TUNIS BUSINESS SCHOOL
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Business Simulation Project

Monte Carlo Simulation of an M/M/1 Queue

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Introduction

Business simulation is widely used to analyze complex operational systems where analytical solutions may be difficult or unrealistic. In this project, we apply **Monte Carlo simulation** to study a queueing system, a common problem in business contexts such as banks, call centers, hospitals, and service operations.

This performance measure is of critical importance in business decision-making. Excessive waiting times can lead to customer dissatisfaction, reduced perceived service quality, and decreased operational efficiency. By estimating this quantity through simulation, managers can better understand system behavior and make informed decisions regarding capacity planning and service improvement.

1. Purpose of the Simulation

The objective of this project is to **analyze the performance of a service system** using **Monte Carlo simulation**, with a focus on **customer waiting time**. Such systems are common in business contexts such as banks, call centers, customer support desks, and service counters.

Specifically, we aim to estimate the **expected waiting time in queue** before service begins, denoted as:

$$\theta = E[W_q]$$

This quantity is crucial in business decision-making because long waiting times negatively affect customer satisfaction, service quality, and operational efficiency.

2. Why a Simulation Approach Was Used

Although the M/M/1 queue has known theoretical formulas, **simulation is used for three key business reasons**:

- Real systems are complex and often violate strict theoretical assumptions.
- Simulation allows experimentation with different system parameters before implementation.
- Monte Carlo methods generalize easily to more realistic systems where no closed-form solution exists.

Thus, this project demonstrates how **simulation complements analytical models** in business analytics.

3. Model Choice: M/M/1 Queue

The system modeled is an **M/M/1 queue**, where:

- **M**: Markovian (Poisson) arrivals
- **M**: Markovian (exponential) service times
- **1**: Single server

Business Interpretation

- Customers arrive randomly over time.
- One server processes customers on a **first-come, first-served** basis.
- There is no limit on queue length.

This model is widely used in **operations management**, **service system design**, and **capacity planning**.

4. Choice of Parameters and Their Justification

Arrival Rate ($\lambda = 0.8$)

- Represents average customer arrivals per unit time.
- A value less than the service rate ensures the system does not overload.
- In business terms, this corresponds to moderate demand pressure.

Service Rate ($\mu = 1.0$)

- Represents the server's processing capacity.
- Chosen slightly higher than the arrival rate to maintain system stability.

Traffic Intensity ($\rho = \lambda/\mu = 0.8$)

- Measures server utilization.
- A utilization of 80% is realistic in business environments and highlights congestion effects without causing system collapse.

Sample Size ($N = 100{,}000$)

- Large enough to ensure high statistical precision.
- Results in a small Monte Carlo Standard Error (MCSE).
- Reflects long-run system behavior, which is important for managerial insights.

5. What Was Simulated and How

Arrival Process

Instead of simulating Poisson counts directly, the simulation uses:

$$A_i \sim \text{Exponential}(\lambda)$$

This approach is mathematically equivalent and computationally simpler. It reflects random, memoryless arrivals common in customer service systems.

Service Process

Service times follow:

$$S_i \sim \text{Exponential}(\mu)$$

This captures variability in service durations.

Queue Dynamics

For each customer:

- Arrival time is computed.
- Service starts either immediately or after waiting.
- Waiting time in queue is recorded.

This event-based simulation mirrors real operational workflows.

6. Monte Carlo Estimation Concept

The Monte Carlo estimator is defined as:

$$\hat{E}[W_q] = \frac{1}{N} \sum_{i=1}^{N} W_{q,i}$$

This estimator relies on the **Law of Large Numbers**, which guarantees convergence to the true expected value as the number of simulations increases.

7. Libraries Used in R and Their Purpose

This project uses **base R only**, which highlights transparency and simplicity.

- `stats` (default):
  - `rexp()` for exponential random variables
  - `mean()`, `sd()`, `median()` for statistics
- `graphics` (default):
  - `plot()` for convergence visualization
  - `hist()` for distribution analysis

No external libraries were required, ensuring reproducibility and robustness.

