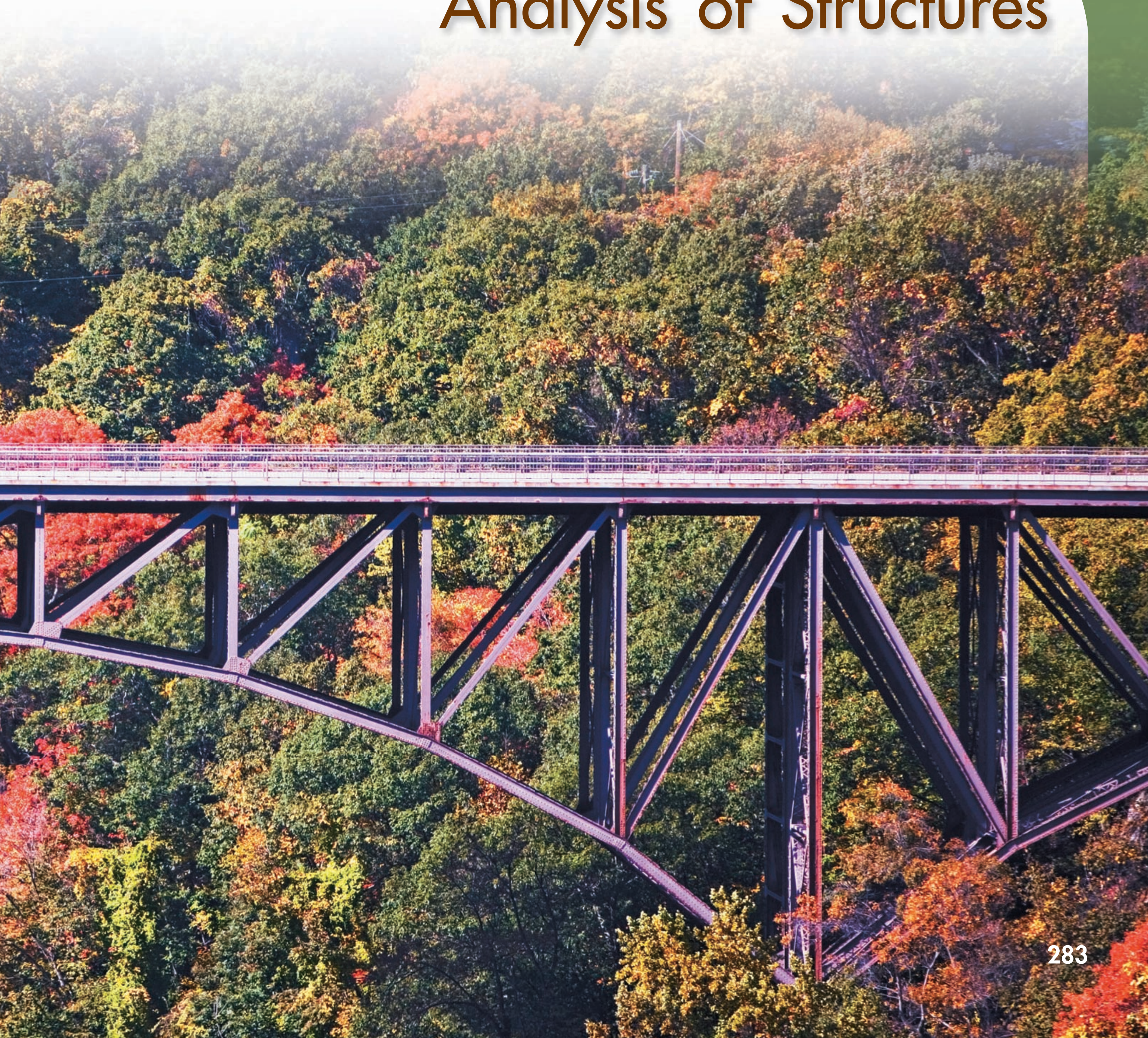
Trusses, such as this Pratt-style cantilever arch bridge in New York State, provide both a practical and an economical solution to many engineering problems.

CHAPTER 6

Analysis of Structures



6.1 INTRODUCTION

The problems considered in the preceding chapters concerned the equilibrium of a single rigid body, and all forces involved were external to the rigid body. We now consider problems dealing with the equilibrium of structures made of several connected parts. These problems call for the determination not only of the external forces acting on the structure but also of the forces which hold together the various parts of the structure. From the point of view of the structure as a whole, these forces are *internal forces*.

Consider, for example, the crane shown in Fig. 6.1*a*, which carries a load W. The crane consists of three beams AD, CF, and BE connected by frictionless pins; it is supported by a pin at A and by a cable DG. The free-body diagram of the crane has been drawn in Fig. 6.1*b*. The external forces, which are shown in the diagram, include the weight $\mathbf{W}$, the two components $\mathbf{A}_x$ and $\mathbf{A}_y$ of the reaction at A, and the force $\mathbf{T}$ exerted by the cable at D. The internal forces holding the various parts of the crane together do not appear in the diagram. If, however, the crane is dismembered and if a free-body diagram is drawn for each of its component parts, the forces holding the three beams together will also be represented, since these forces are external forces from the point of view of each component part (Fig. 6.1*c*).

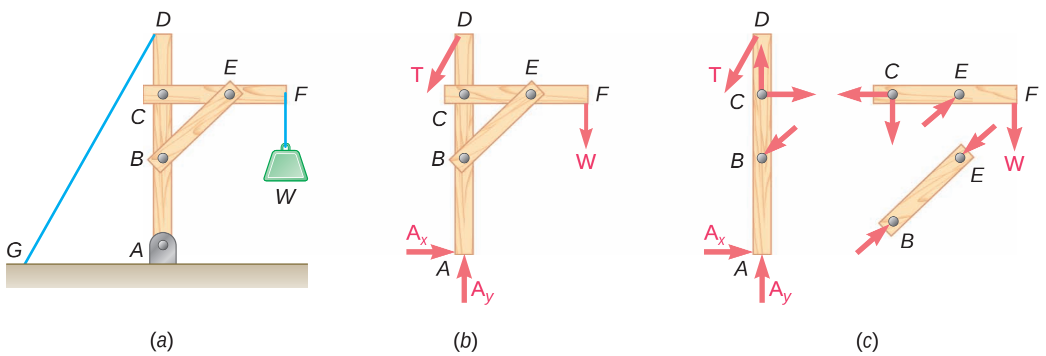


Fig. 6.1

It will be noted that the force exerted at B by member BE on member AD has been represented as equal and opposite to the force exerted at the same point by member AD on member BE; the force exerted at E by BE on CF is shown equal and opposite to the force exerted by CF on BE; and the components of the force exerted at C by CF on AD are shown equal and opposite to the components of the force exerted by AD on CF. This is in conformity with Newton's third law, which states that *the forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite sense*. As pointed out in Chap. 1, this law, which is based on experimental evidence, is one of the six fundamental principles of elementary mechanics, and its application is essential to the solution of problems involving connected bodies.

In this chapter, three broad categories of engineering structures will be considered:

1. *Trusses,* which are designed to support loads and are usually stationary, fully constrained structures. Trusses consist exclusively of straight members connected at joints located at the ends of each member. Members of a truss, therefore, are *two-force members,* i.e., members acted upon by two equal and opposite forces directed along the member.
2. *Frames,* which are also designed to support loads and are also usually stationary, fully constrained structures. However, like the crane of Fig. 6.1, frames always contain at least one *multiforce member,* i.e., a member acted upon by three or more forces which, in general, are not directed along the member.
3. *Machines,* which are designed to transmit and modify forces and are structures containing moving parts. Machines, like frames, always contain at least one multiforce member.

Photo 6.1 Shown is a pin-jointed connection on the approach span to the San Francisco–Oakland Bay Bridge.

TRUSSES

6.2 DEFINITION OF A TRUSS

The truss is one of the major types of engineering structures. It provides both a practical and an economical solution to many engineering situations, especially in the design of bridges and buildings. A typical truss is shown in Fig. 6.2*a*. A truss consists of straight members connected at joints. Truss members are connected at their extremities only; thus no member is continuous through a joint. In Fig. 6.2*a*, for example, there is no member *AB*; there are instead two distinct members *AD* and *DB*. Most actual structures are made of several trusses joined together to form a space framework. Each truss is designed to carry those loads which act in its plane and thus may be treated as a two-dimensional structure.

In general, the members of a truss are slender and can support little lateral load; all loads, therefore, must be applied to the various joints, and not to the members themselves. When a concentrated load is to be applied between two joints, or when a distributed load is to be supported by the truss, as in the case of a bridge truss, a floor system must be provided which, through the use of stringers and floor beams, transmits the load to the joints (Fig. 6.3).

The weights of the members of the truss are also assumed to be applied to the joints, half of the weight of each member being applied to each of the two joints the member connects. Although the members are actually joined together by means of welded, bolted, or riveted connections, it is customary to assume that the members are pinned together; therefore, the forces acting at each end of a member reduce to a single force and no couple. Thus, the only forces assumed to be applied to a truss member are a single

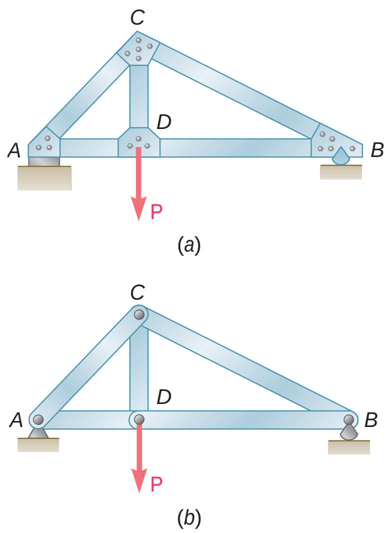


Fig. 6.2

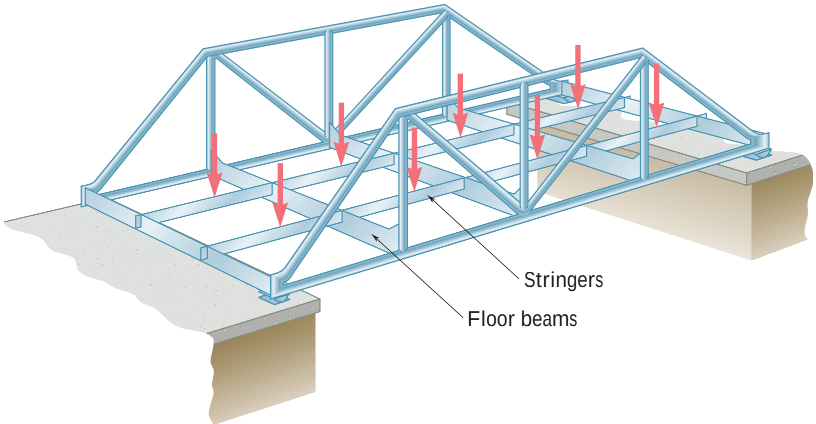


Fig. 6.3

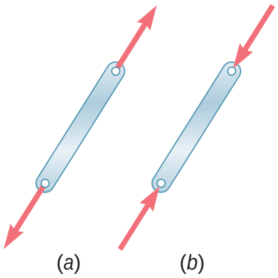


Fig. 6.4

force at each end of the member. Each member can then be treated as a two-force member, and the entire truss can be considered as a group of pins and two-force members (Fig. 6.2*b*). An individual member can be acted upon as shown in either of the two sketches of Fig. 6.4. In Fig. 6.4*a*, the forces tend to pull the member apart, and the member is in tension; in Fig. 6.4*b*, the forces tend to compress the member, and the member is in compression. A number of typical trusses are shown in Fig. 6.5.

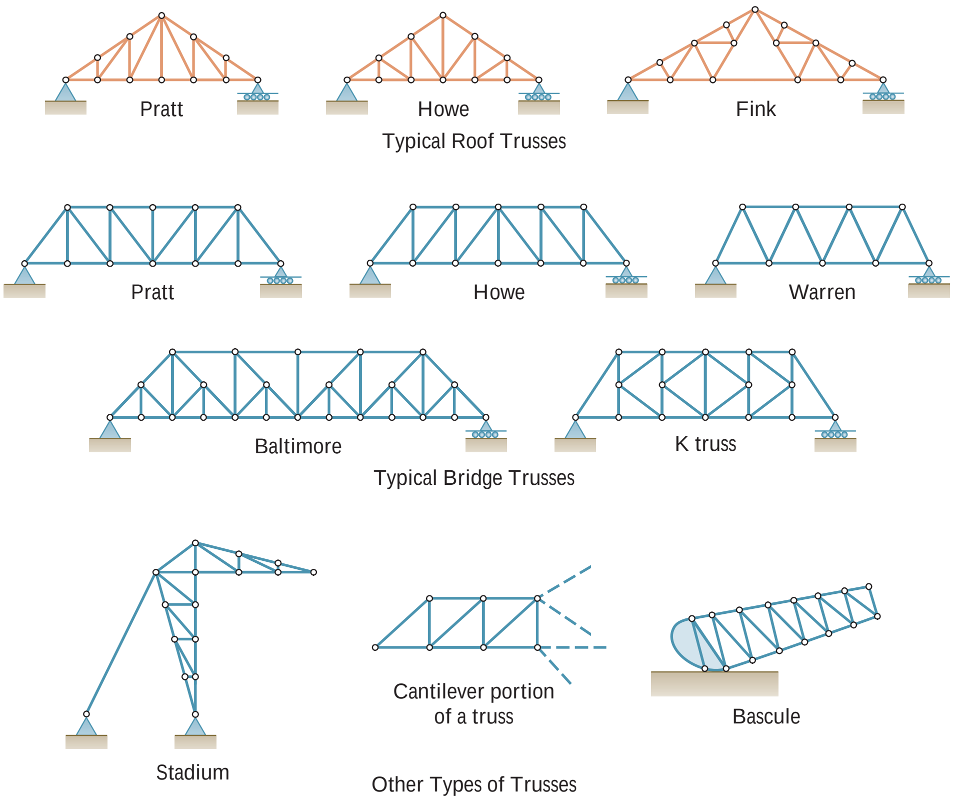


Fig. 6.5

6.3 SIMPLE TRUSSES

Consider the truss of Fig. 6.6*a*, which is made of four members connected by pins at A, B, C, and D. If a load is applied at B, the truss will greatly deform, completely losing its original shape. In contrast, the truss of Fig. 6.6*b*, which is made of three members connected by pins at A, B, and C, will deform only slightly under a load applied at B. The only possible deformation for this truss is one involving small changes in the length of its members. The truss of Fig. 6.6*b* is said to be a *rigid truss*, the term rigid being used here to indicate that the truss *will not collapse*.

