وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 09
Frequency Response (2)

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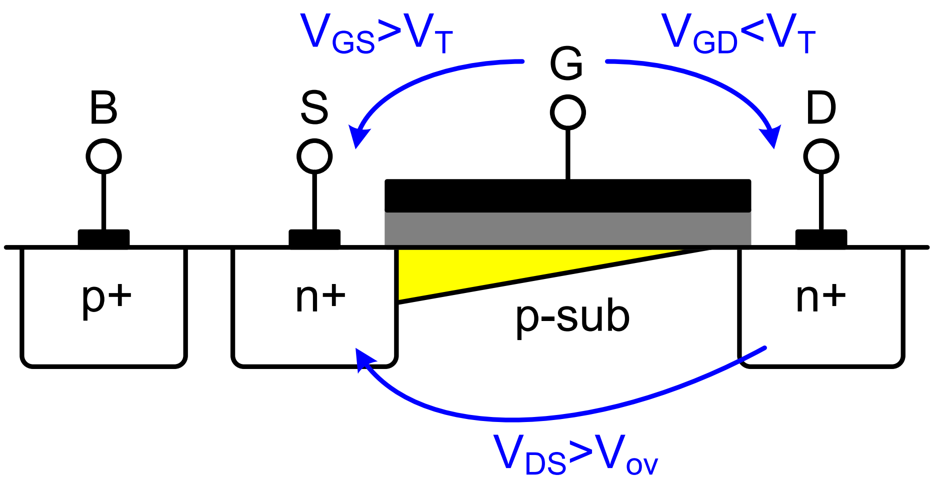
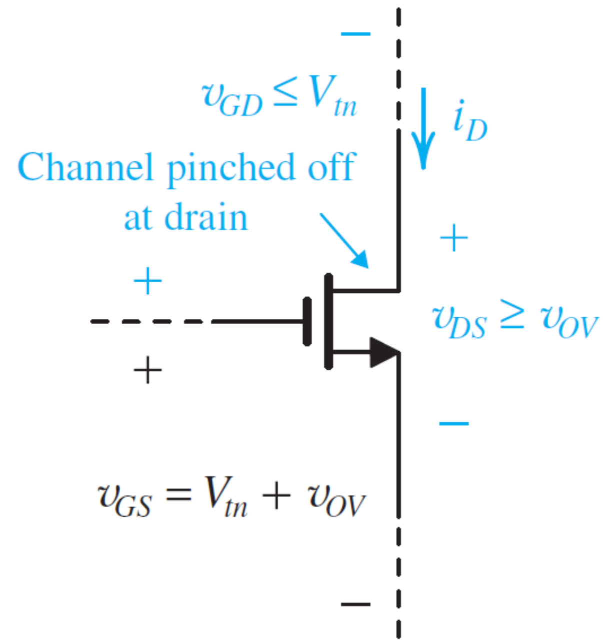
MOSFET in Saturation

- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2 (1 + \lambda V_{DS})$$





Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_T$

ON

$V_{GS} > V_T$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_T$

$$I_D = \mu C_{ox} \frac{W}{L}\left(V_{ov}V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_T$

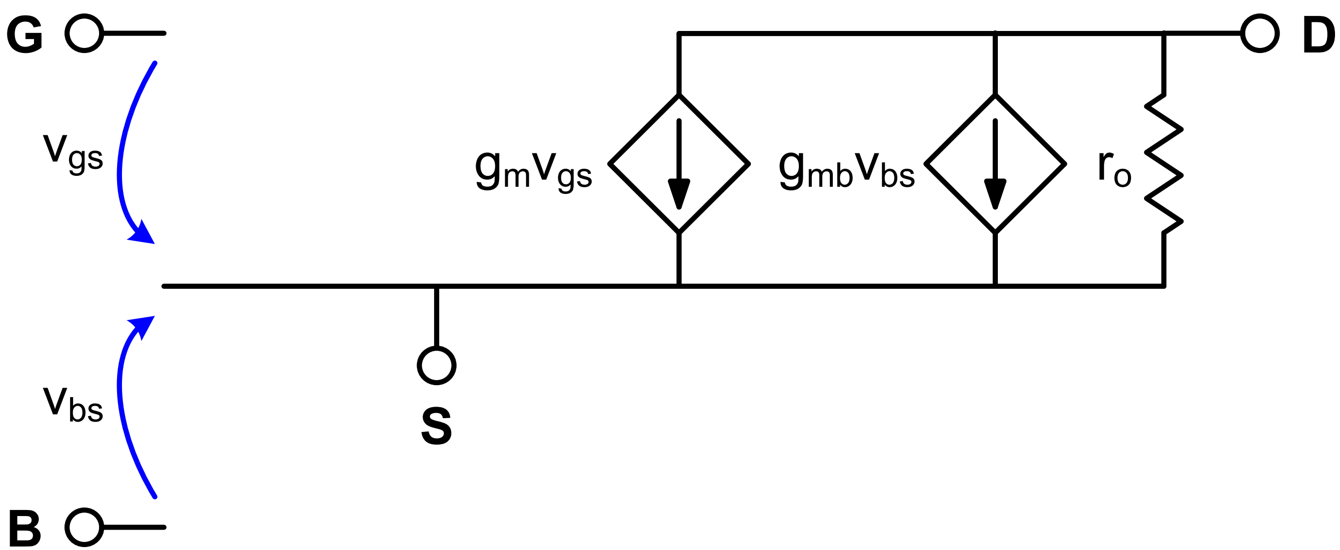
$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}V_{ov}^2(1 + \lambda V_{DS})$$

Low-Frequency Small-Signal Model

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} V_{ov} = \sqrt{\mu C_{ox} \frac{W}{L} \cdot 2 I_D} = \frac{2 I_D}{V_{ov}}$$

$$g_{mb} = \eta g_m, \qquad \eta \approx 0.1 - 0.25$$

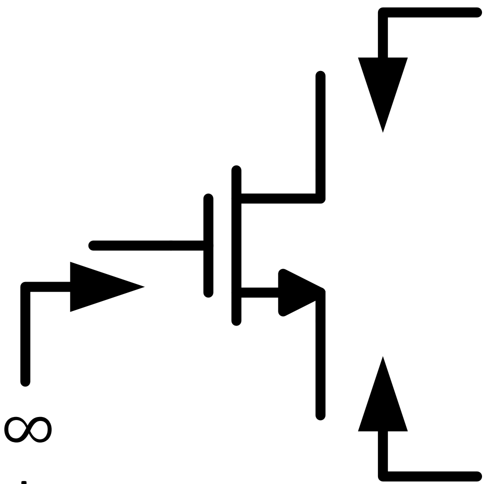
$$r_o = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}} = \frac{1}{\lambda I_D}, \qquad \lambda \propto \frac{1}{L}$$



Rin/out Shortcuts Summary

$$r_o[1 + (g_m + g_{mb})R_S]$$

H.I.N.



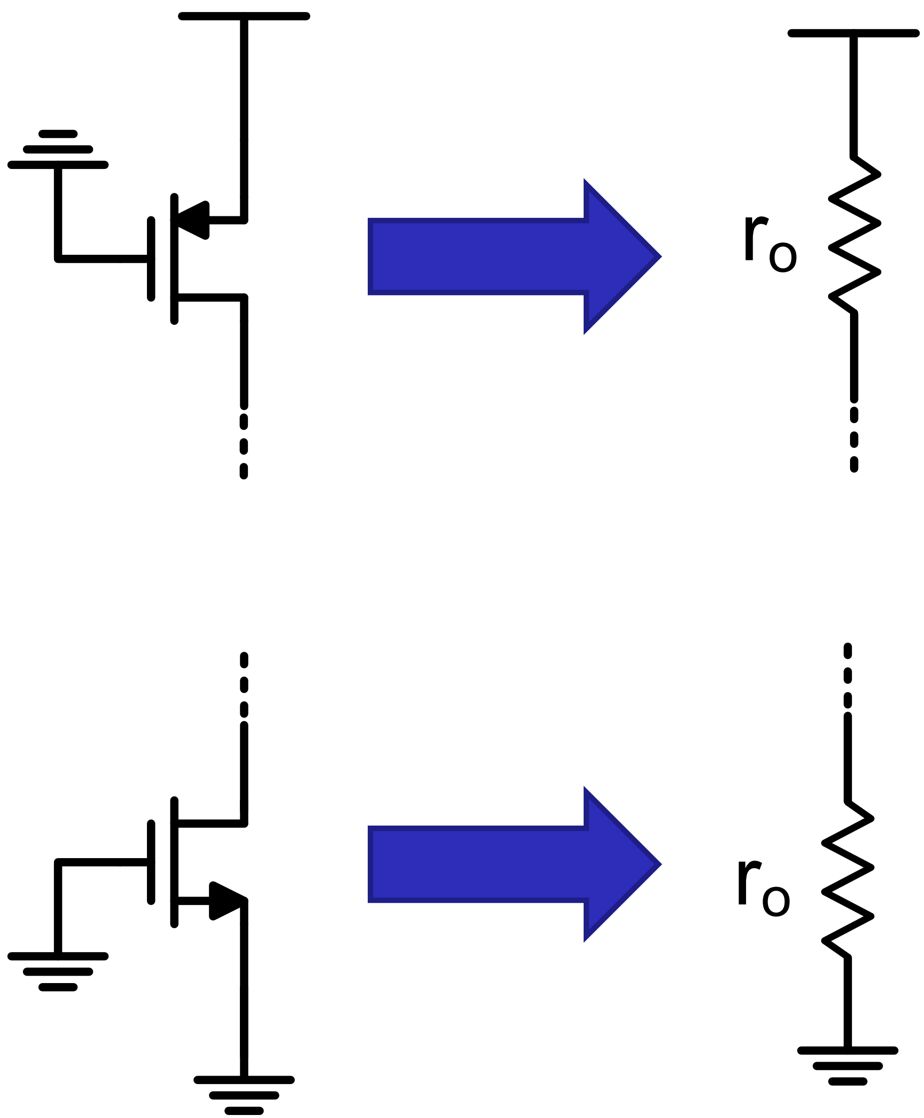
∞

At low frequencies **ONLY**

$$\frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

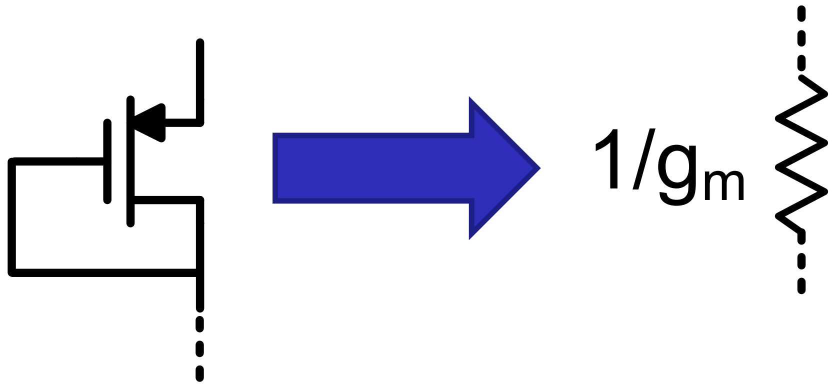
L.I.N.

