

**Bezout's Theorem**  
**- Extended**  
**Euclidean Algorithm**  
**iterative**

**Arabic Animated**  
**intuition**



## Extended Euclidean algorithm iterative

$$ax+by=\gcd(a,b) \quad (1)$$

$$ax_{\text{prev}}+by_{\text{prev}}=r_{\text{prev}} \quad (2)$$

$$ax+by=r_{\text{cur}} \quad (3)$$

$\gcd(r_{\text{prev}}, r_{\text{cur}})$   $r_{\text{prev}}$  from the first gcd call

$$a=r_{\text{prev}}, b=r_{\text{cur}}$$

$$r_{\text{new}}=r_{\text{prev}}-Q*r_{\text{cur}} \quad (4)$$

$$r_{\text{new}}=(ax_{\text{prev}}+by_{\text{prev}})-q(ax+by)$$

$$a(x_{\text{prev}}-qx)+b(y_{\text{prev}}-qy)=r_{\text{new}} \quad (5)$$

from the Euclidean algorithm we know that

$$\gcd(a,b)=\gcd(b,a\%b)$$

$$\gcd(a,b)=\gcd(b,r)$$

$r$  is the remainder  $(R)$   
 $r=a-b*Q$

so we are continually trading in  
the gcd of a pair for  
the gcd of a smaller pair.

At the last step, we have gcd

$$\gcd(a,b)=\gcd(b,a\%b)$$

$$\gcd(r_{\text{prev}}, r_{\text{cur}})=\gcd(r_{\text{cur}}, 0)=r_{\text{cur}}$$

## Extended Euclidean algorithm iterative

$$ax+by=\gcd(a,b) \quad (1)$$

$$ax_{\text{prev}}+by_{\text{prev}}=r_{\text{prev}} \quad (2)$$

$$ax+by=r_{\text{cur}} \quad (3)$$

$$r=a-q*b$$

$$a=r_{\text{prev}}, b=r_{\text{cur}}$$

$$r_{\text{new}}=r_{\text{prev}}-q*r_{\text{cur}} \quad (4)$$

$$r_{\text{new}}=(ax_{\text{prev}}+by_{\text{prev}})-q(ax+ay)$$

$$a(x_{\text{prev}}-qx)+b(y_{\text{prev}}-qy)=r_{\text{new}} \quad (5)$$

from the Euclidean algorithm we know that

$$\gcd(a,b)=\gcd(b,a\%b)$$

$$\gcd(r_0,r_1)=\gcd(r_1,r_0\%r_1)$$

$$\gcd(r_{\text{last}},0)=r_{\text{last}}$$

$$\gcd(r_{\text{prev}},r_{\text{cur}})=\gcd(r_{\text{cur}},0)=r_{\text{cur}}$$

$$a(1)+b(0)=a$$

$$a(0)+b(1)=b$$

$$x_{\text{new}}=x_{\text{prev}}-qx$$

$$y_{\text{new}}=y_{\text{prev}}-qy$$

$$\gcd(a,b)=\gcd(b,a\%b)$$