Monte Carlo Simulation of an M/M/1 Queue: Waiting Time Analysis

Course: Business Simulation

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Motivation

Many business systems involve waiting lines.

Examples:

- Banks
- Call centers
- Hospitals
- Customer service desks

Waiting time strongly affects:

- Customer satisfaction
- Perceived service quality
- Operational efficiency

Enhanced Insight

Reducing waiting time is a strategic business decision, not only an operational one.

Project Objective

- Apply **Monte Carlo simulation** to model a service system.
- Analyze **customer waiting time in the queue** as a key performance indicator.

Main objective:

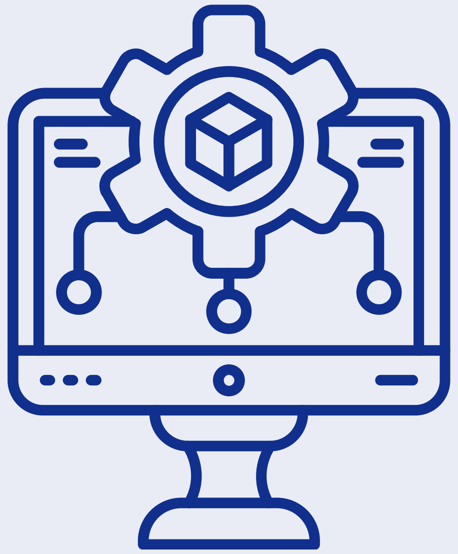
- Estimate the expected waiting time before service

Use this measure to support **operational efficiency** and **capacity planning** decisions.

$$\theta=E[Wq]$$

Why Use Simulation Instead of Only Theory?

- Real business systems often violate theoretical assumptions.
- Simulation allows:
    - –Experimentation without real-world risk
    - –Testing alternative configurations
- Monte Carlo methods are flexible and scalable.
- Simulation complements analytical queueing models.



Model Choice: M/M/1 Queue

M/M/1 Queue Model

1. Markovian (Poisson) **arrivals**
2. Markovian (exponential) **service** times
3. Single server

Assumptions

- First-come, first-served
- Unlimited queue capacity

arrivals: λ

service: μ

Business Interpretation of the Model

- Customers arrive randomly over time.
- A single employee or machine provides service.
- Model is widely used in capacity planning
- The model captures the trade-off between utilization and congestion.