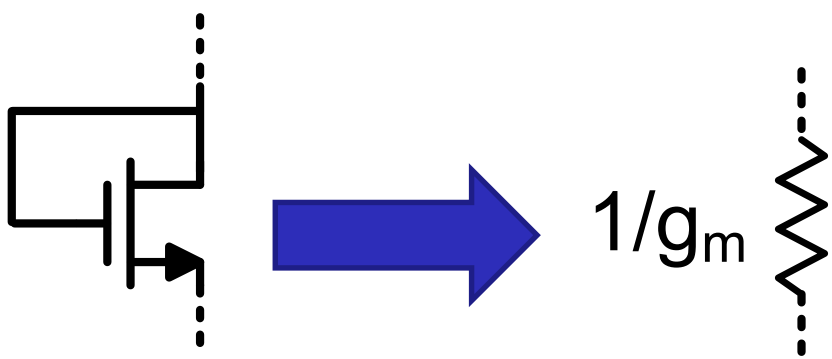
Active Load (Source OFF)



Diode Connected (Source Absorption)

- Always in saturation
- Bulk effect: $g_m \rightarrow g_m + g_{mb}$





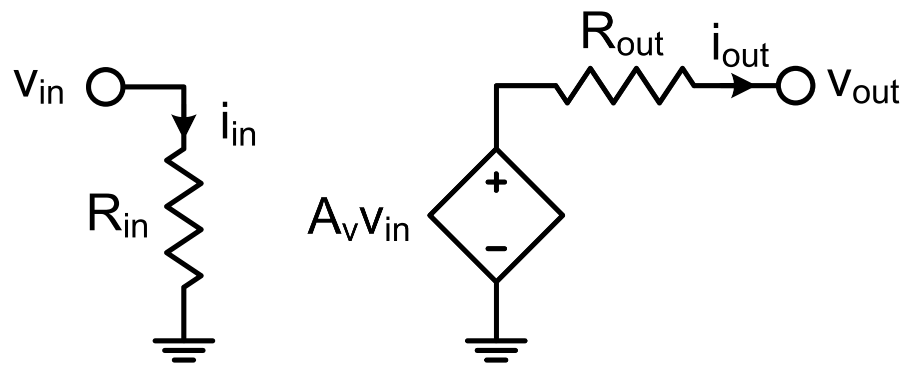
Why GmRout?

$$R_{out} = \frac{v_x}{i_x} \; @ \; v_{in} = 0$$

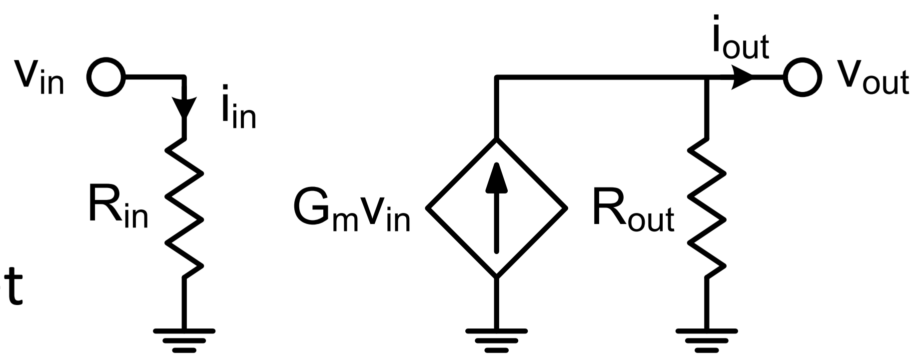
$$G_m = \frac{i_{out,sc}}{v_{in}}$$

$$A_v = G_m R_{out}$$

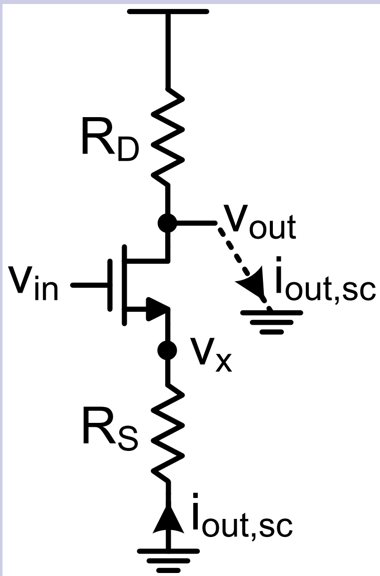
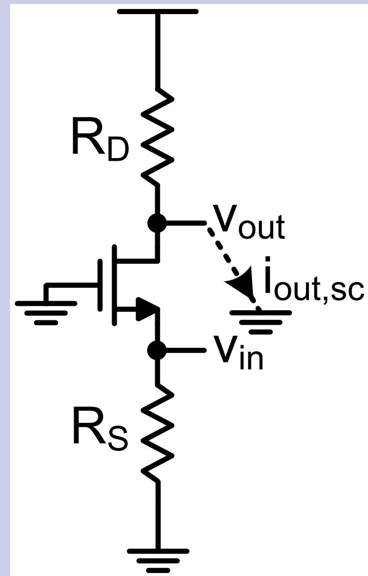
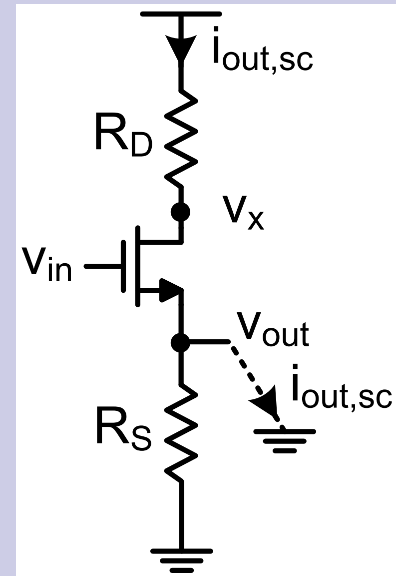
$$A_i = G_m R_{in}$$



- Divide and conquer
  - Rout simplified: vin=0
  - Gm simplified: vout=0
  - We already need Rin/out
  - We can quickly and easily get Rin/out from the shortcuts

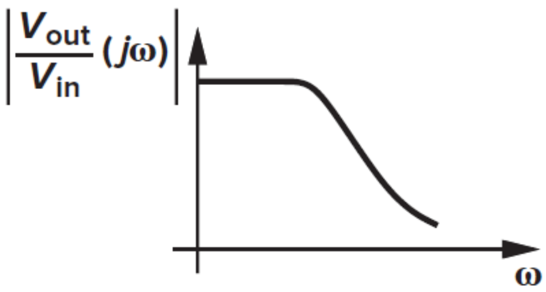


Summary of Basic Topologies

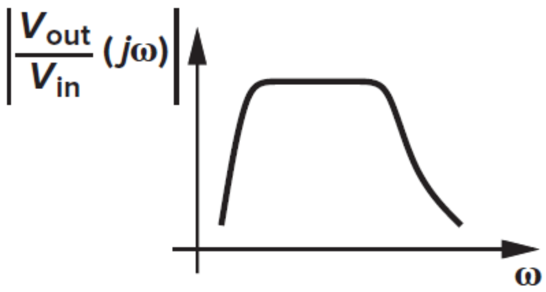
	CS	CG	CD (SF)
			
	Voltage & current amplifier	Current buffer	Voltage buffer
Rin	∞	$R_S // \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$	∞
Rout	$R_D // r_o[1 + (g_m + g_{mb})R_S]$	$R_D // r_o$	$R_S // \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$
Gm	$\frac{-g_m}{1 + (g_m + g_{mb})R_S}$	$g_m + g_{mb}$	$\frac{g_m}{1 + R_D/r_o}$

Frequency Response

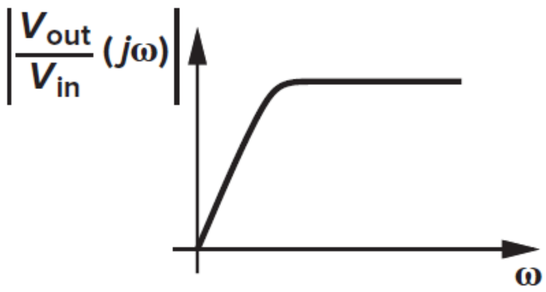
LPF



BPF



HPF



Poles and Zeros

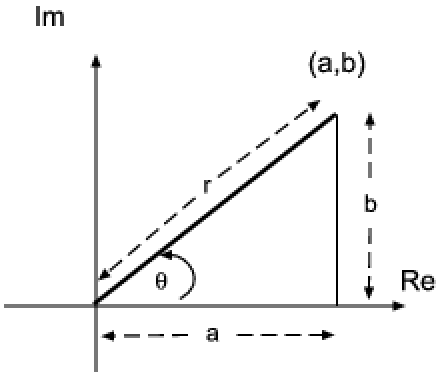
- Transfer function

$$H(s) = \frac{N(s)}{D(s)}$$

- Zeros: roots of numerator => $N(s)$
- Poles: roots of denominator => $D(s)$
- Frequency response: $s \Rightarrow j\omega$

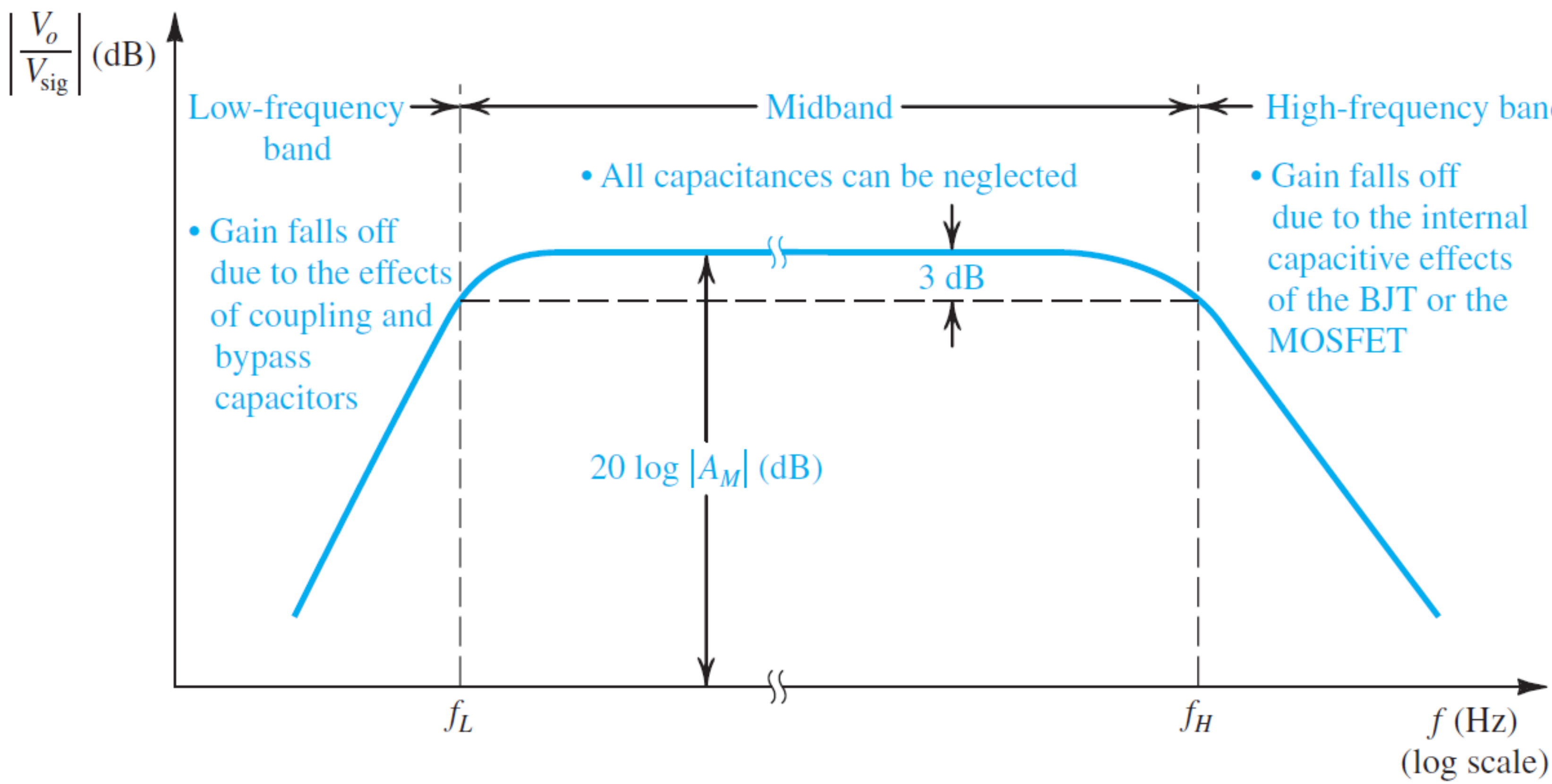
$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = |H(j\omega)|e^{j\phi}$$

- Magnitude$(a + jb) = \sqrt{a^2 + b^2}$
- Phase$(a + jb) = \tan^{-1}\frac{b}{a}$



[math.tutorvista.com]

Frequency Response



Coupling and bypass	✓	s.c.	s.c.
Intrinsic and load	o.c.	o.c.	✓

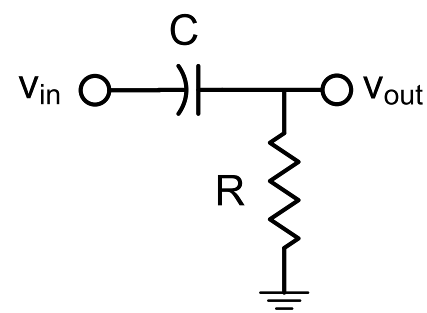
SCTC and OCTC Techniques

- Low-frequency range (LFR) => Not common in IC design
  - Only consider one cap at a time
  - Assume other caps are s.c.
  - s.c. time constant (SCTC) technique
  - $\omega_L \approx \omega_{L1} + \omega_{L2} + \cdots$
  - Highest pole dominates (L.I.N. dominates)
- **High-frequency range (HFR) => More important in IC design**
  - **Only consider one cap at a time**
  - **Assume other caps are o.c.**
  - **o.c. time constant (OCTC) technique**
  - $\boldsymbol{\omega_H \approx \omega_{H1} // \omega_{H2} // \cdots}$
  - **Lowest pole dominates (H.I.N. dominates)**
- Both provide good approx if one pole is dominant (and poles are real)