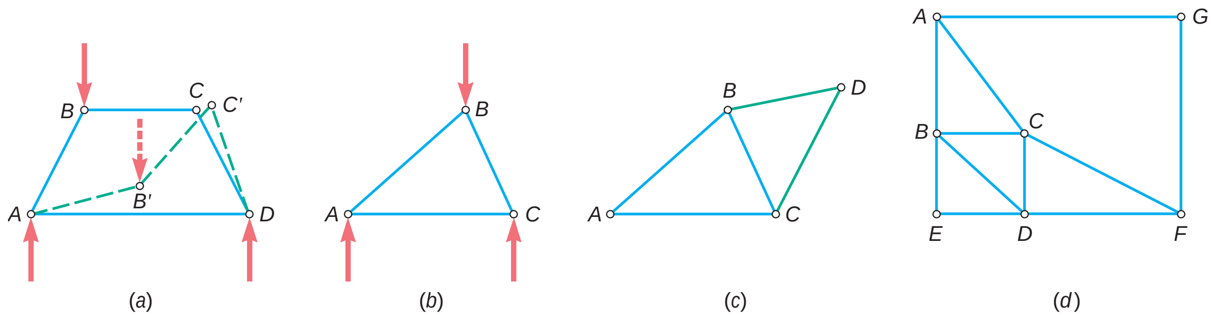


Fig. 6.6

As shown in Fig. 6.6*c*, a larger rigid truss can be obtained by adding two members BD and CD to the basic triangular truss of Fig. 6.6*b*. This procedure can be repeated as many times as desired, and the resulting truss will be rigid if each time two new members are added, they are attached to two existing joints and connected at a new joint.† A truss which can be constructed in this manner is called a *simple truss*.

It should be noted that a simple truss is not necessarily made only of triangles. The truss of Fig. 6.6*d*, for example, is a simple truss which was constructed from triangle ABC by adding successively the joints D, E, F, and G. On the other hand, rigid trusses are not always simple trusses, even when they appear to be made of triangles. The Fink and Baltimore trusses shown in Fig. 6.5, for instance, are not simple trusses, since they cannot be constructed from a single triangle in the manner described above. All the other trusses shown in Fig. 6.5 are simple trusses, as may be easily checked. (For the K truss, start with one of the central triangles.)

Returning to Fig. 6.6, we note that the basic triangular truss of Fig. 6.6*b* has three members and three joints. The truss of Fig. 6.6*c* has two more members and one more joint, i.e., five members and four joints altogether. Observing that every time two new members are added, the number of joints is increased by one, we find that in a simple truss the total number of members is $m = 2n - 3$, where n is the total number of joints.

Photo 6.2 Two K trusses were used as the main components of the movable bridge shown which moved above a large stockpile of ore. The bucket below the trusses picked up ore and redeposited it until the ore was thoroughly mixed. The ore was then sent to the mill for processing into steel.

†The three joints must not be in a straight line.

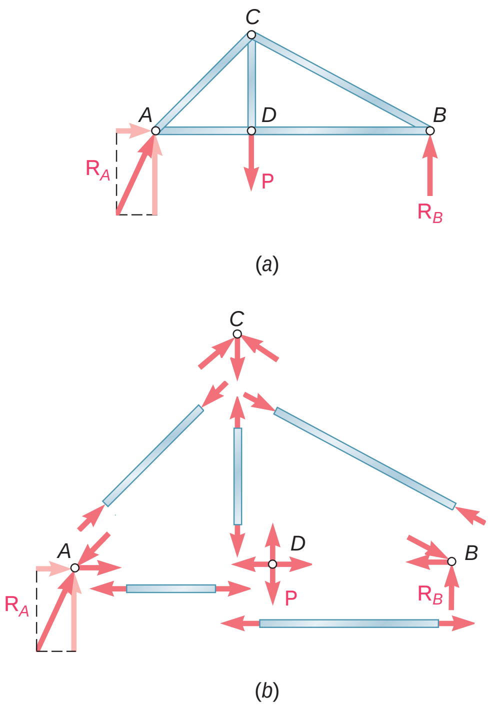


Fig. 6.7

Photo 6.3 Because roof trusses, such as those shown, require support only at their ends, it is possible to construct buildings with large, unobstructed floor areas.

6.4 ANALYSIS OF TRUSSES BY THE METHOD OF JOINTS

We saw in Sec. 6.2 that a truss can be considered as a group of pins and two-force members. The truss of Fig. 6.2, whose free-body diagram is shown in Fig. 6.7*a*, can thus be dismembered, and a free-body diagram can be drawn for each pin and each member (Fig. 6.7*b*). Each member is acted upon by two forces, one at each end; these forces have the same magnitude, same line of action, and opposite sense (Sec. 4.6). Furthermore, Newton's third law indicates that the forces of action and reaction between a member and a pin are equal and opposite. Therefore, the forces exerted by a member on the two pins it connects must be directed along that member and be equal and opposite. The common magnitude of the forces exerted by a member on the two pins it connects is commonly referred to as the *force in the member* considered, even though this quantity is actually a scalar. Since the lines of action of all the internal forces in a truss are known, the analysis of a truss reduces to computing the forces in its various members and to determining whether each of its members is in tension or in compression.

Since the entire truss is in equilibrium, each pin must be in equilibrium. The fact that a pin is in equilibrium can be expressed by drawing its free-body diagram and writing two equilibrium equations (Sec. 2.9). If the truss contains n pins, there will, therefore, be $2n$ equations available, which can be solved for $2n$ unknowns. In the case of a simple truss, we have $m = 2n - 3$, that is, $2n = m + 3$, and the number of unknowns which can be determined from the free-body diagrams of the pins is thus $m + 3$. This means that the forces in all the members, the two components of the reaction $\mathbf{R}_A$, and the reaction $\mathbf{R}_B$ can be found by considering the free-body diagrams of the pins.

The fact that the entire truss is a rigid body in equilibrium can be used to write three more equations involving the forces shown in the free-body diagram of Fig. 6.7*a*. Since they do not contain any new information, these equations are not independent of the equations associated with the free-body diagrams of the pins. Nevertheless, they can be used to determine the components of the reactions at the supports. The arrangement of pins and members in a simple truss is such that it will then always be possible to find a joint involving only two unknown forces. These forces can be determined by the methods of Sec. 2.11 and their values transferred to the adjacent joints and treated as known quantities at these joints. This procedure can be repeated until all unknown forces have been determined.

As an example, the truss of Fig. 6.7 will be analyzed by considering the equilibrium of each pin successively, starting with a joint at which only two forces are unknown. In the truss considered, all pins are subjected to at least three unknown forces. Therefore, the reactions at the supports must first be determined by considering the entire truss as a free body and using the equations of equilibrium of a rigid body. We find in this way that $\mathbf{R}_A$ is vertical and determine the magnitudes of $\mathbf{R}_A$ and $\mathbf{R}_B$.

The number of unknown forces at joint A is thus reduced to two, and these forces can be determined by considering the equilibrium of pin A. The reaction $\mathbf{R}_A$ and the forces $\mathbf{F}_{AC}$ and $\mathbf{F}_{AD}$ exerted on pin A by members AC and AD, respectively, must form a force triangle. First we draw $\mathbf{R}_A$ (Fig. 6.8); noting that $\mathbf{F}_{AC}$ and $\mathbf{F}_{AD}$ are directed along AC and AD, respectively, we complete the triangle and determine the magnitude and sense of $\mathbf{F}_{AC}$ and $\mathbf{F}_{AD}$. The magnitudes F_{AC} and F_{AD} represent the forces in members AC and AD. Since $\mathbf{F}_{AC}$ is directed down and to the left, that is, *toward* joint A, member AC pushes on pin A and is in compression. Since $\mathbf{F}_{AD}$ is directed *away* from joint A, member AD pulls on pin A and is in tension.

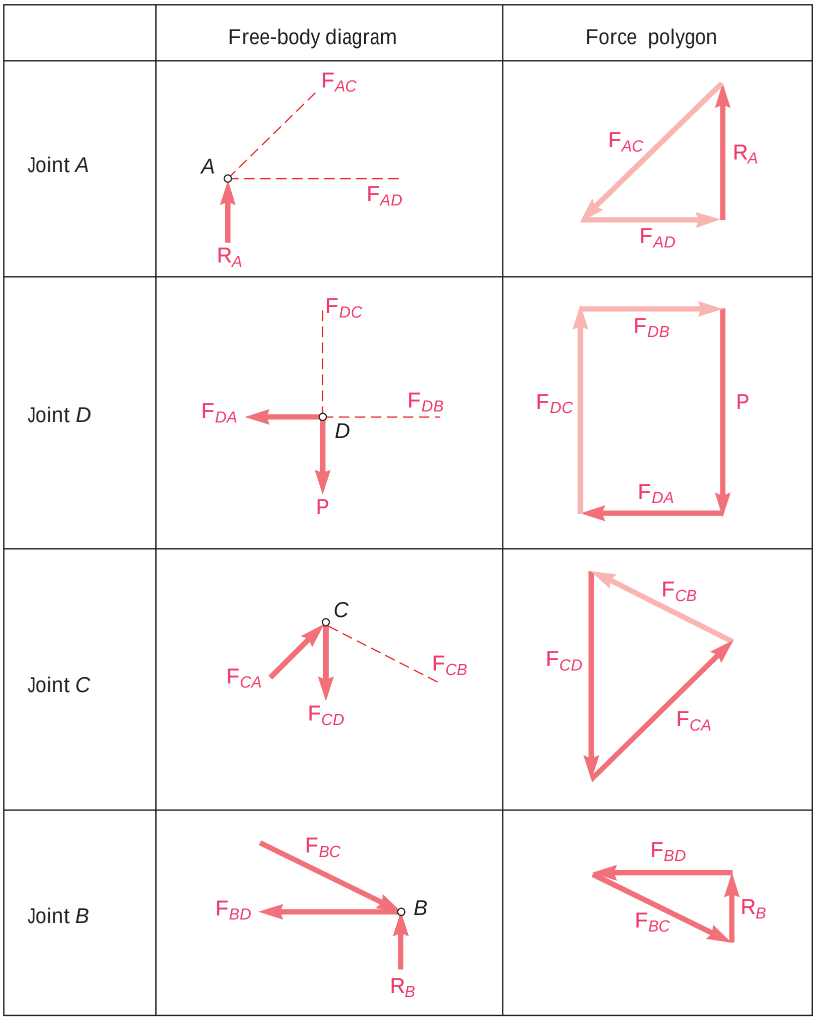


Fig. 6.8

We can now proceed to joint D, where only two forces, $\mathbf{F}_{DC}$ and $\mathbf{F}_{DB}$, are still unknown. The other forces are the load $\mathbf{P}$, which is given, and the force $\mathbf{F}_{DA}$ exerted on the pin by member AD. As indicated above, this force is equal and opposite to the force $\mathbf{F}_{AD}$ exerted by the same member on pin A. We can draw the force polygon corresponding to joint D, as shown in Fig. 6.8, and determine the forces

$\mathbf{F}_{DC}$ and $\mathbf{F}_{DB}$ from that polygon. However, when more than three forces are involved, it is usually more convenient to solve the equations of equilibrium $\Sigma F_x = 0$ and $\Sigma F_y = 0$ for the two unknown forces. Since both of these forces are found to be directed away from joint D, members DC and DB pull on the pin and are in tension.

Next, joint C is considered; its free-body diagram is shown in Fig. 6.8. It is noted that both $\mathbf{F}_{CD}$ and $\mathbf{F}_{CA}$ are known from the analysis of the preceding joints and that only $\mathbf{F}_{CB}$ is unknown. Since the equilibrium of each pin provides sufficient information to determine two unknowns, a check of our analysis is obtained at this joint. The force triangle is drawn, and the magnitude and sense of $\mathbf{F}_{CB}$ are determined. Since $\mathbf{F}_{CB}$ is directed toward joint C, member CB pushes on pin C and is in compression. The check is obtained by verifying that the force $\mathbf{F}_{CB}$ and member CB are parallel.

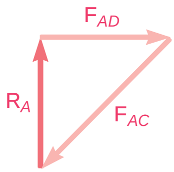


Fig. 6.9

At joint B, all of the forces are known. Since the corresponding pin is in equilibrium, the force triangle must close and an additional check of the analysis is obtained.

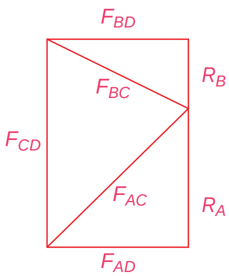


Fig. 6.10

It should be noted that the force polygons shown in Fig. 6.8 are not unique. Each of them could be replaced by an alternative configuration. For example, the force triangle corresponding to joint A could be drawn as shown in Fig. 6.9. The triangle actually shown in Fig. 6.8 was obtained by drawing the three forces $\mathbf{R}_A$, $\mathbf{F}_{AC}$, and $\mathbf{F}_{AD}$ in tip-to-tail fashion in the order in which their lines of action are encountered when moving clockwise around joint A. The other force polygons in Fig. 6.8, having been drawn in the same way, can be made to fit into a single diagram, as shown in Fig. 6.10. Such a diagram, known as *Maxwell's diagram,* greatly facilitates the *graphical analysis* of truss problems.

*6.5 JOINTS UNDER SPECIAL LOADING CONDITIONS

Consider Fig. 6.11*a*, in which the joint shown connects four members lying in two intersecting straight lines. The free-body diagram of Fig. 6.11*b* shows that pin A is subjected to two pairs of directly opposite forces. The corresponding force polygon, therefore, must be a parallelogram (Fig. 6.11*c*), and *the forces in opposite members must be equal*.

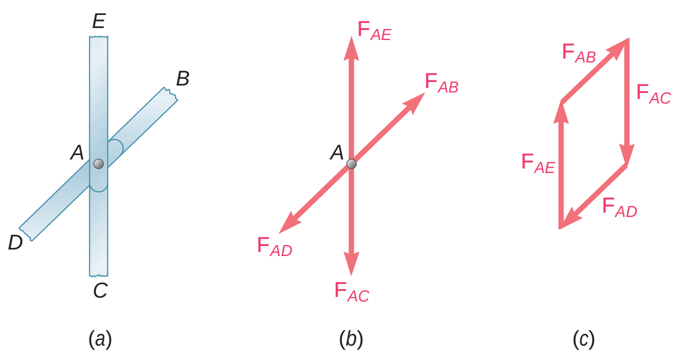


Fig. 6.11

Consider next Fig. 6.12*a*, in which the joint shown connects three members and supports a load **P**. Two of the members lie in the same line, and the load **P** acts along the third member. The free-body diagram of pin *A* and the corresponding force polygon will be as shown in Fig. 6.11*b* and *c*, with $\mathbf{F}_{AE}$ replaced by the load **P**. Thus, *the forces in the two opposite members must be equal, and the force in the other member must equal P*. A particular case of special interest is shown in Fig. 6.12*b*. Since, in this case, no external load is applied to the joint, we have $P = 0$, and the force in member *AC* is zero. Member *AC* is said to be a *zero-force member*.

Consider now a joint connecting two members only. From Sec. 2.9, we know that a particle which is acted upon by two forces will be in equilibrium if the two forces have the same magnitude, same line of action, and opposite sense. In the case of the joint of Fig. 6.13*a*, which connects two members *AB* and *AD* lying in the same line, *the forces in the two members must be equal* for pin *A* to be in equilibrium. In the case of the joint of Fig. 6.13*b*, pin *A* cannot be in equilibrium unless the forces in both members are zero. Members connected as shown in Fig. 6.13*b*, therefore, must be *zero-force members*.