Bank teller

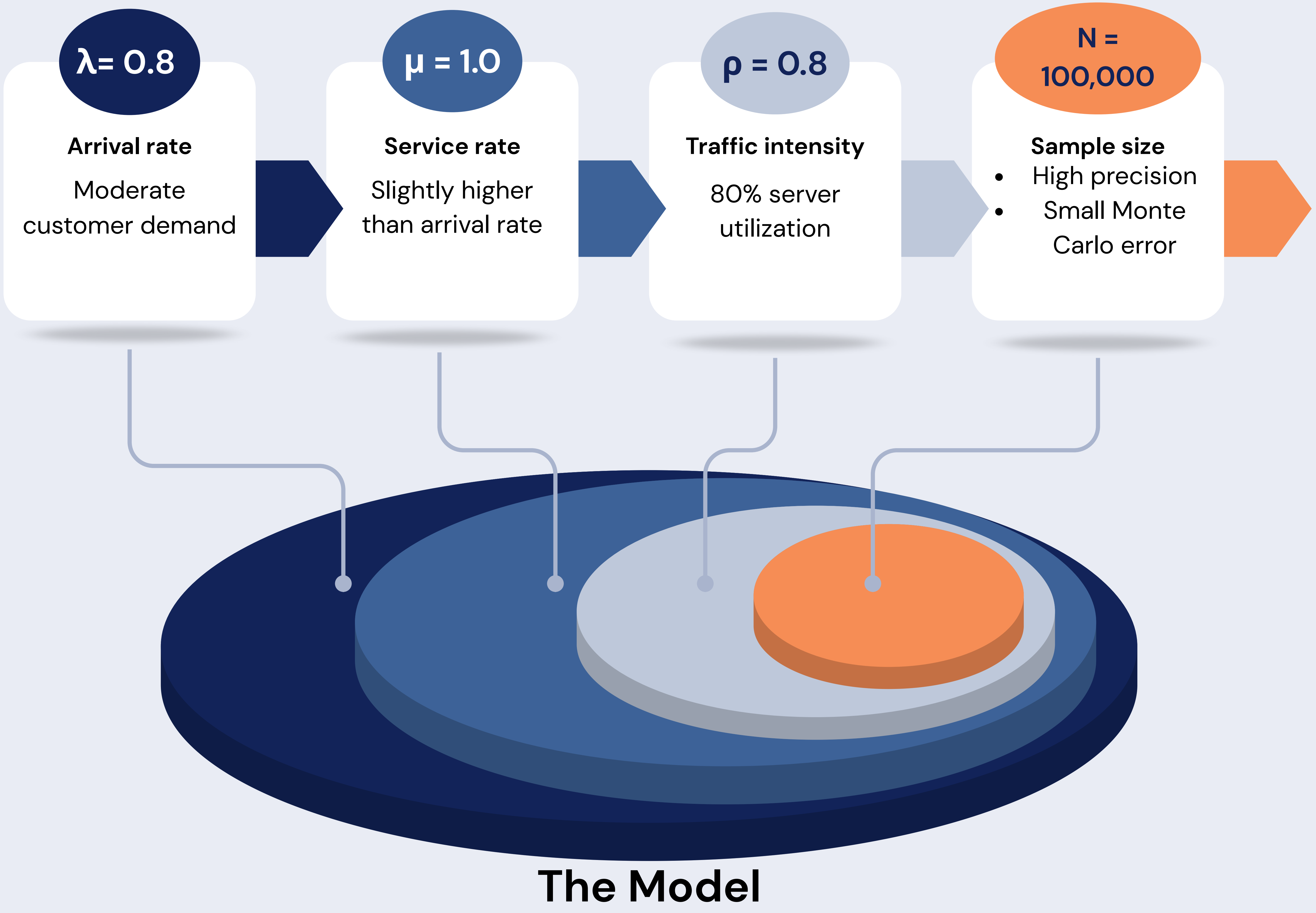
Help desk agent

Checkout counter

Enhanced Insight

Simulation bridges the gap between theory and real operations.

Model Parameters



Simulation Mechanism

Arrival times:

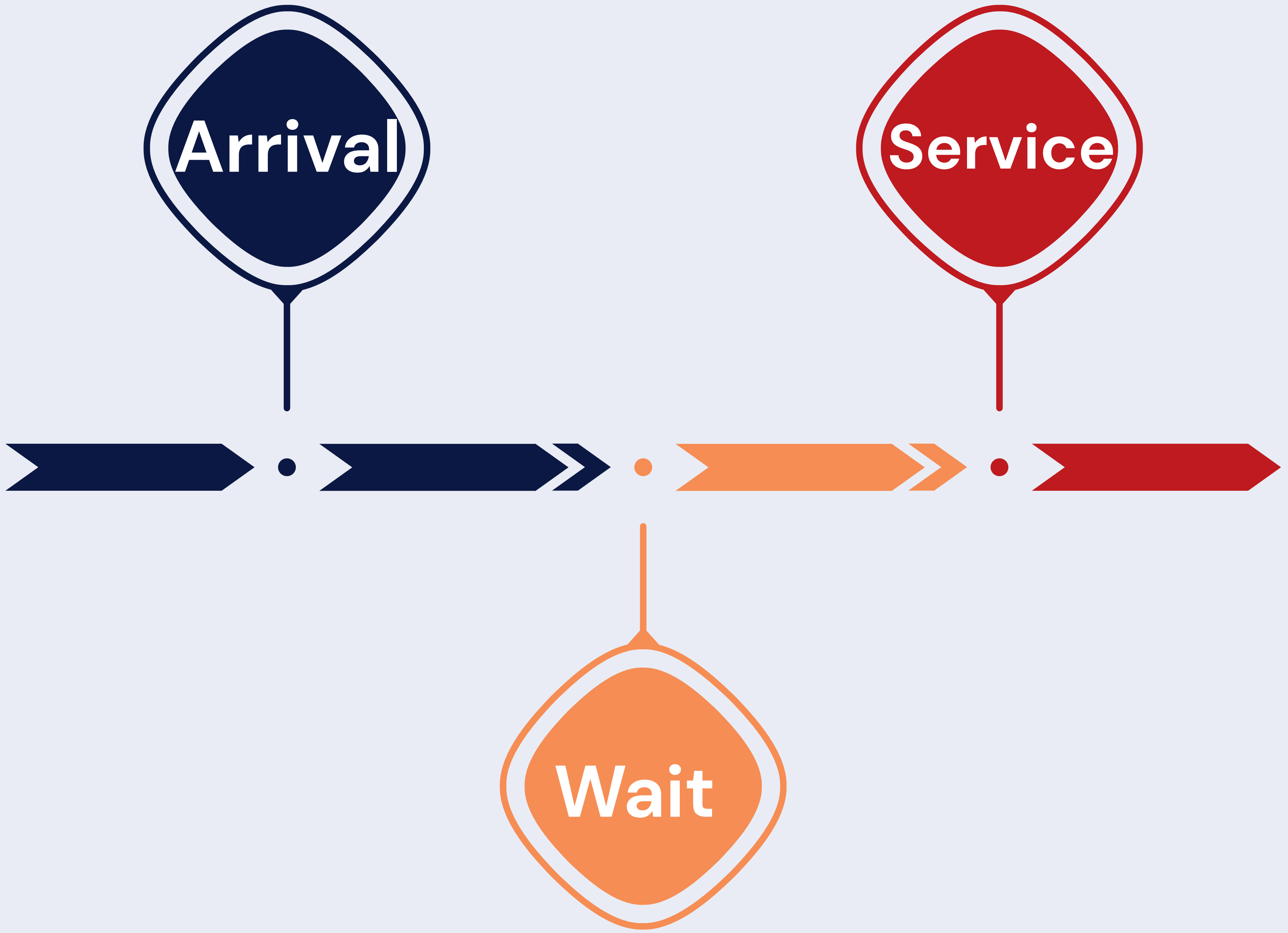
Exponentially distributed

Service times:

Exponentially distributed

For each customer:

- Arrival time
- Service start time
- Waiting time in queue recorded

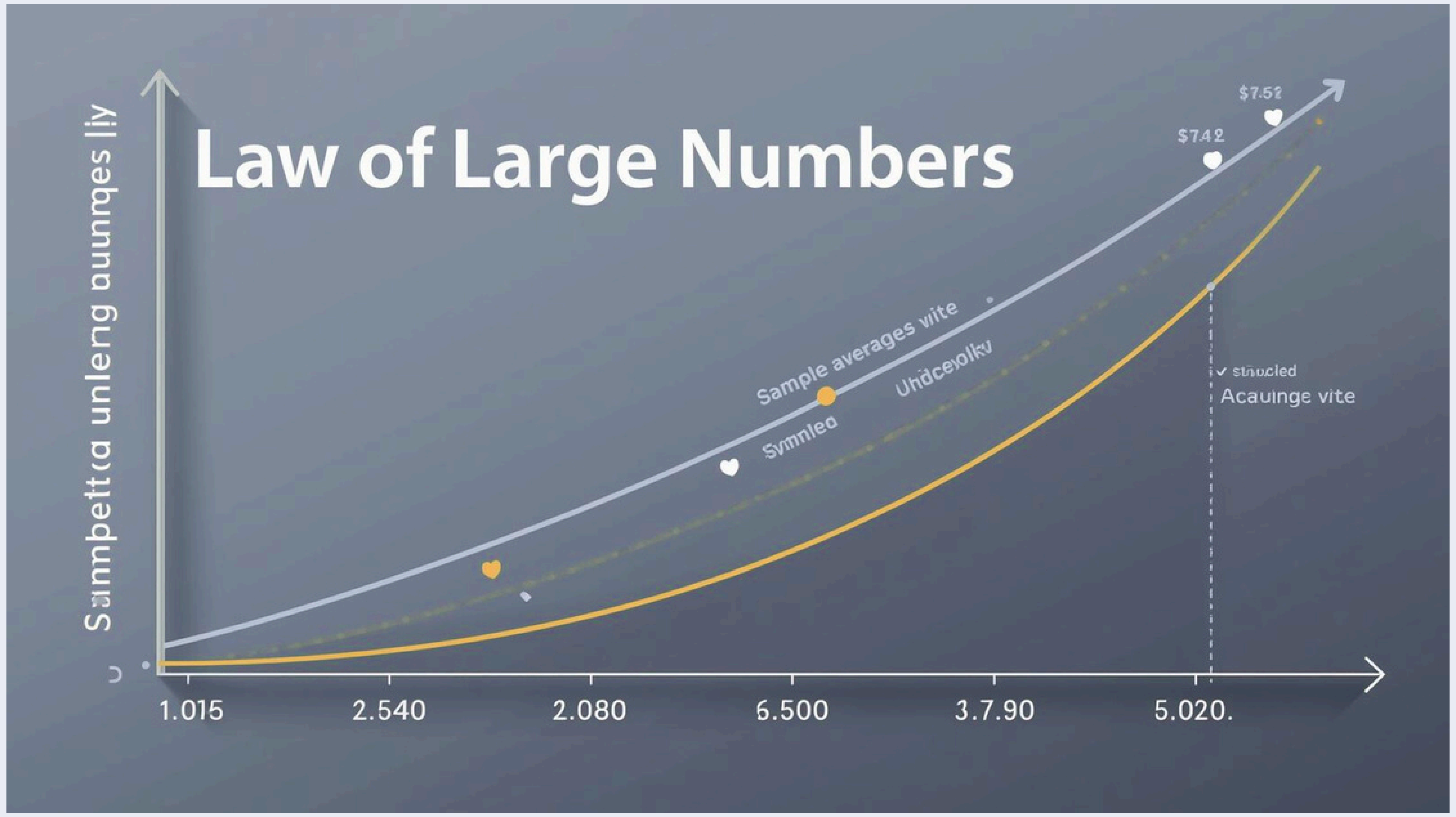


Monte Carlo Estimation

- Waiting time estimator:

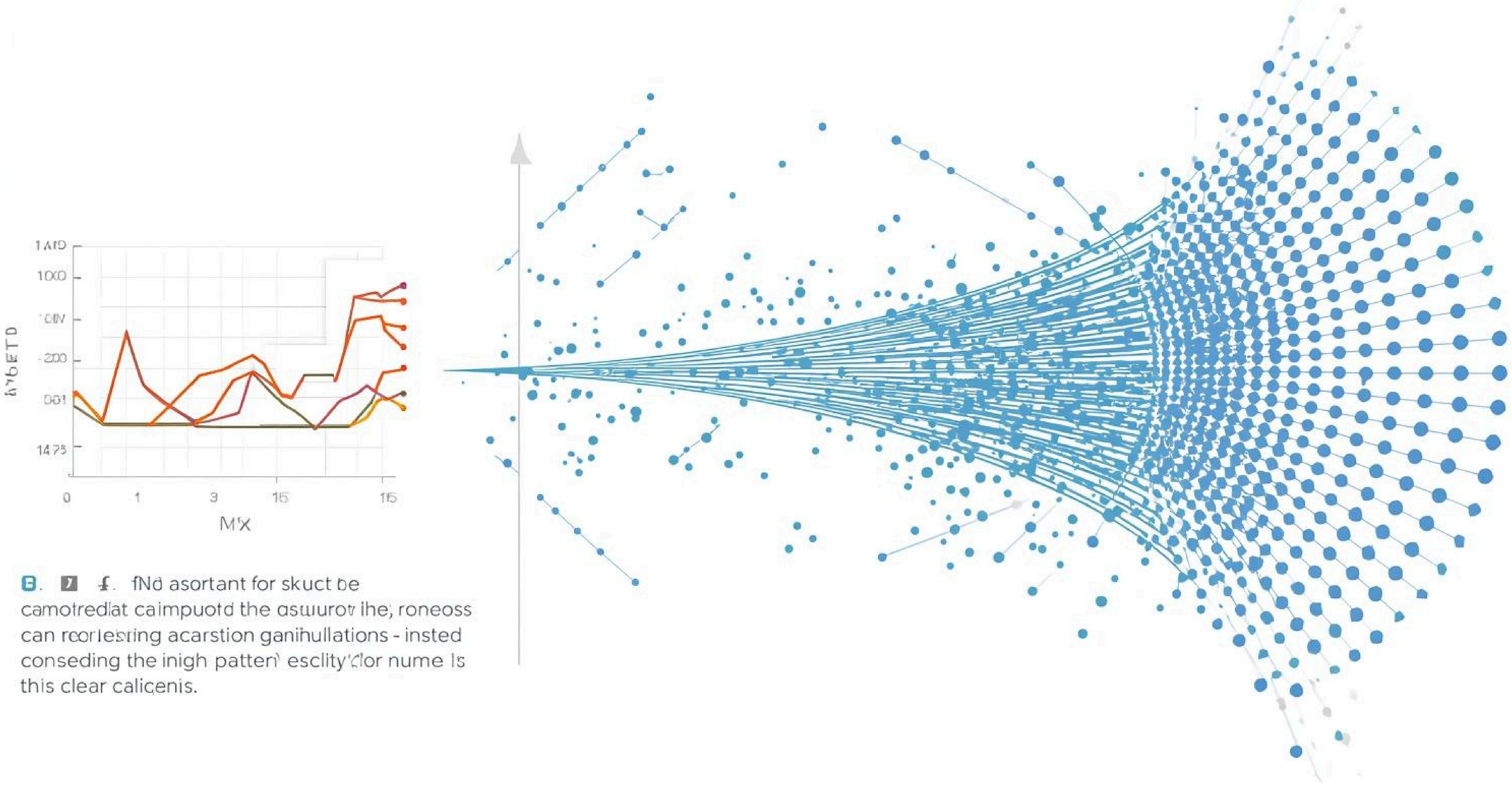
$$\hat{E}[W_q]=1/N \sum^{N} W_{q,i}$$

- Based on the Law of Large Numbers.