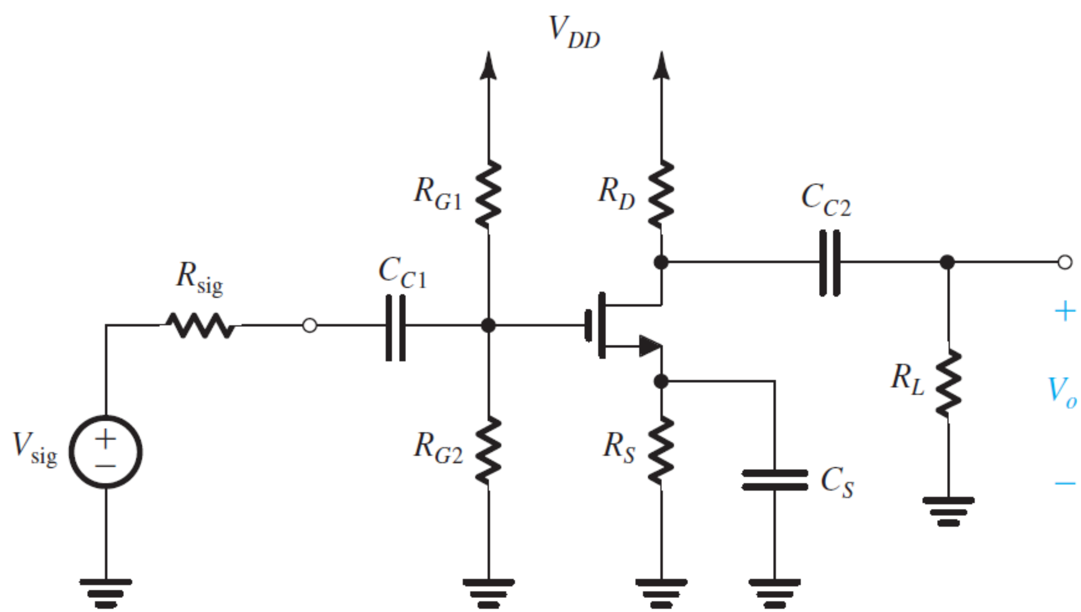
Calculating Zeros by Inspection

1. Find the value $s = s_z$ that makes $H(s) = 0 \Rightarrow v_{out} = 0$

- ❑ Examples:



- ❑ $C_{c1}: v_o = 0 \; if \; Z_{C_1} = \infty$
  - $Z_{C_1} = \frac{1}{sC_1}$
  - $\Rightarrow s_{z1} = 0$

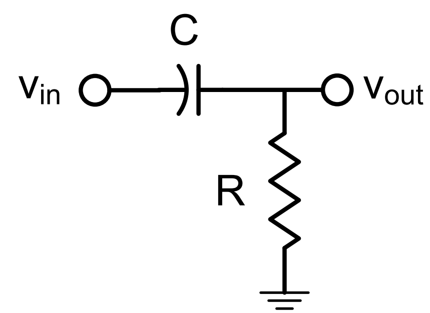


- ❑ C_S: $v_o = 0 \; if \; Z_S = \infty$
  - $Z_S = \frac{R_S}{1+sR_SC_S}$
  - $\Rightarrow s_{z2} = -\frac{1}{R_SC_S}$

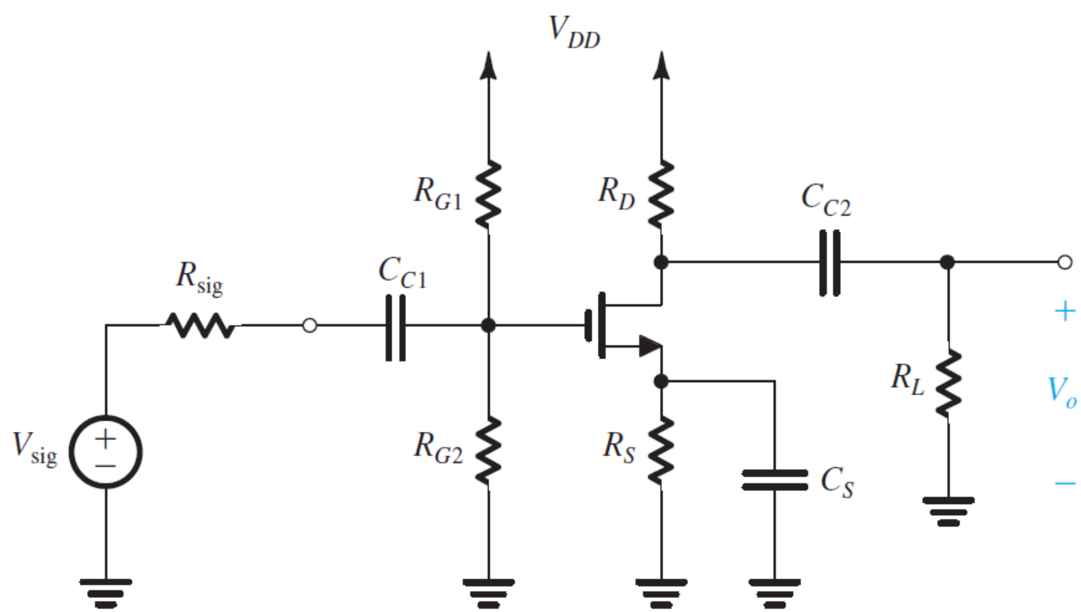
[Sedra/Smith, 2015]

Calculating Poles by Inspection

1. Set $v_{sig} = 0$
2. Calculate thevenin resistance $\left(R_{th,i}\right)$ seen by each cap (C_i)
3. $s_{p,i} = -\frac{1}{R_{th,i}C_i}$



- Examples:



- $C_{c1}: R_{th1} = R_{sig} + R_G$
  - $\Rightarrow s_{p1} = -\frac{1}{(R_{sig}+R_G)C_{c1}}$
- $C_S: R_{th2} \approx R_S // \frac{1}{g_m}$
  - $\Rightarrow s_{p2} = -\frac{1}{\left(R_S//\frac{1}{g_m}\right)C_S}$

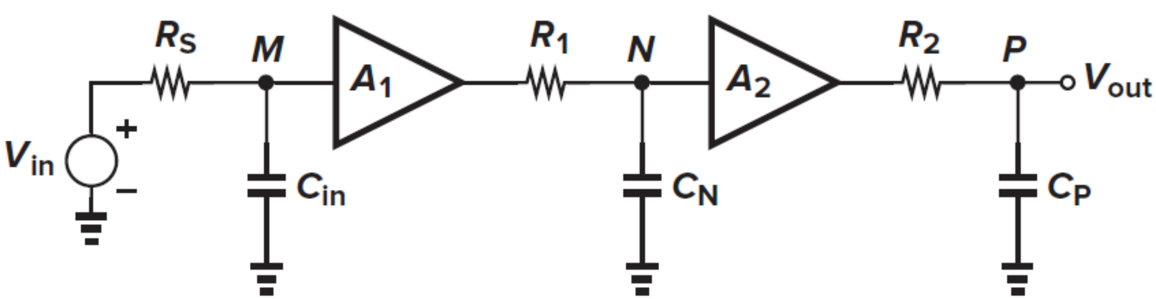
[Sedra/Smith, 2015]

Calculating Poles by Inspection

1. Set $v_{sig} = 0$
2. Calculate thevenin resistance $\left(R_{th,i}\right)$ seen by each cap (C_i)
3. $s_{p,i} = -\frac{1}{R_{th,i}C_i}$

- Examples:
  - Each node is associated with a pole
  - H.I.N. dominates

$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + R_S C_{in} s} \cdot \frac{A_2}{1 + R_1 C_N s} \cdot \frac{1}{1 + R_2 C_P s}$$



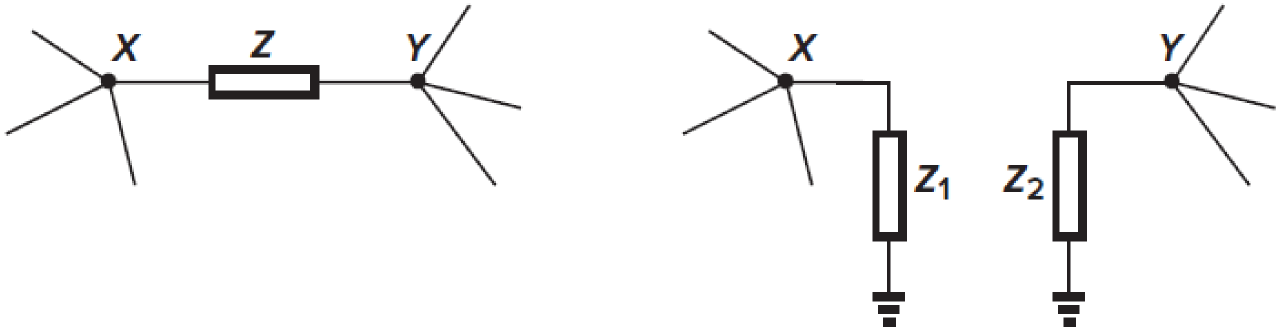
Miller's Theorem

$$A_v = \frac{V_Y}{V_X}$$

$$\frac{V_X - V_Y}{Z} = \frac{V_X}{Z_1} = -\frac{V_Y}{Z_2}$$

$$Z_1 = \frac{Z}{1 - \frac{V_Y}{V_X}} = \frac{Z}{1 - A_v}$$

$$Z_2 = \frac{Z}{1 - \frac{V_X}{V_Y}} = \frac{Z}{1 - \frac{1}{A_v}}$$



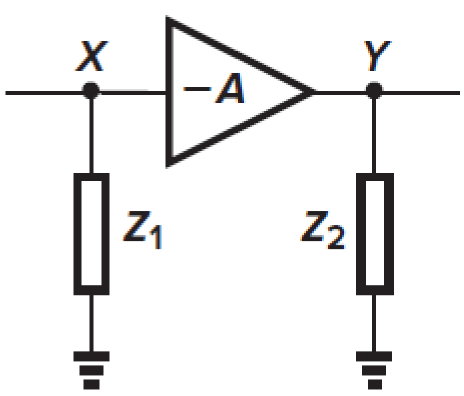
- Note: Miller's Theorem cannot be used for Z_{out} calculation

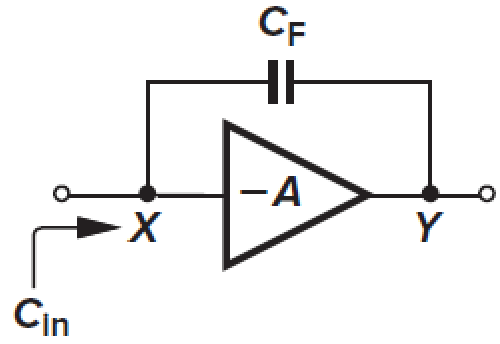
Miller Effect

- Capacitance multiplication: $Z_F = 1/sC_F$

$$Z_1 = \frac{Z_F}{1 - A_v} \approx \frac{Z_F}{A} = \frac{1}{sAC_F} \Rightarrow C_{in} = AC_F$$

$$Z_2 = \frac{Z_F}{1 - \frac{1}{A_v}} \approx Z_F = \frac{1}{sC_F} \Rightarrow C_{out} \approx C_F$$





Miller's Approximation

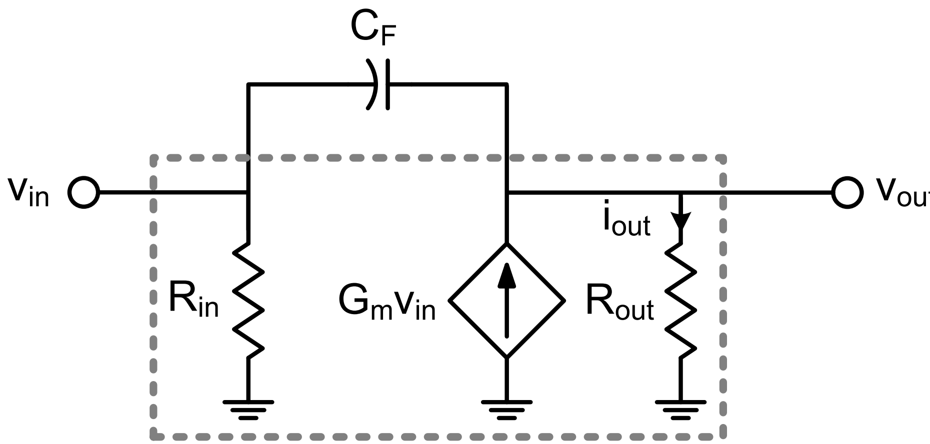
$$Z_1 = \frac{Z}{1 - A_v} \;\&\; Z_2 = \frac{Z}{1 - \frac{1}{A_v}}$$

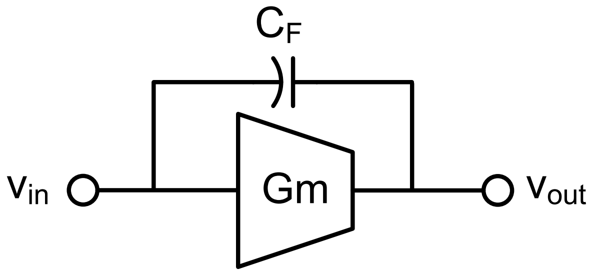
- But A_v is a function of frequency!
- Miller's Approximation: Substitute with the low frequency gain
  - $A_v(s) \approx A_o$
  - It does not tell about the feedforward zero (next slide)
  - **Gives good approx ONLY if the i/p pole is dominant**
- See Example 6.4 in [Razavi, 2017]