Spotting the joints which are under the special loading conditions listed above will expedite the analysis of a truss. Consider, for example, a Howe truss loaded as shown in Fig. 6.14. All of the members represented by green lines will be recognized as zero-force members. Joint *C* connects three members, two of which lie in the same line, and is not subjected to any external load; member *BC* is thus a zero-force member. Applying the same reasoning to joint *K*, we find that member *JK* is also a zero-force member. But joint *J* is now in the same situation as joints *C* and *K*, and member *IJ* must be a zero-force member. The examination of joints *C*, *J*, and *K* also shows that the forces in members *AC* and *CE* are equal, that the forces in members *HJ* and *JL* are equal, and that the forces in members *IK* and *KL* are equal. Turning our attention to joint *I*, where the 20-kN load and member *HI* are collinear, we note that the force in member *HI* is 20 kN (tension) and that the forces in members *GI* and *IK* are equal. Hence, the forces in members *GI*, *IK*, and *KL* are equal.

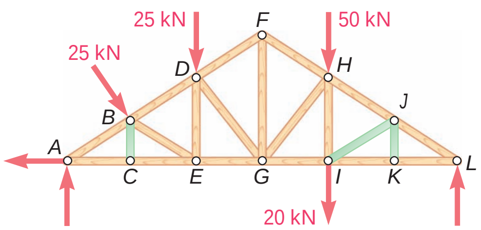


Fig. 6.14

Note that the conditions described above do not apply to joints *B* and *D* in Fig. 6.14, and it would be wrong to assume that the force in member *DE* is 25 kN or that the forces in members *AB* and *BD* are equal. The forces in these members and in all remaining members should be found by carrying out the analysis of joints *A*, *B*, *D*, *E*, *F*, *G*, *H*, and *L* in the usual manner. Thus, until you have become thoroughly familiar with the conditions under which the rules established in this

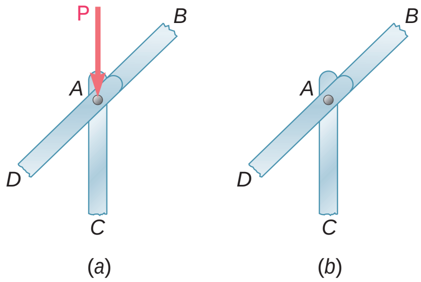


Fig. 6.12

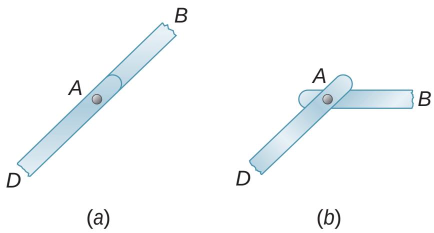


Fig. 6.13

Photo 6.4 Three-dimensional or space trusses are used for broadcast and power transmission line towers, roof framing, and spacecraft applications, such as components of the *International Space Station*.

section can be applied, you should draw the free-body diagrams of all pins and write the corresponding equilibrium equations (or draw the corresponding force polygons) whether or not the joints being considered are under one of the special loading conditions described above.

A final remark concerning zero-force members: These members are not useless. For example, although the zero-force members of Fig. 6.14 do not carry any loads under the loading conditions shown, the same members would probably carry loads if the loading conditions were changed. Besides, even in the case considered, these members are needed to support the weight of the truss and to maintain the truss in the desired shape.

*6.6 SPACE TRUSSES

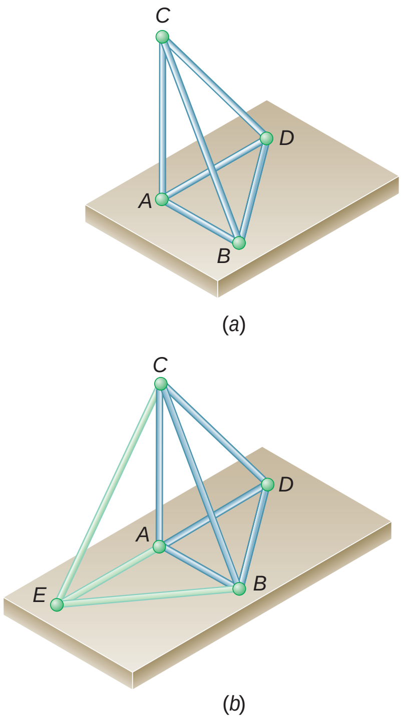


Fig. 6.15

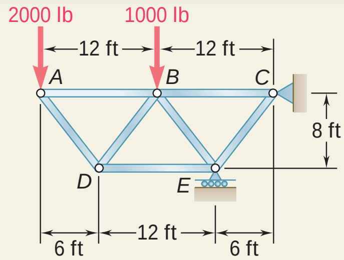
When several straight members are joined together at their extremities to form a three-dimensional configuration, the structure obtained is called a *space truss*.

We recall from Sec. 6.3 that the most elementary two-dimensional rigid truss consisted of three members joined at their extremities to form the sides of a triangle; by adding two members at a time to this basic configuration, and connecting them at a new joint, it was possible to obtain a larger rigid structure which was defined as a simple truss. Similarly, the most elementary rigid space truss consists of six members joined at their extremities to form the edges of a tetrahedron *ABCD* (Fig. 6.15*a*). By adding three members at a time to this basic configuration, such as *AE*, *BE*, and *CE*, attaching them to three existing joints, and connecting them at a new joint,† we can obtain a larger rigid structure which is defined as a *simple space truss* (Fig. 6.15*b*). Observing that the basic tetrahedron has six members and four joints and that every time three members are added, the number of joints is increased by one, we conclude that in a simple space truss the total number of members is $m = 3n - 6$, where n is the total number of joints.

If a space truss is to be completely constrained and if the reactions at its supports are to be statically determinate, the supports should consist of a combination of balls, rollers, and balls and sockets which provides six unknown reactions (see Sec. 4.9). These unknown reactions may be readily determined by solving the six equations expressing that the three-dimensional truss is in equilibrium.

Although the members of a space truss are actually joined together by means of bolted or welded connections, it is assumed that each joint consists of a ball-and-socket connection. Thus, no couple will be applied to the members of the truss, and each member can be treated as a two-force member. The conditions of equilibrium for each joint will be expressed by the three equations $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma F_z = 0$. In the case of a simple space truss containing n joints, writing the conditions of equilibrium for each joint will thus yield $3n$ equations. Since $m = 3n - 6$, these equations suffice to determine all unknown forces (forces in m members and six reactions at the supports). However, to avoid the necessity of solving simultaneous equations, care should be taken to select joints in such an order that no selected joint will involve more than three unknown forces.

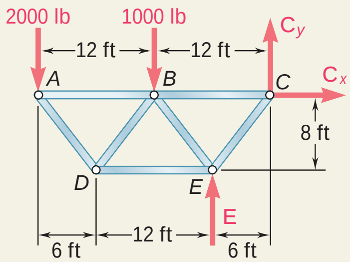
†The four joints must not lie in a plane.



SAMPLE PROBLEM 6.1

Using the method of joints, determine the force in each member of the truss shown.

SOLUTION



Free-Body: Entire Truss. A free-body diagram of the entire truss is drawn; external forces acting on this free body consist of the applied loads and the reactions at C and E. We write the following equilibrium equations.

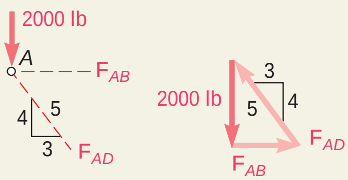
$$+\circlearrowleft \Sigma M_C = 0: \quad (2000 \text{ lb})(24 \text{ ft}) + (1000 \text{ lb})(12 \text{ ft}) - E(6 \text{ ft}) = 0$$

$$E = +10{,}000 \text{ lb} \qquad \mathbf{E} = 10{,}000 \text{ lb} \uparrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \qquad \mathbf{C}_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad -2000 \text{ lb} - 1000 \text{ lb} + 10{,}000 \text{ lb} + C_y = 0$$

$$C_y = -7000 \text{ lb} \qquad \mathbf{C}_y = 7000 \text{ lb} \downarrow$$

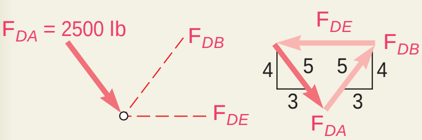


Free-Body: Joint *A*. This joint is subjected to only two unknown forces, namely, the forces exerted by members AB and AD. A force triangle is used to determine $\mathbf{F}_{AB}$ and $\mathbf{F}_{AD}$. We note that member AB pulls on the joint and thus is in tension and that member AD pushes on the joint and thus is in compression. The magnitudes of the two forces are obtained from the proportion

$$\frac{2000 \text{ lb}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$F_{AB} = 1500 \text{ lb } T \blacktriangleleft$$

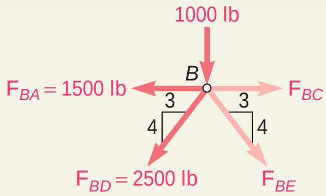
$$F_{AD} = 2500 \text{ lb } C \blacktriangleleft$$



Free-Body: Joint *D*. Since the force exerted by member AD has been determined, only two unknown forces are now involved at this joint. Again, a force triangle is used to determine the unknown forces in members DB and DE.

$$F_{DB} = F_{DA} \qquad F_{DB} = 2500 \text{ lb } T \blacktriangleleft$$

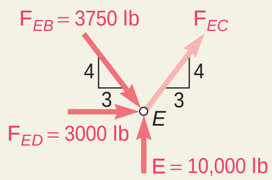
$$F_{DE} = 2(\tfrac{3}{5})F_{DA} \qquad F_{DE} = 3000 \text{ lb } C \blacktriangleleft$$



Free-Body: Joint *B*. Since more than three forces act at this joint, we determine the two unknown forces $\mathbf{F}_{BC}$ and $\mathbf{F}_{BE}$ by solving the equilibrium equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. We arbitrarily assume that both unknown forces act away from the joint, i.e., that the members are in tension. The positive value obtained for F_{BC} indicates that our assumption was correct; member BC is in tension. The negative value of F_{BE} indicates that our assumption was wrong; member BE is in compression.

$$+\uparrow \Sigma F_y = 0: \qquad -1000 - \tfrac{4}{5}(2500) - \tfrac{4}{5}F_{BE} = 0$$
$$F_{BE} = -3750 \text{ lb} \qquad F_{BE} = 3750 \text{ lb } C \quad ◀$$

$$\xrightarrow{+} \Sigma F_x = 0: \qquad F_{BC} - 1500 - \tfrac{3}{5}(2500) - \tfrac{3}{5}(3750) = 0$$
$$F_{BC} = +5250 \text{ lb} \qquad F_{BC} = 5250 \text{ lb } T \quad ◀$$

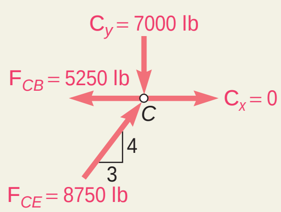


Free-Body: Joint *E*. The unknown force $\mathbf{F}_{EC}$ is assumed to act away from the joint. Summing x components, we write

$$\xrightarrow{+} \Sigma F_x = 0: \qquad \tfrac{3}{5}F_{EC} + 3000 + \tfrac{3}{5}(3750) = 0$$
$$F_{EC} = -8750 \text{ lb} \qquad F_{EC} = 8750 \text{ lb } C \quad ◀$$

Summing y components, we obtain a check of our computations:

$$\begin{aligned} +\uparrow \Sigma F_y &= 10{,}000 - \tfrac{4}{5}(3750) - \tfrac{4}{5}(8750) \\ &= 10{,}000 - 3000 - 7000 = 0 \qquad \text{(checks)} \end{aligned}$$



Free-Body: Joint *C*. Using the computed values of $\mathbf{F}_{CB}$ and $\mathbf{F}_{CE}$, we can determine the reactions $\mathbf{C}_x$ and $\mathbf{C}_y$ by considering the equilibrium of this joint. Since these reactions have already been determined from the equilibrium of the entire truss, we will obtain two checks of our computations. We can also simply use the computed values of all forces acting on the joint (forces in members and reactions) and check that the joint is in equilibrium:

$$\xrightarrow{+} \Sigma F_x = -5250 + \tfrac{3}{5}(8750) = -5250 + 5250 = 0 \qquad \text{(checks)}$$
$$+\uparrow \Sigma F_y = -7000 + \tfrac{4}{5}(8750) = -7000 + 7000 = 0 \qquad \text{(checks)}$$

SOLVING PROBLEMS ON YOUR OWN

In this lesson you learned to use the *method of joints* to determine the forces in the members of a *simple truss,* that is, a truss that can be constructed from a basic triangular truss by adding to it two new members at a time and connecting them at a new joint.

Your solution will consist of the following steps:

1. Draw a free-body diagram of the entire truss, and use this diagram to determine the reactions at the supports.

2. Locate a joint connecting only two members, and draw the free-body diagram of its pin. Use this free-body diagram to determine the unknown force in each of the two members. If only three forces are involved (the two unknown forces and a known one), you will probably find it more convenient to draw and solve the corresponding force triangle. If more than three forces are involved, you should write and solve the equilibrium equations for the pin, $\Sigma F_x = 0$ and $\Sigma F_y = 0$, assuming that the members are in tension. A positive answer means that the member is in tension, a negative answer that the member is in compression. Once the forces have been found, enter their values on a sketch of the truss, with T for tension and C for compression.

3. Next, locate a joint where the forces in only two of the connected members are still unknown. Draw the free-body diagram of the pin and use it as indicated above to determine the two unknown forces.

4. Repeat this procedure until the forces in all the members of the truss have been found. Since you previously used the three equilibrium equations associated with the free-body diagram of the entire truss to determine the reactions at the supports, you will end up with three extra equations. These equations can be used to check your computations.

5. Note that the choice of the first joint is not unique. Once you have determined the reactions at the supports of the truss, you can choose either of two joints as a starting point for your analysis. In Sample Prob. 6.1, we started at joint A and proceeded through joints D, B, E, and C, but we could also have started at joint C and proceeded through joints E, B, D, and A. On the other hand, having selected a first joint, you may in some cases reach a point in your analysis beyond which you cannot proceed. You must then start again from another joint to complete your solution.

Keep in mind that the analysis of a *simple truss* can always be carried out by the method of joints. Also remember that it is helpful to outline your solution *before* starting any computations.

