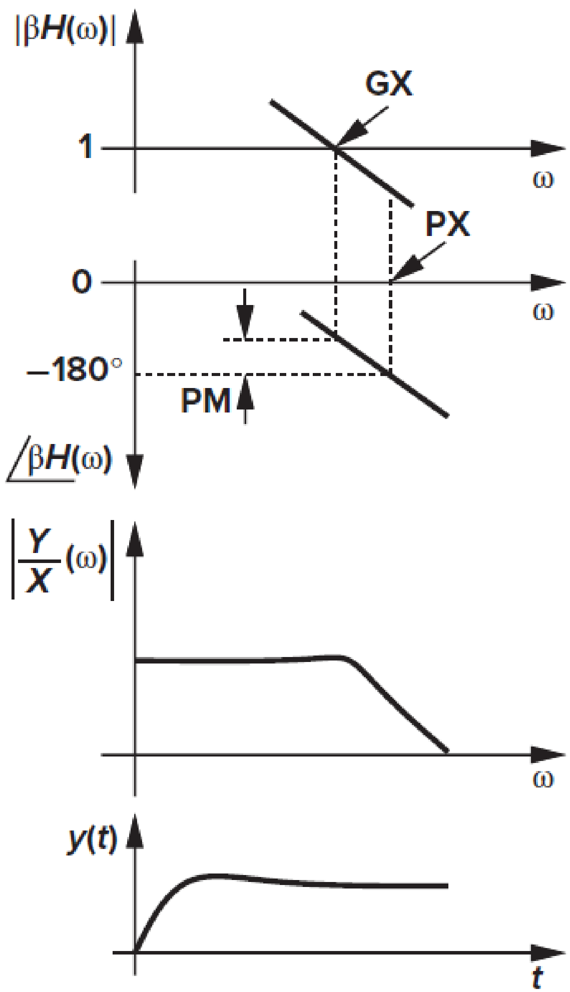
Three-Pole System

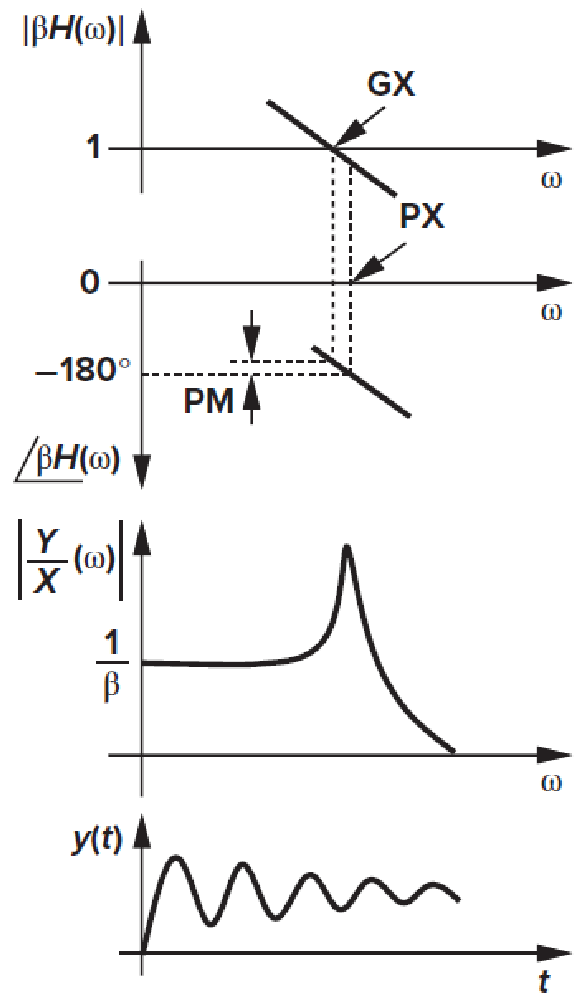


[Razavi, 2017]

Phase Margin (PM)

- PM > 0 → stable, but…
  - Low PM → frequency domain peaking → time domain ringing