The Feedforward Zero

- ❑ $v_{out} = 0 \rightarrow i_{out} = 0$
- ❑ $v_{in} s C_F = -G_m v_{in}$

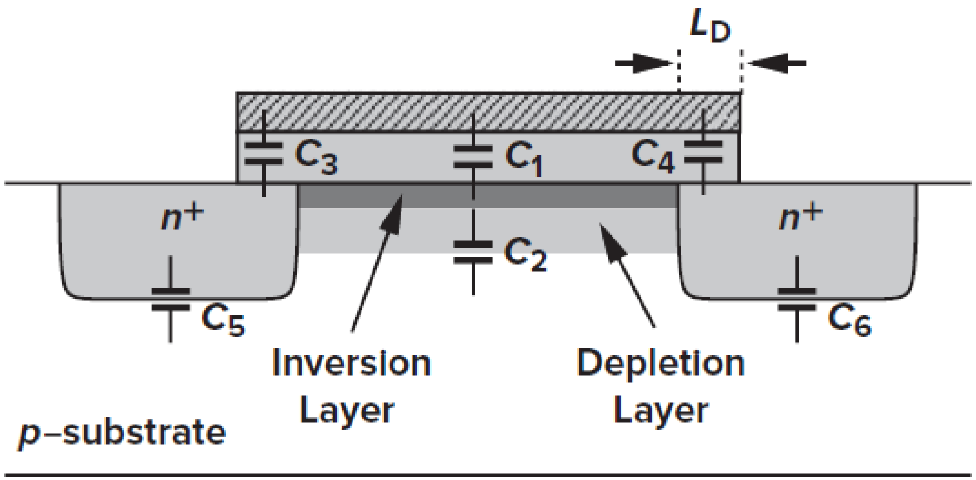
$$\boxed{s_z = -\frac{G_m}{C_F}}$$

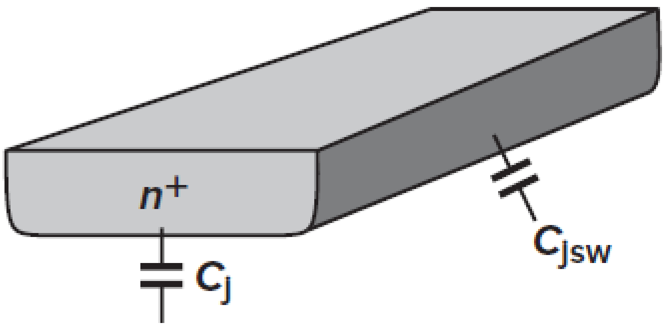


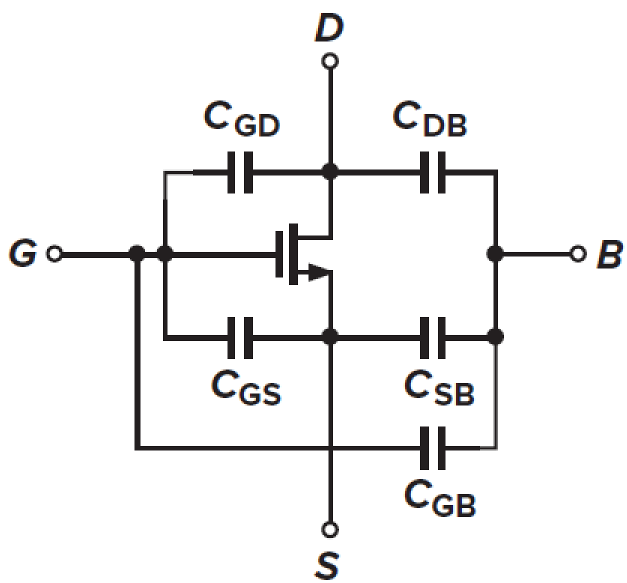


- ❑ **RHP zero if G_m is –ve (e.g. CS)**
  - **Mag inc and phase drops**
  - **Very bad for loop stability**
  - **More on this when we study OTAs**

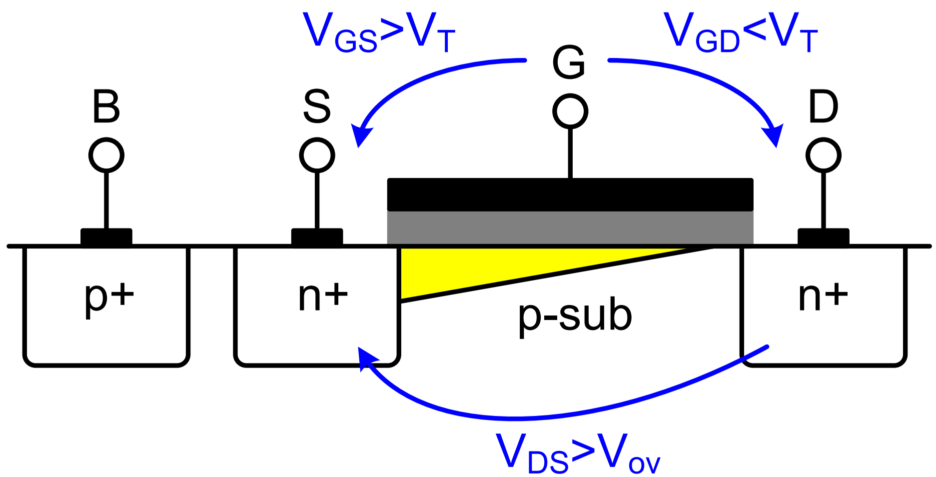
MOSFET Capacitances



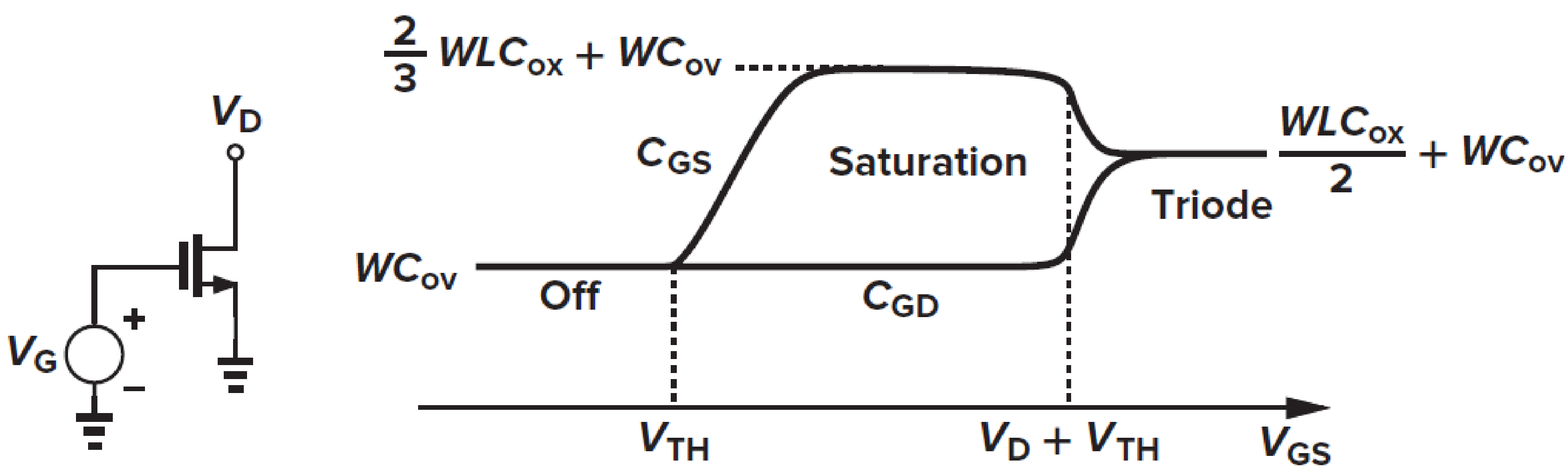




C_{gs} and C_{gd}

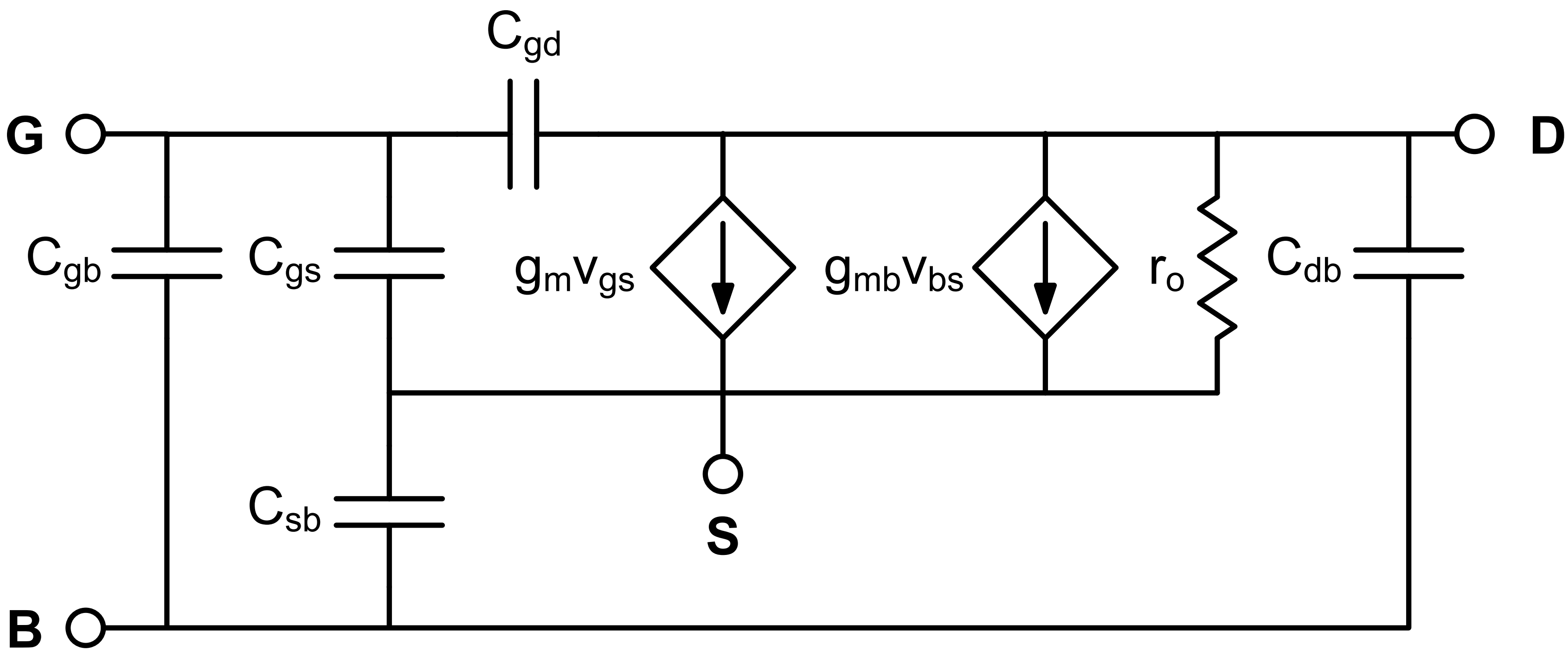


$V_{GS}>V_T$
G
$V_{GD}<V_T$
B
S
D
p+
n+
p-sub
n+
$V_{DS}>V_{ov}$



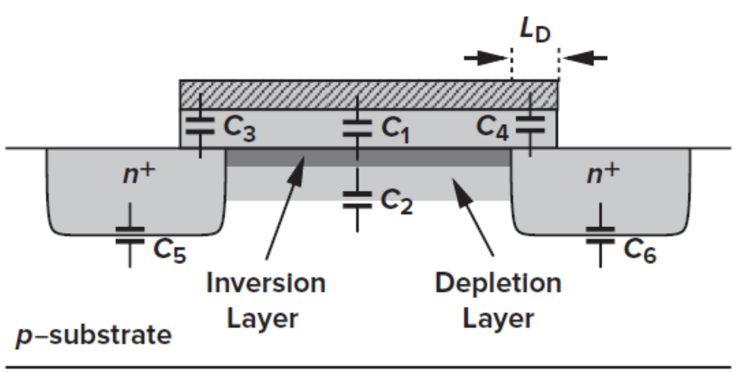
V_D
V_G
+
−
$\frac{2}{3}WLC_{ox} + WC_{ov}$
C_{GS}
Saturation
$\frac{WLC_{ox}}{2} + WC_{ov}$
Triode
WC_{ov}
Off
C_{GD}
V_{TH}
$V_D + V_{TH}$
V_{GS}