PROBLEMS

6.1 through 6.8 Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

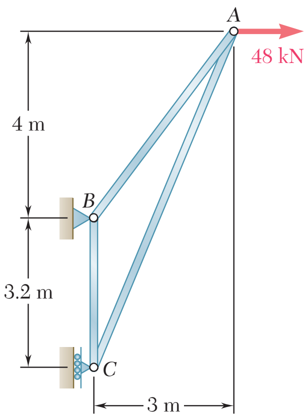


Fig. P6.1

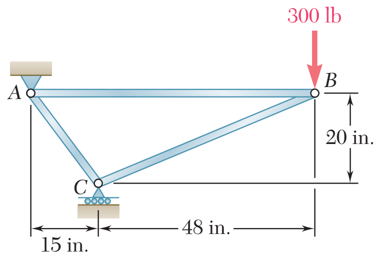


Fig. P6.2

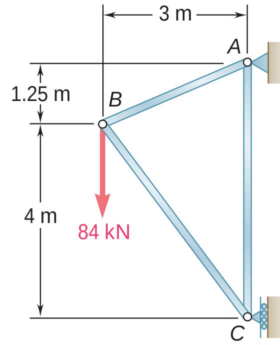


Fig. P6.3

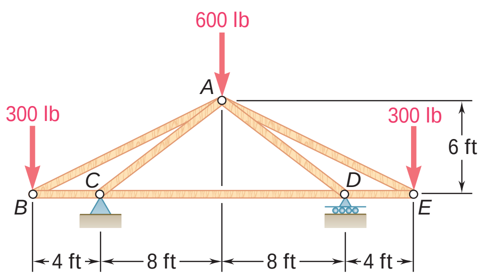


Fig. *P6.4*

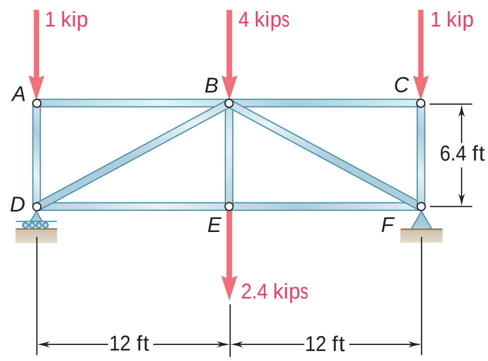


Fig. P6.5

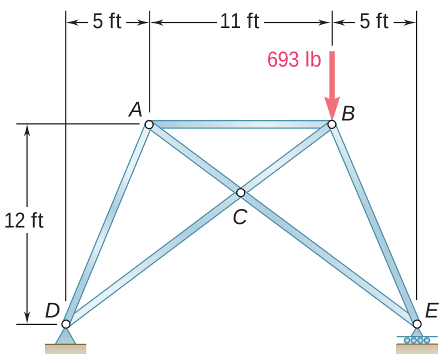


Fig. *P6.6*

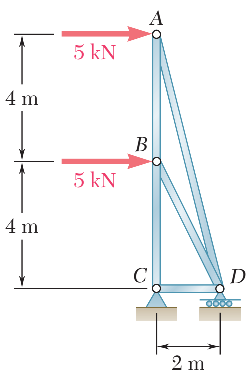


Fig. P6.7

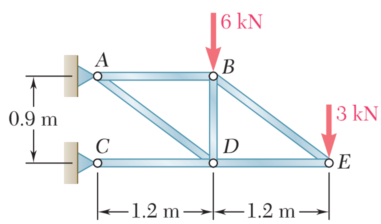


Fig. P6.8

6.9 Determine the force in each member of the Gambrel roof truss shown. State whether each member is in tension or compression.

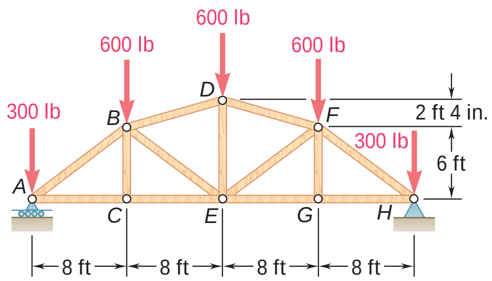


Fig. P6.9

6.10 Determine the force in each member of the Howe roof truss shown. State whether each member is in tension or compression.

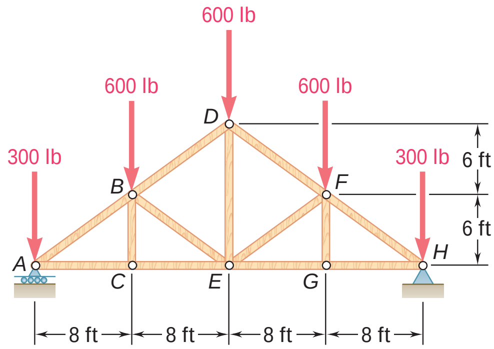


Fig. P6.10

6.11 Determine the force in each member of the Pratt roof truss shown. State whether each member is in tension or compression.

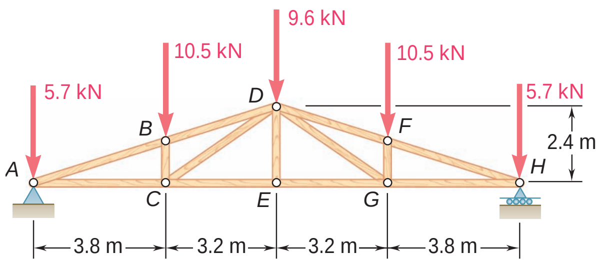


Fig. P6.11

6.12 Determine the force in each member of the Fink roof truss shown. State whether each member is in tension or compression.

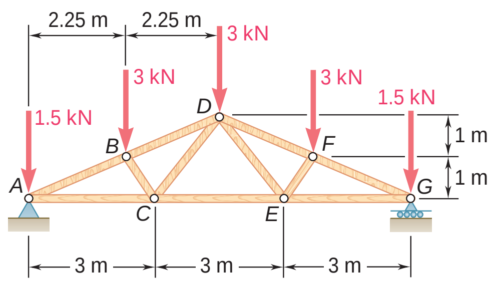


Fig. P6.12

6.13 Determine the force in each member of the double-pitch roof truss shown. State whether each member is in tension or compression.

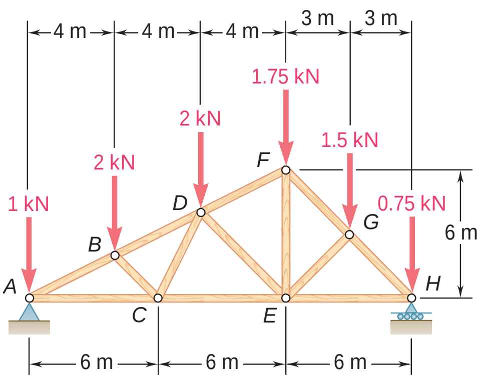


Fig. P6.13

6.14 The truss shown is one of several supporting an advertising panel. Determine the force in each member of the truss for a wind load equivalent to the two forces shown. State whether each member is in tension or compression.

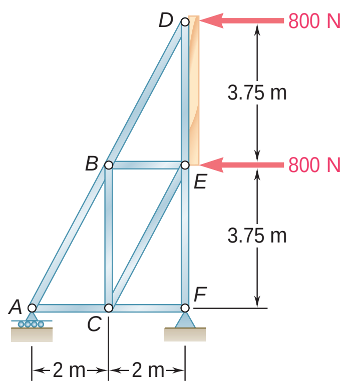


Fig. P6.14

6.15 Determine the force in each of the members located to the left of line *FGH* for the studio roof truss shown. State whether each member is in tension or compression.

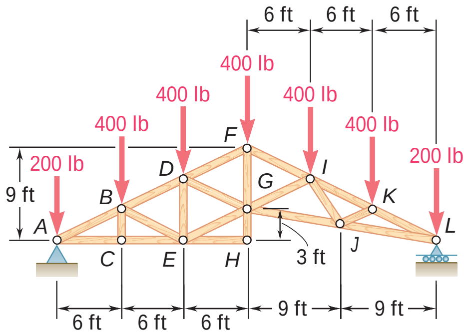


Fig. P6.15 and *P6.16*

6.16 Determine the force in member *FG* and in each of the members located to the right of *FG* for the studio roof truss shown. State whether each member is in tension or compression.

6.17 Determine the force in each of the members located to the left of *FG* for the scissors roof truss shown. State whether each member is in tension or compression.

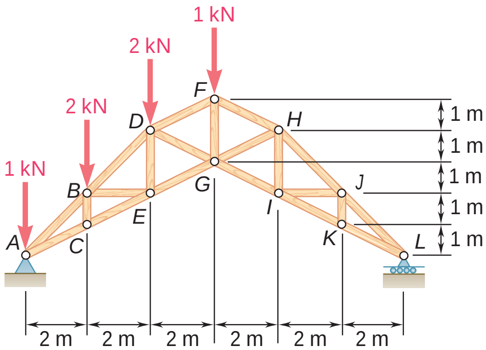


Fig. P6.17 and P6.18

6.18 Determine the force in member *FG* and in each of the members located to the right of *FG* for the scissors roof truss shown. State whether each member is in tension or compression.

6.19 Determine the force in each member of the Warren bridge truss shown. State whether each member is in tension or compression.

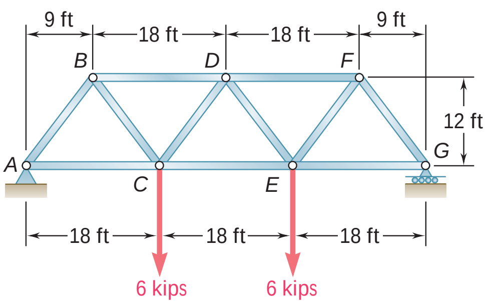


Fig. P6.19

6.20 Solve Prob. 6.19 assuming that the load applied at *E* has been removed.

6.21 Determine the force in each member of the Pratt bridge truss shown. State whether each member is in tension or compression.

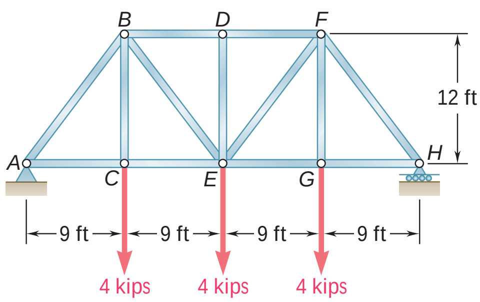


Fig. P6.21

6.22 Solve Prob. 6.21 assuming that the load applied at *G* has been removed.

6.23 The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

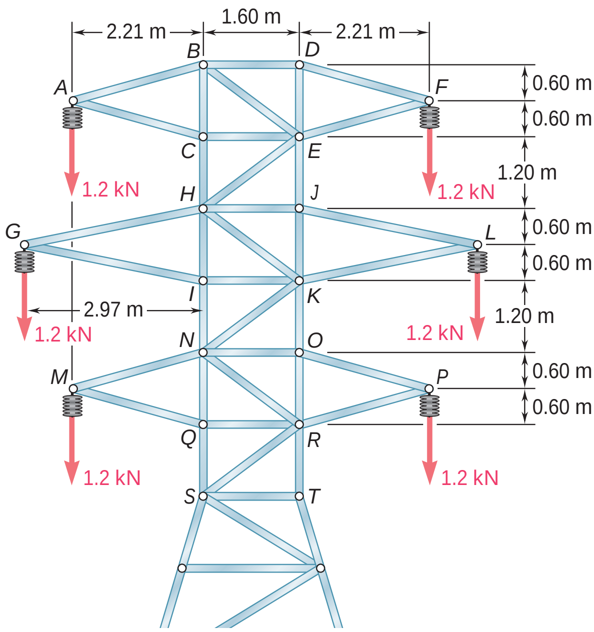


Fig. P6.23

6.24 For the tower and loading of Prob. 6.23 and knowing that $F_{CH} = F_{EJ} = 1.2$ kN *C* and $F_{EH} = 0$, determine the force in member *HJ* and in each of the members located between *HJ* and *NO*. State whether each member is in tension or compression.

6.25 Solve Prob. 6.23 assuming that the cables hanging from the right side of the tower have fallen to the ground.

6.26 Determine the force in each of the members connecting joints *A* through *F* of the vaulted roof truss shown. State whether each member is in tension or compression.

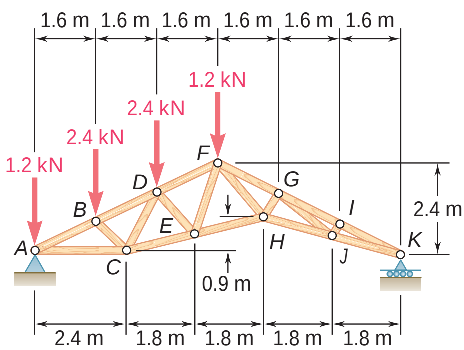


Fig. P6.26

6.27 Determine the force in each member of the truss shown. State whether each member is in tension or compression.

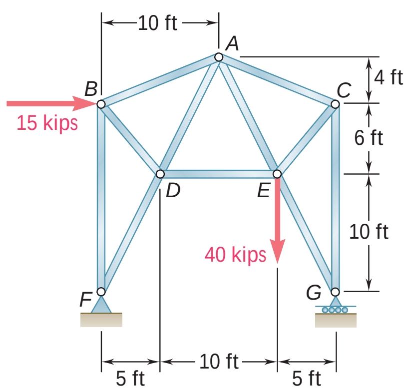


Fig. P6.27

6.28 Determine the force in each member of the truss shown. State whether each member is in tension or compression.

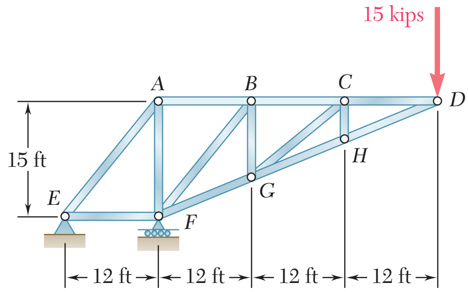


Fig. P6.28

6.29 Determine whether the trusses of Probs. 6.31*a*, 6.32*a*, and 6.33*a* are simple trusses.

6.30 Determine whether the trusses of Probs. 6.31*b*, 6.32*b*, and 6.33*b* are simple trusses.

6.31 For the given loading, determine the zero-force members in each of the two trusses shown.

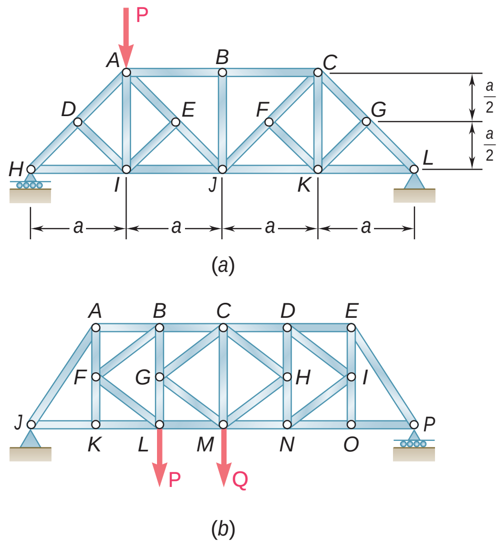


Fig. P6.31

6.32 For the given loading, determine the zero-force members in each of the two trusses shown.

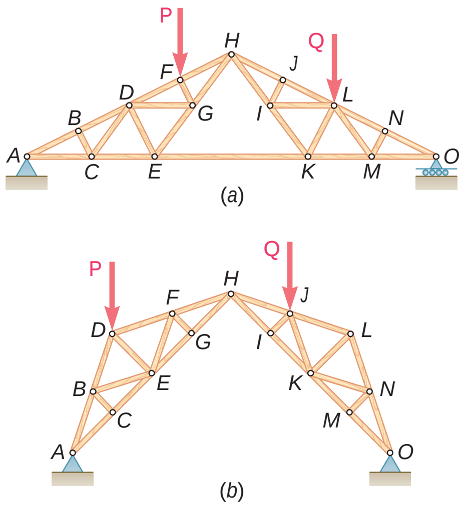


Fig. P6.32

6.33 For the given loading, determine the zero-force members in each of the two trusses shown.

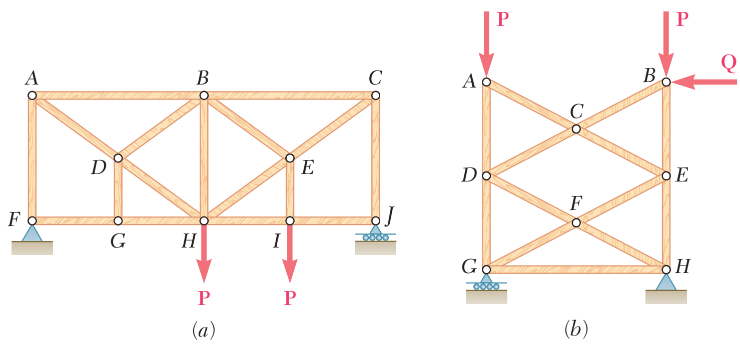


Fig. P6.33

6.34 Determine the zero-force members in the truss of (*a*) Prob. 6.26, (*b*) Prob. 6.28.

***6.35** The truss shown consists of six members and is supported by a short link at A, two short links at B, and a ball and socket at D. Determine the force in each of the members for the given loading.

