وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 06
Basic Amplifier Stages

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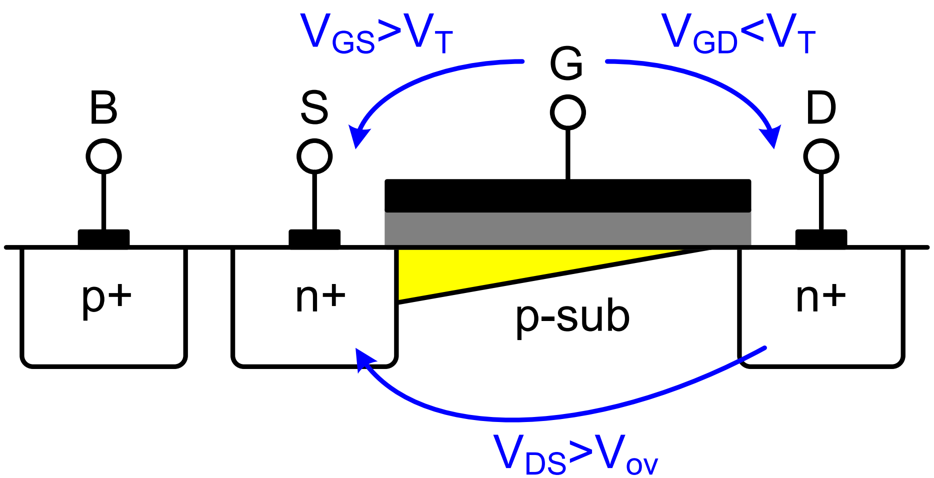
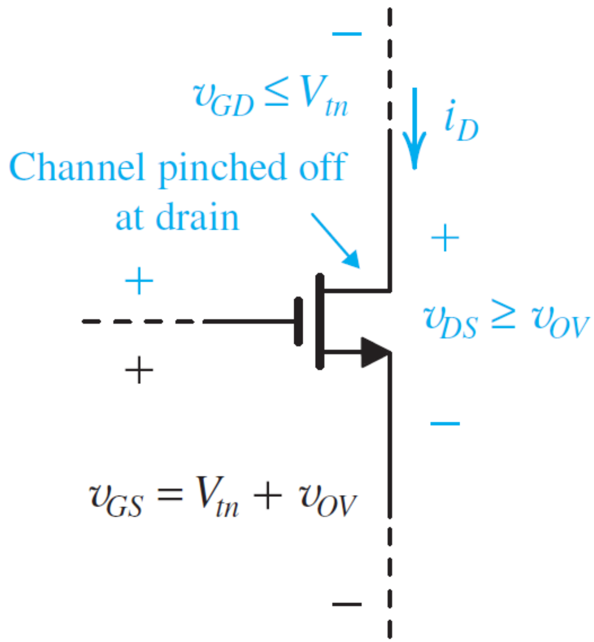
MOSFET in Saturation

- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L} \cdot V_{ov}^2(1 + \lambda V_{DS})$$





 [Sedra/Smith, 2015]

Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_T$

ON

$V_{GS} > V_T$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_T$

$$I_D = \mu C_{ox} \frac{W}{L} \left(V_{ov} V_{DS} - \frac{V_{DS}^2}{2} \right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_T$

$$I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} V_{ov}^2 (1 + \lambda V_{DS})$$

Long-Channel Square-Law Assumptions

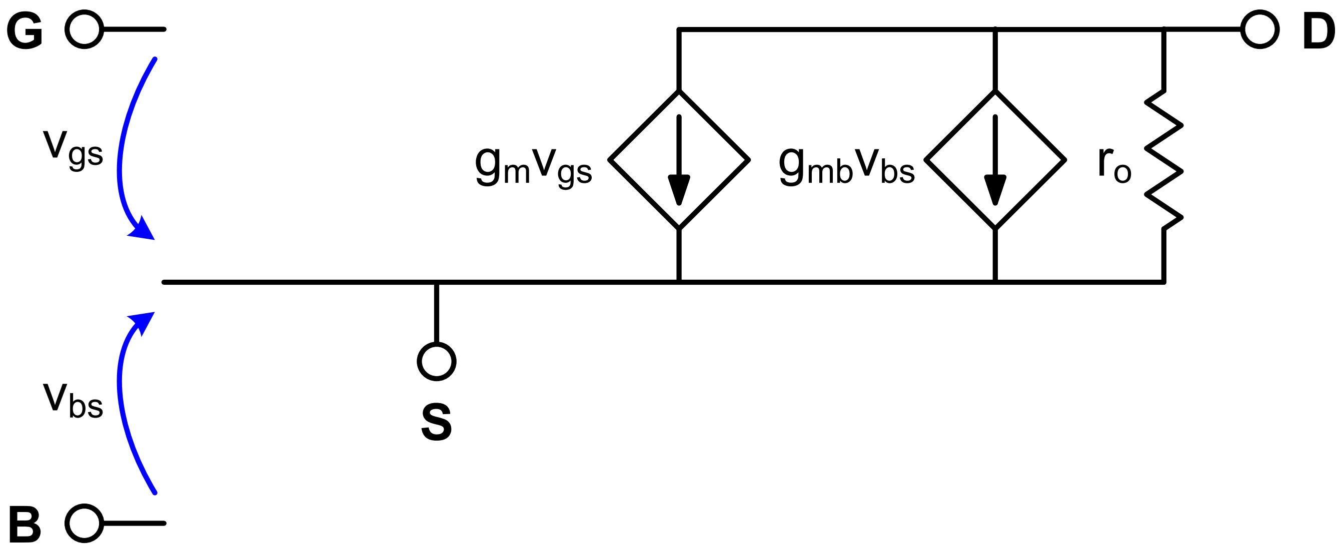
- Square-Law: $I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} V_{ov}^2 (1 + \lambda V_{DS})$
- Valid "relatively" if
  - Relatively long channel length
  - Moderate overdrive voltage (e.g., $V_{ov} \approx 100 - 300mV$)
    - For small V_{ov}: weak inversion and subthreshold operation
      - ID-VGS relation becomes exponential
    - For large V_{ov}: velocity saturation happens before pinch-off
      - ID-VGS relation becomes linear
- If the above assumptions are not valid
  - Use gm/ID design methodology
- Actually, better to use gm/ID even if the above assumptions are valid!

Low-Frequency Small-Signal Model

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} V_{ov} = \sqrt{\mu C_{ox} \frac{W}{L} \cdot 2 I_D} = \frac{2 I_D}{V_{ov}}$$

$$g_{mb} = \eta g_m, \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}} = \frac{1}{\lambda I_D}, \qquad \lambda \propto \frac{1}{L}$$



Large and Small Signal Analysis

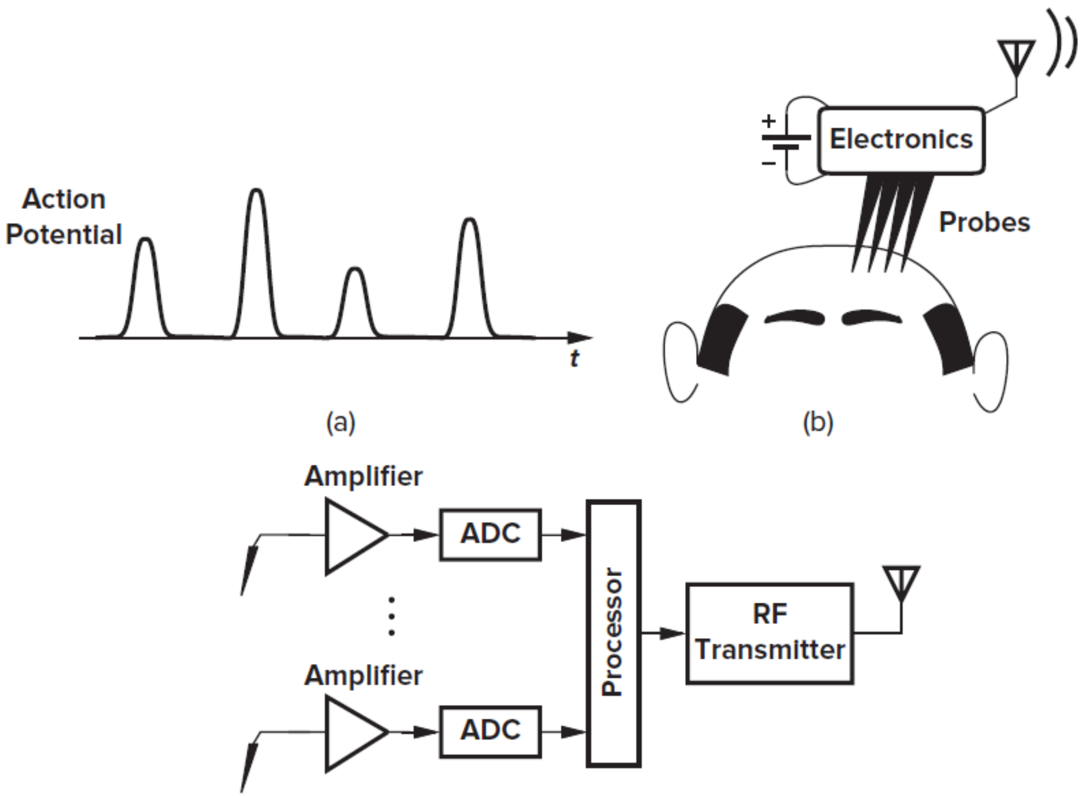
	Large Signal Analysis	Small Signal Analysis
Model	Large signal model	Small signal model
Linearity	Non-linear	Linear
Simulation	DC and transient analysis	AC analysis
Purpose	Calculate bias point, signal swing, distortion, etc.	Calculate $A_v, R_{in}, R_{out}, BW, etc.$
VDC	✓	s.c.
IDC	✓	o.c.
Capacitor	o.c. (in DC)	?
Inductor	s.c. (in DC)	?