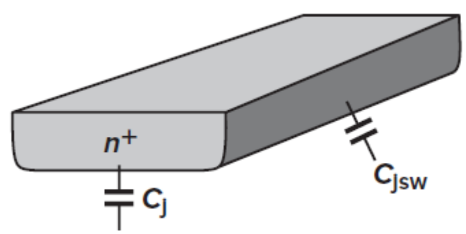
High Frequency Small Signal Model

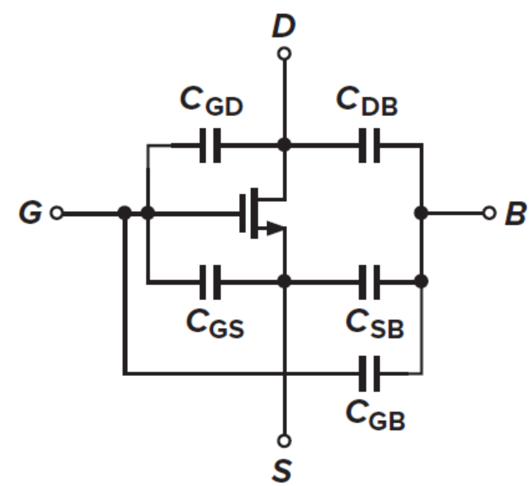


MOSFET Capacitances Summary

- Usually $C_{gs} \gg C_{gd}$
- Usually $C_{sb} \gg C_{db}$





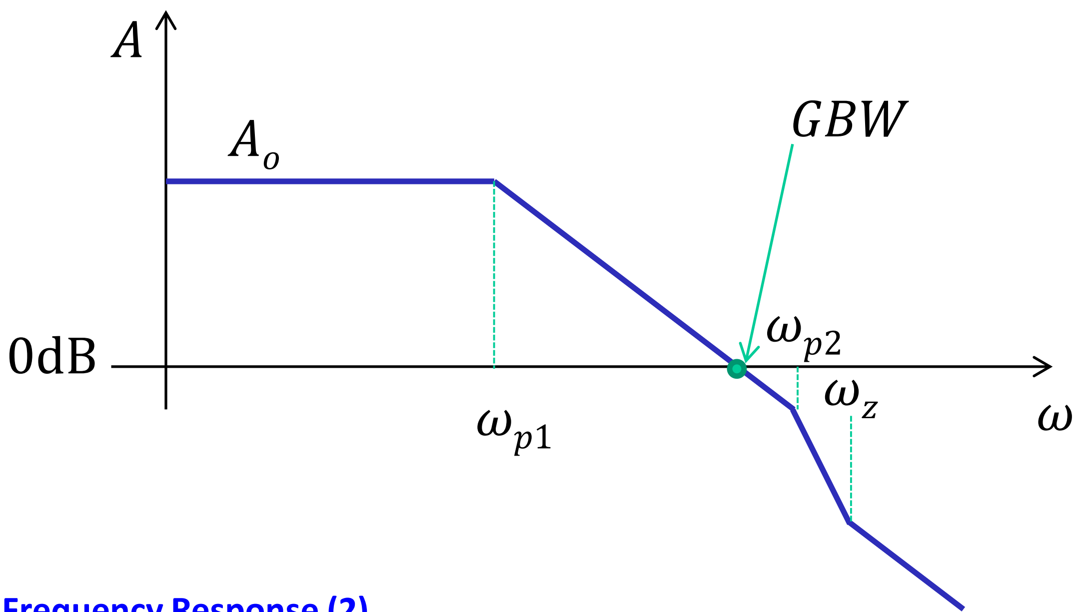


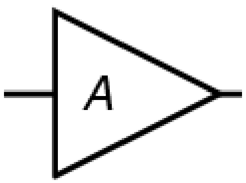
	C_{gb}	C_{gs}	C_{gd}	C_{sb}	C_{db}
Cutoff	$\leq WLC_{ox}$	WC_{ov}	WC_{ov}	$A_S C_j + P_S C_{j,SW}$	$A_D C_j + P_D C_{j,SW}$
Triode	0	$\frac{1}{2}WLC_{ox}+WC_{ov}$	$\frac{1}{2}WLC_{ox}+WC_{ov}$	$\left(A_S + \frac{WL}{2}\right) C_j + P_S C_{j,SW}$	$\left(A_D + \frac{WL}{2}\right) C_j + P_D C_{j,SW}$
Saturation	0	$\frac{2}{3}WLC_{ox}+WC_{ov}$	WC_{ov}	$(A_S + WL)\, C_j + P_S C_{j,SW}$	$A_D C_j + P_D C_{j,SW}$

[Johns and Martin, 2012]

IC Amplifier Frequency Response

- **A_o** is the midband gain (or DC gain) of the amplifier
- $\boldsymbol{\omega_{p1}}$ is the 3dB bandwidth $= \mathrm{BW} = \omega_{3\mathrm{dB}}$
- $\boldsymbol{\omega_{p2}}$ is the non-dominant pole
- **Gain-Bandwidth Product (GBW)** is the frequency at which gain is unity (1 = 0dB) (a.k.a. unity gain frequency: ω_u)
- Usually, we design the amplifier such that ω_{p2} and ωz > GBW





$$\frac{V_{out}}{V_{in}} = \frac{A_o(1 + s/\omega_z)}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}$$

$$GBW = A_o \omega_{p1}$$

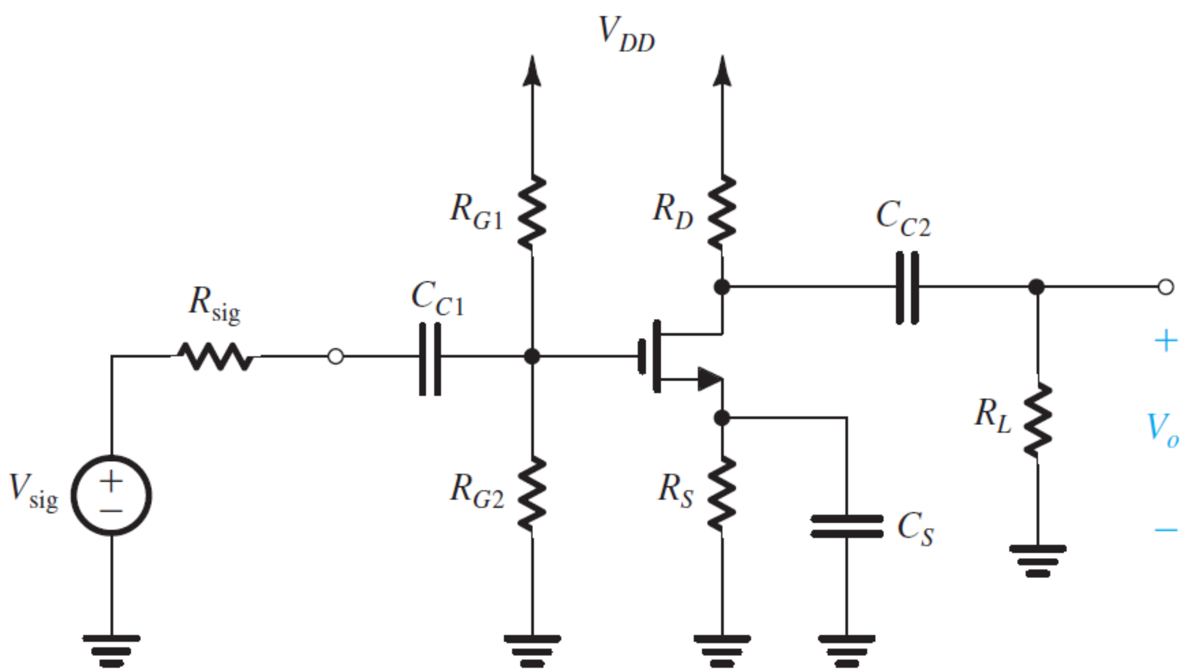
[M. El-Nozahi, Ain Shams Univ.]

Frequency Response of CS: Midband

$$A_v = \frac{v_{in}}{v_{sig}} \cdot \frac{v_o}{v_{in}}$$

$$\frac{v_o}{v_{in}} = G_m R_{out} = -g_m(R_D||R_L||r_o)$$

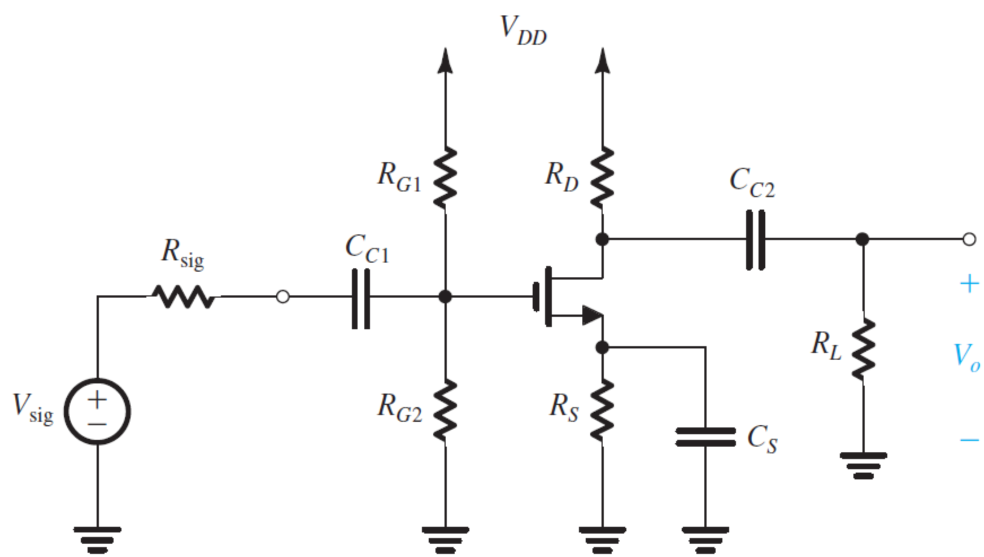
$$\frac{v_{in}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}}, R_{in} = R_G = R_{G1}||R_{G2}$$



[Sedra/Smith, 2015]

Frequency Response of CS: LFR

- C_{C1}: $R_{th} = R_{sig} + R_G \rightarrow \omega_{p,C_{C1}} = \frac{1}{(R_{sig}+R_G)C_{C1}}$ & $\omega_{z,C_{C1}} = 0$
- C_{C2}: $R_{th} = R_L + R_D||r_o \rightarrow \omega_{p,C_{C2}} = \frac{1}{(R_L+R_D||r_o)C_{C2}}$ & $\omega_{z,C_{C2}} = 0$
- C_S: $R_{th} = R_S||R_{LFS} \rightarrow \omega_{p,C_S} = \frac{1}{(R_S||R_{LFS})C_S}$ & $\omega_{z,C_S} = \frac{1}{R_S C_S}$
- Usually ω_{p,C_S} is dominant: $\omega_L \approx \omega_{p,C_S}$ (why?)
- Note that for ICs we usually use direct coupling (no LFR)

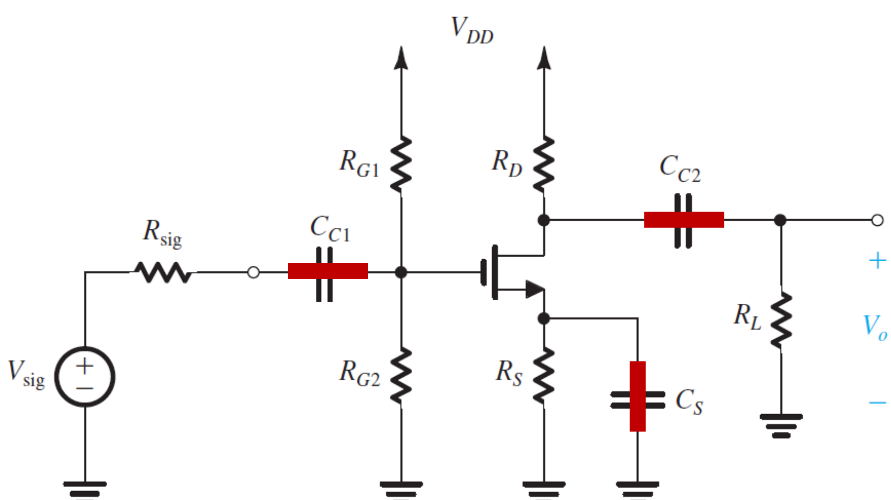


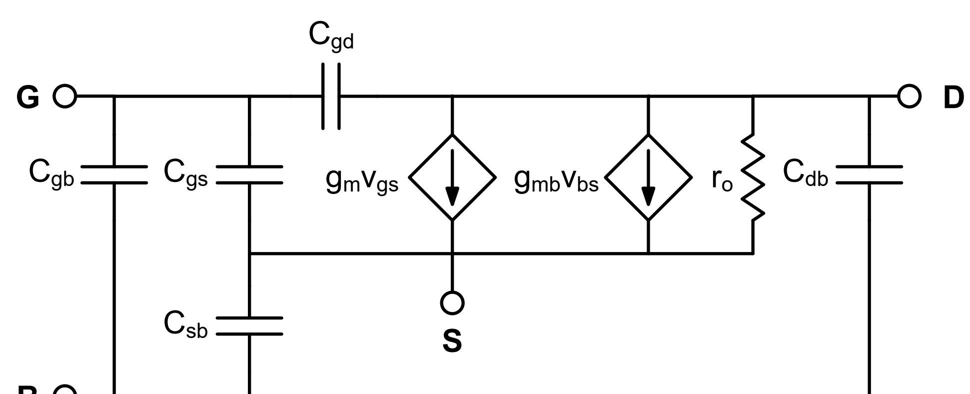
[Sedra/Smith, 2015]

Frequency Response of CS: HFR

- Break the feedback capacitance (C_{gd}) using Miller
  - Read the exact analysis w/o Miller approx in [Razavi, 2017, S6.2]
- Each node is associated with a pole
  - i/p node → i/p pole $(\omega_{p,in})$ & o/p node → o/p pole $(\omega_{p,out})$
- Don't forget the RHP feedforward zero

$$\omega_{z,C_{gd}} = \frac{g_m}{C_{gd}}$$ → Usually $\omega_{z,C_{gd}}$ is very high (why?)





