

# Telco Simulation Report

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## I. PROBLEM DEFINITION

### A. The Scenario

THE managers of a telco want to improve one of their company's 24/7 customer assistance centers (CAC). Since experiments with the actual system are too time and cost intensive and might alienate customers [1], the executives decide to reach out to a group of data scientists. The scientists are hired to design a computer simulation which is able to model the traffic of the CAC and to report back with strategies that optimise internal processes. The scientists are confident that analysing data retrieved in the simulation process allows them to come up with a recommended set of actions. The output measures of interest are the average waiting time of customers, the percentage of customers exceeding the performance bounds and the costs.

The managers can gain leverage by influencing the schedule of the Customer Service Agents (CSAs) and the policy regarding call allocation. A given strategy consists of:

- the amount of CSAs for each shift
- the distribution of the two types of CSAs for each shift
- a clear policy regarding the degree of involvement of corporate CSAs with consumer calls

### B. Objective

The objective of this report is to develop a simulation environment which can be run with various sets of input and offers an interface for data analysis of collected data. The data analysis in turn uncovers strategies which can be used interchangeably if needed. From this, two leading questions are derived:

- 1) *Does one strategy lead to better better performance given the same amount of corporate and customer agents for each shift?*
- 2) *Can we say with confidence that a chosen roster fulfils all performance requirements and has a certain average waiting time for customers?*

All findings in this report are based on the given description of the system which is presented in the next section II.

## II. SYSTEM ATTRIBUTES

### A. Employee and costumer types

There exist two types of costumers for the CAC, corporate users and consumer users. The service for corporate users is more profitable, however the consumers

cannot be disregarded.

The CAC employs two different kind of service agents. Corporate agents  $CSA_{corp}$  are trained to work with incoming calls from both corporate users and consumers, whereas the consumer agents  $CSA_{cons}$  are only allowed to take consumer calls. The cost per hour for one  $CSA_{corp}$  is 60 euros, whereas the cost for a  $CSA_{cons}$  is 35 euros.

### B. Shifts and Performance Guarantees

The CSAs work in three shifts:

- 1) Shift 1: (6am - 2pm)
- 2) Shift 2: (2pm - 10pm)
- 3) Shift 3: (10pm - 6am)

The calling costumers should be assisted in a specific time frame. This is reflected in the following performance requirements:

- 95% of corporate users within 3 min
- 99% of corporate users within 7 min
- 90% of consumers within 5 min
- 95% of consumers within 10 min

### C. Arrival Rate Distribution and Service Times

- Incoming corporate calls are  $\sim Pois(\lambda_t)$ , where

$$\lambda_t = \begin{cases} 8 < t < 18, & \lambda_t = 1/60 \\ 18 < t < 8, & \lambda_t = 1/300 \end{cases} \quad (1)$$

- Incoming consumer calls are  $\sim Pois(\lambda_t)$ , where  $\lambda_t$  changes according to a sinusoid poisson process,

$$\lambda_t = 1.8 \sin \left( \left( \frac{2\pi}{24} \right) (x + 15) \right) + 2 \quad (2)$$

- The service time in seconds for corporate costumers is  $\sim \psi(\bar{\mu} = 216, \bar{\sigma} = 72, a = 45, b = +\infty; x)$ , so

$$\psi(\bar{\mu}, \bar{\sigma}, a, b; x) = \begin{cases} 0 & \text{if } x \leq 45 \\ \frac{1}{\bar{\sigma}} \cdot \frac{\phi(\bar{\mu}, \bar{\sigma}; x)}{1 - \Phi(\bar{\mu}, \bar{\sigma}; 45)} & \text{if } 45 < x < b \end{cases} \quad (3)$$

- The service time in seconds of consumers is  $\sim \psi(\bar{\mu} = 72, \bar{\sigma} = 35, a = 25, b = +\infty; x)$ , so

$$\psi(\bar{\mu}, \bar{\sigma}, a, b; x) = \begin{cases} 0 & \text{if } x \leq 25 \\ \frac{1}{\bar{\sigma}} \cdot \frac{\phi(\bar{\mu}, \bar{\sigma}; x)}{1 - \Phi(\bar{\mu}, \bar{\sigma}; 25)} & \text{if } 25 < x < b \end{cases} \quad (4)$$

### III. MODEL

#### A. Sampling the Distributions

As described in II-C, the service times of consumer and corporate customers follows a truncated Normal distribution with truncation at  $a=25$  and  $a=45$  respectively. To generate samples from this desired distribution, the Box-Muller method [2] is first used to generate standard normally distributed data random variates which are then scaled to the desired distribution parameters, if the r.v. is below the truncation point, then repeat the procedure. The algorithm is:

- 1) Generate 2 Uniform(0,1) random variates  $U_1, U_2$
- 2) Use  $U_1$  and  $U_2$  to generate pairs  $Z_1, Z_2$  of independent, standard, normally distributed (zero expectation, 1 variance) random numbers with to Box-Muller transform method.
- 3) Transform the 2 newly created r.v.s to normal distribution with desired parameters according to ( $X = \text{this.mean} + \text{this.std} * Z$ )
- 4) Do until  $a \leq (X_1, X_2) \leq b$  (within truncation range)

The algorithm is efficient in the simulation context, as the truncation interval represents a large part of the normal probability mass for the given parameters and truncation ranges: For corporate service times the truncated interval  $>45$  represents 99.12 percent of normal probability mass. For consumer service times the truncated interval  $>25$  represents 91.03 percent of normal probability mass. This means that it is very unlikely that the algorithm needs to loop more than once in order to find a valid sample with the desired underlying truncated normal distribution. Hence, the algorithm is more efficient than a 'pure' Acceptance-Rejection method and exploits the fact that we can make use of the Box-Muller transform by scaling the output according to desired parameters and reject if the generated values are outside of the truncation range. For generating the arrivals according to the desired rate for corporate customers, inverse transform with the exponential distribution is used. Generating the arrivals of consumers is more complex and involves sampling of the rate  $\lambda$  with the thinning algorithm in order to achieve a non-stationary poisson process.

#### B. Queues

In the model that is used, there are two queues. These are both FIFO (first in, first out) queues. However, the source from which they get customers is different. One queue only receives consumers calls, while the other only takes in corporate customers calls.

#### C. Customer Service Agents (CSAs)

There are in total two types of customer service agents; one which can handle consumer customer calls, and one which can handle corporate customer calls.

Two different strategies of the corporate customer service agent that have been tested. One of these strategies lets the corporate CSAs always take in consumer customers, at the moment they become idle, under the condition that the corporate queue is empty.

The other strategy always keeps a certain amount of corporate CSAs idle  $K_c$ , even if the corporate queue is empty. Then, if there are more idle CSAs than there need to be, the number of CSAs that exceed the number  $K_c$  needed can help out consumer customers. In a case in which  $K_c$  is defined to be zero, this strategy would be the same as the previously described strategy.

These strategies are visualised in the figure below, which shows the separate queues and how the CSAs interact with it.

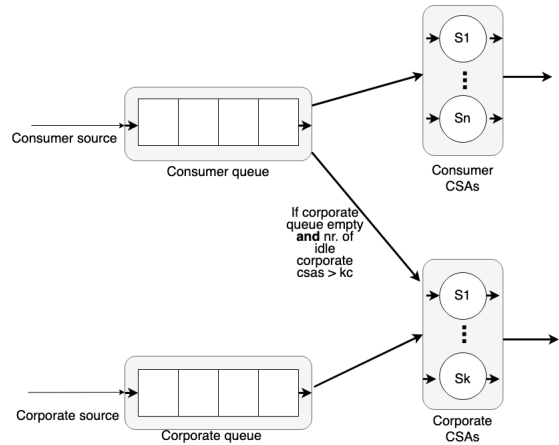


Fig. 1: Diagram of queues and CSAs

#### D. Events

There are three different types of events, excluding the event of shift changes. These are: arrivals, the start of a call, and the end of a call.

Times of arrivals are decided by the formulas as described in section II-C. The two different customers come from different sources, which adhere to these formulas. Customers which have only arrived, but have not been accepted yet, are in their respective queue.

When a customers' call is accepted by a CSA, it begins the process of being handled. This gets marked as the start of a call. At the moment a call is started, the service time gets calculated (given the formulas in section II-C, which uses a truncated normal distribution). The CSA which is handling this call, is set to busy.

Lastly, after the call is handled, another event takes place. This is the end of the call. When that happens, the CSA, which handled the call, are idle again. The call gets stored in the sink.

#### E. Shifts

A day in the simulation is comprised of three shifts; a morning, afternoon and evening shift.

As it was outlined earlier, shift changes are an event. When this happens CSAs get removed when their last call, which started before the shift change, ends. Other CSAs get added according to the roster that is specified. Afterwards, the simulation continues as normal.