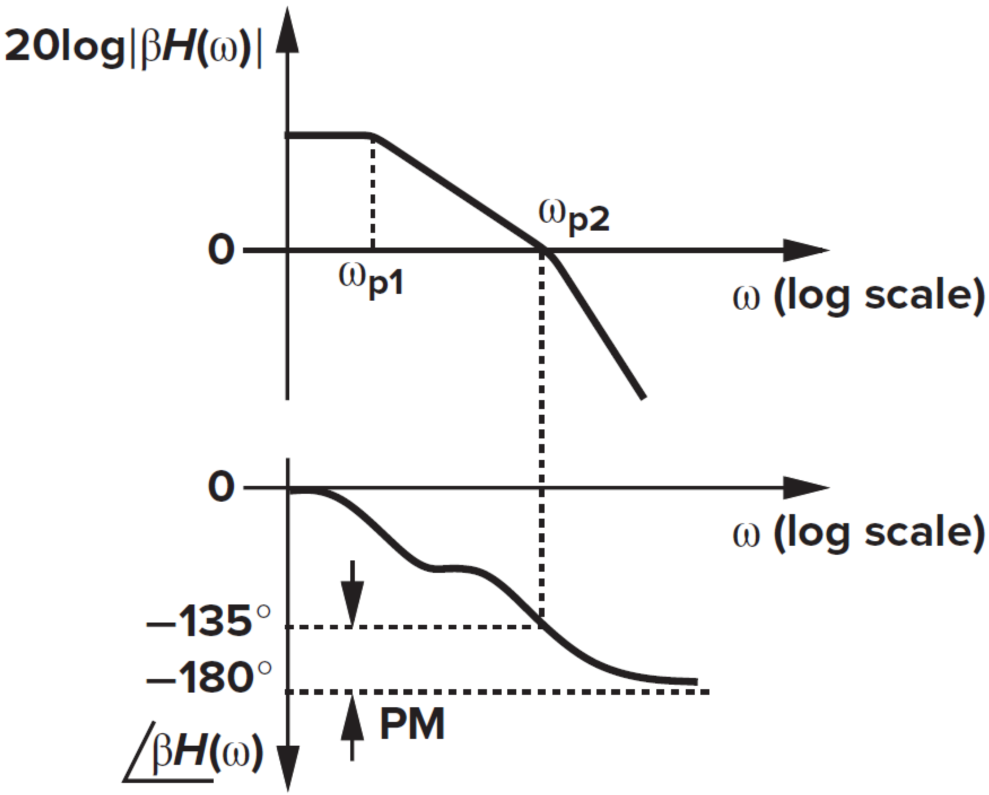


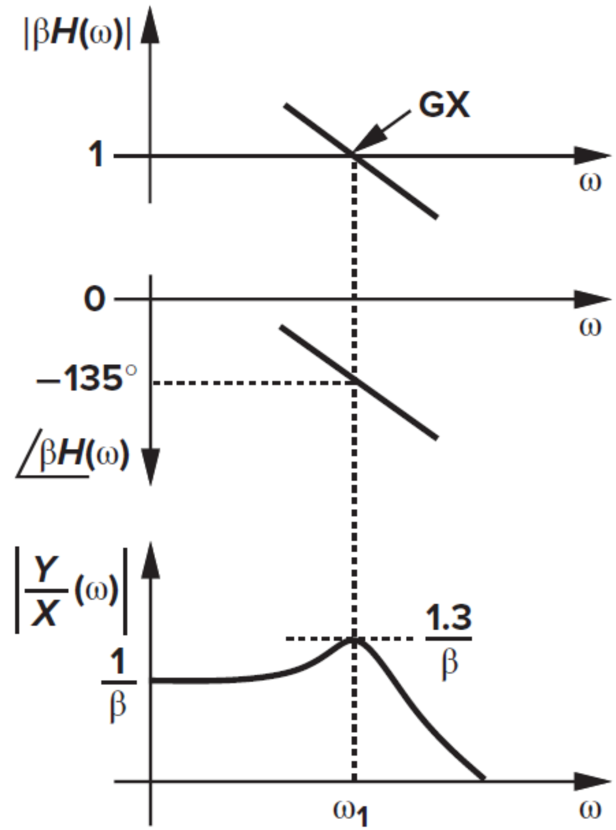


[Razavi, 2017]