Monte Carlo Estimation

- Accuracy increases with the number of simulations.



Tools and Libraries

Random Generation

```
A <- rexp(N, rate = lambda)   # interarrival times ~ Exp( )
S <- rexp(N, rate = mu)       # service times ~ Exp( )
```

For Statistics

```
theta_hat <- mean(Wq)   # simulation estimate
mcse <- sd(Wq) / sqrt(N)   # Monte Carlo standard error
```

For Visualization

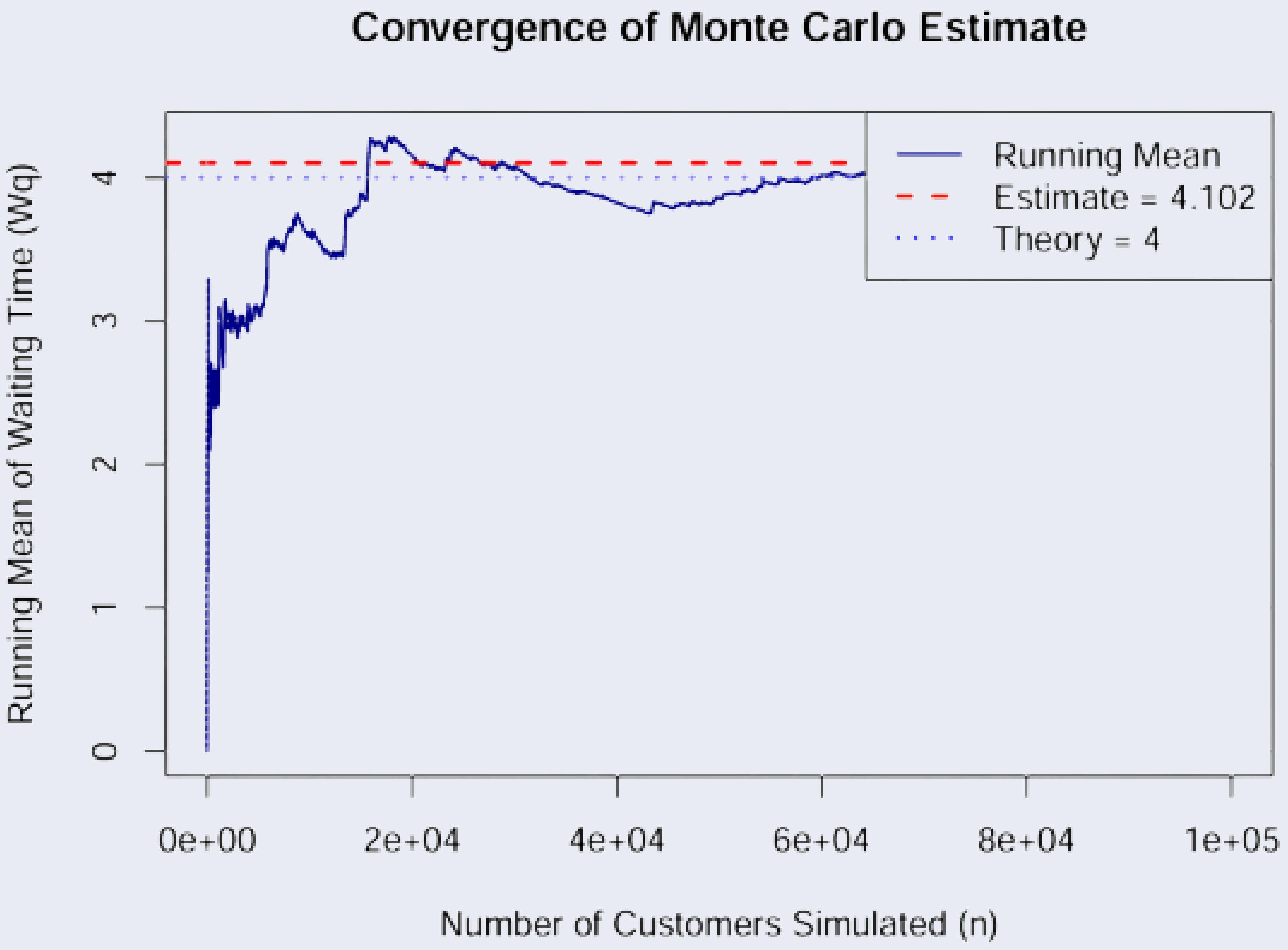
```
hist(Wq, breaks = 100,
```

```
plot(running_mean[1:50000], type = "l", lwd = 1.5,
```