### IV. TRANSIENT

As the system should run all the time, and the simulation starts at a moment in which there are no customers in the system, there will be a transient. This transient needs to be cut off, in order to find the steady state of the system. To do this, a visual approach to replication-deletion is used. After plotting the data, the transient can be found, which will be the part in which it does not appear to be in the steady state yet. Below, ?? plot the waiting results for consumers agents, while ?? plot the waiting results for the corporate agents, when the simulation is run with the setup described in section V, for the first strategy.

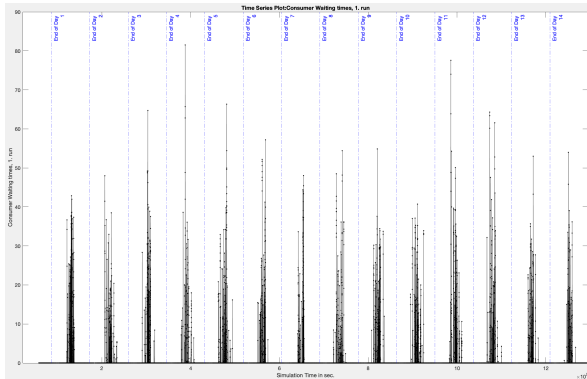


Fig. 2: Consumer waiting times, one run

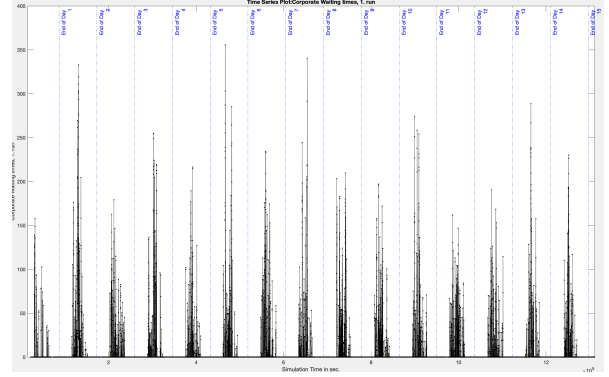


Fig. 3: Corporate waiting times, one run

Moreover, for 5 different runs the waiting times have been plotted, to show that these runs have the same shape

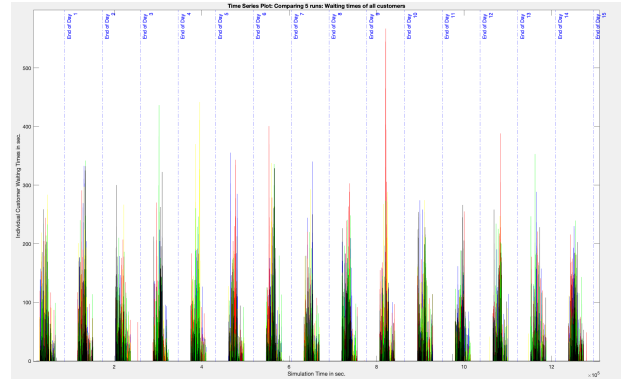


Fig. 4: Combined waiting times, multiple runs

From these results there cannot be an obviously ascertained cut-off point from the transient to the steady state. However, as a precautionary measure a cut-off was made after two days of simulation, in order to ensure that any data used was truly from the steady state.

### V. EXPERIMENT SETUP

The two different strategies are compared with the same roster shown by table 1. We run each strategy twenty times independently to get an estimate of the true distribution and create a confidence interval of the average waiting times and the performance requirements. The  $k_c$  for strategy 2 is 3.

CSA type	Shift 1	Shift 2	Shift 3
Consumer	5	8	5
Corporate	6	8	6

TABLE I: Roster for both strategies

## VI. DISTRIBUTIONS OF DATA

In the following two diagrams, the data that is found by running the simulation is compared to the distribution that was implemented, as a way to ensure that the right distribution is being used. This is done as a general measure of finding whether this data uses the distribution needed.

In the figures below, the consumer service times are and its distribution are compared, as well as the corporate service time and its distribution.