Today

- Why Amplifiers?
- Rin/out Shortcuts
- GmRout Method
- Basic Amplifier Topologies
  - Common Source
  - Common Gate
  - Common Drain

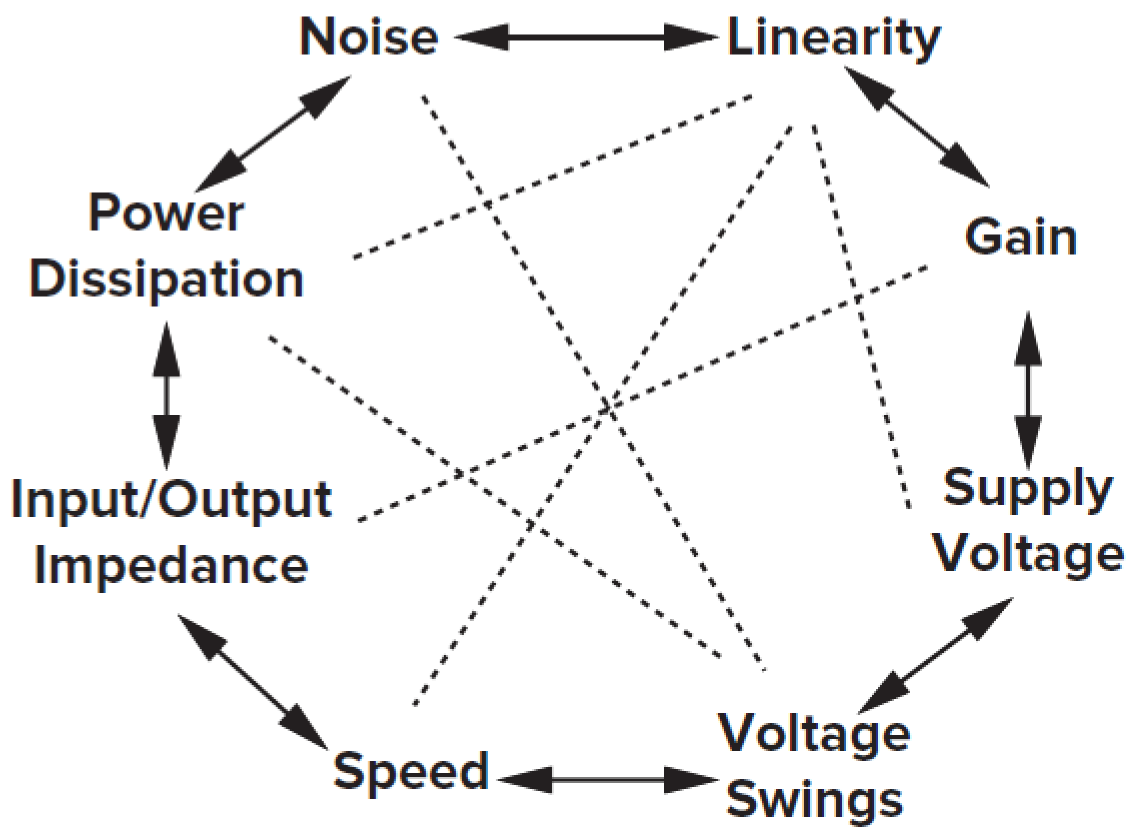
Why Amplifiers?

- All the physical signals in the world around us are analog
  - Voice, light, temperature, pressure, etc.
- We (will) always need an “analog” interface circuit to connect between our physical world and our digital electronics



 [Razavi, 2017]

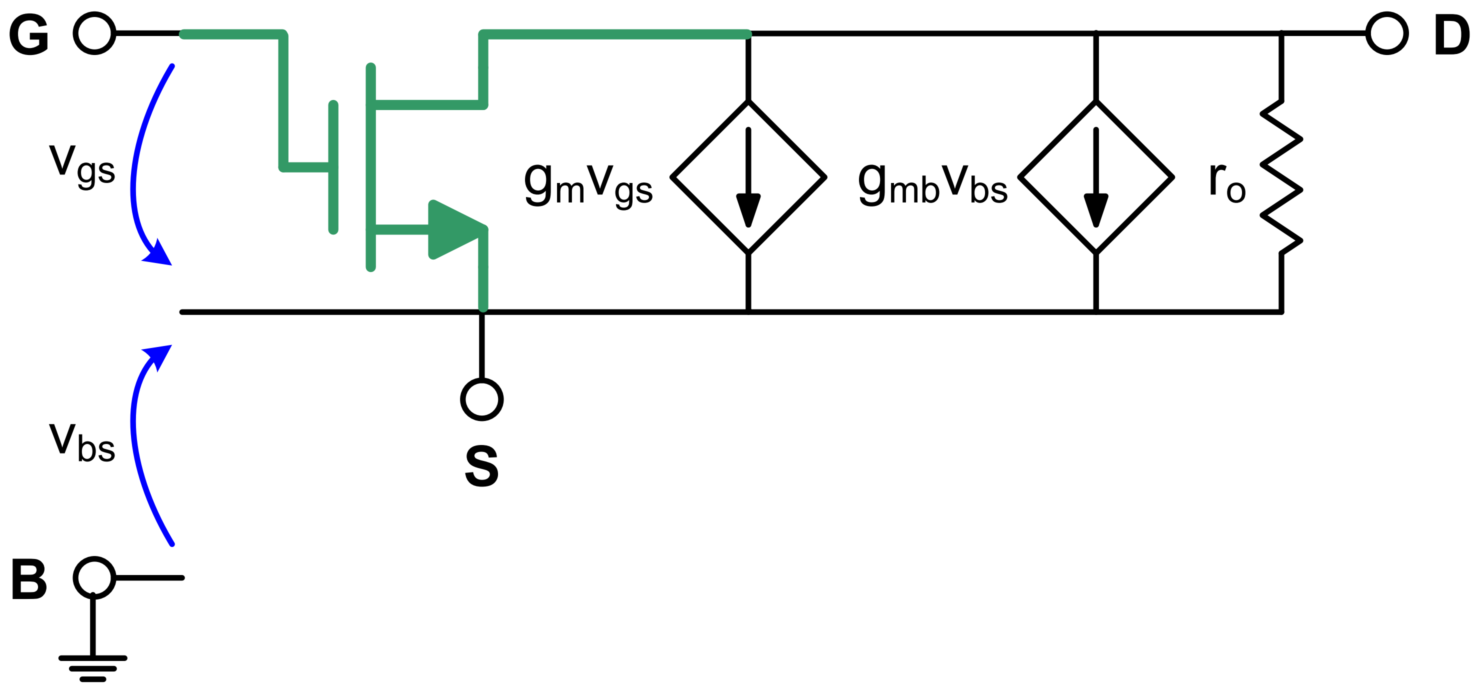
Analog Design Octagon



[Razavi, 2017]

Direct Analysis on Schematics

- No need to draw the small signal model every time
- Just remember we have two VCCSs and r_o between D and S



Intrinsic Gain

$$v_{out} = -(g_m v_{in}) r_o$$

$$|A_v| = \left|\frac{v_{out}}{v_{in}}\right| = g_m r_o$$

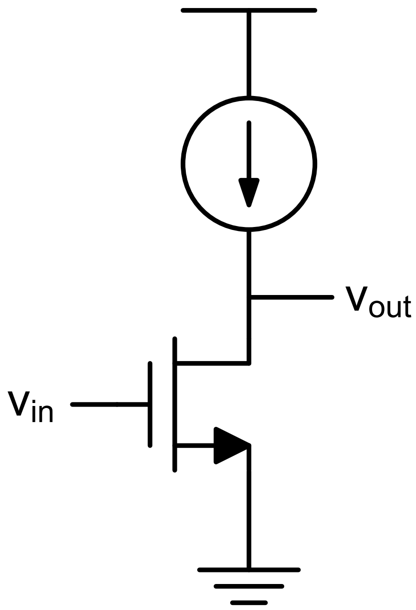
- $g_m r_o$ is the max gain that can be obtained from a single transistor
- Common approximations that we usually use

$$g_m r_o \gg 1$$

$$r_o \gg \frac{1}{g_m}$$

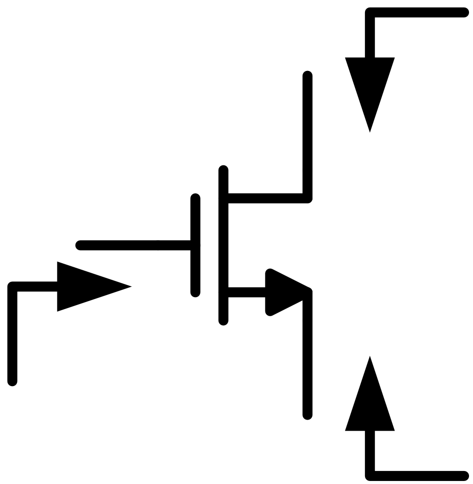
$$r_o + \frac{1}{g_m} \approx r_o$$

$$r_o // \frac{1}{g_m} \approx \frac{1}{g_m}$$



Rin/out Shortcuts

- Find equivalent impedance looking from Gate, Source, and Drain



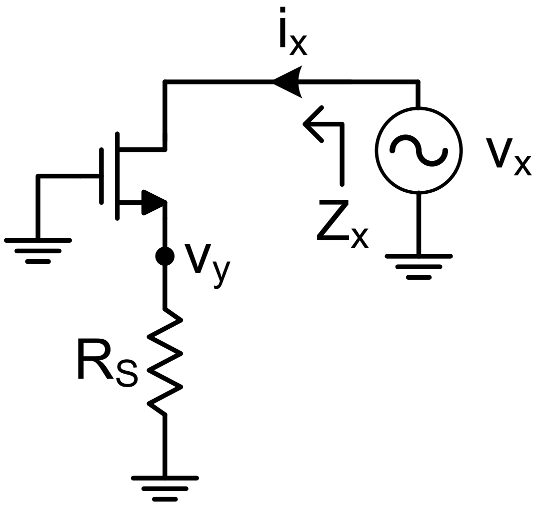
Looking From Drain

- Apply KCL at v_y (note that $v_{gs} = -v_y$)

$$\frac{v_y}{R_S} + \frac{v_y - v_x}{r_o} + (g_m + g_{mb})v_y = 0$$

- But $v_y = i_x R_S$ and $g_m r_o \gg 1$

$$R_x = \frac{v_x}{i_x} \approx r_o[1 + (g_m + g_{mb})R_S]$$



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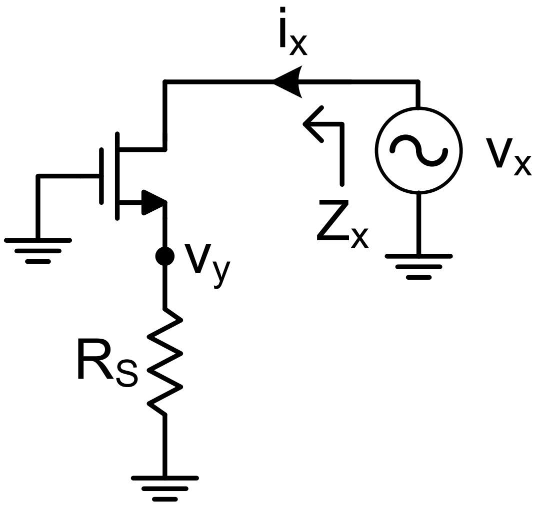
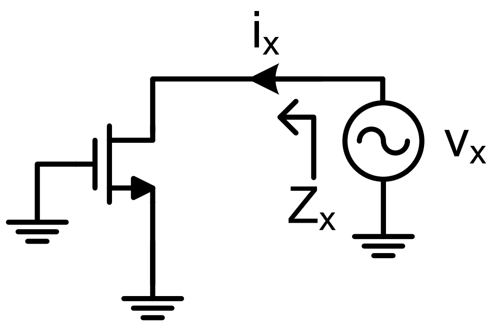


- Special case: $R_S = 0$ (G and S ac s.c.)