8. Interpretation of Numerical Results

Expected Waiting Time

- Simulation estimate: **4.1021**
- Theoretical value: **4**

The close agreement confirms correct model implementation and valid Monte Carlo estimation.

Monte Carlo Standard Error (MCSE = 0.0162)

- Indicates high precision.
- Small relative to the estimated mean.
- Demonstrates adequacy of the chosen sample size.

Distribution Statistics

- Minimum waiting time = 0 (immediate service).
- Long right tail indicates occasional congestion.
- Typical of real service systems.

9. Interpretation of Graphs

Convergence Plot

- Shows the running mean stabilizing as simulations increase.
- Confirms consistency and convergence of the estimator.
- Convergence occurs around 20,000–30,000 customers.

Business insight: long-run averages provide stable performance metrics.

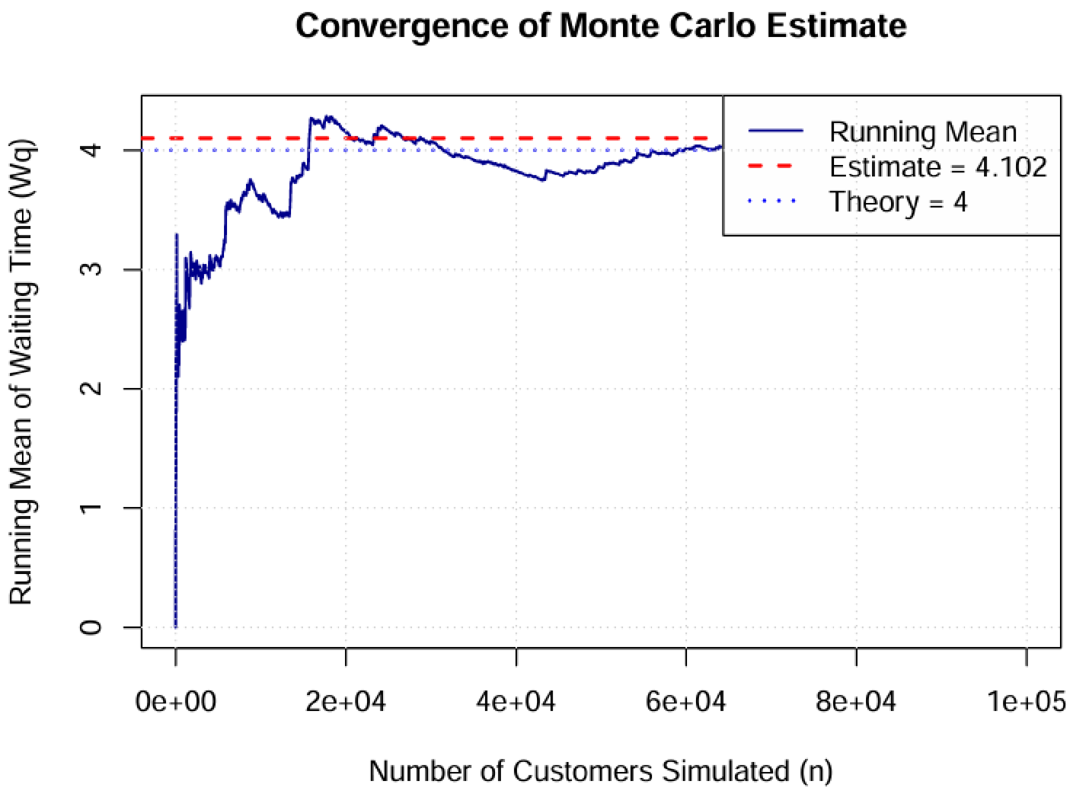

Figure 1: Convergence graph

Histogram of Waiting Times

- Right-skewed distribution.
- Most customers experience short waits.
- A small fraction experiences long delays.

Business insight: while averages are acceptable, extreme waits may require managerial attention.