Phase Margin: Ultimate GBW

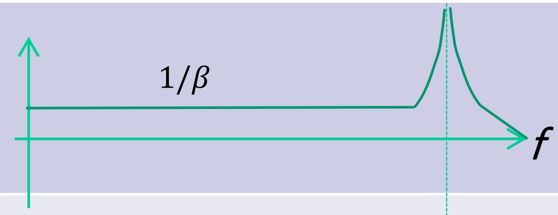
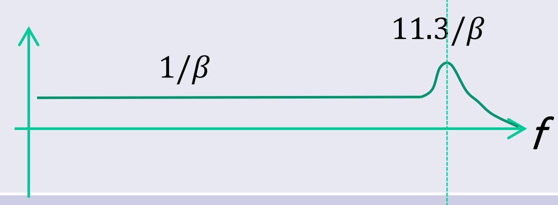
- If $\omega_{p2} = \omega_u$: PM = 45° → typically inadequate (peaking/ringing)
- The ultimate ω_u cannot exceed ω_{p2} → $\omega_{p1} < \omega_u < \omega_{p2}$
  - For $\omega < \omega_u$ the Bode plot is similar to a 1st order system





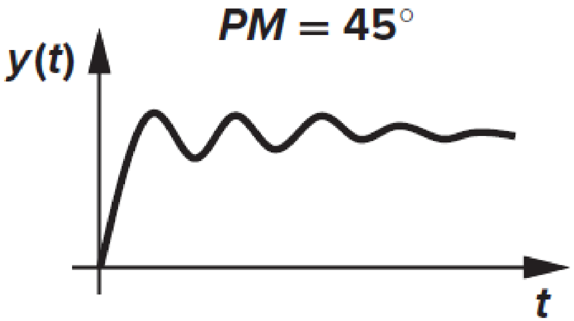
[Razavi, 2017]

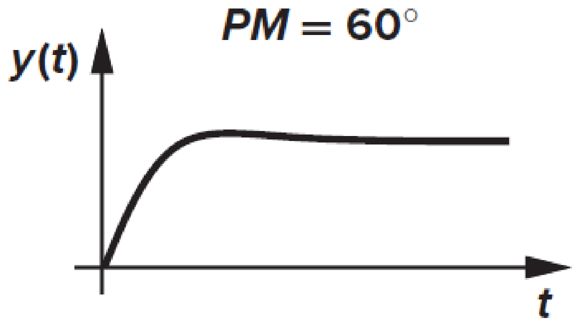
Optimum Phase Margin

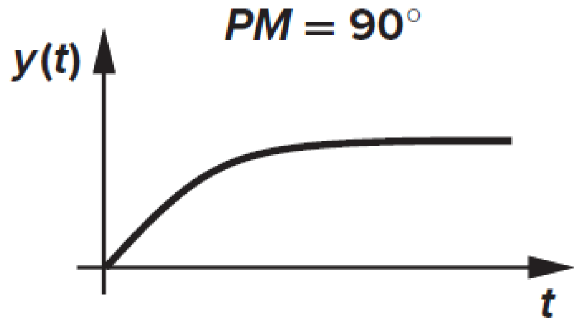
PM	Peaking	Closed Loop Response
0	∞	
5	$\frac{11.5}{\beta}$	
45	$\frac{1.3}{\beta}$	
60	$\frac{1}{\beta}$	
90	$\frac{0.707}{\beta}$	

Optimum Phase Margin

- PM = 60° is optimum
- But we must take some extra margin to account for variations