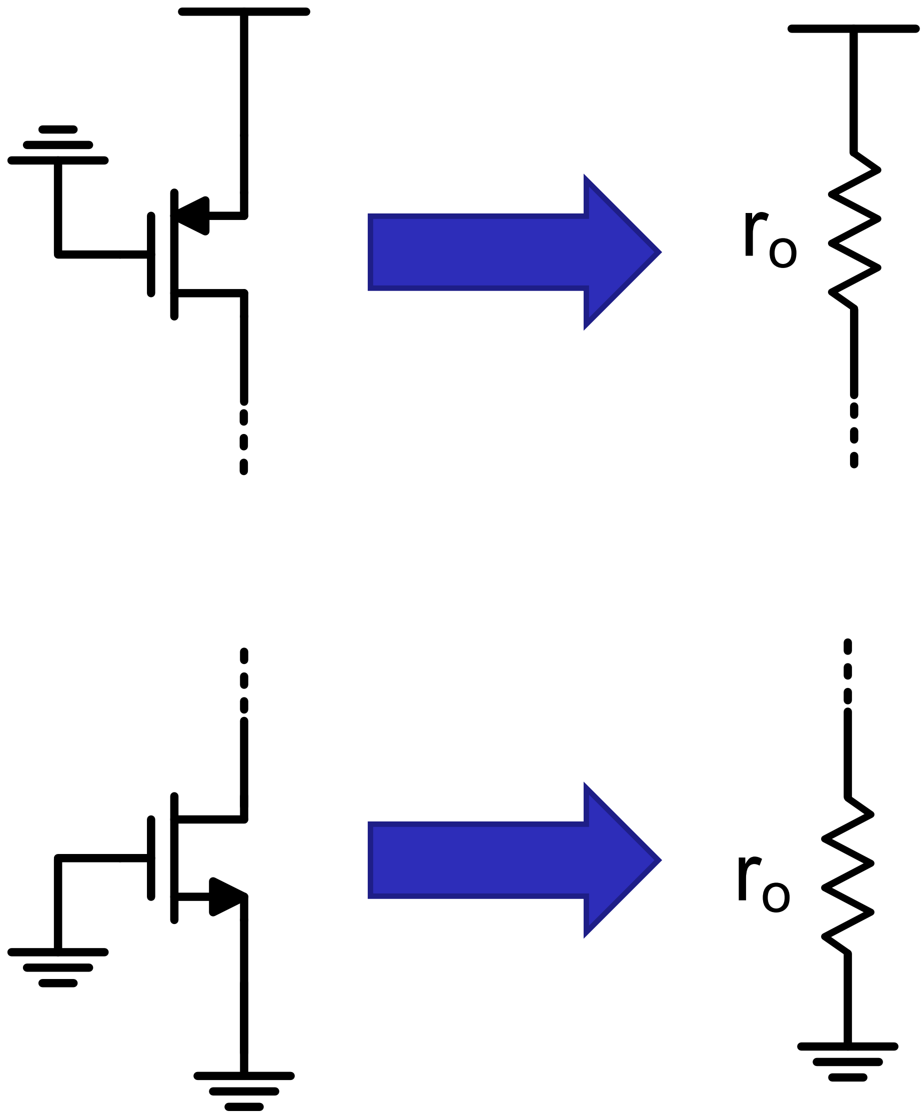
$$R_x \approx r_o$$

(Active load)

- **Drain is a high-impedance node (H.I.N.)**

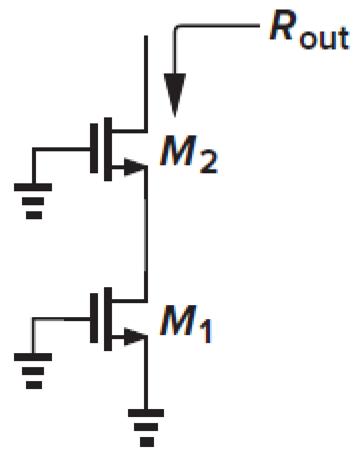


Active Load (Source OFF)



Bonus Question

- Assume M1 and M2 have the same gm and ro, and neglect body effect
- Find Rout



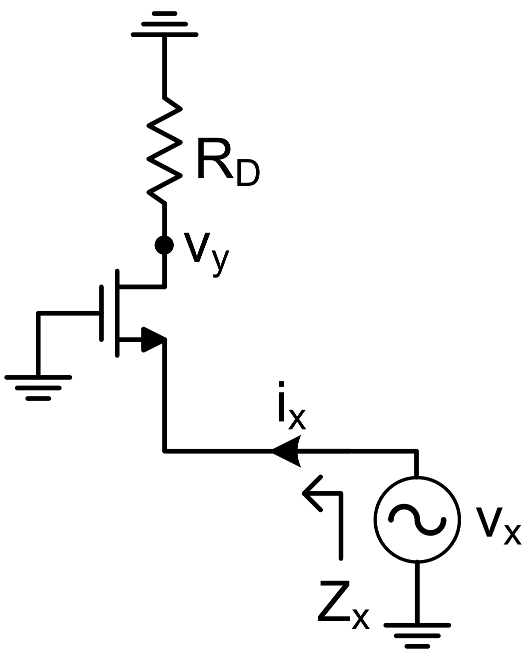
Looking From Source

- Apply KCL at v_y (note that $v_{gs} = -v_x$)

$$\frac{v_y}{R_D} + \frac{v_y - v_x}{r_o} - (g_m + g_{mb})v_x = 0$$

- But $v_y = i_x R_D$ and $g_m r_o \gg 1$

$$R_x = \frac{v_x}{i_x} \approx \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$



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- But $v_y = i_x R_D$ and $g_m r_o \gg 1$

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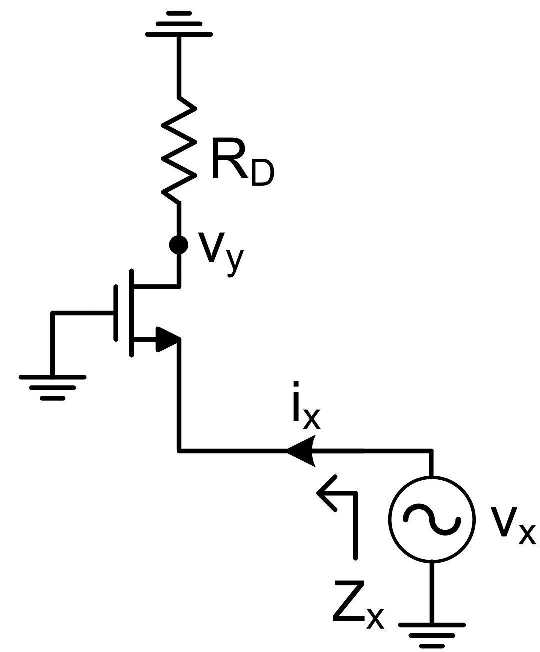
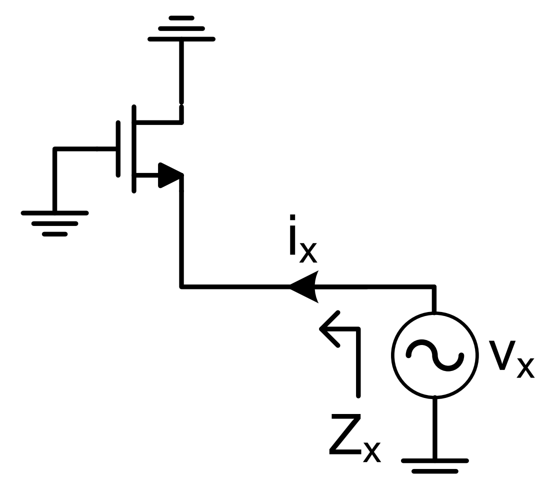


- Special case: $R_D = 0$ (G & D ac s.c.)

$$R_x \approx \frac{1}{g_m + g_{mb}}$$

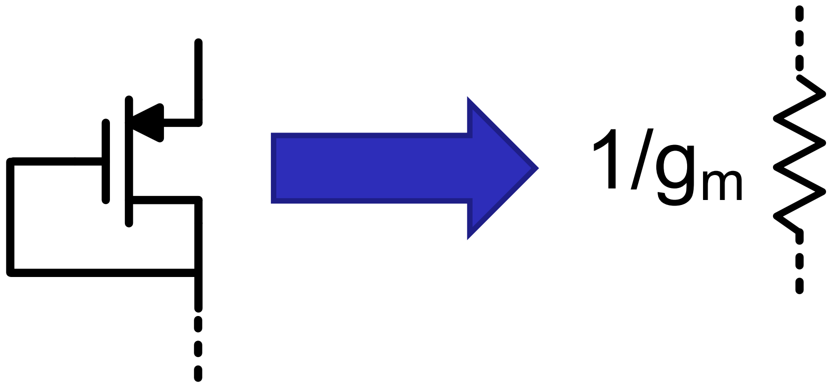
(Diode connected)

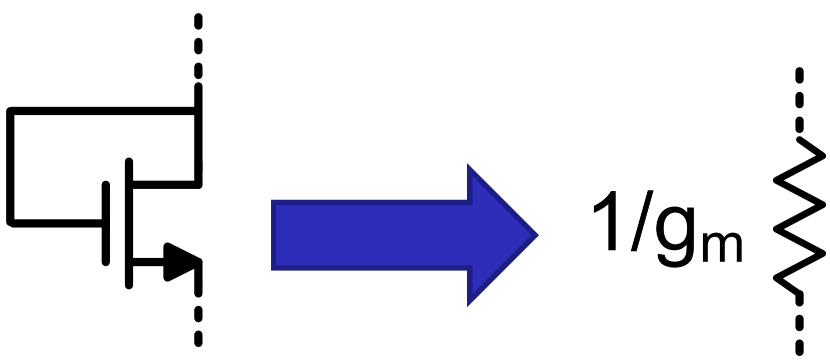
- **Source is a low impedance node (L.I.N.)**



Diode Connected (Source Absorption)

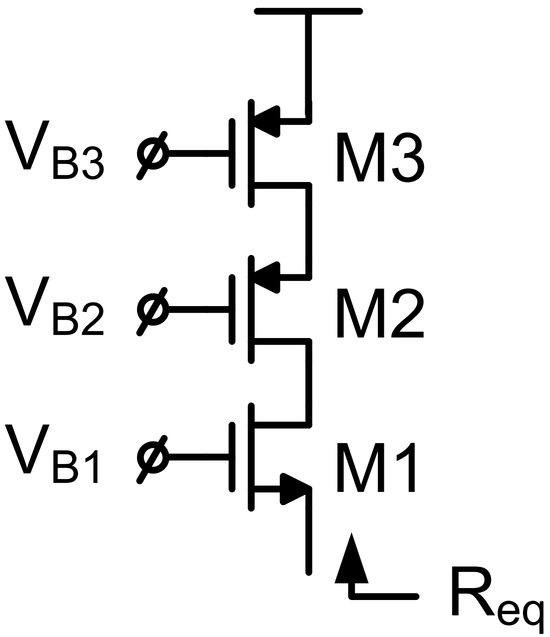
- Always in saturation
- Bulk effect: $g_m \rightarrow g_m + g_{mb}$



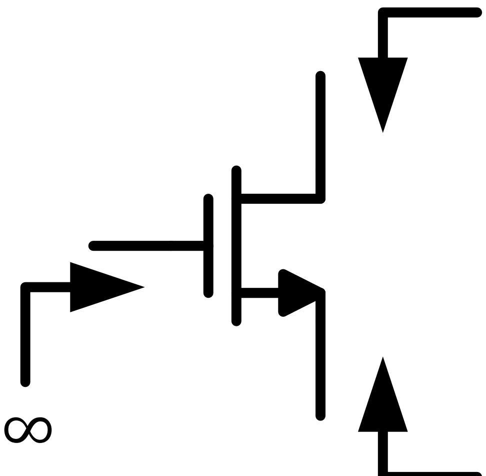


Bonus Question

- Assume M1, M2, and M3 have the same gm and ro. Ignore body effect.
- Find Req



Rin/out Shortcuts Summary



$$r_o[1 + (g_m + g_{mb})R_S]$$

H.I.N.