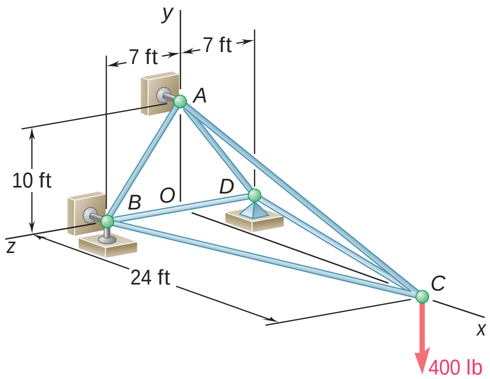


Fig. P6.35

***6.36** The truss shown consists of six members and is supported by a ball and socket at B, a short link at C, and two short links at D. Determine the force in each of the members for $\mathbf{P} = (-2184 \text{ N})\mathbf{j}$ and $\mathbf{Q} = 0$.

***6.37** The truss shown consists of six members and is supported by a ball and socket at B, a short link at C, and two short links at D. Determine the force in each of the members for $\mathbf{P} = 0$ and $\mathbf{Q} = (2968 \text{ N})\mathbf{i}$.

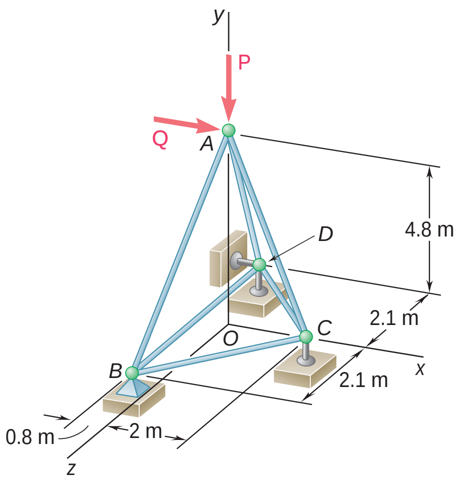


Fig. P6.36 and P6.37

***6.38** The truss shown consists of nine members and is supported by a ball and socket at A, two short links at B, and a short link at C. Determine the force in each of the members for the given loading.

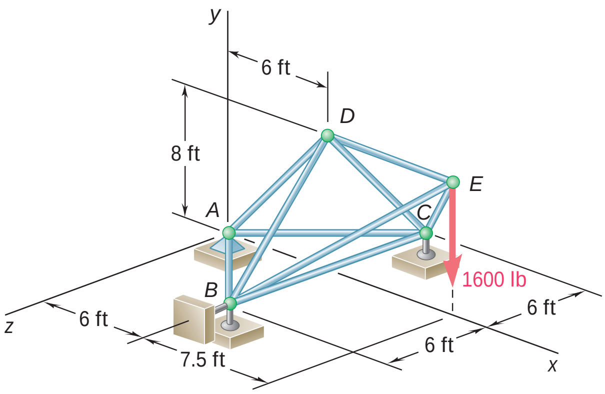


Fig. P6.38

***6.39** The truss shown consists of nine members and is supported by a ball and socket at B, a short link at C, and two short links at D. (*a*) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (*b*) Determine the force in each member for $\mathbf{P} = (-1200 \text{ N})\mathbf{j}$ and $\mathbf{Q} = 0$.

***6.40** Solve Prob. 6.39 for $\mathbf{P} = 0$ and $\mathbf{Q} = (-900 \text{ N})\mathbf{k}$.

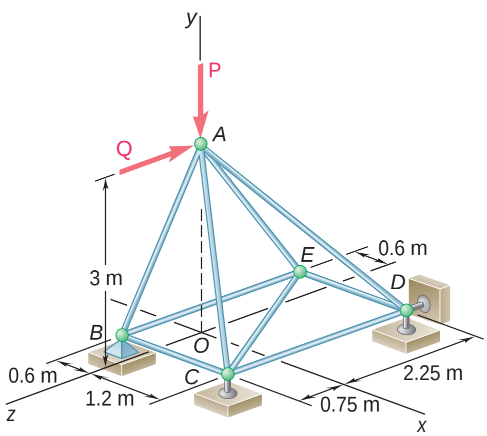


Fig. P6.39

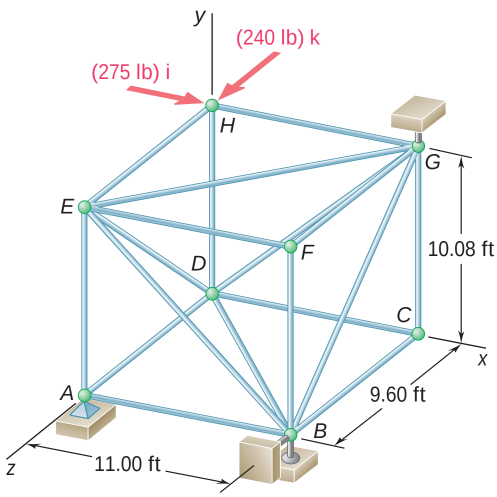


Fig. *P6.41* and *P6.42*

***6.41** The truss shown consists of 18 members and is supported by a ball and socket at A, two short links at B, and one short link at G. (*a*) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (*b*) For the given loading, determine the force in each of the six members joined at E.

***6.42** The truss shown consists of 18 members and is supported by a ball and socket at A, two short links at B, and one short link at G. (*a*) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (*b*) For the given loading, determine the force in each of the six members joined at G.

6.7 ANALYSIS OF TRUSSES BY THE METHOD OF SECTIONS

The method of joints is most effective when the forces in all the members of a truss are to be determined. If, however, the force in only one member or the forces in a very few members are desired, another method, the method of sections, is more efficient.

Assume, for example, that we want to determine the force in member BD of the truss shown in Fig. 6.16*a*. To do this, we must determine the force with which member BD acts on either joint B or joint D. If we were to use the method of joints, we would choose either joint B or joint D as a free body. However, we can also choose as a free body a larger portion of the truss, composed of several joints and members, provided that the desired force is one of the external forces acting on that portion. If, in addition, the portion of the truss is chosen so that there is a total of only three unknown forces acting upon it, the desired force can be obtained by solving the equations of equilibrium for this portion of the truss. In practice, the portion of the truss to be utilized is obtained by *passing a section* through three members of the truss, one of which is the desired member, i.e., by drawing a line which divides the truss into two completely separate parts but does not intersect more than three members. Either of the two portions of the truss obtained after the intersected members have been removed can then be used as a free body.†

In Fig. 6.16*a*, the section nn has been passed through members BD, BE, and CE, and the portion ABC of the truss is chosen as the free body (Fig. 6.16*b*). The forces acting on the free body are the loads $\mathbf{P}_1$ and $\mathbf{P}_2$ at points A and B and the three unknown forces $\mathbf{F}_{BD}$, $\mathbf{F}_{BE}$, and $\mathbf{F}_{CE}$. Since it is not known whether the members removed were in tension or compression, the three forces have been arbitrarily drawn away from the free body as if the members were in tension.

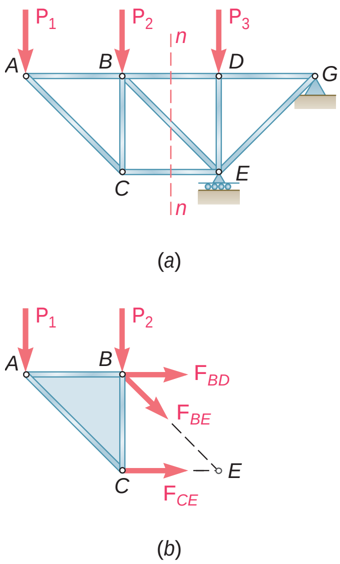


Fig. 6.16

†In the analysis of certain trusses, sections are passed which intersect more than three members; the forces in one, or possibly two, of the intersected members may be obtained if equilibrium equations can be found, each of which involves only one unknown (see Probs. 6.61 through 6.64).

The fact that the rigid body ABC is in equilibrium can be expressed by writing three equations which can be solved for the three unknown forces. If only the force $\mathbf{F}_{BD}$ is desired, we need write only one equation, provided that the equation does not contain the other unknowns. Thus the equation $\Sigma M_E = 0$ yields the value of the magnitude F_{BD} of the force $\mathbf{F}_{BD}$ (Fig. 6.16*b*). A positive sign in the answer will indicate that our original assumption regarding the sense of $\mathbf{F}_{BD}$ was correct and that member BD is in tension; a negative sign will indicate that our assumption was incorrect and that BD is in compression.

On the other hand, if only the force $\mathbf{F}_{CE}$ is desired, an equation which does not involve $\mathbf{F}_{BD}$ or $\mathbf{F}_{BE}$ should be written; the appropriate equation is $\Sigma M_B = 0$. Again a positive sign for the magnitude F_{CE} of the desired force indicates a correct assumption, that is, tension; and a negative sign indicates an incorrect assumption, that is, compression.

If only the force $\mathbf{F}_{BE}$ is desired, the appropriate equation is $\Sigma F_y = 0$. Whether the member is in tension or compression is again determined from the sign of the answer.

When the force in only one member is determined, no independent check of the computation is available. However, when all the unknown forces acting on the free body are determined, the computations can be checked by writing an additional equation. For instance, if $\mathbf{F}_{BD}$, $\mathbf{F}_{BE}$, and $\mathbf{F}_{CE}$ are determined as indicated above, the computation can be checked by verifying that $\Sigma F_x = 0$.

*6.8 TRUSSES MADE OF SEVERAL SIMPLE TRUSSES

Consider two simple trusses ABC and DEF. If they are connected by three bars BD, BE, and CE as shown in Fig. 6.17*a*, they will form together a rigid truss $ABDF$. The trusses ABC and DEF can also be combined into a single rigid truss by joining joints B and D into a single joint B and by connecting joints C and E by a bar CE (Fig. 6.17*b*). The truss thus obtained is known as a *Fink truss*. It should be noted that the trusses of Fig. 6.17*a* and *b* are *not* simple trusses; they cannot be constructed from a triangular truss by adding successive pairs of members as prescribed in Sec. 6.3. They are rigid trusses, however, as we can check by comparing the systems of connections used to hold the simple trusses ABC and DEF together (three bars in Fig. 6.17*a*, one pin and one bar in Fig. 6.17*b*) with the systems of supports discussed in Secs. 4.4 and 4.5. Trusses made of several simple trusses rigidly connected are known as *compound trusses*.

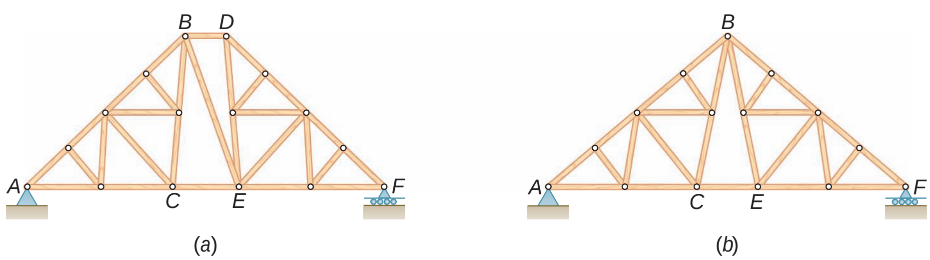
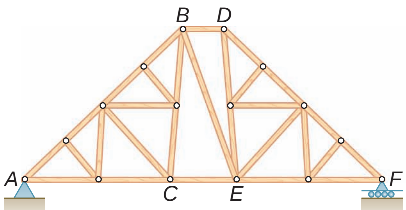


Fig. 6.17

In a compound truss the number of members m and the number of joints n are still related by the formula $m = 2n - 3$. This can be verified by observing that, if a compound truss is supported by a frictionless pin and a roller (involving three unknown reactions), the total number of unknowns is $m + 3$, and this number must be equal to the number $2n$ of equations obtained by expressing that the n pins are in equilibrium; it follows that $m = 2n - 3$. Compound trusses supported by a pin and a roller, or by an equivalent system of supports, are *statically determinate, rigid,* and *completely constrained*. This means that all of the unknown reactions and the forces in all the members can be determined by the methods of statics, and that the truss will neither collapse nor move. The forces in the members, however, cannot all be determined by the method of joints, except by solving a large number of simultaneous equations. In the case of the compound truss of Fig. 6.17*a*, for example, it is more efficient to pass a section through members BD, BE, and CE to determine the forces in these members.



(*a*)

Fig. 6.17 (*repeated*)

Suppose, now, that the simple trusses ABC and DEF are connected by *four* bars BD, BE, CD, or CE (Fig. 6.18). The number of members m is now larger than $2n - 3$; the truss obtained is *overrigid*, and one of the four members BD, BE, CD, or CE is said to be *redundant*. If the truss is supported by a pin at A and a roller at F, the total number of unknowns is $m + 3$. Since $m > 2n - 3$, the number $m + 3$ of unknowns is now larger than the number $2n$ of available independent equations; the truss is *statically indeterminate*.

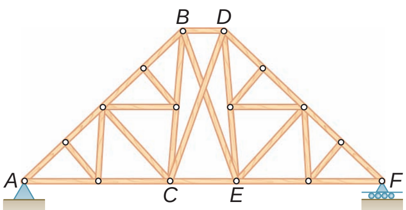
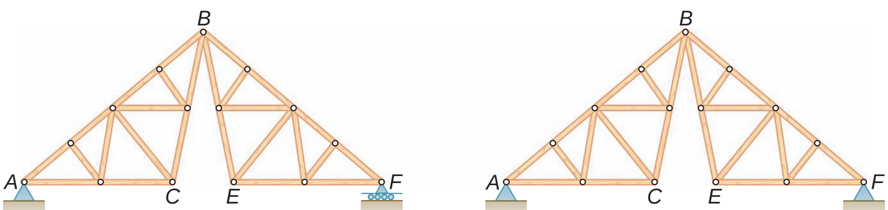


Fig. 6.18

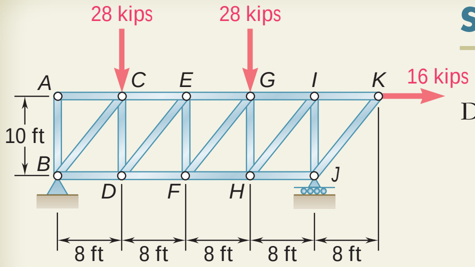
Finally, let us assume that the two simple trusses ABC and DEF are joined by a pin as shown in Fig. 6.19*a*. The number of members m is smaller than $2n - 3$. If the truss is supported by a pin at A and a roller at F, the total number of unknowns is $m + 3$. Since $m < 2n - 3$, the number $m + 3$ of unknowns is now smaller than the number $2n$ of equilibrium equations which should be satisfied; the truss is *nonrigid* and will collapse under its own weight. However, if two pins are used to support it, the truss becomes *rigid* and will not collapse (Fig. 6.19*b*). We note that the total number of unknowns is now $m + 4$ and is equal to the number $2n$ of equations. More generally, if the reactions at the supports involve r unknowns, the condition for a compound truss to be statically determinate, rigid, and completely constrained is $m + r = 2n$. However, while necessary this condition is not sufficient for the equilibrium of a structure which ceases to be rigid when detached from its supports (see Sec. 6.11).