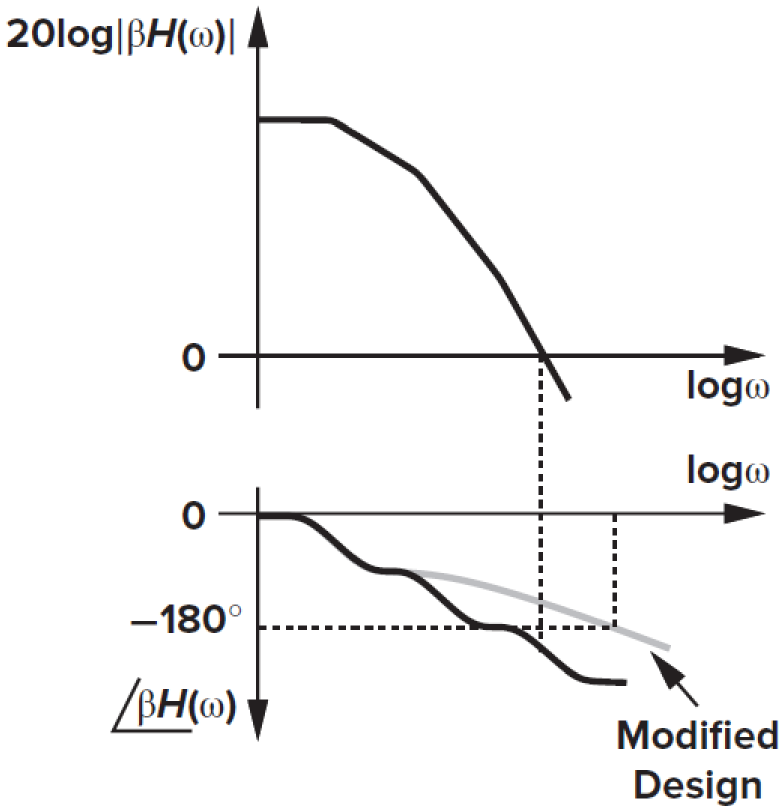


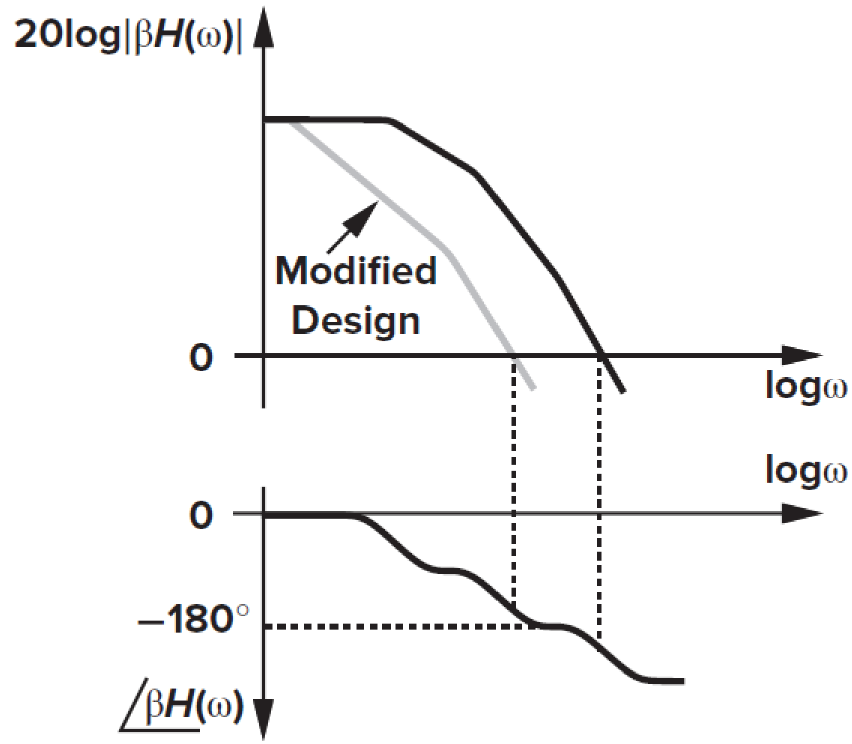
ω_{p2}/ω_u	PM
1	45^o
2	60^o
3	$\mathbf{72^o}$

[Razavi, 2017]

Frequency Compensation

- We need GX much smaller than PX
- Push PX outwards: minimize poles → minimize nodes/stages
- Push GX inwards: lower GBW





[Razavi, 2017]

Thank you!