∞

At low frequencies **ONLY**

$$\frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

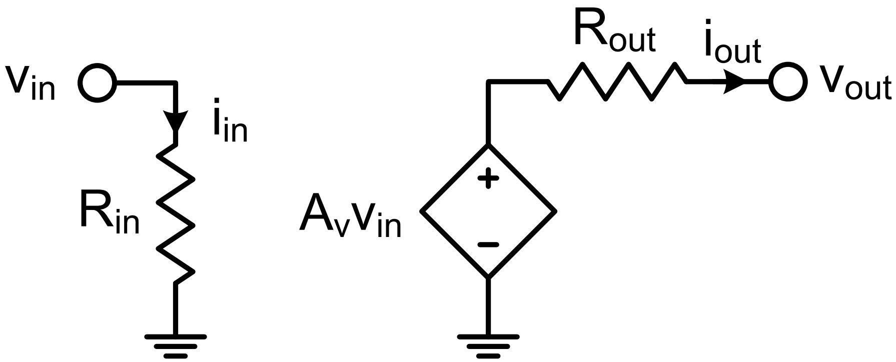
L.I.N.

Amplifier Model

- O.C. voltage gain

$$v_{out,oc} = A_v v_{in}$$

$$A_v = \frac{v_{out,oc}}{v_{in}}$$



- Rin/out

$$R_{in} = \frac{v_{in}}{i_{in}}$$

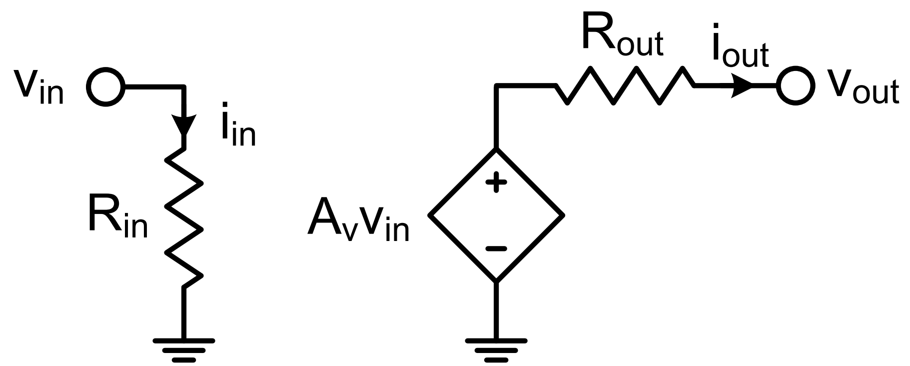
$$R_{out} = \frac{v_x}{i_x} \ @\ v_{in} = 0$$

Amplifier Model

- Transconductance

$$i_{out,sc} = G_m v_{in}$$

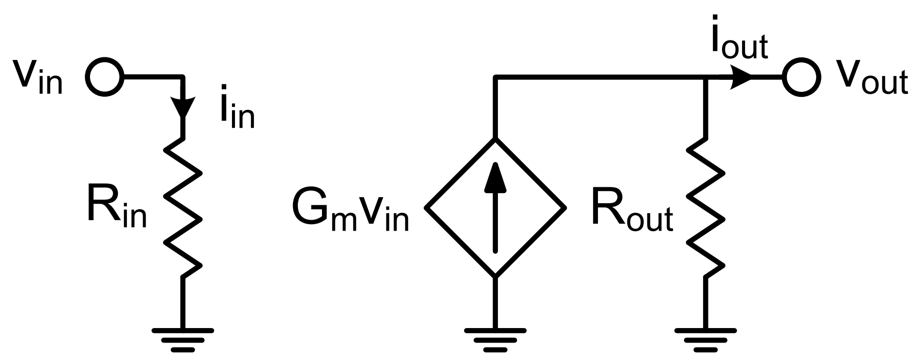
$$G_m = \frac{i_{out,sc}}{v_{in}}$$



- Thevenin ⇔ Norton

$$A_v v_{in} = (G_m v_{in}) R_{out}$$

$$\boxed{A_v = \frac{v_{out,oc}}{v_{in}} = G_m R_{out}}$$



- S.C. Current Gain

$$\boxed{A_i = \frac{i_{out,sc}}{i_{in}} = G_m R_{in}}$$

Why GmRout?

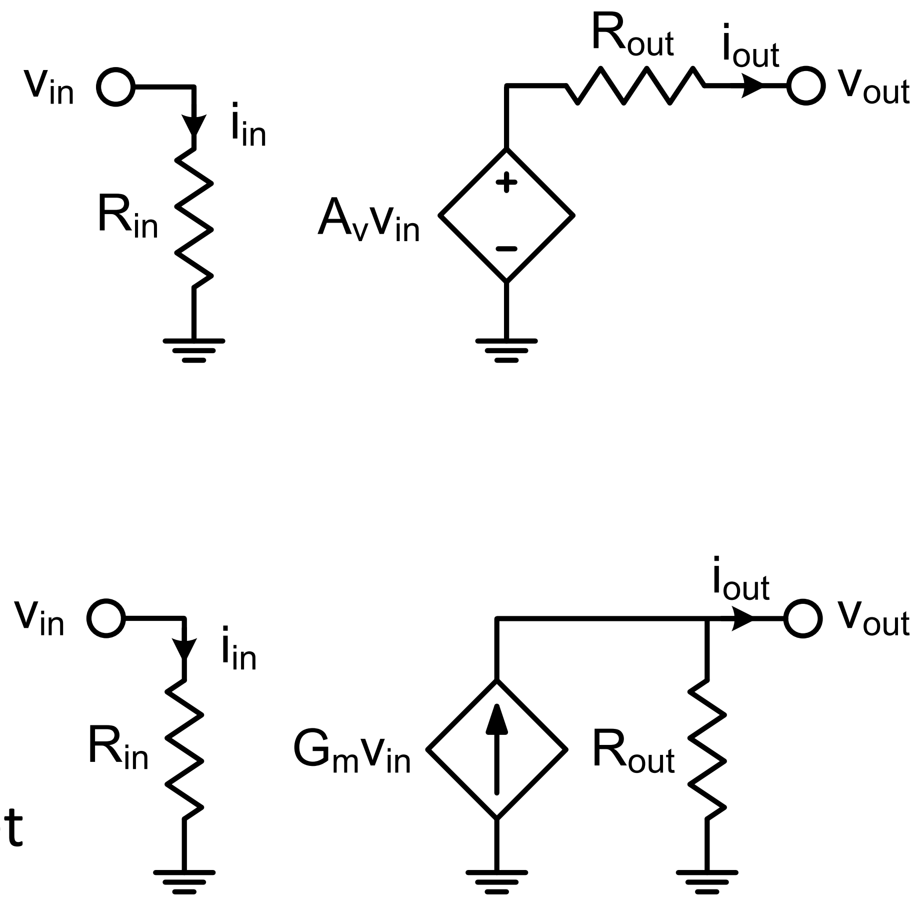
$$R_{out} = \frac{v_x}{i_x} \ @\ v_{in} = 0$$

$$G_m = \frac{i_{out,sc}}{v_{in}}$$

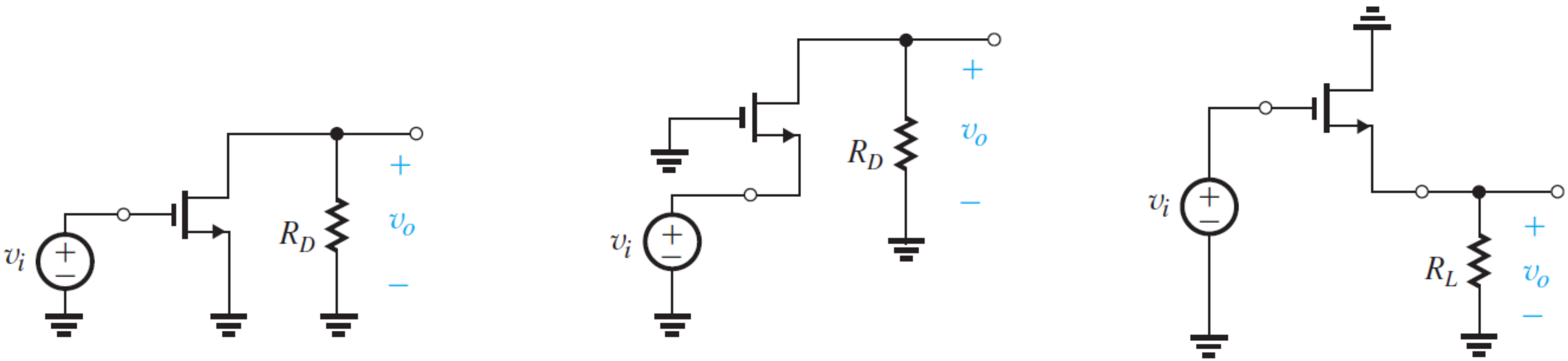
$$A_v = G_m R_{out}$$

$$A_i = G_m R_{in}$$

- Divide and conquer
  - Rout simplified: vin=0
  - Gm simplified: vout=0
  - We already need Rin/out
  - We can quickly and easily get Rin/out from the shortcuts



Basic Amplifier Topologies



(a) Common Source (CS)

(b) Common Gate (CG)

(c) Common Drain (CD) or Source Follower

Topology	Input	Output
Common-Source	Gate	Drain
Common-Gate	Source	Drain
Common-Drain (Source-Follower)	Gate	Source

[Sedra/Smith, 2015]

Common Source (CS)

- KCL at v_x

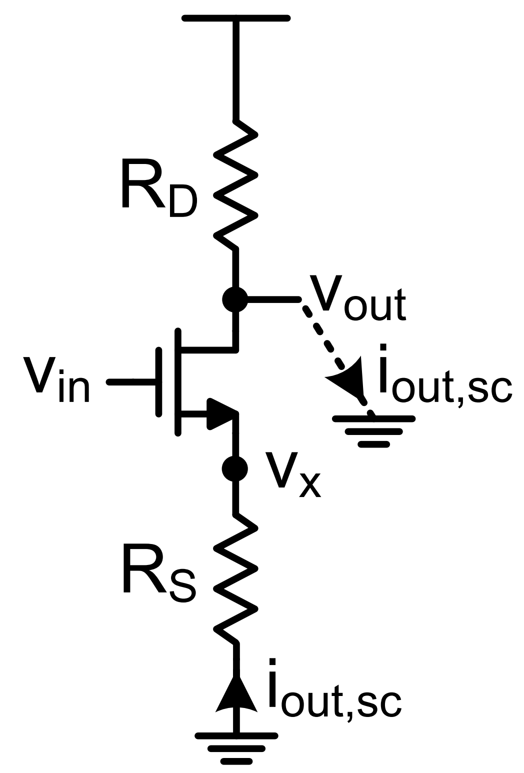
$$i_{out,sc} + g_m(v_{in} - v_x) + g_{mb}(-v_x) - \frac{v_x}{r_o} = 0$$

- But $v_x = -i_{out,sc}R_S$ and $g_m r_o \gg 1$

$$\boxed{G_m = \frac{i_{out,sc}}{v_{in}} \approx \frac{-g_m}{1 + (g_m + g_{mb})R_S}}$$

$$R_{out} \approx R_D//r_o[1 + (g_m + g_{mb})R_S]$$

$$A_v = G_m R_{out}$$



Common Source (CS)