[Sedra/Smith, 2015]

Frequency Response of CS: HFR

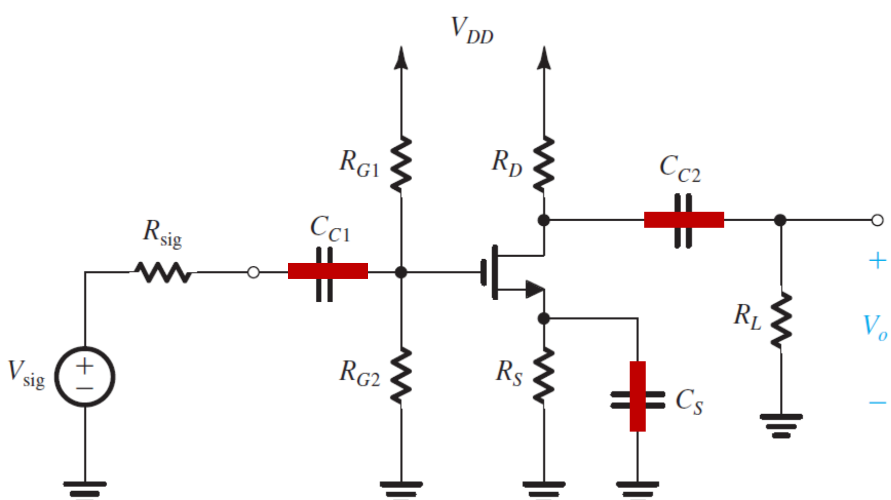
- i/p pole: **suffers from Miller effect**

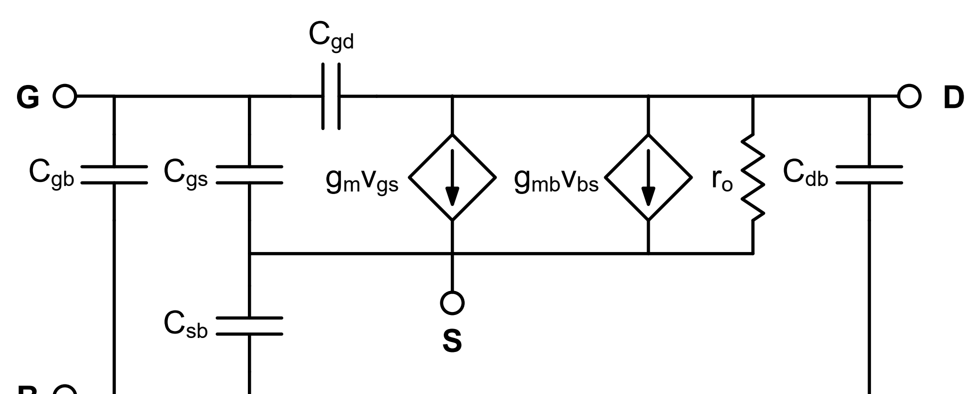
$$R_{th,in} = R_{sig}||R_G \rightarrow \omega_{p,in} \approx \frac{1}{(R_{sig}||R_G)\left(C_{gs}+\boldsymbol{C_{gd}(1+A_o)}\right)}, A_o = \left|\frac{v_o}{v_{in}}\right|$$

- o/p pole:

$$R_{th,out} = R_L||R_D||r_o \rightarrow \omega_{p,out} \approx \frac{1}{(R_L||R_D||r_o)\left(C_L+C_{db}+C_{gd}(1+1/A_o)\right)}$$

- Usually i/p pole is dominant: $\omega_H \approx \omega_{p,in}$ (why?), unless $R_{sig} \downarrow\downarrow$





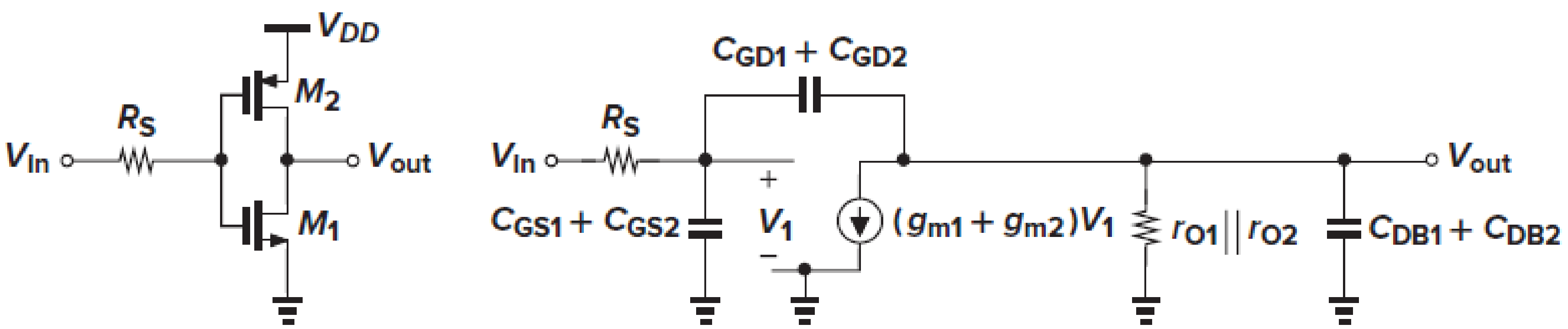
[Sedra/Smith, 2015]

Frequency Response of CS: HFR

- If $\omega_{p,out}$ is dominant (e.g., if R_{sig} ↓↓ or C_L ↑↑)
  - Multiplying C_{gd} by the low frequency gain is **NOT** accurate (why?)
  - $\omega_{p,in} \approx \dfrac{1}{(R_{sig}||R_G)\left(C_{gs}+\boldsymbol{C_{gd}(1+A_v)}\right)} \approx \dfrac{1}{(R_{sig}||R_G)(C_{gs}+\boldsymbol{C_{gd}})}$
  - $\omega_H \approx \omega_{p,out} = \dfrac{1}{R_{out}C_{out}}$
  - $GBW = A_v\omega_H = G_m R_{out} \cdot \dfrac{1}{R_{out}C_{out}} = \dfrac{G_m}{C_{out}}$ → ind. of R_{out}!

Frequency Response of CS: HFR

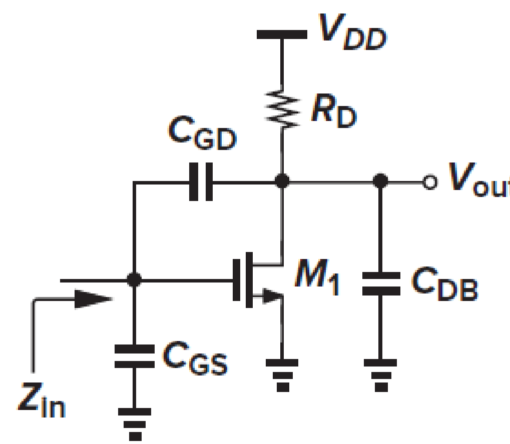
- Note that we must consider the parasitic capacitors of all transistors in the circuit
- Example:



Frequency Response of CS: Z_{in}

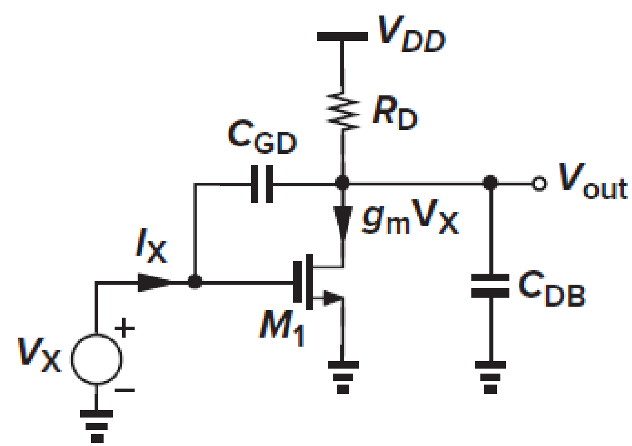
- With Miller approx.

$$Z_{in} = \frac{1}{[C_{GS} + (1 + g_m R_D)C_{GD}]s}$$



- Exact Analysis (C_{gs} adds in parallel)

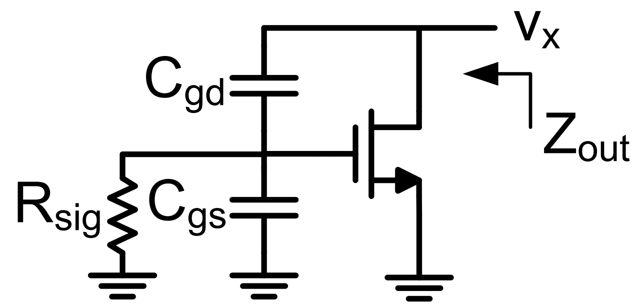
$$\frac{V_X}{I_X} = \frac{1 + R_D(C_{GD} + C_{DB})s}{C_{GD}s(1 + g_m R_D + R_D C_{DB}s)}$$



  - Extra pole and zero at high frequency
  - At relatively low frequency the exact solution reduces to Miller approx.

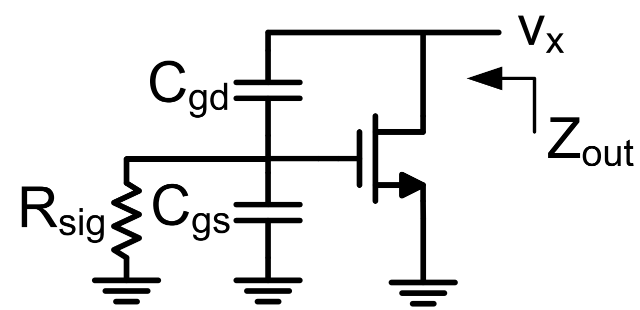
Frequency Response of CS: Z_{out}

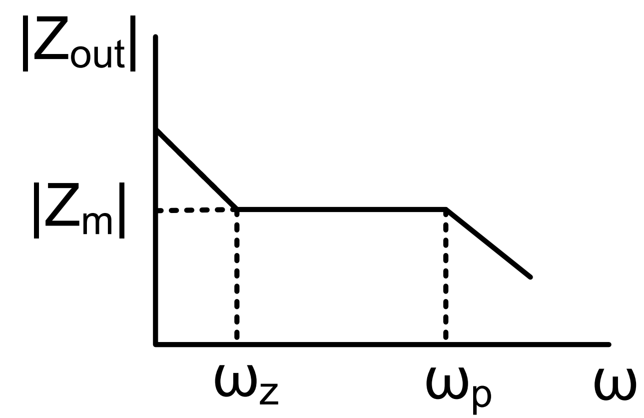
- Can we use Miller?
- $i_x = f(v_{gs}) =$
- $v_x = f(v_{gs}) =$
- $Z_{out} = \frac{v_x}{i_x} =$
- r_o and C_{db} add in parallel
- Important special case: If we have a large capacitor parallel to C_{gd}
  - We will need this case when we study Miller OTA



Frequency Response of CS: Z_{out}

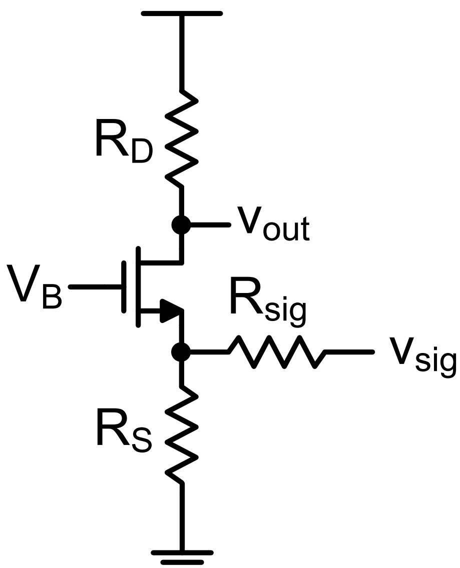
- Can we use Miller?
- $i_x = f(v_{gs}) = g_m v_{gs} + \frac{v_{gs}(1+R_{sig}C_{gs})}{R_{sig}}$
- $v_x = f(v_{gs}) = v_{gs} + \frac{v_{gs}(1+R_{sig}C_{gs})}{sR_{sig}C_{gd}}$
- $Z_{out} = \frac{v_x}{i_x} \approx \frac{1+sR_{sig}(C_{gs}+C_{gd})}{sC_{gd}g_m R_{sig}\left(1+s\frac{C_{gs}}{g_m}\right)}$
- r_o and C_{db} add in parallel
- Important special case: If we have a large capacitor parallel to C_{gd}
  - $|Z_m| \approx 1/g_m$
  - We will need this case when we study Miller OTA