(*a*) (*b*)

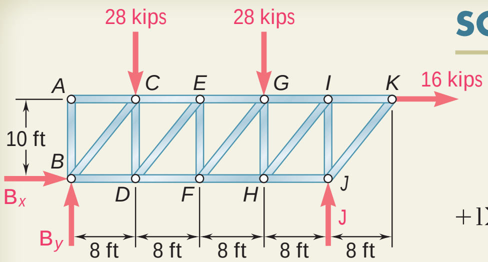
Fig. 6.19

SAMPLE PROBLEM 6.2



Determine the force in members *EF* and *GI* of the truss shown.

SOLUTION

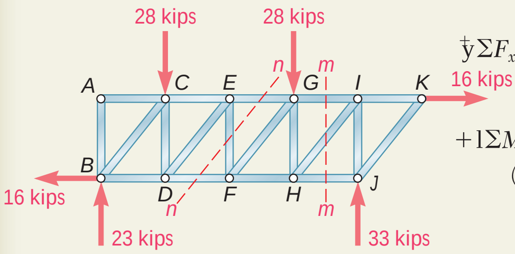


Free-Body: Entire Truss. A free-body diagram of the entire truss is drawn; external forces acting on this free body consist of the applied loads and the reactions at *B* and *J*. We write the following equilibrium equations.

$+\circlearrowleft \Sigma M_B = 0$:

$$-(28 \text{ kips})(8 \text{ ft}) - (28 \text{ kips})(24 \text{ ft}) - (16 \text{ kips})(10 \text{ ft}) + J(32 \text{ ft}) = 0$$

$$J = +33 \text{ kips} \qquad \mathbf{J} = 33 \text{ kips} \uparrow$$



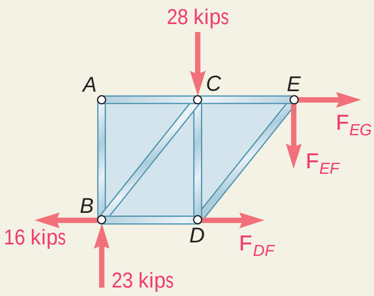
$\xrightarrow{+} \Sigma F_x = 0$: $\qquad B_x + 16 \text{ kips} = 0$

$$B_x = -16 \text{ kips} \qquad \mathbf{B}_x = 16 \text{ kips} \leftarrow$$

$+\circlearrowleft \Sigma M_J = 0$:

$$(28 \text{ kips})(24 \text{ ft}) + (28 \text{ kips})(8 \text{ ft}) - (16 \text{ kips})(10 \text{ ft}) - B_y(32 \text{ ft}) = 0$$

$$B_y = +23 \text{ kips} \qquad \mathbf{B}_y = 23 \text{ kips} \uparrow$$



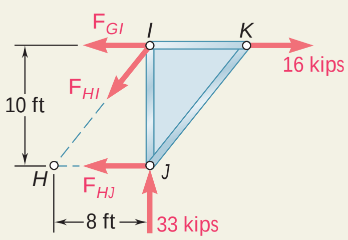
Force in Member *EF*. Section *nn* is passed through the truss so that it intersects member *EF* and only two additional members. After the intersected members have been removed, the left-hand portion of the truss is chosen as a free body. Three unknowns are involved; to eliminate the two horizontal forces, we write

$+\uparrow \Sigma F_y = 0$: $\qquad +23 \text{ kips} - 28 \text{ kips} - F_{EF} = 0$

$$F_{EF} = -5 \text{ kips}$$

The sense of $\mathbf{F}_{EF}$ was chosen assuming member *EF* to be in tension; the negative sign obtained indicates that the member is in compression.

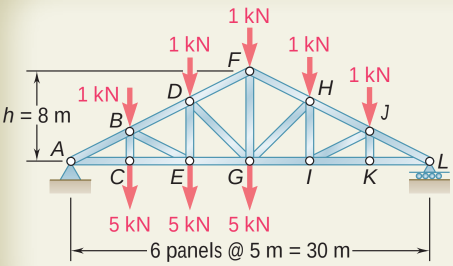
$$F_{EF} = 5 \text{ kips } C \quad \blacktriangleleft$$



Force in Member *GI*. Section *mm* is passed through the truss so that it intersects member *GI* and only two additional members. After the intersected members have been removed, we choose the right-hand portion of the truss as a free body. Three unknown forces are again involved; to eliminate the two forces passing through point *H*, we write

$+\circlearrowleft \Sigma M_H = 0$: $\qquad (33 \text{ kips})(8 \text{ ft}) - (16 \text{ kips})(10 \text{ ft}) + F_{GI}(10 \text{ ft}) = 0$

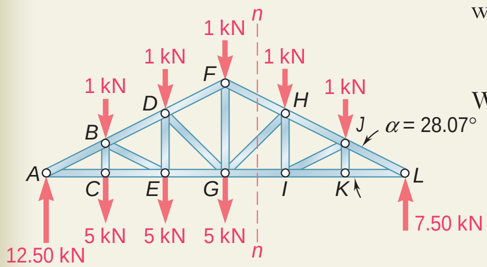
$$F_{GI} = -10.4 \text{ kips} \qquad F_{GI} = 10.4 \text{ kips } C \quad \blacktriangleleft$$



SAMPLE PROBLEM 6.3

Determine the force in members *FH*, *GH*, and *GI* of the roof truss shown.

SOLUTION

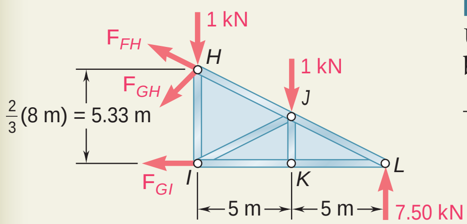


Free Body: Entire Truss. From the free-body diagram of the entire truss, we find the reactions at *A* and *L*:

$$\mathbf{A} = 12.50 \text{ kN}\uparrow \qquad \mathbf{L} = 7.50 \text{ kN}\uparrow$$

We note that

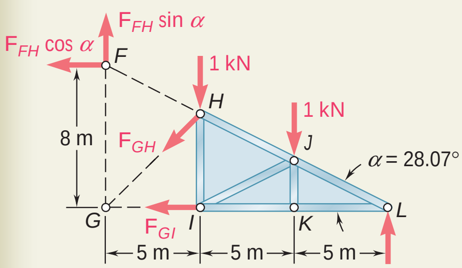
$$\tan \alpha = \frac{FG}{GL} = \frac{8 \text{ m}}{15 \text{ m}} = 0.5333 \qquad \alpha = 28.07°$$



Force in Member *GI*. Section *nn* is passed through the truss as shown. Using the portion *HLI* of the truss as a free body, the value of F_{GI} is obtained by writing

$$+\circlearrowleft \Sigma M_H = 0: \qquad (7.50 \text{ kN})(10 \text{ m}) - (1 \text{ kN})(5 \text{ m}) - F_{GI}(5.33 \text{ m}) = 0$$

$$F_{GI} = +13.13 \text{ kN} \qquad F_{GI} = 13.13 \text{ kN } T$$

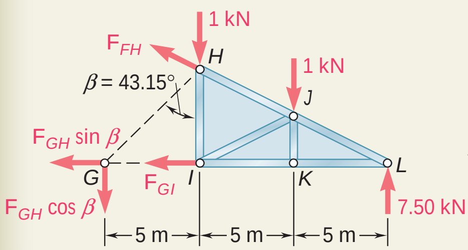


Force in Member *FH*. The value of F_{FH} is obtained from the equation $\Sigma M_G = 0$. We move $\mathbf{F}_{FH}$ along its line of action until it acts at point *F*, where it is resolved into its *x* and *y* components. The moment of $\mathbf{F}_{FH}$ with respect to point *G* is now equal to $(F_{FH} \cos \alpha)(8 \text{ m})$.

$$+\circlearrowleft \Sigma M_G = 0:$$

$$(7.50 \text{ kN})(15 \text{ m}) - (1 \text{ kN})(10 \text{ m}) - (1 \text{ kN})(5 \text{ m}) + (F_{FH} \cos \alpha)(8 \text{ m}) = 0$$

$$F_{FH} = -13.81 \text{ kN} \qquad F_{FH} = 13.81 \text{ kN } C$$



Force in Member *GH*. We first note that

$$\tan \beta = \frac{GI}{HI} = \frac{5 \text{ m}}{\frac{2}{3}(8 \text{ m})} = 0.9375 \qquad \beta = 43.15°$$

The value of F_{GH} is then determined by resolving the force $\mathbf{F}_{GH}$ into *x* and *y* components at point *G* and solving the equation $\Sigma M_L = 0$.

$$+\circlearrowleft \Sigma M_L = 0: \qquad (1 \text{ kN})(10 \text{ m}) + (1 \text{ kN})(5 \text{ m}) + (F_{GH} \cos \beta)(15 \text{ m}) = 0$$

$$F_{GH} = -1.371 \text{ kN} \qquad F_{GH} = 1.371 \text{ kN } C$$

SOLVING PROBLEMS ON YOUR OWN

The *method of joints* that you studied earlier is usually the best method to use when the forces *in all the members* of a simple truss are to be found. However, the method of sections, which was covered in this lesson, is more effective when the force *in only one member* or the forces *in a very few members* of a simple truss are desired. The method of sections must also be used when the truss *is not a simple truss*.

A. To determine the force in a given truss member by the method of sections, you should follow these steps:

1. Draw a free-body diagram of the entire truss, and use this diagram to determine the reactions at the supports.

2. Pass a section through three members of the truss, one of which is the desired member. After you have removed these members, you will obtain two separate portions of truss.

3. Select one of the two portions of truss you have obtained, and draw its free-body diagram. This diagram should include the external forces applied to the selected portion as well as the forces exerted on it by the intersected members before these members were removed.

4. You can now write three equilibrium equations which can be solved for the forces in the three intersected members.

5. An alternative approach is to write a single equation, which can be solved for the force in the desired member. To do so, first observe whether the forces exerted by the other two members on the free body are parallel or whether their lines of action intersect.

a. If these forces are parallel, they can be eliminated by writing an equilibrium equation involving *components in a direction perpendicular* to these two forces.

b. If their lines of action intersect at a point H, they can be eliminated by writing an equilibrium equation involving *moments about* H.

6. Keep in mind that the section you use must intersect three members only. This is because the equilibrium equations in step 4 can be solved for three unknowns only. However, you can pass a section through more than three members to find the force in one of those members if you can write an equilibrium equation containing only that force as an unknown. Such special situations are found in Probs. 6.61 through 6.64.

(continued)

B. About completely constrained and determinate trusses:

1. First note that any simple truss which is simply supported is a completely constrained and determinate truss.

2. To determine whether any other truss is or is not completely constrained and determinate, you first count the number m of its members, the number n of its joints, and the number r of the reaction components at its supports. You then compare the sum $m + r$ representing the number of unknowns and the product $2n$ representing the number of available independent equilibrium equations.

a. If $m + r < 2n$, there are fewer unknowns than equations. Thus, some of the equations cannot be satisfied; the truss is only *partially constrained*.

b. If $m + r > 2n$, there are more unknowns than equations. Thus, some of the unknowns cannot be determined; the truss is *indeterminate*.

c. If $m + r = 2n$, there are as many unknowns as there are equations. This, however, does not mean that all the unknowns can be determined and that all the equations can be satisfied. To find out whether the truss is *completely* or *improperly constrained,* you should try to determine the reactions at its supports and the forces in its members. If all can be found, the truss is *completely constrained and determinate*.

PROBLEMS

6.43 Determine the force in members *CD* and *DF* of the truss shown.

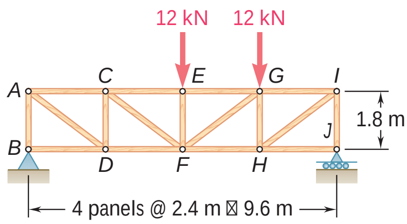


Fig. P6.43 and P6.44

6.44 Determine the force in members *FG* and *FH* of the truss shown.

6.45 A Warren bridge truss is loaded as shown. Determine the force in members *CE*, *DE*, and *DF*.

6.46 A Warren bridge truss is loaded as shown. Determine the force in members *EG*, *FG*, and *FH*.

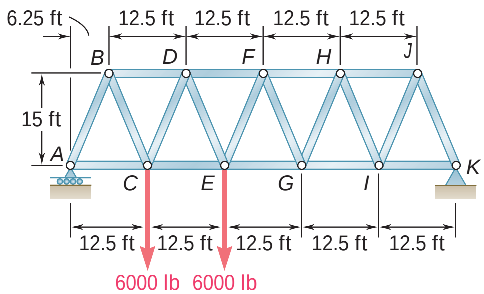


Fig. P6.45 and P6.46

6.47 Determine the force in members *DF*, *EF*, and *EG* of the truss shown.

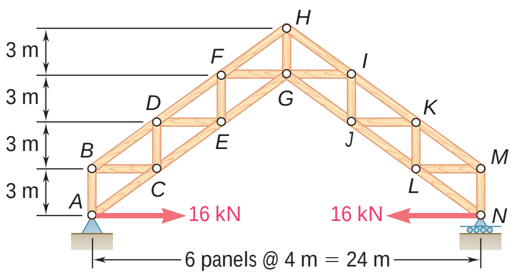


Fig. *P6.47* and *P6.48*

6.48 Determine the force in members *GI*, *GJ*, and *HI* of the truss shown.

6.49 Determine the force in members *AD*, *CD*, and *CE* of the truss shown.

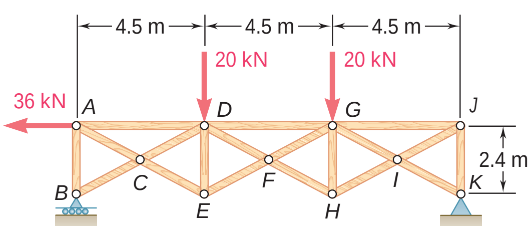


Fig. P6.49 and P6.50

6.50 Determine the force in members *DG*, *FG*, and *FH* of the truss shown.

6.51 A stadium roof truss is loaded as shown. Determine the force in members AB, AG, and FG.

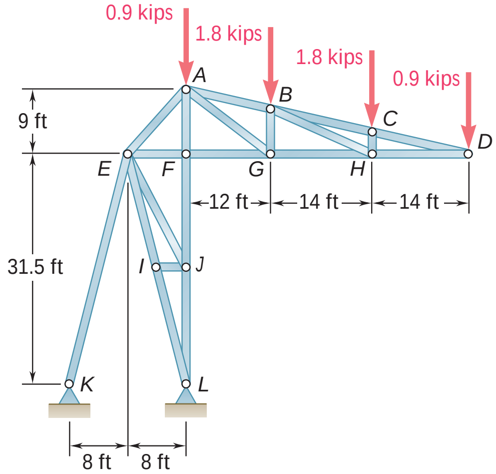


Fig. P6.51 and P6.52

6.52 A stadium roof truss is loaded as shown. Determine the force in members AE, EF, and FJ.

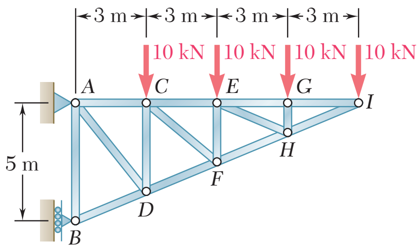


Fig. P6.53 and P6.54

6.53 Determine the force in members CD and DF of the truss shown.

6.54 Determine the force in members CE and EF of the truss shown.

6.55 The truss shown was designed to support the roof of a food market. For the given loading, determine the force in members FG, EG, and EH.