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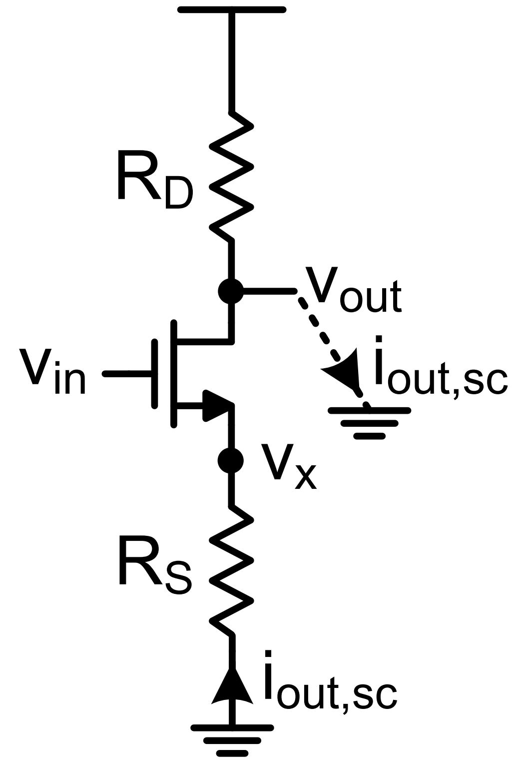
- But $v_x = -i_{out,sc}R_S$ and $g_m r_o \gg 1$

$$\boxed{G_m = \frac{i_{out,sc}}{v_{in}} \approx \frac{-g_m}{1 + (g_m + g_{mb})R_S}}$$

$$R_{out} \approx R_D // r_o[1 + (g_m + g_{mb})R_S]$$

$$A_v = G_m R_{out}$$



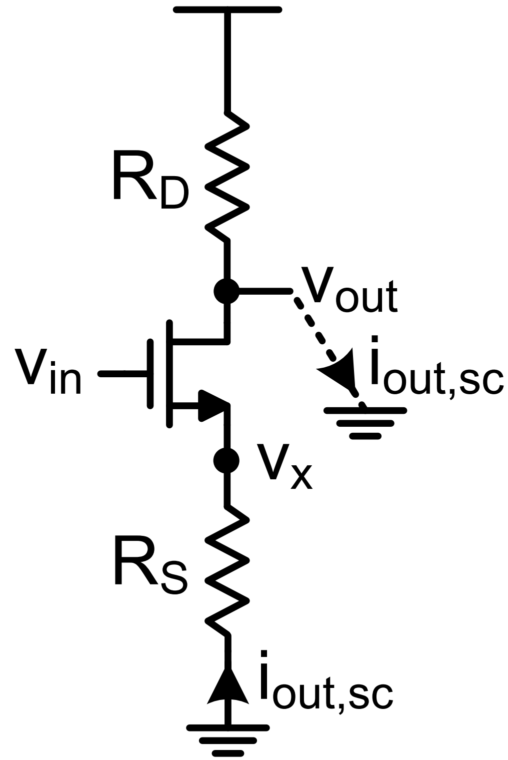
- If R_D is ac o.c.: $A_v = -g_m r_o$
- If $R_D \ll R_{LFD}$: $A_v \approx \frac{-g_m R_D}{1+(g_m+g_{mb})R_S}$
- If $R_S = 0$: $A_v = -g_m(R_D//r_o)$

Common Source (CS)

$$\boxed{G_m = \frac{i_{out,sc}}{v_{in}} \approx \frac{-g_m}{1 + (g_m + g_{mb})R_S}}$$

$$R_{out} \approx R_D // r_o[1 + (g_m + g_{mb})R_S]$$

$$A_v = G_m R_{out}$$



- If S and B are connected (for PMOS):

$$A_v \approx \frac{-R_D//R_{LFD}}{\frac{1}{g_m} + R_S} = -\frac{Drain\ Res.}{\frac{1}{g_m} + Source\ Res}$$

- If $R_S \gg \frac{1}{g_m}$ & $R_D \ll R_{LFD}$: $A_v \approx \frac{-R_D}{R_S} \rightarrow Linear$
- R_S reduces G_m → **Source degeneration**
  - But improves linearity

Common Gate (CG)

- KCL at v_{out}

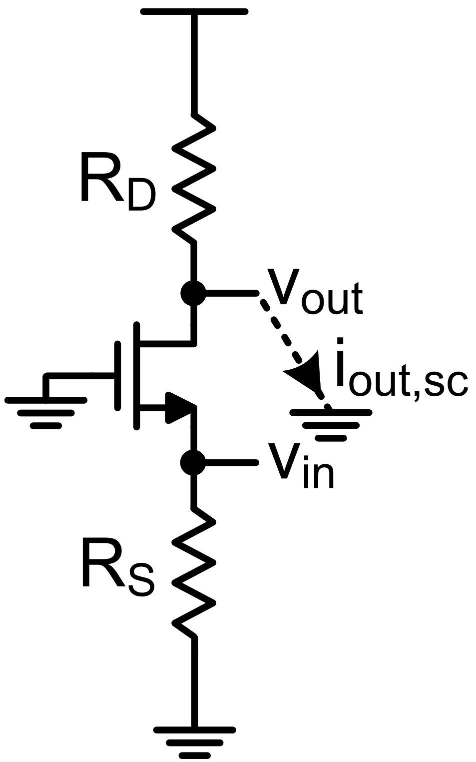
$$i_{out,sc} + (g_m + g_{mb})(-v_{in}) - \frac{v_{in}}{r_o} = 0$$

- But $g_m r_o \gg 1$

$$\boxed{G_m = \frac{i_{out,sc}}{v_{in}} \approx g_m + g_{mb}}$$

$$R_{out} \approx R_D // r_o \ (\textbf{why}?)$$

$$A_v = G_m R_{out}$$



Common Gate (CG)

- KCL at v_{out}

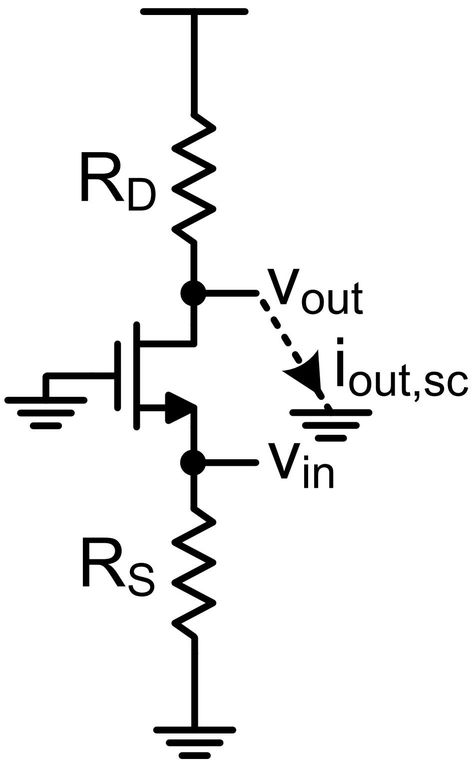
$$i_{out,sc} + (g_m + g_{mb})(-v_{in}) - \frac{v_{in}}{r_o} = 0$$

- But $g_m r_o \gg 1$

$$\boxed{G_m = \frac{i_{out,sc}}{v_{in}} \approx g_m + g_{mb}}$$

$$R_{out} \approx R_D // r_o \ (\mathbf{why}?)$$

$$A_v = G_m R_{out}$$



- If R_D is ac o.c.: $A_v = (g_m + g_{mb}) r_o$
- If $R_D \ll r_o$: $A_v \approx (g_m + g_{mb}) \boldsymbol{R_D}$

- For PMOS connect S and B: $(g_m + g_{mb}) \rightarrow g_m$
- Note that $A_i = G_m R_{in} \approx$ **1 (Current Buffer)**

Common Drain (CD) – Source Follower

- KCL at v_x

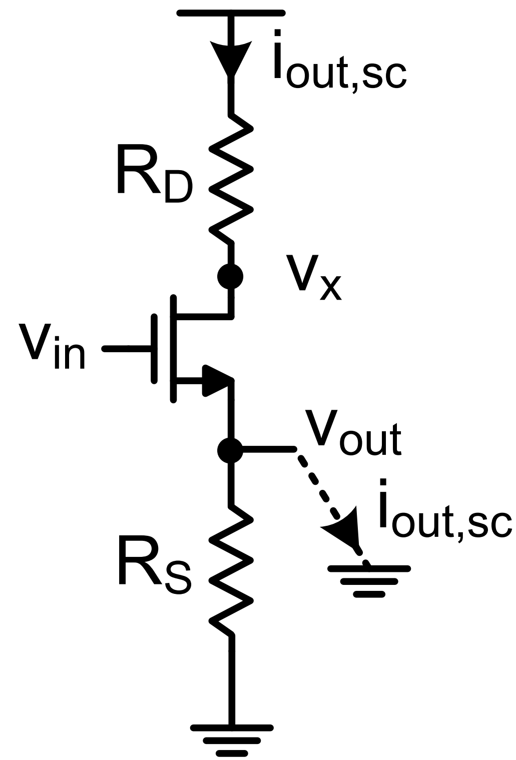
$$i_{out,sc} - g_m v_{in} - \frac{v_x}{r_o} = 0$$

- But $v_x = -i_{out,sc} R_D$ and $g_m r_o \gg 1$

$$\boxed{\boldsymbol{G_m = \frac{i_{out,sc}}{v_{in}} \approx \frac{g_m}{1 + R_D/r_o}}}$$

$$R_{out} \approx R_S // \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

$$A_v = G_m R_{out}$$



Common Drain (CD) – Source Follower

- KCL at v_x

$$i_{out,sc} - g_m v_{in} - \frac{v_x}{r_o} = 0$$

- But $v_x = -i_{out,sc} R_D$ and $g_m r_o \gg 1$

$$\boxed{\boldsymbol{G_m = \frac{i_{out,sc}}{v_{in}} \approx \frac{g_m}{1 + R_D/r_o}}}$$