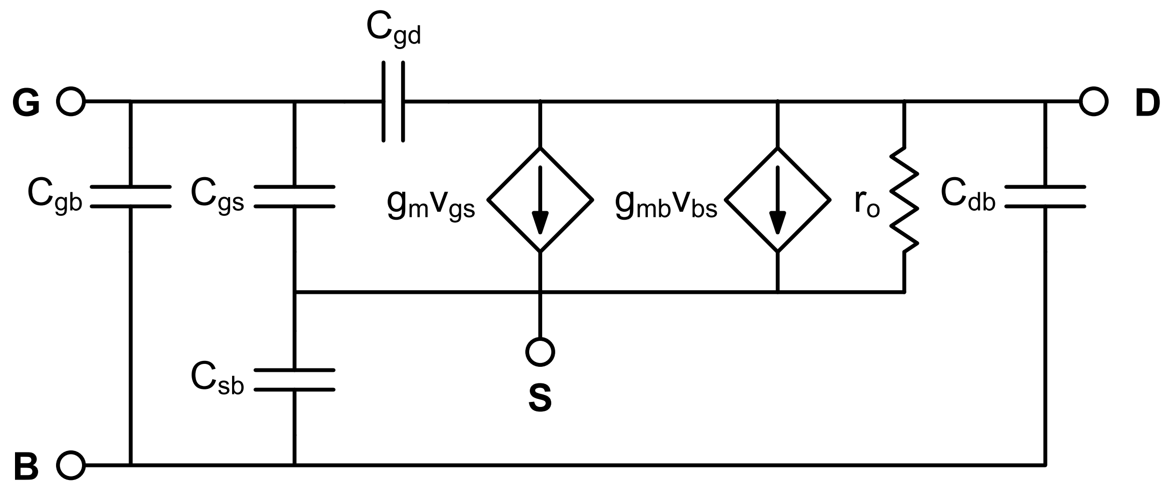




Frequency Response of CG: HFR

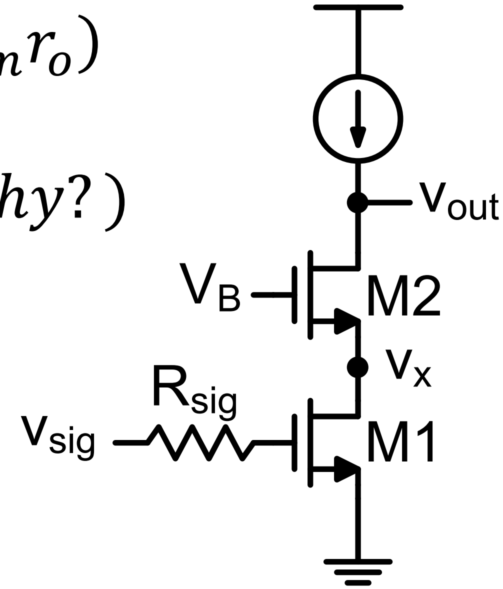
- i/p pole: $\omega_{p,in} = \dfrac{1}{(R_{sig}||R_S||R_{LFS})(C_{gs}+C_{sb})}$
- o/p pole: $\omega_{p,out} = \dfrac{1}{(R_D||R_{LFD})(C_L+C_{db}+C_{gd})}$
- Usually o/p pole is dominant: $\omega_H \approx \omega_{p,out}$ (why?)
- No FB cap → No Miller effect → $\boldsymbol{BW_{CG} \gg BW_{CS}}$





Frequency Response of Cascode: HFR

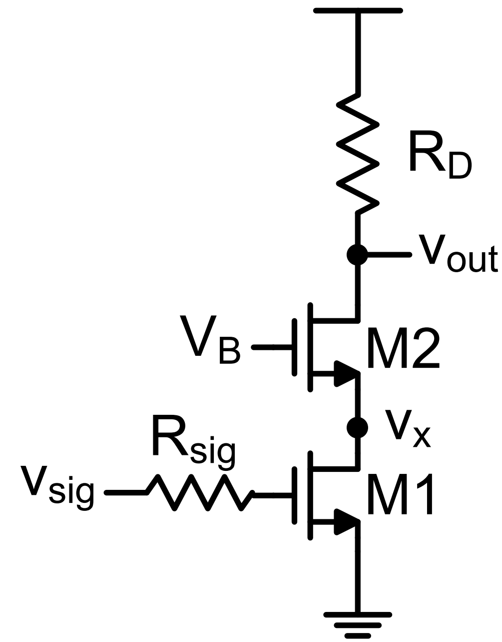
Case 1: BW limited by o/p pole (R_D ↑↑ R_{sig} ↓↓) (cascode for gain)



- $A_v = \frac{v_{out}}{v_{sig}} \approx (g_{m1} r_{o1})(g_{m2} r_{o2}) = A_{v,CS} \cdot (g_m r_o)$
- $A_{o1} = \left|\frac{v_x}{v_{in}}\right| = g_{m1}(r_{o1} || R_{LFS}),\ R_{LFS} \neq \infty\ (why?)$
- $\omega_{p,in} = \frac{1}{R_{sig}\left(C_{gs1} + C_{gd1}(1 + A_{o1})\right)} = \omega_{p,in,CS}$
  - Still a bit suffering from Miller effect
- $\omega_{p,x} = \frac{1}{r_{o1}\left(C_{gs2} + C_{sb2} + C_{db1} + C_{gd1}(1 + 1/A_{o1})\right)}$
- $\omega_{p,out} \approx \frac{1}{r_{o1}(g_{m2} r_{o2})(C_L + C_{db2} + C_{gd2})} = \frac{\omega_{p,out,CS}}{g_m r_o}$ → Dominant
- $GBW = A_v \omega_{p,out} = A_{v,CS}\omega_{p,out,CS} = \frac{G_m}{C_{out}}$ → Same as CS!

Frequency Response of Cascode: HFR

Case 2: BW limited by i/p pole ($\boldsymbol{R_D} \downarrow\downarrow \boldsymbol{R_{sig}} \uparrow\uparrow$) (cascode for BW)

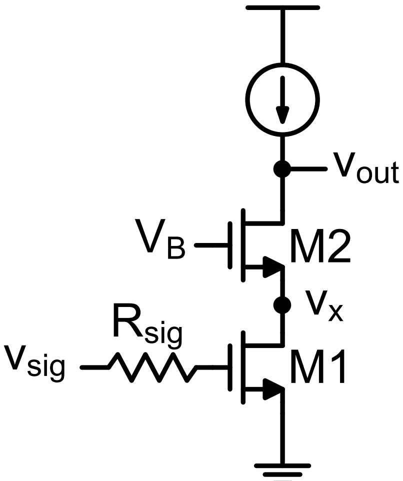


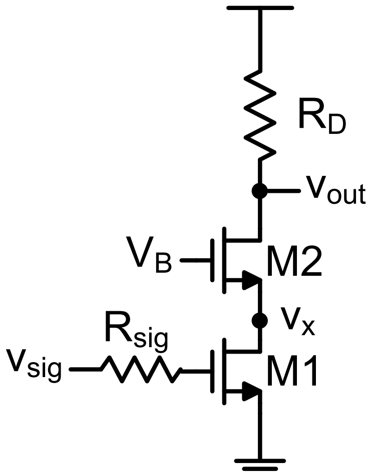
- $A_v = \frac{v_{out}}{v_{sig}} \approx g_{m1}R_D \approx A_{v,CS}$ → Similar to CS!
- $A_{o1} = \left|\frac{v_x}{v_{in}}\right| = g_{m1}(r_{o1}||\frac{1}{g_{m2}}) \approx 1$
- $\omega_{p,in} \approx \frac{1}{R_{sig}(C_{gs1}+2C_{gd1})}$
  - **Miller effect reduced → BW extension!**
- $\omega_{p,x} \approx \frac{1}{(r_{o1}||\frac{1}{g_{m2}})(C_{gs2}+C_{sb2}+C_{db1}+2C_{gd1})}$
- $\omega_{p,out} \approx \frac{1}{R_D(C_L+C_{db2}+C_{gd2})} \approx \omega_{p,out,CS}$ → Similar to CS!
- $GBW = A_v\omega_{p,in} > GBW$ of CS

Frequency Response of Cascode: HFR

Summary

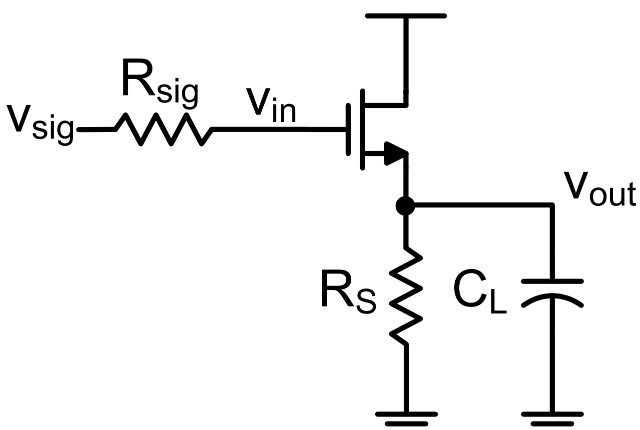
- If BW is limited by o/p pole
  - $GBW = A_v \omega_{p,out} = G_m R_{out} \cdot \frac{1}{R_{out} C_{out}} = \frac{G_m}{C_{out}}$
  - Cascode can be used **to trade gain for bandwidth** by modifying R_{out}
  - But the GBW remain unchanged @ $\frac{G_m}{C_{out}}$
- If BW is limited by i/p pole
  - Cascode can provide higher BW (Miller ↓)
  - The gain may be higher as well
  - $A_v \omega_{p,out}$ remains unchanged but GBW ↑
- See Example 10.10 in Sedra/Smith 7th ed.





Frequency Response of CD: HFR

- The poles can be complex and nearby → OCTC cannot be used ☹
  - Simple circuit, but complicated and lengthy analysis!
- Special case 1: $R_{sig} = 0$
- Special case 2: $R_S \rightarrow IDC,\ C_L = 0,\ g_{mb}\ and\ r_o\ neglected, A_v \equiv 1$



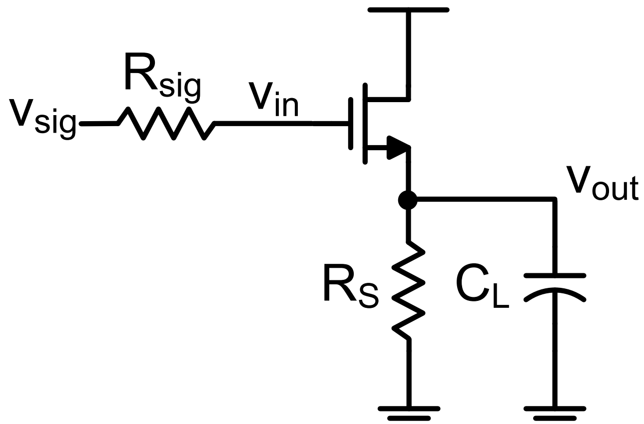
Frequency Response of CD: HFR

- Special case 1: $R_{sig} = 0$
  - Only one pole at the output

$$\omega_{p,out} = \frac{1}{\left(R_S // \frac{1}{g_m + g_{mb}}\right)(C_L + C_{gs} + C_{sb})}$$ → Large BW (why?)