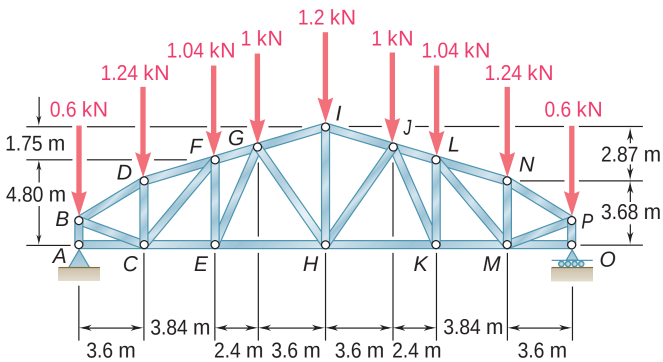


Fig. P6.55 and P6.56

6.56 The truss shown was designed to support the roof of a food market. For the given loading, determine the force in members KM, LM, and LN.

6.57 A Polynesian, or duopitch, roof truss is loaded as shown. Determine the force in members *DF*, *EF*, and *EG*.

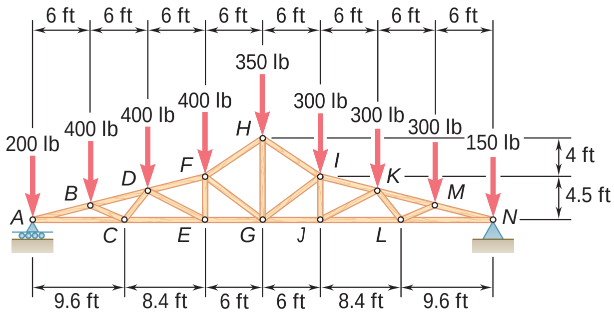


Fig. *P6.57* and *P6.58*

6.58 A Polynesian, or duopitch, roof truss is loaded as shown. Determine the force in members *HI*, *GI*, and *GJ*.

6.59 Determine the force in members *DE* and *DF* of the truss shown when $P = 20$ kips.

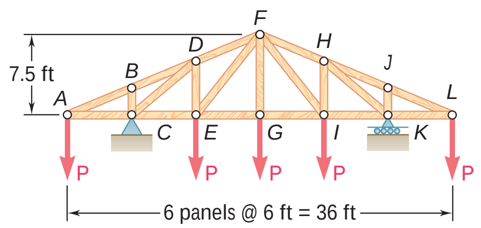


Fig. P6.59 and P6.60

6.60 Determine the force in members *EG* and *EF* of the truss shown when $P = 20$ kips.

6.61 Determine the force in members *EH* and *GI* of the truss shown. (*Hint:* Use section *aa*.)

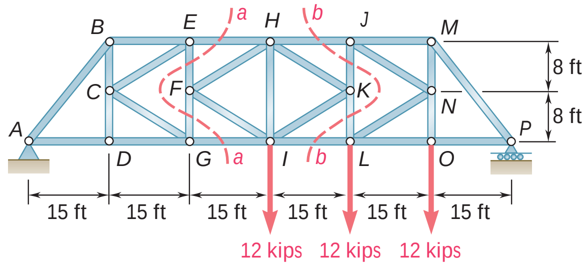


Fig. *P6.61* and *P6.62*

6.62 Determine the force in members *HJ* and *IL* of the truss shown. (*Hint:* Use section *bb*.)

6.63 Determine the force in members *DG* and *FI* of the truss shown. (*Hint:* Use section *aa*.)

6.64 Determine the force in members *GJ* and *IK* of the truss shown. (*Hint:* Use section *bb*.)

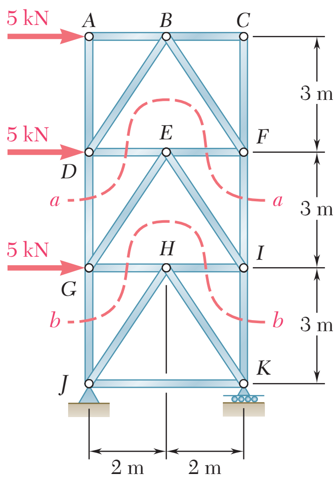


Fig. P6.63 and P6.64

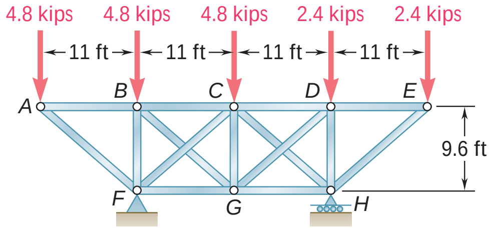


Fig. P6.65

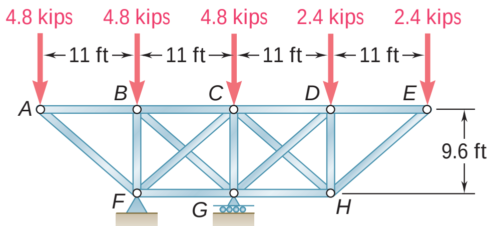


Fig. P6.66

6.65 and 6.66 The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the forces in the counters that are acting under the given loading.

6.67 and 6.68 The diagonal members in the center panels of the power transmission line tower shown are very slender and can act only in tension; such members are known as *counters*. For the given loading, determine (*a*) which of the two counters listed below is acting, (*b*) the force in that counter.

6.67 Counters *CJ* and *HE*.

6.68 Counters *IO* and *KN*.

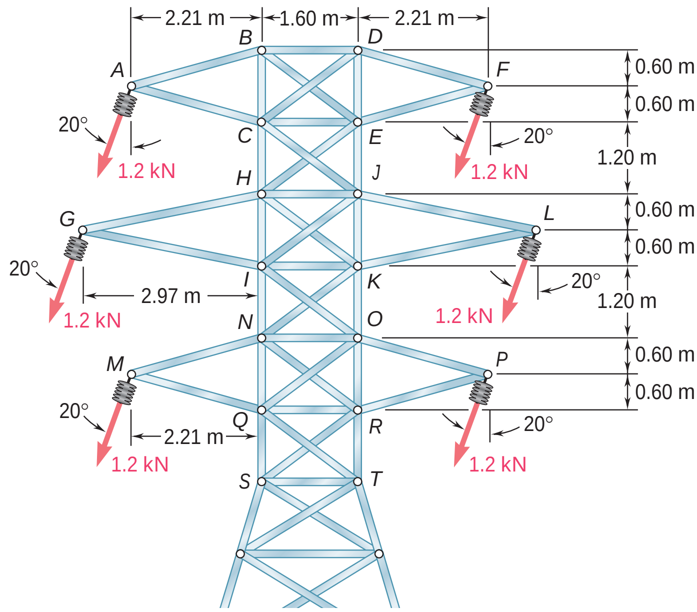


Fig. P6.67 and P6.68

6.69 Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)

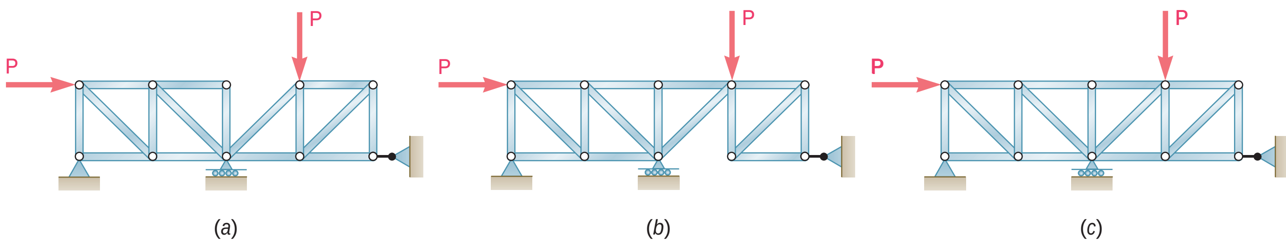


Fig. P6.69

6.70 through 6.74 Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)

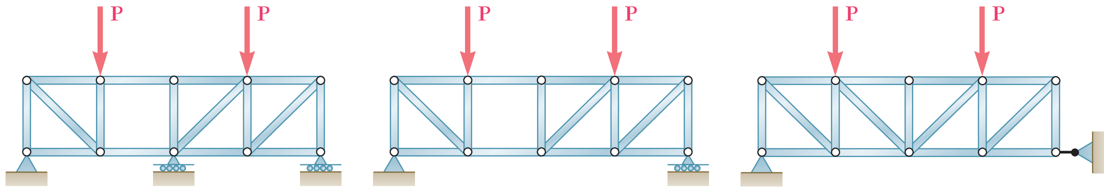


Fig. P6.70 (*a*) (*b*) (*c*)

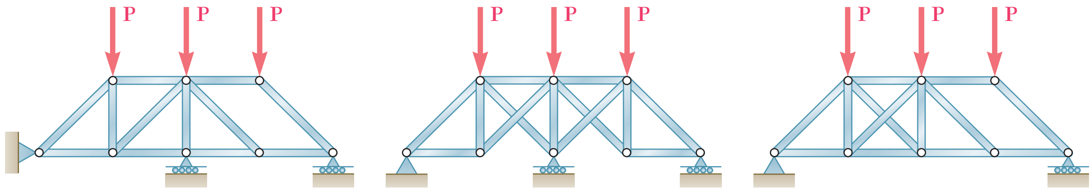


Fig. P6.71 (*a*) (*b*) (*c*)

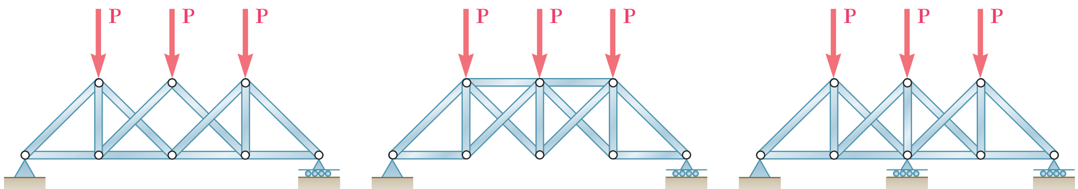


Fig. P6.72 (*a*) (*b*) (*c*)

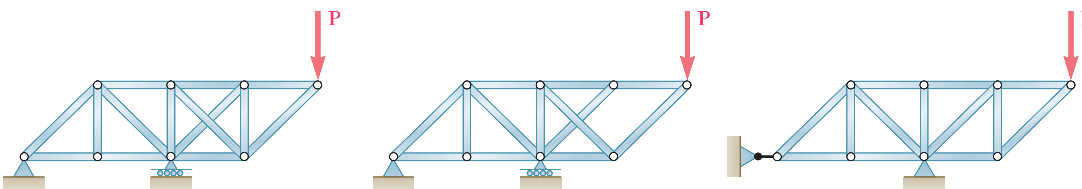


Fig. *P6.73* (*a*) (*b*) (*c*)

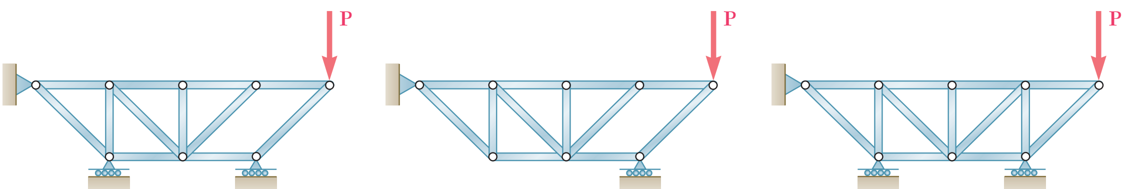


Fig. *P6.74* (*a*) (*b*) (*c*)

FRAMES AND MACHINES

6.9 STRUCTURES CONTAINING MULTIFORCE MEMBERS

Under trusses, we have considered structures consisting entirely of pins and straight two-force members. The forces acting on the two-force members were known to be directed along the members themselves. We now consider structures in which at least one of the members is a *multiforce* member, i.e., a member acted upon by three or more forces. These forces will generally not be directed along the members on which they act; their direction is unknown, and they should be represented therefore by two unknown components.

Frames and machines are structures containing multiforce members. *Frames* are designed to support loads and are usually stationary, fully constrained structures. *Machines* are designed to transmit and modify forces; they may or may not be stationary and will always contain moving parts.

6.10 ANALYSIS OF A FRAME

As a first example of analysis of a frame, the crane described in Sec. 6.1, which carries a given load W (Fig. 6.20*a*), will again be considered. The free-body diagram of the entire frame is shown in Fig. 6.20*b*. This diagram can be used to determine the external forces acting on the frame. Summing moments about A, we first determine the force $\mathbf{T}$ exerted by the cable; summing x and y components, we then determine the components $\mathbf{A}_x$ and $\mathbf{A}_y$ of the reaction at the pin A.

In order to determine the internal forces holding the various parts of a frame together, we must dismember the frame and draw a free-body diagram for each of its component parts (Fig. 6.20*c*). First, the two-force members should be considered. In this frame, member BE is the only two-force member. The forces acting at each end of this member must have the same magnitude, same line of action, and opposite sense (Sec. 4.6). They are therefore directed along BE and will be denoted, respectively, by $\mathbf{F}_{BE}$ and $-\mathbf{F}_{BE}$. Their sense will be arbitrarily assumed as shown in Fig. 6.20*c*; later the sign obtained for the common magnitude F_{BE} of the two forces will confirm or deny this assumption.

