
Source Code

```
using System;

using System.Diagnostics;

namespace MyApp // Note: actual namespace depends on the project name.
{
    internal class Program
    {

        public struct node
        {
            public int last_move;
            public int distance;
            public int index_of_zero;
            public List<int> grid;
            public node(int last_move, int distance, int index_of_zero, List<int> grid)
            {
                this.last_move = last_move;
                this.index_of_zero = index_of_zero;
                this.distance = distance;
                this.grid = grid;
            }
        }

        static int getInvCount(int[,] ar)
        {
            int inv_count = 0;

            List<int> arr = new List<int>();

            for (int i = 0; i < N; i++)
```

```

{
    for (int j = 0; j < N; j++)
    {
        arr.Add(ar[i, j]);
    }
}
for (int i = 0; i < N * N - 1; i++)
{
    for (int j = i + 1; j < N * N; j++)
    {

        if (arr[j] != 0 && arr[i] != 0 && arr[i] > arr[j])
            inv_count++;
    }
}
return inv_count;
}
static int findXPosition(int[,] puzzle)
{
    for (int i = N - 1; i >= 0; i--)
        for (int j = N - 1; j >= 0; j--)
            if (puzzle[i, j] == 0)
                return N - i;
    return 0;
}
static bool isSolvable(int[,] puzzle)
{

```

```
int invCount = getInvCount(puzzle);
```

```
if (N % 2 != 0)
```

```
    return !(invCount % 2 != 0);
```

```
else
```

```
{
```

```
    int pos = findXPosition(puzzle);
```

```
    if (pos % 2 != 0)
```

```
        return !(invCount % 2 != 0);
```

```
    else
```

```
        return invCount % 2 != 0;
```

```
}
```

```
}
```

```
static bool have_solution(List<int> grid)
```

```
{
```

```
    int[,] g = new int[M, M];
```

```
    int x = 0;
```

```
    for (int i = 0; i < N; i++)
```

```
    {
```

```
        for (int j = 0; j < N; j++)
```

```
        {
```

```
            g[i, j] = grid[x++];
```

```
        }
```

```
    }
```

```
    return (isSolvable(g));  
}
```

```
static bool valid(int x)  
{  
    return x >= 0 && x < N * N;  
}
```

```
static void pre()  
{  
    goal = new List<int>();  
    for (int i = 0; i < N * N - 1; i++)  
        goal.Add(i + 1);  
    goal.Add(0);  
}
```

```
static void print(List<int> v)  
{  
    int x = 0;  
    for (int i = 0; i < N; i++)  
    {  
        for (int j = 0; j < N; j++)  
        {  
            Console.Write(v[x++] + " ");  
        }  
        Console.WriteLine();  
    }  
}
```

```
static int calc_hamming(List<int> grid)
```

```

{
    int c = 1;

    int res = 0;

    for (int i = 0; i < N * N - 1; i++)
    {
        if (grid[i] != c)

            res++;

        c++;

    }

    return res;
}

static int calc_Manhattan(List<int> grid)
{
    int c = 1;

    int res = 0;

    int cx = 0, cy = 0;

    for (int i = 0; i < N * N; i++)
    {
        if (grid[i] != c && grid[i] != 0)
        {
            int x = (grid[i] - 1) / (N);

            int y = Math.Abs(grid[i] - (1 + (x * N)));

            res += Math.Abs(cx - x);

            res += Math.Abs(cy - y);

        }

        cy++;

        if (cy == N)
        {

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        cy = 0;

        cx++;

    }

    c++;

}

return res;

}

static int calc_Heuristic(List<int> grid, int option)
{
    if (option == 1)
    {
        return calc_hamming(grid);
    }
    else
    {
        return calc_Manhattan(grid);
    }
}

static void get_solution(List<int> cur, List<int> term_node, Dictionary<List<int>, List<int>>> parent,
int dis)
{

    Console.WriteLine("solution is found");

    Console.WriteLine("*****");

    Console.WriteLine("number of moves = " + dis);

    if (N == 3)
    {
        List<int> cc = cur;

        List<List<int>> ans = new List<List<int>>>();

```

```

int sz = 0;

while (cc != term_node)
{
    ans.Add(cc);

    sz++;

    cc = parent[cc];
}

ans.Reverse();

foreach (var vec in ans)
{
    print(vec);

    Console.WriteLine("=====");
}

}

}

static void A_star_algorithm(List<int> grid, int option)
{
    int ix = -1;

    PriorityQueue<node,int> not_vis = new PriorityQueue<node, int>();

    for (int i = 0; i < N * N; i++)
    {
        if (grid[i] == 0)
        {
            ix = i;

            break;
        }
    }

    Dictionary<List<int>, int> vis = new Dictionary<List<int>, int>();