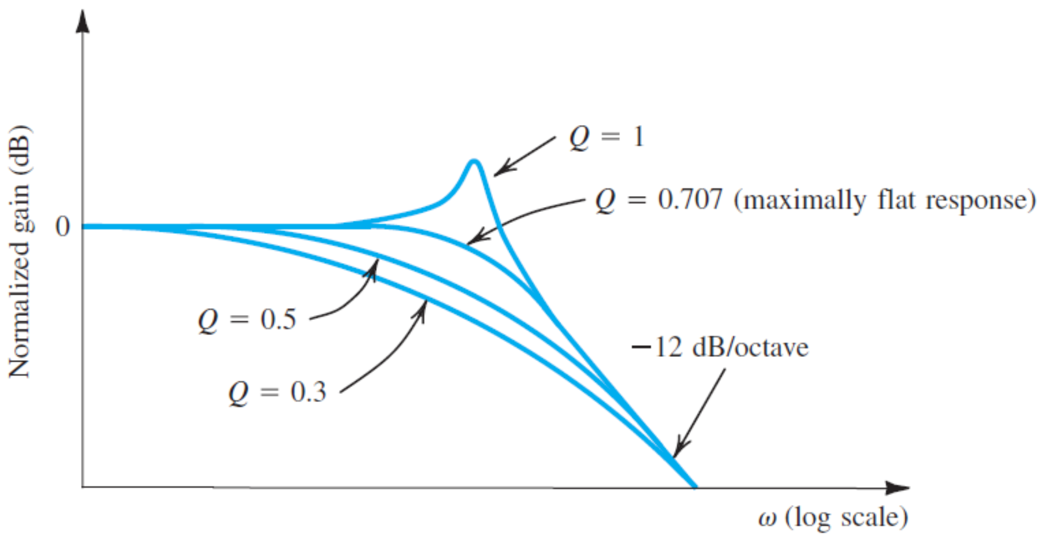
- Special case 2: $R_S \rightarrow IDC,\ C_L = 0,\ g_{mb}\ and\ r_o\ neglected, A_v \equiv 1$
  - Apply Miller: C_{gs} is **bootstrapped**

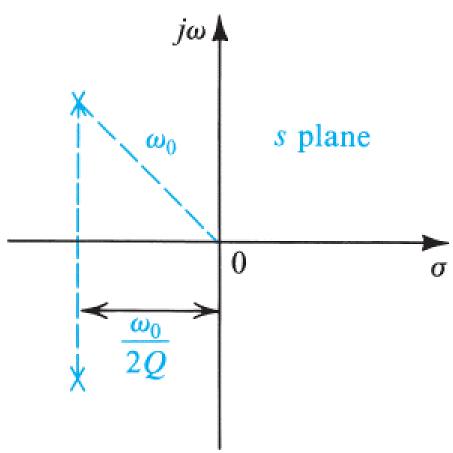
$$\omega_{p,in} = \frac{1}{R_{sig} C_{gd}}$$ → Large BW (why?)

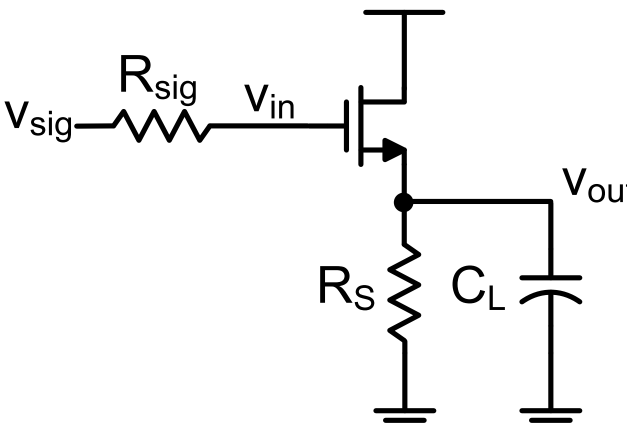


Frequency Response of CD: HFR

- The poles can be complex and nearby → OCTC cannot be used ☹
  - Simple circuit, but complicated and lengthy analysis!







[Sedra/Smith, 2015]

Frequency Response of CD: Z_{in}

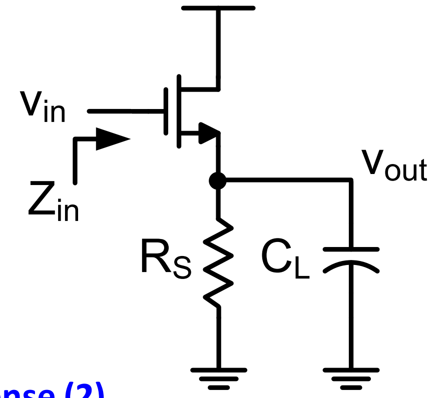
- $i_{in} = f(v_{gs}) = v_{gs} s C_{gs}$
- $v_{in} = f(v_{gs}) = v_{gs} + (v_{gs} s C_{gs} + g_m v_{gs})\left(R_S // r_o // \frac{1}{g_{mb}} // \frac{1}{sC_L}\right)$

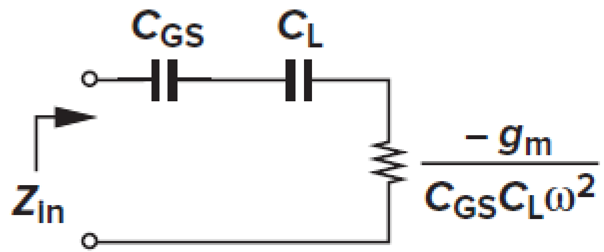
$$Z_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{sC_{gs}} + \left(1 + \frac{g_m}{sC_{gs}}\right)\left(R_S // r_o // \frac{1}{g_{mb}} // \frac{1}{sC_L}\right)$$

- If $\frac{1}{sC_L}$ is dominant (e.g., CD driving large cap load, or @ high freq)

$$Z_{in} \approx \frac{1}{sC_{gs}} + \frac{1}{sC_L} + \frac{g_m}{s^2 C_{gs} C_L} = \frac{1}{j\omega C_{gs}} + \frac{1}{j\omega C_L} - \frac{g_m}{\omega^2 C_{gs} C_L}$$ → -ve res?!!

- Can be used in oscillators, and may make amplifiers unstable!



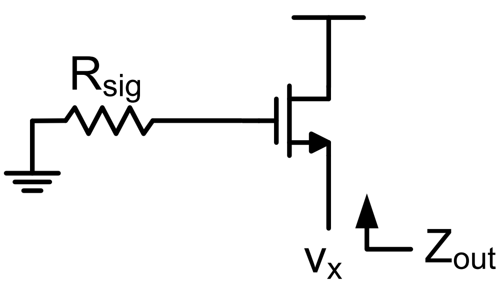


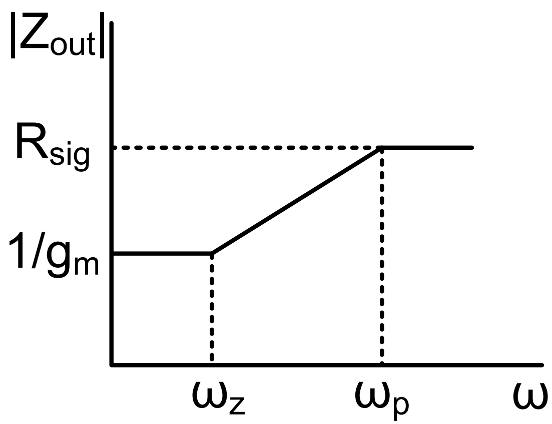
Frequency Response of CD: Z_{out}

- $i_x = f(v_{gs}) = -v_{gs}sC_{gs} - g_m v_{gs}$
- $v_x = f(v_{gs}) = -v_{gs} - v_{gs}sC_{gs}R_{sig}$

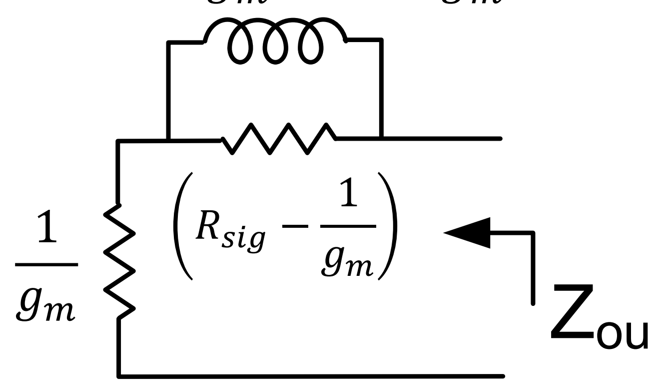
$$Z_{out} = \frac{v_x}{i_x} = \frac{1+sC_{gs}R_{sig}}{sC_{gs}+g_m} = \frac{1}{g_m}\left(\frac{1+sR_{sig}C_{gs}}{1+s\frac{C_{gs}}{g_m}}\right) \rightarrow @\ \omega \uparrow\uparrow: Z_{out} \approx R_{sig}$$

- Usually $R_{sig} > \frac{1}{g_m}$ (buffer) → inductive rise
- Body resistance $\left(\frac{1}{g_{mb}}\right)$ and r_o add in parallel



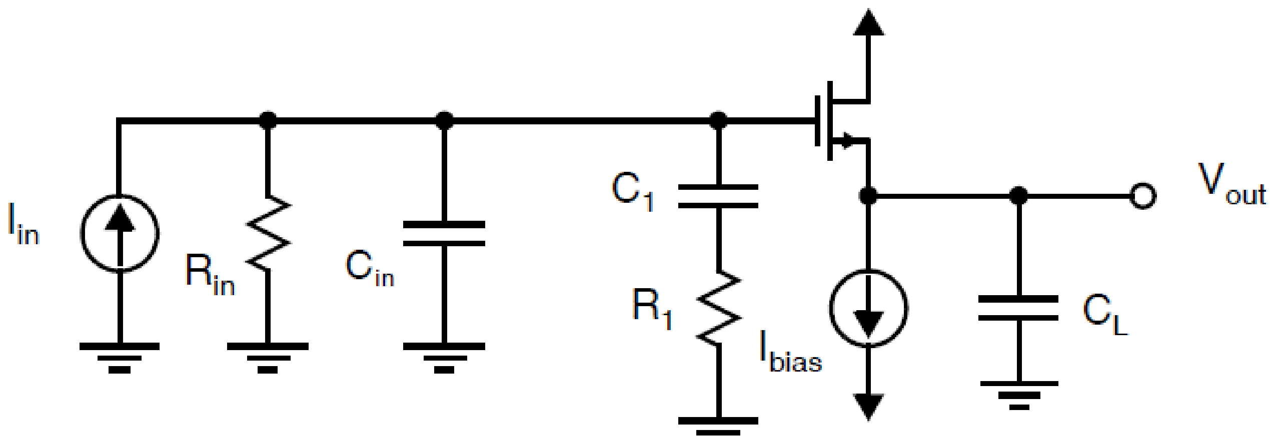


$$L = \frac{C_{gs}}{g_m}\left(R_{sig} - \frac{1}{g_m}\right)$$



Frequency Response of CD: HFR

- A compensation network (R_1 and C_1) can be used to compensate for the negative input impedance and prevent overshoots
- See Johns and Martin Section 4.4 for more details



Thank you!

Frequency Response of CD: HFR

$$\frac{v_{out}}{v_{sig}} = A_M \frac{1+\frac{s}{\omega_z}}{1+b_1 s+b_2 s^2} = A_M \frac{1+\frac{s}{\omega_z}}{1+\frac{1}{Q}\frac{s}{\omega_o}+\frac{s^2}{\omega_o^2}}$$

$$\frac{v_{out}}{v_{sig}} = A_M \frac{1+\frac{s}{\omega_z}}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)} = A_M \frac{1+\frac{s}{\omega_z}}{1+\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)s+\frac{s^2}{\omega_{p1}\omega_{p2}}}$$

$$\boldsymbol{\omega_z} = \frac{\boldsymbol{g_m}}{\boldsymbol{C_{gs}}}, \omega_o = \frac{1}{\sqrt{b_2}}, Q = \frac{\sqrt{b_2}}{b_1}, R_{Leff} = R_S // r_o // \frac{1}{g_{mb}}$$

$$b_1 = \left(C_{gd} + \frac{C_{gs}}{g_m R_{Leff}+1}\right) R_{sig} + \left(\frac{C_{gs}+C_L}{g_m R_{Leff}+1}\right) R_{Leff}$$

$$b_2 = \left(\frac{\left(C_{gs}+C_{gd}\right)C_L + C_{gs}C_{gd}}{g_m R_{Leff}+1}\right) R_{sig} R_{Leff}$$

[Sedra/Smith, 2015]

Frequency Response of CD: HFR

- Special case: R_{Leff} ↑↑

$$\frac{v_{out}}{v_{sig}} = A_M \frac{1+\frac{s}{\omega_z}}{1+b_1 s+b_2 s^2} A_M = \frac{1+\frac{s}{\omega_z}}{1+\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)s+\frac{s^2}{\omega_{p1}\omega_{p2}}}$$

$$b_1 = C_{gd}R_{sig} + \frac{C_{gs}+C_L}{g_m}$$

$$b_2 = \left(\frac{\left(C_{gs}+C_{gd}\right)C_L + C_{gs}C_{gd}}{g_m}\right)R_{sig}$$

- If $C_L = 0$

$$b_1 = C_{gd}R_{sig} + \frac{C_{gs}}{g_m} = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} \quad \Rightarrow \quad \boldsymbol{\omega_{p2} = \omega_z}$$

$$b_2 = C_{gd}R_{sig} \cdot \frac{C_{gs}}{g_m} = \frac{1}{\omega_{p1}}\frac{1}{\omega_{p2}}$$

Frequency Response of CD: HFR

- Special case: ω_{p1} *is dominant*

$$\frac{v_{out}}{v_{sig}} = A_M \frac{1+\frac{s}{\omega_z}}{1+b_1 s+b_2 s^2} \approx A_M \frac{1+\frac{s}{\omega_z}}{1+\left(\frac{1}{\omega_{p1}}\right)s+\frac{s^2}{\omega_{p1}\omega_{p2}}}$$

$$\omega_{p1} \approx \frac{1}{b_1}$$

$$\omega_{p2} \approx \frac{b_1}{b_2}$$