Numerical Results: Simulation vs Theory

Measure	Monte Carlo Simulation	Theoretical Value
Expected Waiting Time (Wq)	4.1021	4.0000
Difference	0.1021	—
Standard Error	0.0162	—

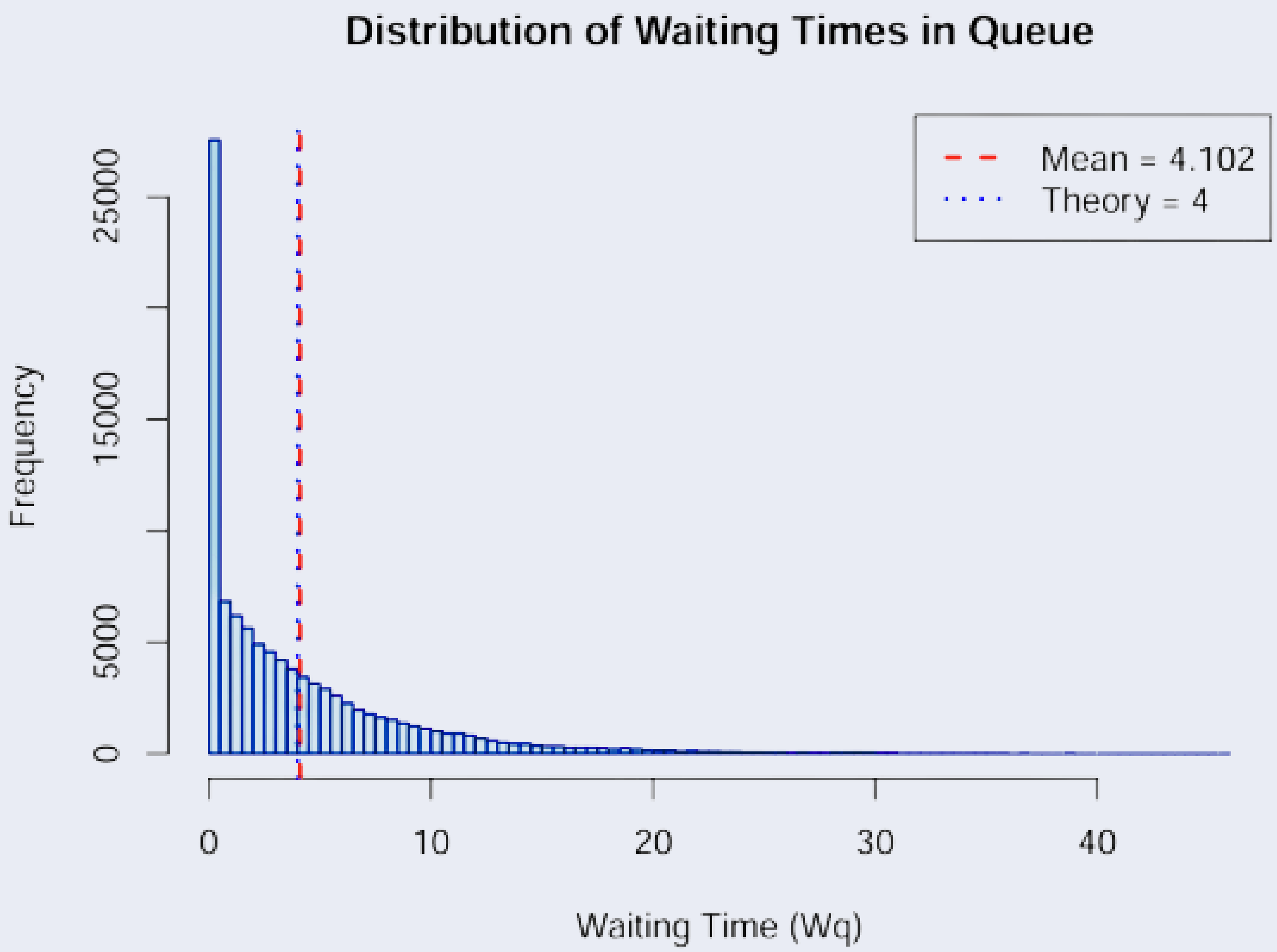
Graphical Analysis



Convergence Plot

- Running mean stabilizes as simulations increase.
- Convergence observed after ~25,000 customers.