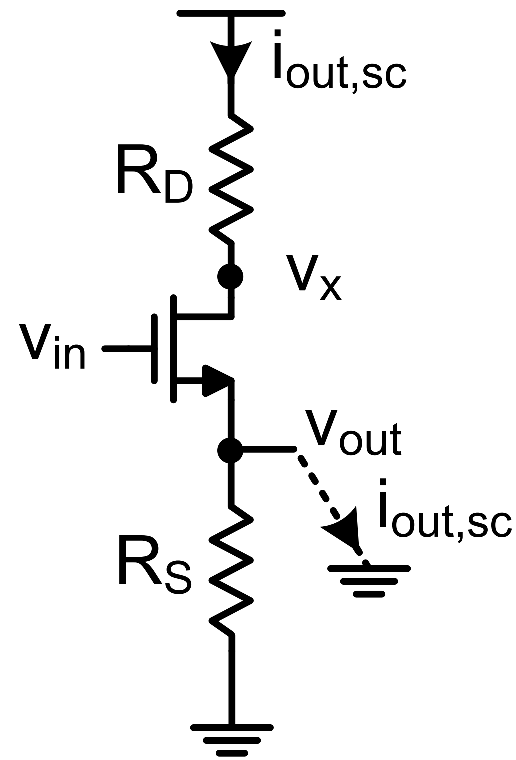
$$R_{out} \approx R_S // \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

$$A_v = G_m R_{out}$$

- If $R_S \gg R_{LFS}$: $A_v \approx \frac{g_m}{g_m + g_{mb}}$

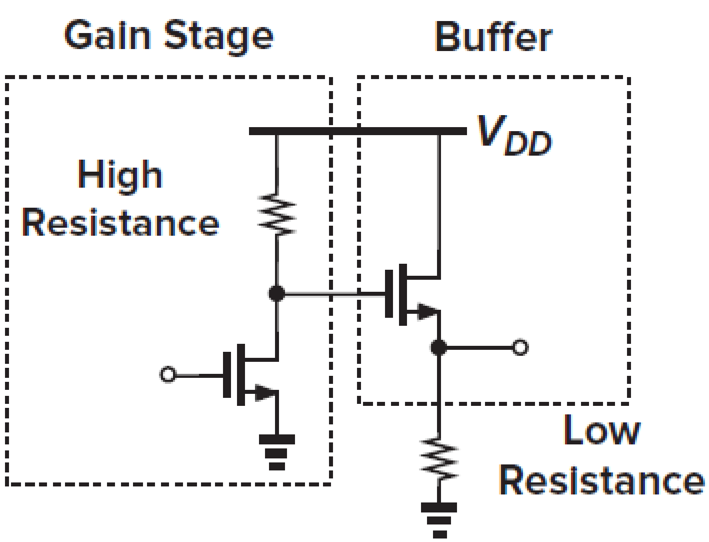
- If S and B are connected (for PMOS):

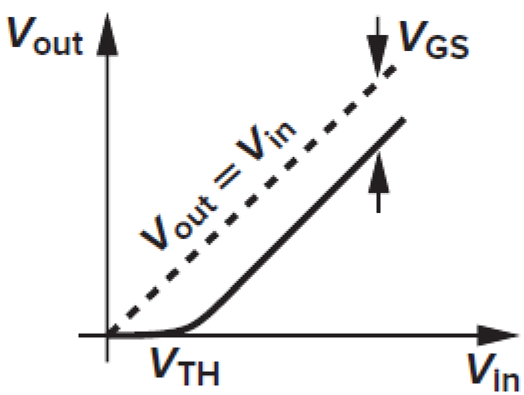
$$A_v \approx \mathbf{1}$$



Why Source Follower?

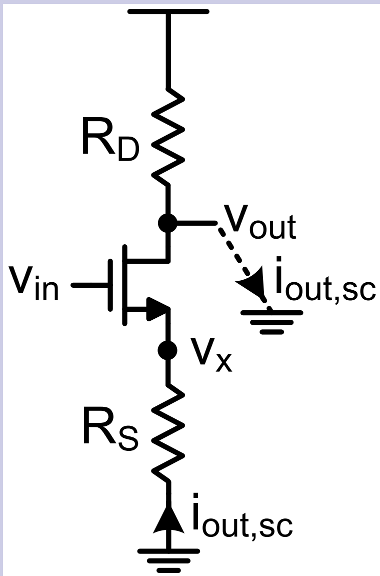
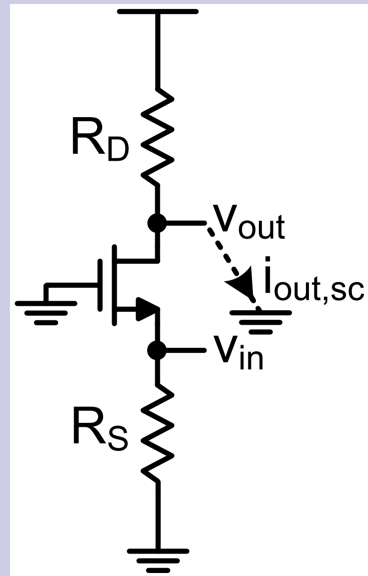
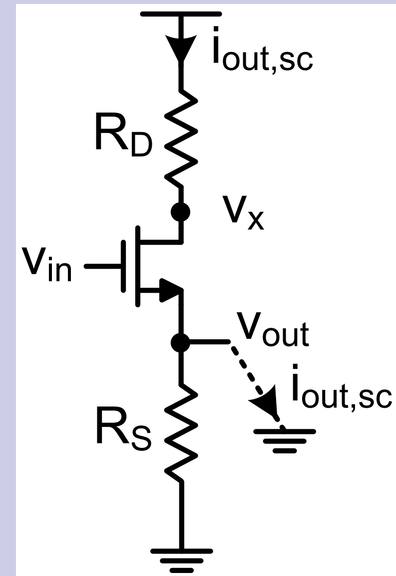
1. Buffer
2. Level-shifter





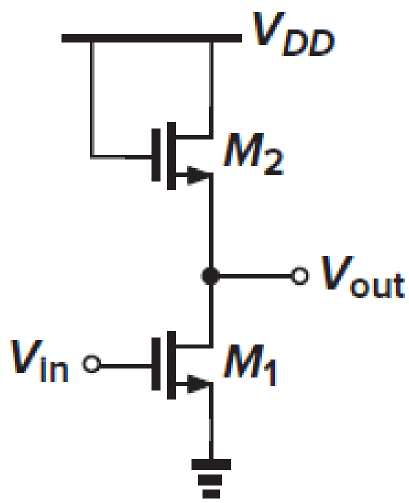
[Razavi, 2017]

Summary of Basic Topologies

	CS	CG	CD (SF)
			
	Voltage & current amplifier	Current buffer	Voltage buffer
Rin	∞	$R_S // \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$	∞
Rout	$R_D // r_o[1 + (g_m + g_{mb})R_S]$	$R_D // r_o$	$R_S // \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$
Gm	$\frac{-g_m}{1 + (g_m + g_{mb})R_S}$	$g_m + g_{mb}$	$\frac{g_m}{1 + R_D/r_o}$

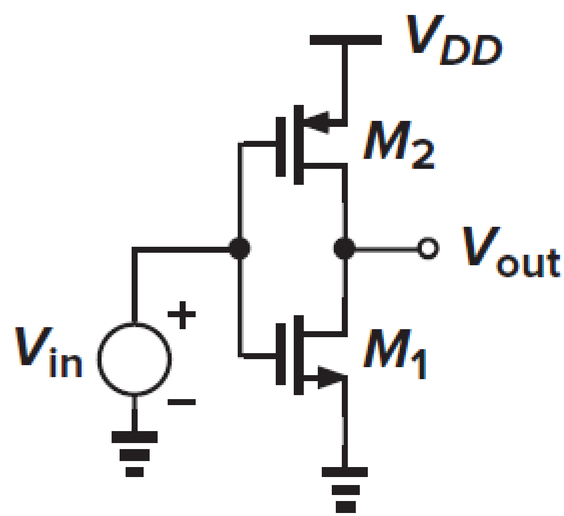
Q: CS With Diode-Connected Load

- Find the gain using GmRout (ignore body effect)
  - Express the gain in terms of $(W/L)_1$ and $(W/L)_2$
- This is a "linear" CS amplifier



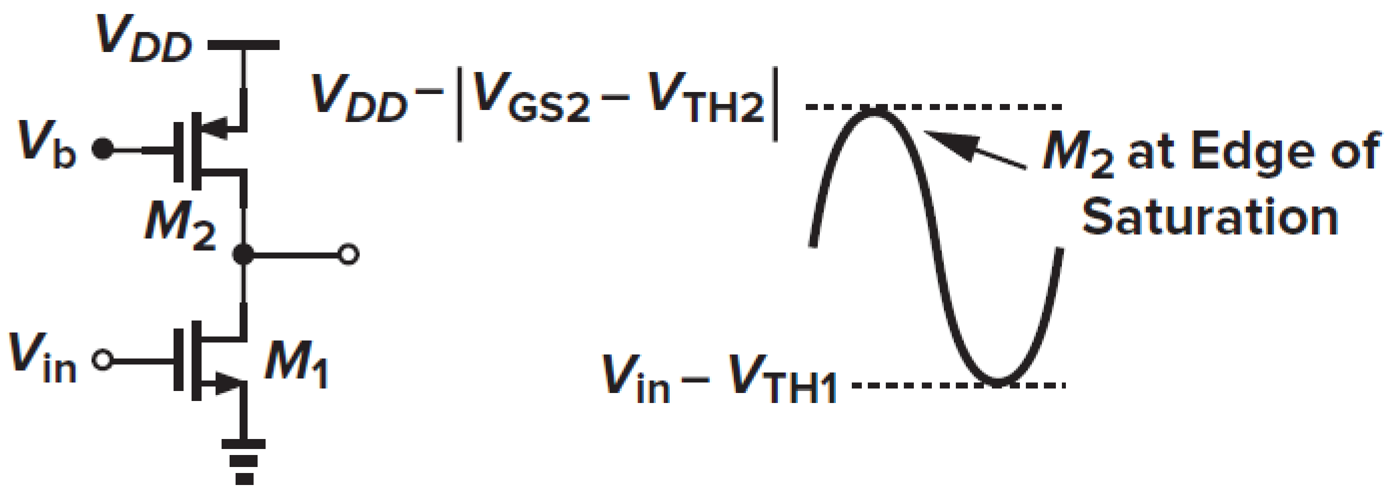
Q: Complementary CS (Inverter Amp)

- Find the gain using GmRout



[Razavi, 2017]