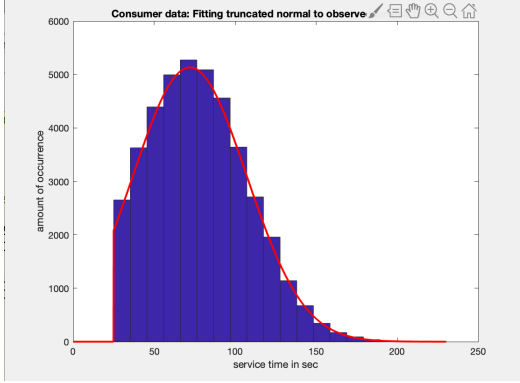


Fig. 5: Fitting consumer service times to the distribution

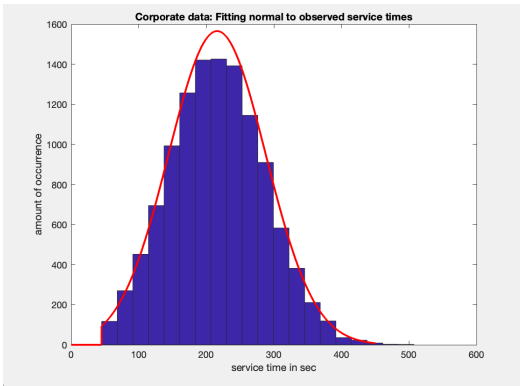


Fig. 6: Fitting corporate service times to the distribution

When the data is compared to its (truncated) distribution, it can be seen that these two align. The normal part of these figures are similar to each other. It also does not contain data that is below the value at which it is truncated. The data does indeed seem to come from this distribution.

The interarrival times for corporate are displayed in figure 7 also compared to their distribution:

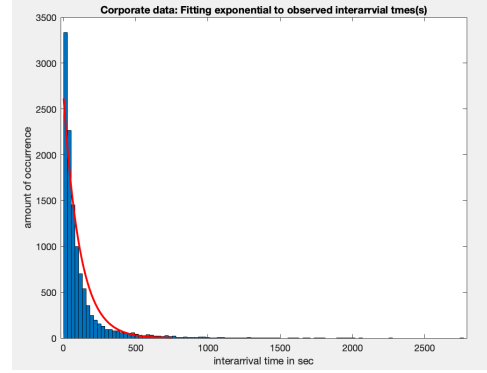


Fig. 7: Fitting corporate interarrival times to the distribution

The data here also aligns with its predefined distribution. It has a very high value close to zero, and then seems to lessen. The data that is found looks the same, so it seems to come from the distribution.

Note that, although the diagrams shown are for one run only, these diagrams look similar for each of the runs. Therefore, all the data seems to come from the distribution it needs to come from.

## VII. RESULTS

Performance Measure	Strategy 1	Strategy 2
Expected Overall Waiting time	3.449 +-0.287	3.611 +-0.807
Expected Consumer Waiting time	0.752	1.222
Expected Corporate Waiting time	13.571	12.612
% Consumers Assisted within 5 min	100 +-0	99.994 +-0.019
% Consumers Assisted within 10 min	100 +-0	100 +-0
% Corporate Assisted within 3 min	98.657 +-0.309	98.974 +-0.326
% Corporate Assisted within 7 min	99.952 +-0.052	99.980 +-0.023

TABLE II: Results with overall 95% confidence interval (cut-off at 100% for percentages).

Performance Measure	Difference	t	p
Expected Overall Waiting time	-1.001/0.677	2.031	0.062
Expected Consumer Waiting time	-1.762/0.821	3.838	0.002
Expected Corporate Waiting time	-2.396/1.454	2.575	0.002

TABLE III: System differences with overall 95% confidence interval

### A. The cost

Using the roster that was predefined in section V, the cost is 14,640 euros for one day, and 102,480 euros for one week. As both of these strategies use the same roster (with another tactic of how to use corporate agents to help consumers), the cost for both of these strategies will be the same.

## VIII. DISCUSSION

Strategy 1 has significantly less expected waiting time for consumer customer, but strategy 2 has significantly less expected waiting time for corporate customers (p-values: 0.002/0.002). As expected this is the case, because we always hold  $k_c$  amount of corporate CSAs back idle to handle incoming corporate calls. The first strategy tends towards having being significantly less overall waiting time (p-value: 0.062).

The confidence intervals of all measurements clearly fall into the performance requirements and the overall waiting time of both strategies is less than 4 seconds. Knowing this, we can assume that it is possible to use rosters with less agents to save cost and still fulfill the performance requirements.

## IX. CONCLUSION

- 1) We can conclude that the strategy of keeping  $k_c$  corporate CSAs idle for incoming corporate calls leads to less waiting time for corporate customers with a trade-off of waiting time for consumer customers.
- 2) For both strategies, we can conclude that the chosen roster fulfills the performance measures for the percentage of customers helped within the designated time frames.

## REFERENCES

- [1] A. M. Law, *Simulation Modeling & Analysis*, 5th ed. New York, NY, USA: McGraw-Hill, 2015.
- [2] G. E. P. Box and M. E. Muller, "A note on the generation of random normal deviates," *Ann. Math. Statist.*, vol. 29, no. 2, pp. 610–611, 06 1958. [Online]. Available: <https://doi.org/10.1214/aoms/1177706645>