```

```

node n = new node(0,0,ix,grid);
not_vis.Enqueue(n, 0);
Dictionary<List<int>, List<int>> parent = new Dictionary<List<int>, List<int>>();
List<int> term = new List<int>();
term.Add(-1);
parent[grid] = term;
int c = calc_Heuristic(grid, option);
if (c == 0)
{
    Console.WriteLine("Time taken: " + before.ElapsedMilliseconds + "ms");
    get_solution(grid, term, parent, 0);
    return;
}
while (true)
{
    var ccc = not_vis.Dequeue();
    List<int> cur = ccc.grid;
    int idx_of_zero = ccc.index_of_zero;
    int vs = ccc.last_move; // move
    int d = ccc.distance; // distance
    List<int> v = new List<int>();
    if (idx_of_zero + N < N * N)
        v.Add(N);
    if (idx_of_zero % N != 0)
        v.Add(-1);
    if ((idx_of_zero + 1) % N != 0)
        v.Add(1);
    if (idx_of_zero - N >= 0)
        v.Add(-1 * N);

```



```

foreach (var j in v)
{
    if (j == vs * -1)
    {
        continue;
    }
    int from = idx_of_zero;
    int to = from + j;
    List<int> next = new List<int>();
    next.AddRange(cur);
    int tmp = next[from];
    next[from] = next[to];
    next[to] = tmp;
    int cost = calc_Heuristic(next, option);//O(N*N)
    if (N == 3)
        parent[next] = cur;
    if (cost == 0)
    {
        float after = int.Parse(before.ElapsedMilliseconds.ToString());
        Console.WriteLine("Time taken: " + after/1000.0 + " Second");
        get_solution(next, term, parent,d +1);
        return;
    }
    node nn = new node(j, d+1, j + idx_of_zero, next);
    not_vis.Enqueue(nn, cost + d+1);
}
}

```

```

}

static void calc_BFS(List<int> grid)
{
    return;
}

static Stopwatch before = Stopwatch.StartNew();

static List<int> grid, GD, goal;

static int N = 0, M = 111;

static void Main(string[] args)
{
    while (true)
    {
        string name= @"C:\Users\EslamSaeed\Desktop\all\";
        Console.WriteLine("Unsolvables puzzles\n");
        Console.WriteLine("[1] 15 Puzzle 1 - Unsolvables");
        Console.WriteLine("[2] 99 Puzzle - Unsolvables Case 1");
        Console.WriteLine("[3] 99 Puzzle - Unsolvables Case 2");
        Console.WriteLine("[4] 9999 Puzzle Unsolvables");
        Console.WriteLine("\nSolvables puzzles\n");
        Console.WriteLine("\nManhattan & Hamming\n");
        Console.WriteLine("[5] 50 Puzzle");
        Console.WriteLine("[6] 99 Puzzle - 1");
        Console.WriteLine("[7] 99 Puzzle - 2");
        Console.WriteLine("[8] 9999 Puzzle");
        Console.WriteLine("\nManhattan Only\n");
        Console.WriteLine("[9] 15 Puzzle 1");
        Console.WriteLine("[10] 15 Puzzle 3");
        Console.WriteLine("[11] 15 Puzzle 4");
        Console.WriteLine("[12] 15 Puzzle 5");
    }
}

```

```
Console.WriteLine("\nV. Large test case\n");
Console.WriteLine("[13] TEST");
int test = int.Parse(Console.ReadLine());
if (test == 1)
{
    name += "15 Puzzle 1 - Unsolvable";
}
else if (test == 2)
{
    name += "99 Puzzle - Unsolvable Case 1";
}
else if (test == 3)
{
    name += "99 Puzzle - Unsolvable Case 2";
}
else if (test == 4)
{
    name += "9999 Puzzle Unsolvable";
}
else if (test == 5)
{
    name += "50 Puzzle";
}
else if (test == 6)
{
    name += "99 Puzzle - 1";
}
else if (test == 7)
{
```

```
        name += "99 Puzzle - 2";
    }
    else if (test == 8)
    {
        name += "9999 Puzzle";
    }
    else if (test == 9)
    {
        name += "15 Puzzle 1";
    }
    else if (test == 10)
    {
        name += "15 Puzzle 3";
    }
    else if (test == 11)
    {
        name += "15 Puzzle 4";
    }
    else if (test == 12)
    {
        name += "15 Puzzle 5";
    }
    else if (test == 13)
    {
        name += "TEST";
    }
    else
    {
        Console.WriteLine("Enter size of grid");
    }
}
```

```

int n = int.Parse(Console.ReadLine());

N = n;

grid = new List<int>();

Console.WriteLine("GRID ? ");

for (int i = 0; i < n; i++)
{
    int[] a = Array.ConvertAll(Console.ReadLine().Split(' '), int.Parse);

    for (int j = 0; j < n; j++)
        grid.Add(a[j]);

}

GD = grid;

if (have_solution(grid) == true)
{
    Console.WriteLine("this grid have a solution");
    Console.WriteLine("[1] use A_star_algorithm");
    Console.WriteLine("[2] use BFS_algorithm");
    int opt = int.Parse(Console.ReadLine());
    if (opt == 1)
    {
        Console.WriteLine("[1] use hamming");
        Console.WriteLine("[2] use Manhattan");

        opt = int.Parse(Console.ReadLine());
        Console.WriteLine("Running.....");
        before = Stopwatch.StartNew();
        A_star_algorithm(grid, opt);

    }
}