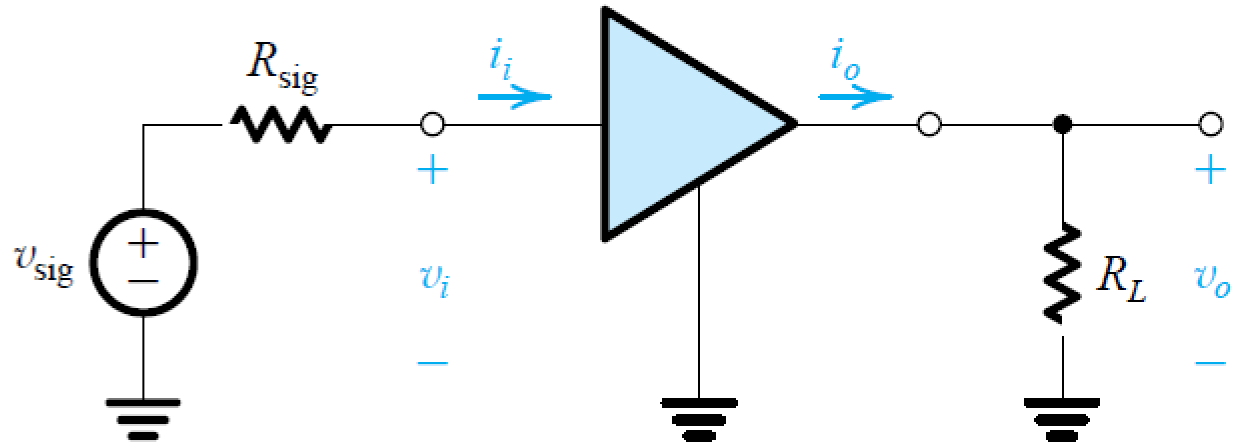
Output Signal Swing



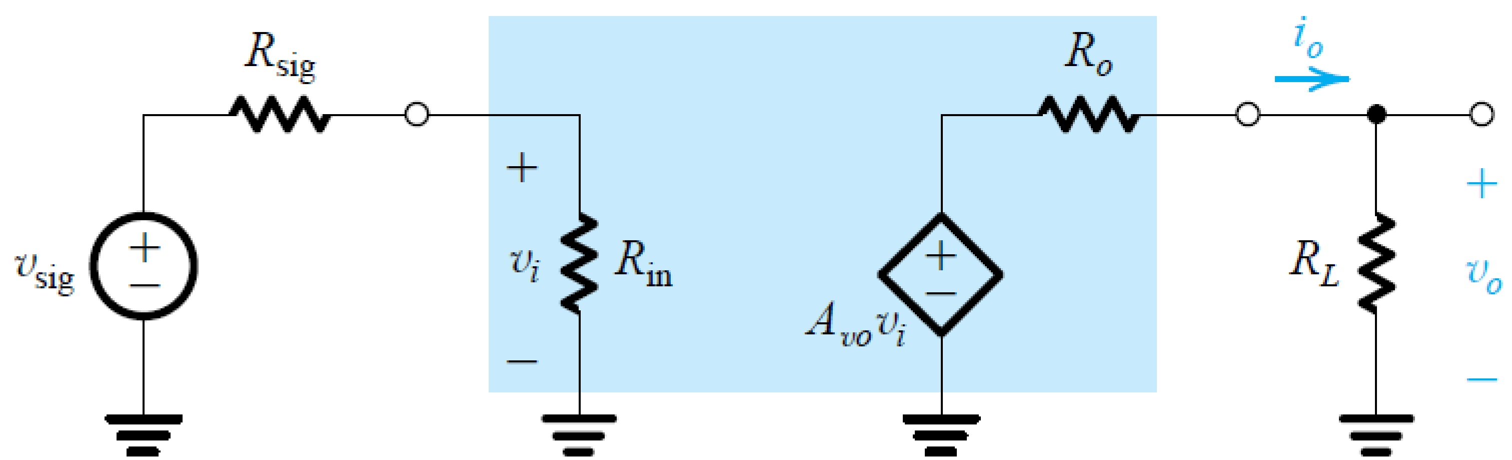
[Razavi, 2017]

Thank you!

Voltage Amplifier Model



(a)



(b)

[Sedra/Smith, 2015]

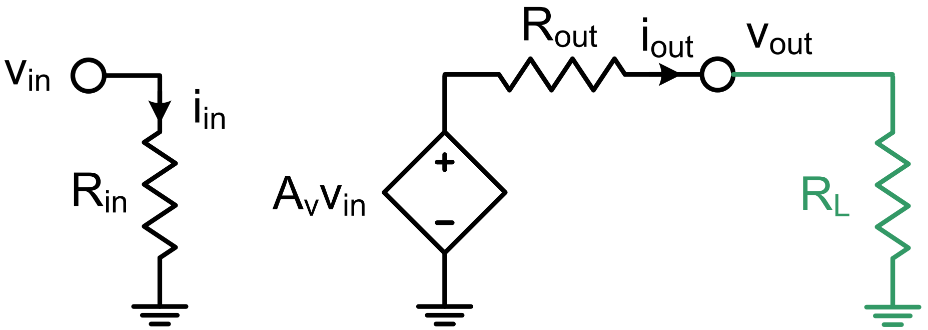
What If the Amplifier Is Loaded?

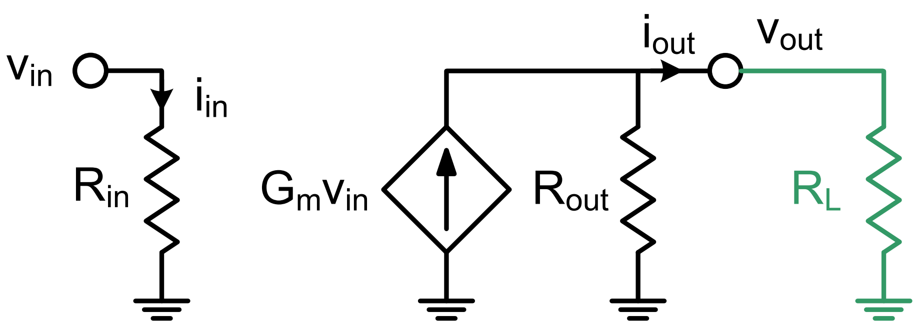
- Voltage gain

$$A_v' = A_v \frac{R_L}{R_{out} + R_L}$$

$$= G_m R_{out} \frac{R_L}{R_{out} + R_L}$$

$$= G_m (R_{out} // R_L)$$

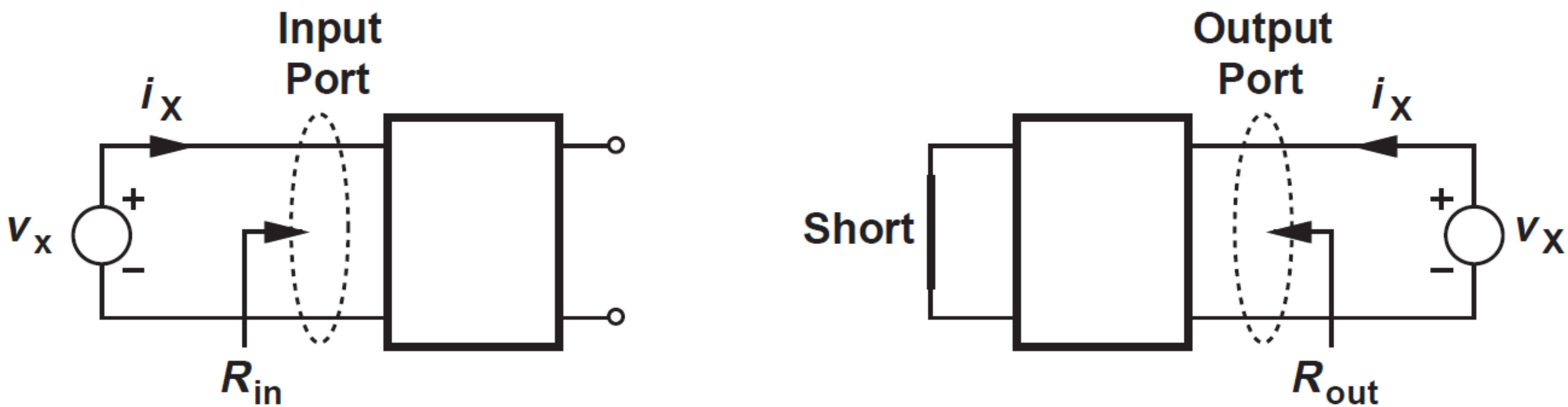




- Current gain

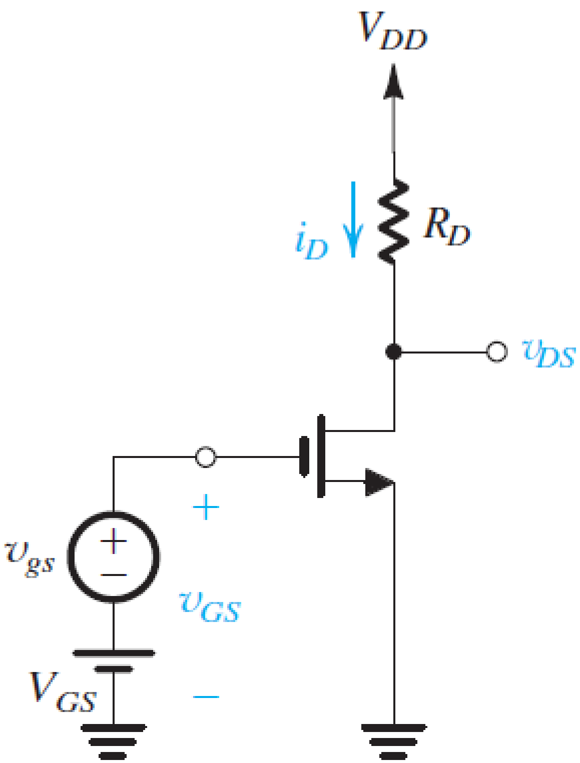
$$A_i' = A_i \frac{R_{out}}{R_{out} + R_L}$$

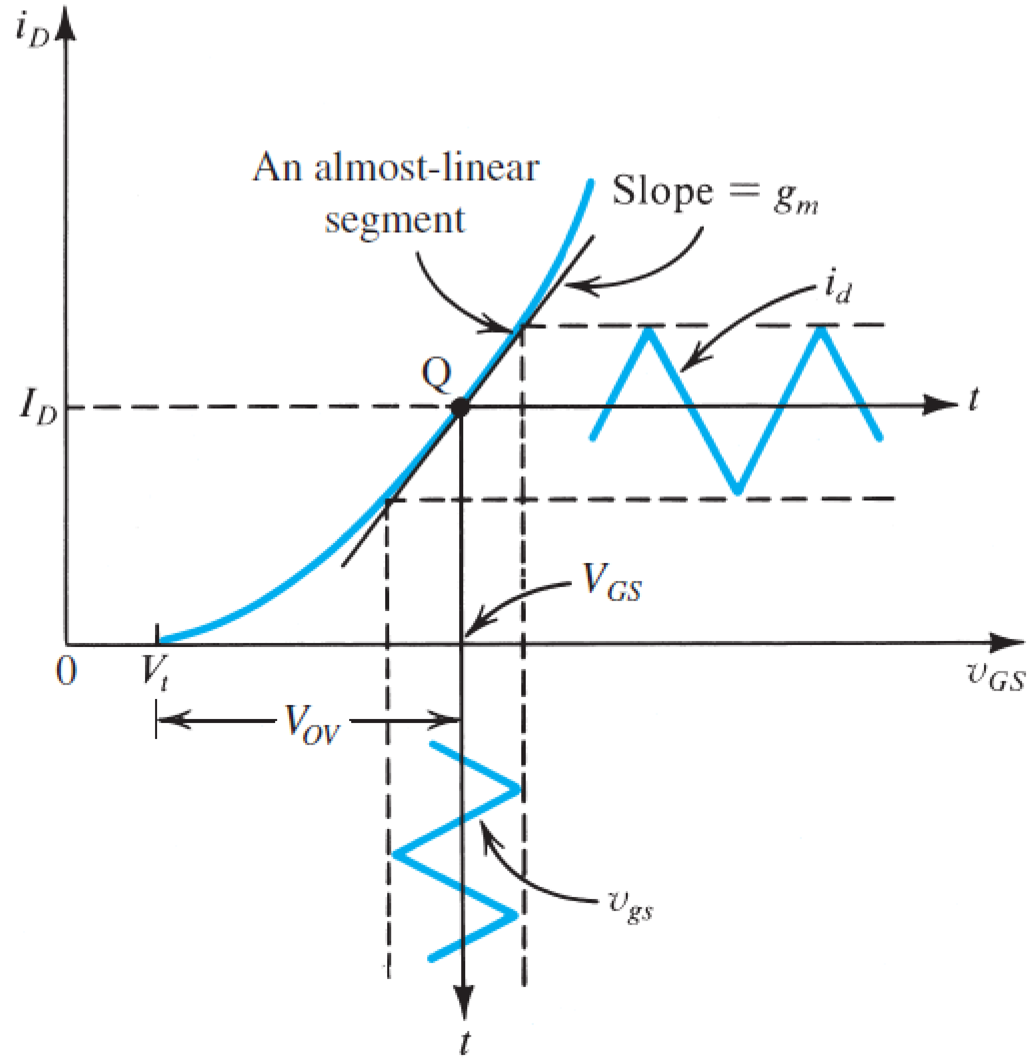
Input and Output Impedances



[Razavi, 2014]