Graphical Analysis

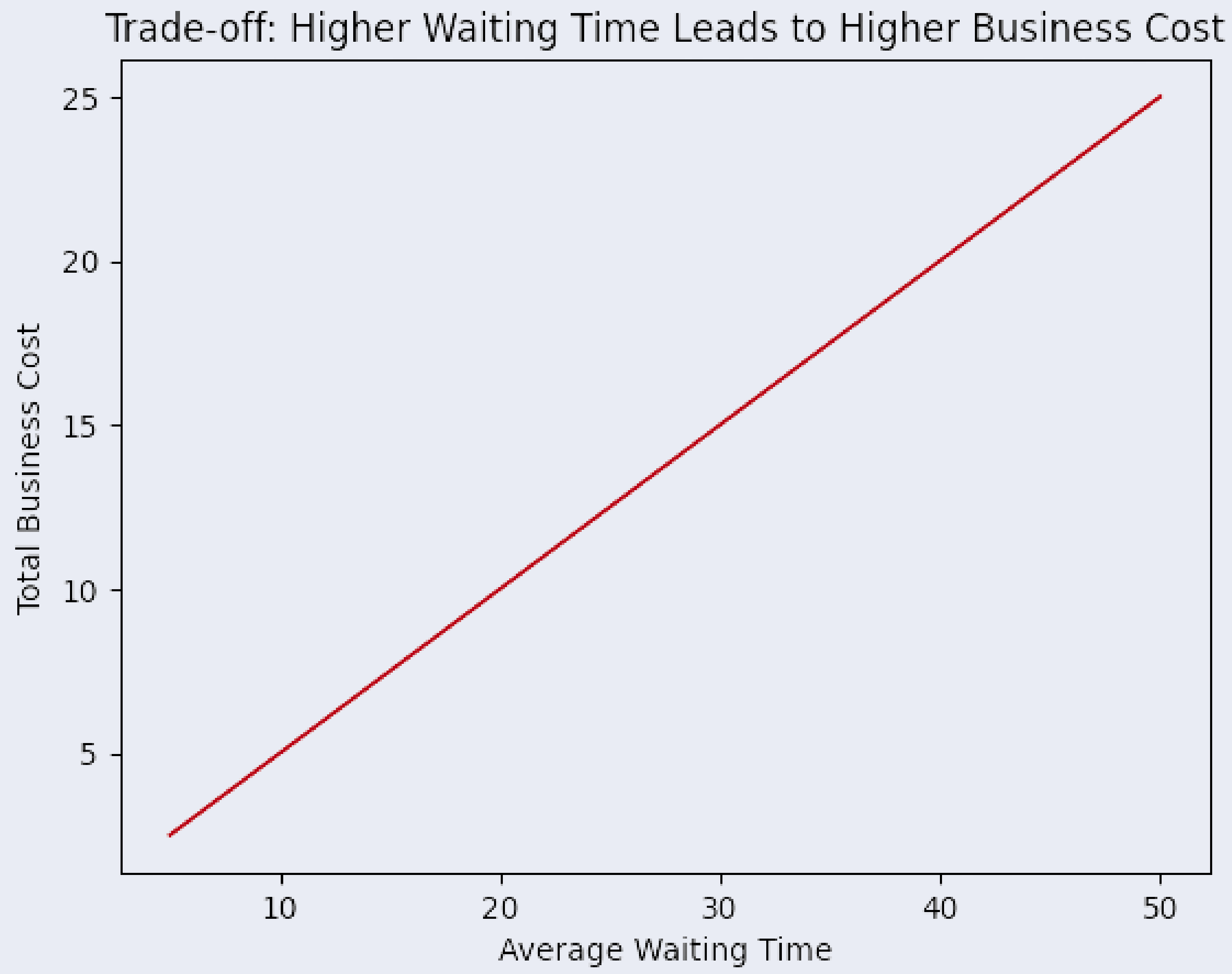


Histogram

- Right-skewed distribution.
- Most customers experience short waits.
- Few customers face long delays.