```

```

        else
        {
            Console.WriteLine("Running.....");

            before = Stopwatch.StartNew();

            calc_BFS(grid);
        }
    }
    else
        Console.WriteLine("there is no solution for this grid");
}

if (test > 13)
    continue;

name += ".txt";

var file_read = File.ReadLines(name);

grid = new List<int>();

N = 0;

foreach(var line in file_read)
{
    if (line.Length == 0)
        continue;

    if (N == 0)
        N = int.Parse(line);
    else
    {
        int[] a = Array.ConvertAll(line.Split(' '), int.Parse);

        for (int j = 0; j < a.Length; j++)
            grid.Add(int.Parse(a[j].ToString()));
    }
}

```

```

GD = grid;
if (have_solution(grid) == true)
{
    Console.WriteLine("this grid have a solution");
    Console.WriteLine("[1] use A_star_algorithm");
    Console.WriteLine("[2] use BFS_algorithm");
    int opt = int.Parse(Console.ReadLine());
    if (opt == 1)
    {
        Console.WriteLine("[1] use hamming");
        Console.WriteLine("[2] use Manhattan");
        opt = int.Parse(Console.ReadLine());
        Console.WriteLine("Running.....");
        before = Stopwatch.StartNew();
        A_star_algorithm(grid, opt);
    }
    else
    {
        Console.WriteLine("Running.....");
        before = Stopwatch.StartNew();
        calc_BFS(grid);
    }
}
else
    Console.WriteLine("there is no solution for this grid");
}
before.Stop();
}
}

```

Analysis of CODE

A_star_algorithm:	$O(E \cdot \log(V))$, E=total number of moves, V=number of states to reach the solution.
Insertion of Priority Queue	$O(\log(N))$, for 1 element. $O(N \log(N))$ for n elements.
calc_hamming	$O(N^2)$
calc_Manhattan	$O(N^2)$
Getting available moves	$O(1)$
GetSolution(Base Case)	$O(1)$, in case $n \neq 3$ $O(V)$, in case $n = 3$, such that : V=number of states to reach the solution.
Removing from priority queue	$O(\log(N))$, for 1 element.
Reading files	$O(N^2)$
Calculate BFS	$O(E+V)$
Have solution	$O(S^2)$, S=size of grid

Comparison Between Hamming and Manhattan using A* Algorithm

Test Name	type	solvable	MIN Number of moves:	Execution time in ms
[1] 15 Puzzle 1	-----	NO	-----	-----
[2] 99 Puzzle	-----	NO	-----	-----
[3] 99 Puzzle	-----	NO	-----	-----
[4] 9999 Puzzle	-----	NO	-----	-----
[5] 50 Puzzle	Hamming	Yes	18	0.153
	Manhattan		18	0.054
[6] 99 Puzzle - 1	Hamming	Yes	18	0
	Manhattan		18	0
[7] 99 Puzzle - 2	Hamming	Yes	38	0
	Manhattan		38	0
[8] 9999 Puzzle	Hamming	Yes	4	0
	Manhattan		4	0.001
[9] 15 Puzzle 1	Manhattan	Yes	46	4.218
[10] 15 Puzzle 3	Manhattan	Yes	38	0.941
[11] 15 Puzzle 4	Manhattan	Yes	44	1.422
[12] 15 Puzzle 5	Manhattan	Yes	45	30.319
[13] V. Large test case	Hamming	NO	56	22.159
	Manhattan	Yes		

Comparison Between A* Algorithm and BFS Algorithm

A* Algorithm	BFS Algorithm
<p>It is considered to be the best algorithm for N PUZZLE.</p> <p>As it is the fastest one that is able to solve any large puzzle, because it uses priority queue, that makes it a greedy algorithm that reach solution directly .</p> <p>And it neglects all other solutions and focuses on the fastest solution that has minimum number of moves to solve the puzzle.</p>	<p>It has a limit to solve the N puzzle ,it can solve only 3*3 puzzle and can't solve what above.</p> <p>As it tries to reach as many as possible of solutions, and that's what makes it slow.</p>