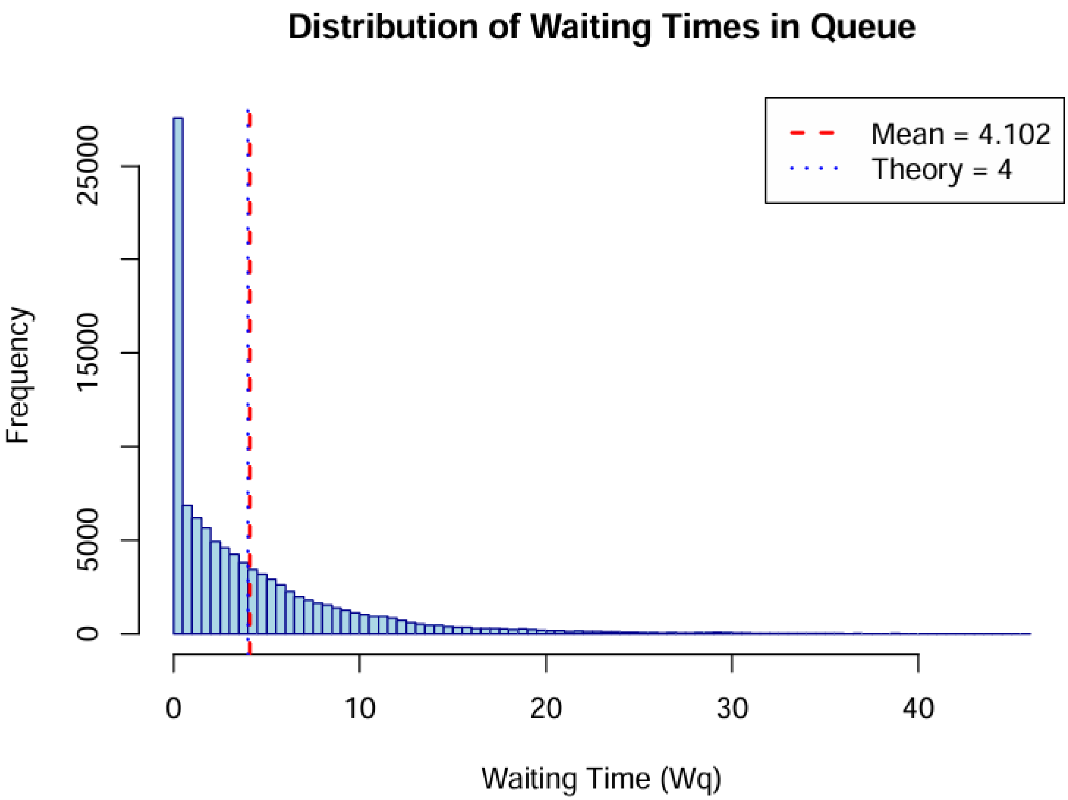

Figure 2: Histogram of waiting times in queue

Figure 2: Convergence graph

10. Business Simulation Concepts Applied

This project applies key concepts from the business simulation field:

- Discrete-event simulation
- Stochastic modeling
- System utilization analysis
- Performance measurement under uncertainty
- Monte Carlo estimation and validation
- Trade-off between efficiency and congestion

11. Practical Implications for Businesses

- At 80% utilization, waiting times increase rapidly.
- Managers can use such simulations to test staffing decisions, evaluate service capacity, and balance cost versus customer satisfaction.

12. Limitations and Extensions

Limitations

- Exponential assumptions may not hold in all real systems.
- Single-server only.

Possible Extensions

- Multiple servers (M/M/c).
- Non-exponential service times.
- Finite queue capacity.
- Time-varying arrival rates.

13. Conclusion

This business simulation project demonstrates how Monte Carlo methods can effectively analyze service systems. The close match between simulation and theory validates the approach, while the graphical and statistical results provide actionable insights for operational decision-making. The methodology serves as a foundation for more complex and realistic business simulations.