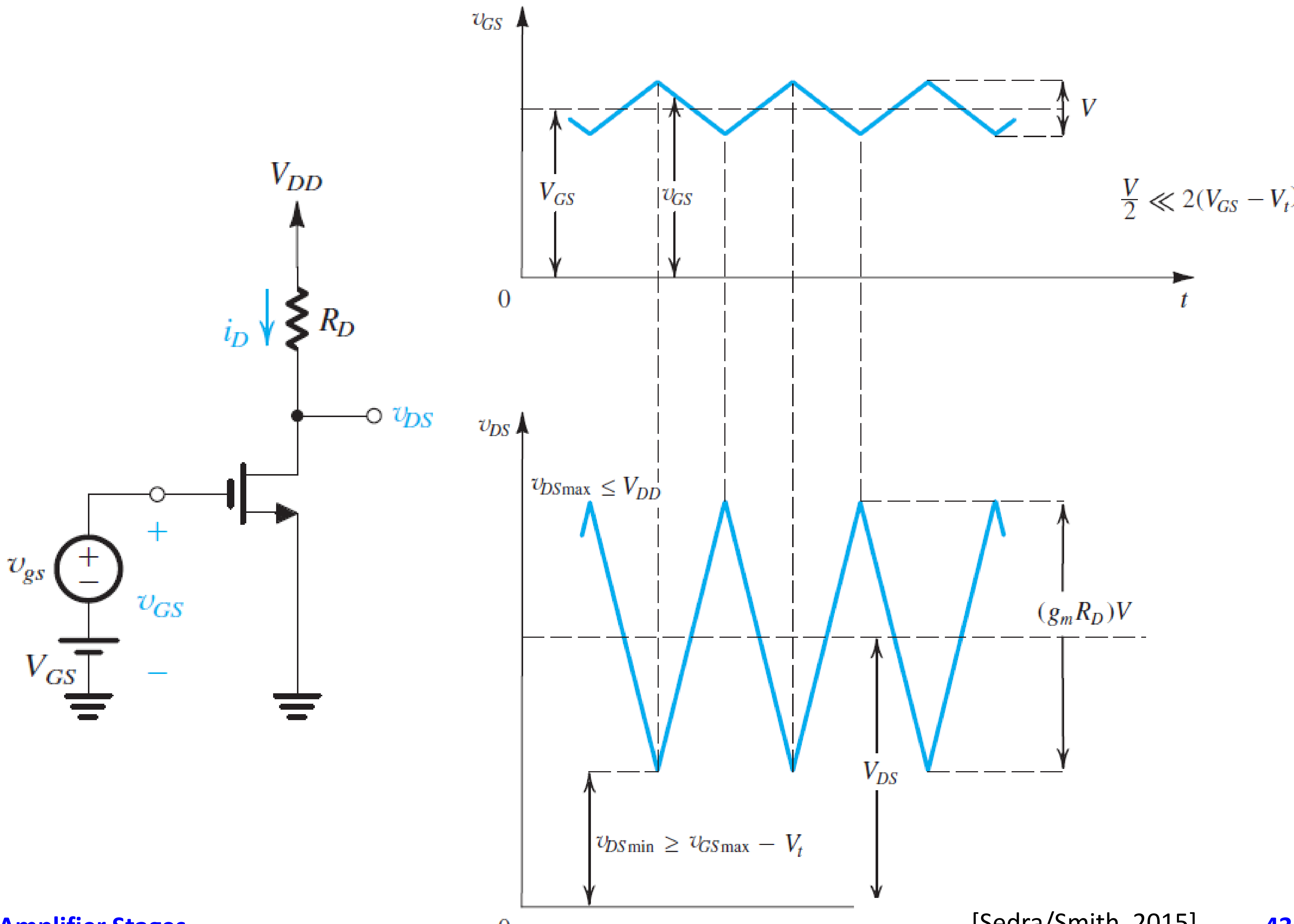
MOSFET Amplifier





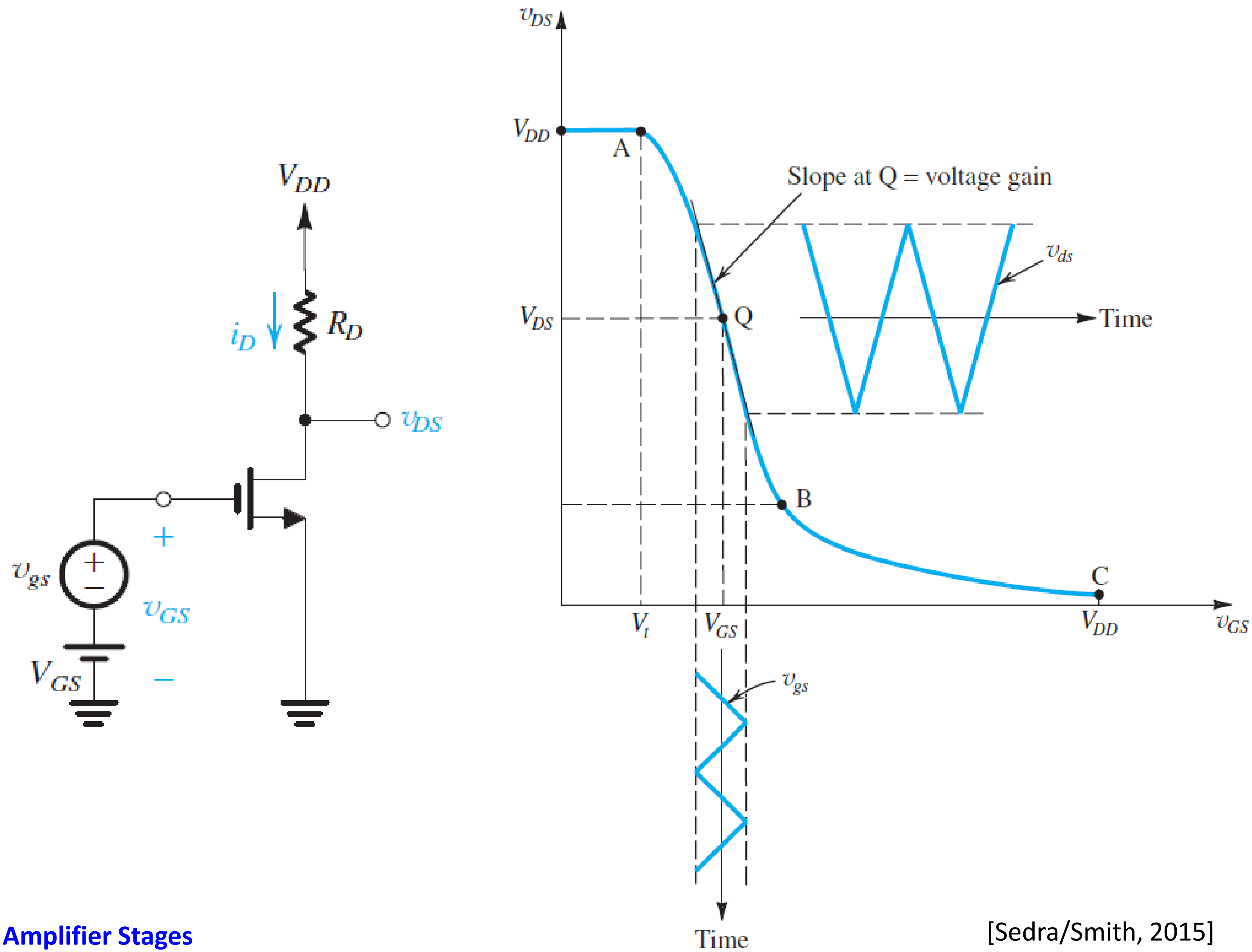
[Sedra/Smith, 2015]

MOSFET Amplifier



[Sedra/Smith, 2015]

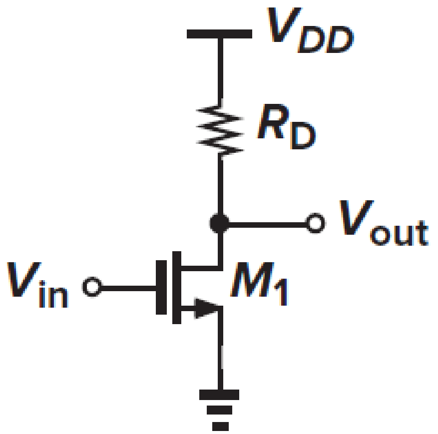
MOSFET Amplifier

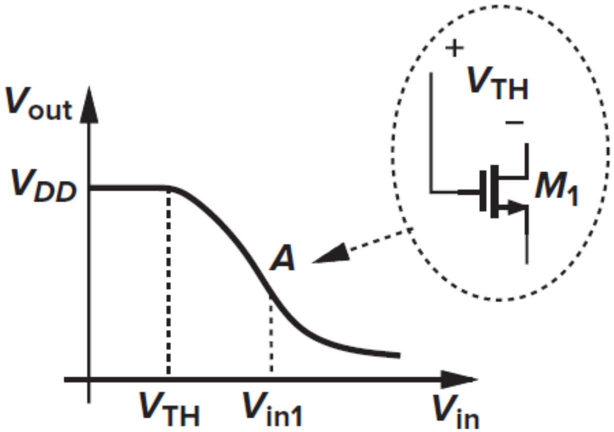


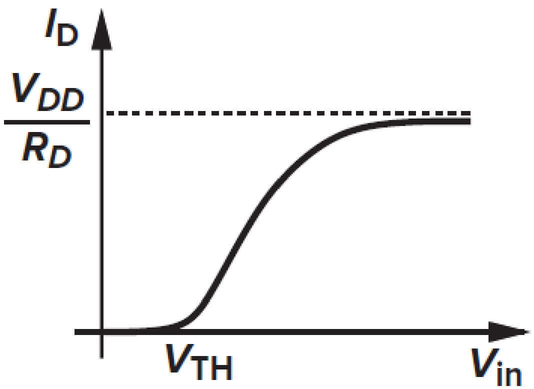
[Sedra/Smith, 2015]

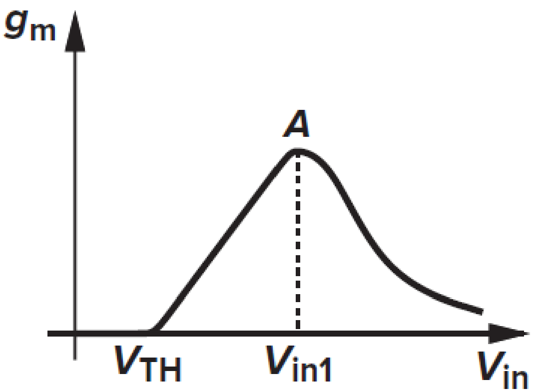
CS Large Signal Behavior

- Large signal gain is non-linear: Av = f(vin)
- Small signal gain is also non-linear: gm = f(vin)
- For linear gain, Av should NOT be f(vin)
- Av and gm are max at edge of triode
  - But they are highly non-linear
  - And the available signal swing vanishes









[Razavi, 2017]