Business Insights

- High utilization leads to rapidly increasing waiting times.
- Average performance can hide extreme delays.
- Simulation helps managers:
  - –Test staffing levels
  - –Evaluate service capacity
  - –Balance cost and customer satisfaction



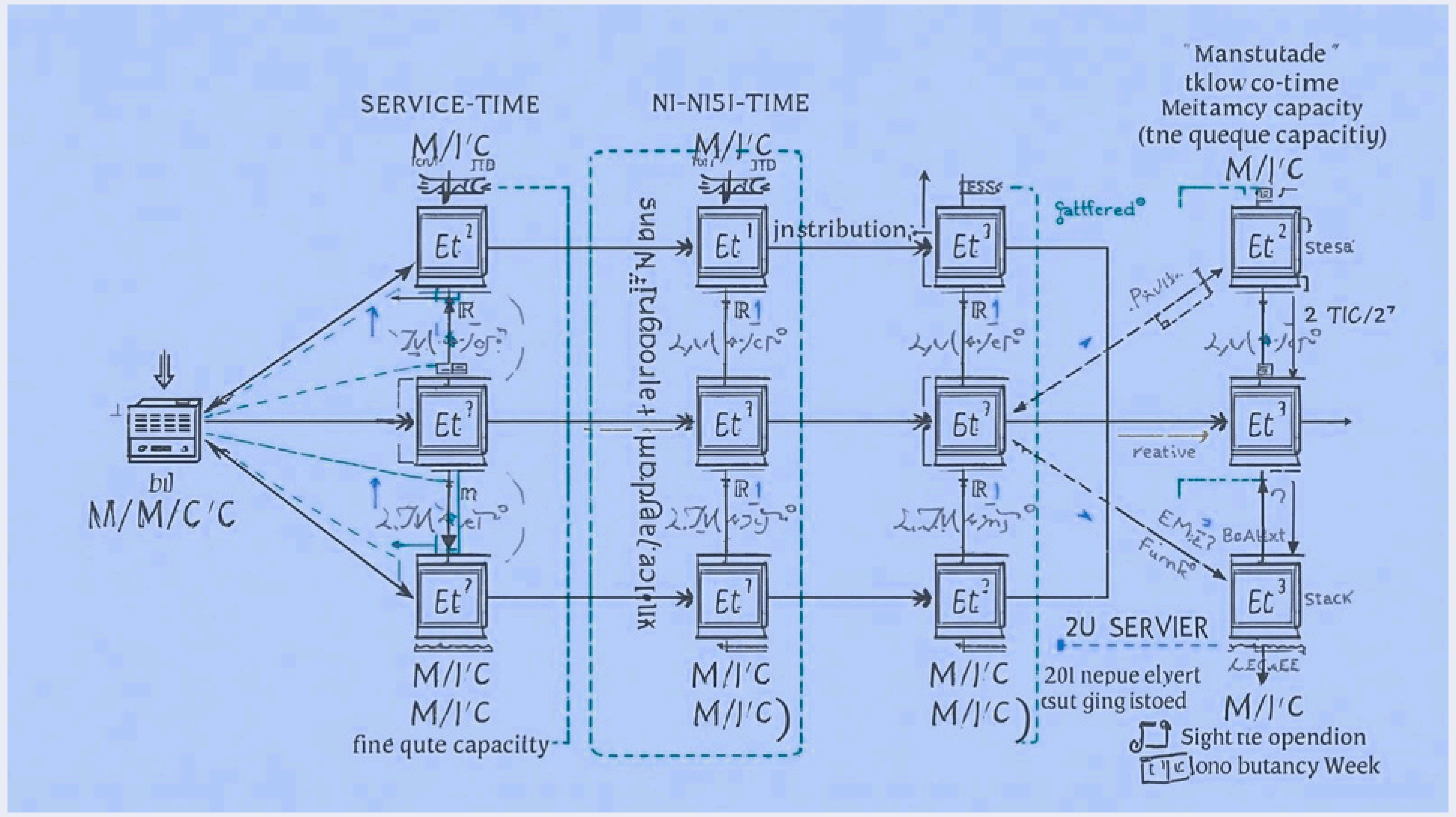
Limitations and Possible Extensions

Limitations

- Exponential assumptions may not always hold.
- Single-server model only.

Extensions

- Multiple servers (M/M/c)
- Different service-time distributions
- Finite queues
- Time-dependent arrival rates



Conclusion

- Monte Carlo simulation is a powerful tool for service analysis.
- Results closely match theoretical expectations.
- Provides practical insights for decision-making.
- Forms a foundation for more realistic business simulations.