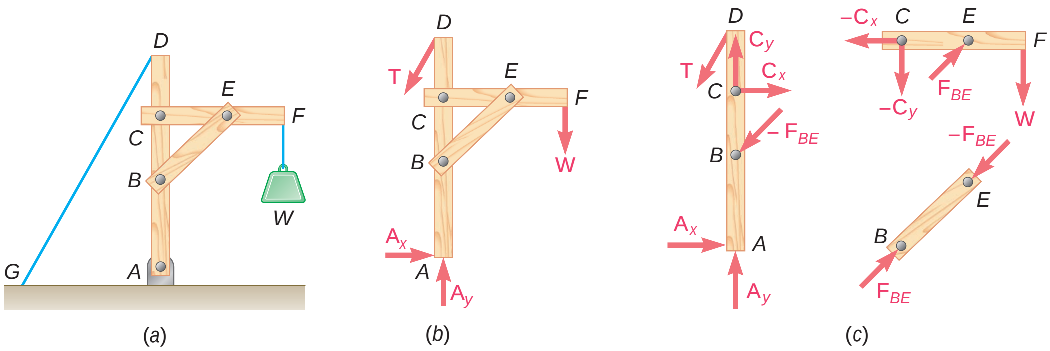


Fig. 6.20

Next, we consider the multiforce members, i.e., the members which are acted upon by three or more forces. According to Newton's third law, the force exerted at B by member BE on member AD must be equal and opposite to the force $\mathbf{F}_{BE}$ exerted by AD on BE. Similarly, the force exerted at E by member BE on member CF must be equal and opposite to the force $-\mathbf{F}_{BE}$ exerted by CF on BE. Thus the forces that the two-force member BE exerts on AD and CF are, respectively, equal to $-\mathbf{F}_{BE}$ and $\mathbf{F}_{BE}$; they have the same magnitude F_{BE} and opposite sense, and should be directed as shown in Fig. 6.20*c*.

At C two multiforce members are connected. Since neither the direction nor the magnitude of the forces acting at C is known, these forces will be represented by their x and y components. The components $\mathbf{C}_x$ and $\mathbf{C}_y$ of the force acting on member AD will be arbitrarily directed to the right and upward. Since, according to Newton's third law, the forces exerted by member CF on AD and by member AD on CF are equal and opposite, the components of the force acting on member CF *must* be directed to the left and downward; they will be denoted, respectively, by $-\mathbf{C}_x$ and $-\mathbf{C}_y$. Whether the force $\mathbf{C}_x$ is actually directed to the right and the force $-\mathbf{C}_x$ is actually directed to the left will be determined later from the sign of their common magnitude C_x, a plus sign indicating that the assumption made was correct, and a minus sign that it was wrong. The free-body diagrams of the multiforce members are completed by showing the external forces acting at A, D, and F.†

The internal forces can now be determined by considering the free-body diagram of either of the two multiforce members. Choosing the free-body diagram of CF, for example, we write the equations $\Sigma M_C = 0$, $\Sigma M_E = 0$, and $\Sigma F_x = 0$, which yield the values of the magnitudes F_{BE}, C_y, and C_x, respectively. These values can be checked by verifying that member AD is also in equilibrium.

It should be noted that the pins in Fig. 6.20 were assumed to form an integral part of one of the two members they connected and so it was not necessary to show their free-body diagram. This assumption can always be used to simplify the analysis of frames and machines. When a pin connects three or more members, however, or when a pin connects a support and two or more members, or when a load is applied to a pin, a clear decision must be made in choosing the member to which the pin will be assumed to belong. (If multiforce members are involved, the pin should be attached to one of these members.) The various forces exerted on the pin should then be clearly identified. This is illustrated in Sample Prob. 6.6.

†It is not strictly necessary to use a minus sign to distinguish the force exerted by one member on another from the equal and opposite force exerted by the second member on the first, since the two forces belong to different free-body diagrams and thus cannot easily be confused. In the Sample Problems, the same symbol is used to represent equal and opposite forces which are applied to different free bodies. It should be noted that, under these conditions, the sign obtained for a given force component will not directly relate the sense of that component to the sense of the corresponding coordinate axis. Rather, a positive sign will indicate that *the sense assumed for that component in the free-body diagram* is correct, and a negative sign will indicate that it is wrong.

6.11 FRAMES WHICH CEASE TO BE RIGID WHEN DETACHED FROM THEIR SUPPORTS

The crane analyzed in Sec. 6.10 was so constructed that it could keep the same shape without the help of its supports; it was therefore considered as a rigid body. Many frames, however, will collapse if detached from their supports; such frames cannot be considered as rigid bodies. Consider, for example, the frame shown in Fig. 6.21*a*, which consists of two members AC and CB carrying loads $\mathbf{P}$ and $\mathbf{Q}$ at their midpoints; the members are supported by pins at A and B and are connected by a pin at C. If detached from its supports, this frame will not maintain its shape; it should therefore be considered as made of *two distinct rigid parts* AC and CB.

The equations $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$ (about any given point) express the conditions for the *equilibrium of a rigid body* (Chap. 4); we should use them, therefore, in connection with the free-body diagrams of rigid bodies, namely, the free-body diagrams of members AC and CB (Fig. 6.21*b*). Since these members are multiforce members, and since pins are used at the supports and at the connection, the reactions at A and B and the forces at C will each be represented by two components. In accordance with Newton's third law, the components of the force exerted by CB on AC and the components of the force exerted by AC on CB will be represented by vectors of the same magnitude and opposite sense; thus, if the first pair of components consists of $\mathbf{C}_x$ and $\mathbf{C}_y$, the second pair will be represented by $-\mathbf{C}_x$ and $-\mathbf{C}_y$. We note that four unknown force components act on free body AC, while only three independent equations can be used to express that the body is in equilibrium; similarly, four unknowns, but only three equations, are associated with CB. However, only six different unknowns are involved in the analysis of the two members, and altogether six equations are available to express that the members are in equilibrium. Writing $\Sigma M_A = 0$ for free body AC and $\Sigma M_B = 0$ for CB, we obtain two simultaneous equations which may be solved for the common magnitude C_x of the components $\mathbf{C}_x$ and $-\mathbf{C}_x$, and for the common magnitude C_y of the components $\mathbf{C}_y$ and $-\mathbf{C}_y$. We then write $\Sigma F_x = 0$ and $\Sigma F_y = 0$ for each of the two free bodies, obtaining, successively, the magnitudes A_x, A_y, B_x, and B_y.

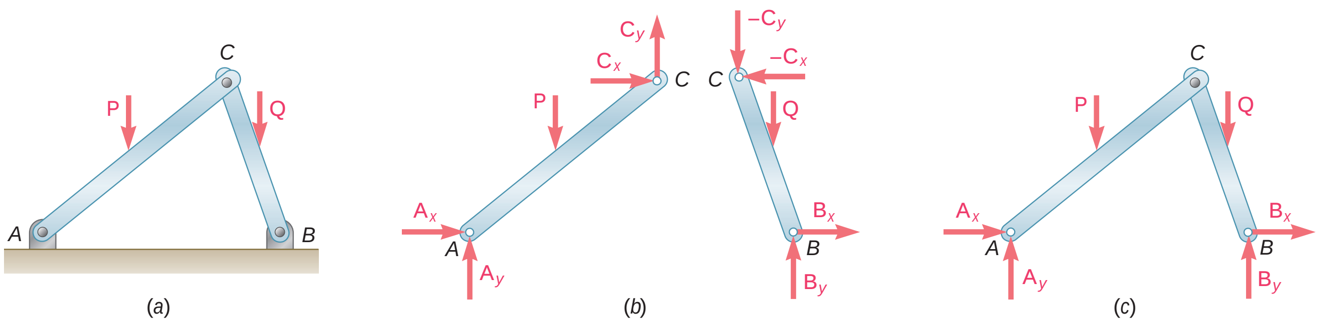


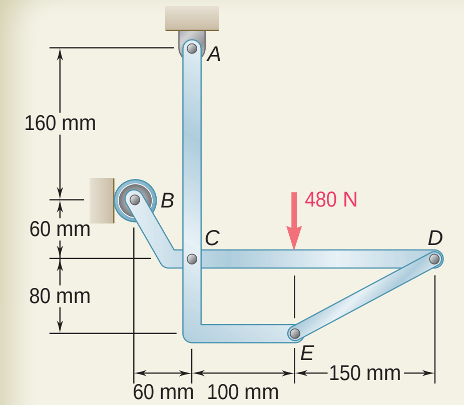
Fig. 6.21

It can now be observed that since the equations of equilibrium $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M = 0$ (about any given point) are satisfied by the forces acting on free body AC, and since they are also satisfied by the forces acting on free body CB, they must be satisfied when the forces acting on the two free bodies are considered simultaneously. Since the internal forces at C cancel each other, we find that the equations of equilibrium must be satisfied by the external forces shown on the free-body diagram of the frame ACB itself (Fig. 6.21*c*), although the frame is not a rigid body. These equations can be used to determine some of the components of the reactions at A and B. We will also find, however, that *the reactions cannot be completely determined from the free-body diagram of the whole frame*. It is thus necessary to dismember the frame and to consider the free-body diagrams of its component parts (Fig. 6.21*b*), even when we are interested in determining external reactions only. This is because the equilibrium equations obtained for free body ACB *are necessary conditions* for the equilibrium of a nonrigid structure, *but are not sufficient conditions*.

The method of solution outlined in the second paragraph of this section involved simultaneous equations. A more efficient method is now presented, which utilizes the free body ACB as well as the free bodies AC and CB. Writing $\Sigma M_A = 0$ and $\Sigma M_B = 0$ for free body ACB, we obtain B_y and A_y. Writing $\Sigma M_C = 0$, $\Sigma F_x = 0$, and $\Sigma F_y = 0$ for free body AC, we obtain, successively, A_x, C_x, and C_y. Finally, writing $\Sigma F_x = 0$ for ACB, we obtain B_x.

We noted above that the analysis of the frame of Fig. 6.21 involves six unknown force components and six independent equilibrium equations. (The equilibrium equations for the whole frame were obtained from the original six equations and, therefore, are not independent.) Moreover, we checked that all unknowns could be actually determined and that all equations could be satisfied. The frame considered is *statically determinate and rigid*.† In general, to determine whether a structure is statically determinate and rigid, we should draw a free-body diagram for each of its component parts and count the reactions and internal forces involved. We should also determine the number of independent equilibrium equations (excluding equations expressing the equilibrium of the whole structure or of groups of component parts already analyzed). If there are more unknowns than equations, the structure is *statically indeterminate*. If there are fewer unknowns than equations, the structure is *nonrigid*. If there are as many unknowns as equations, *and if all unknowns can be determined and all equations satisfied* under general loading conditions, the structure is *statically determinate and rigid*. If, however, due to an *improper arrangement* of members and supports, all unknowns cannot be determined and all equations cannot be satisfied, the structure is *statically indeterminate and nonrigid*.

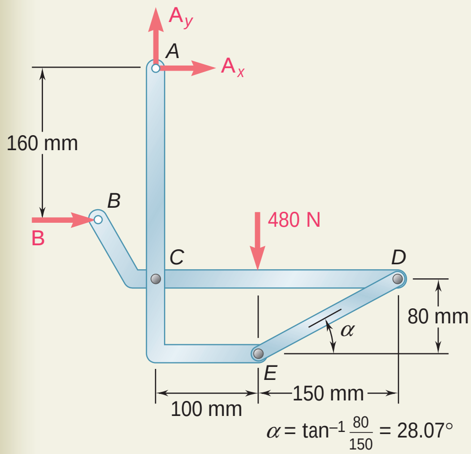
†The word "rigid" is used here to indicate that the frame will maintain its shape as long as it remains attached to its supports.



SAMPLE PROBLEM 6.4

In the frame shown, members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*.

SOLUTION



Free Body: Entire Frame. Since the external reactions involve only three unknowns, we compute the reactions by considering the free-body diagram of the entire frame.

$+\text{x}\Sigma F_y = 0$: $\quad A_y - 480\text{ N} = 0 \qquad A_y = +480\text{ N} \qquad \mathbf{A}_y = 480\text{ N}\,\text{x}$

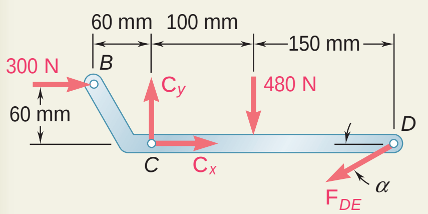
$+1\Sigma M_A = 0$: $\quad -(480\text{ N})(100\text{ mm}) + B(160\text{ mm}) = 0$

$B = +300\text{ N} \qquad \mathbf{B} = 300\text{ N y}$

$\overset{+}{\text{y}}\Sigma F_x = 0$: $\quad B + A_x = 0$

$300\text{ N} + A_x = 0 \qquad A_x = -300\text{ N} \qquad \mathbf{A}_x = 300\text{ N z}$

Members. We now dismember the frame. Since only two members are connected at *C*, the components of the unknown forces acting on *ACE* and *BCD* are, respectively, equal and opposite and are assumed directed as shown. We assume that link *DE* is in tension and exerts equal and opposite forces at *D* and *E*, directed as shown.



Free Body: Member BCD. Using the free body *BCD*, we write

$+\text{i}\Sigma M_C = 0$:

$$(F_{DE}\sin \text{a})(250\text{ mm}) + (300\text{ N})(80\text{ mm}) + (480\text{ N})(100\text{ mm}) = 0$$

$$F_{DE} = -561\text{ N} \qquad F_{DE} = 561\text{ N } C$$

$\overset{+}{\text{y}}\Sigma F_x = 0$: $\quad C_x - F_{DE}\cos \text{a} + 300\text{ N} = 0$

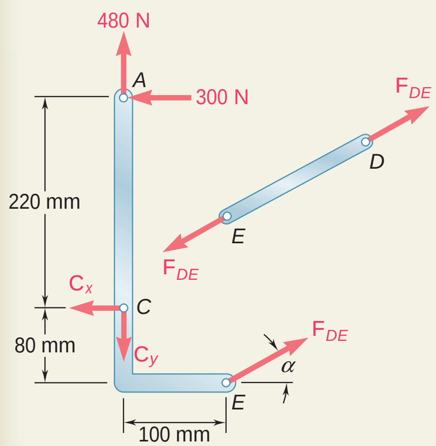
$C_x - (-561\text{ N})\cos 28.07° + 300\text{ N} = 0 \qquad C_x = -795\text{ N}$

$+\text{x}\Sigma F_y = 0$: $\quad C_y - F_{DE}\sin \text{a} - 480\text{ N} = 0$

$C_y - (-561\text{ N})\sin 28.07° - 480\text{ N} = 0 \qquad C_y = +216\text{ N}$

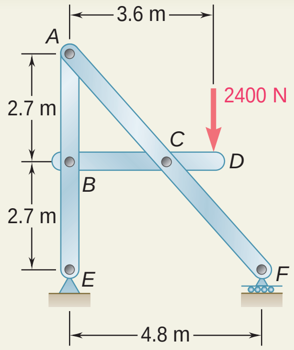
From the signs obtained for C_x and C_y we conclude that the force components $\mathbf{C}_x$ and $\mathbf{C}_y$ exerted on member *BCD* are directed, respectively, to the left and up. We have

$$C_x = 795\text{ N z},\ C_y = 216\text{ N x}$$



Free Body: Member ACE (Check). The computations are checked by considering the free body *ACE*. For example,

$$+1\Sigma M_A = (F_{DE}\cos \text{a})(300\text{ mm}) + (F_{DE}\sin \text{a})(100\text{ mm}) - C_x(220\text{ mm})$$
$$= (-561\cos \text{a})(300) + (-561\sin \text{a})(100) - (-795)(220) = 0$$

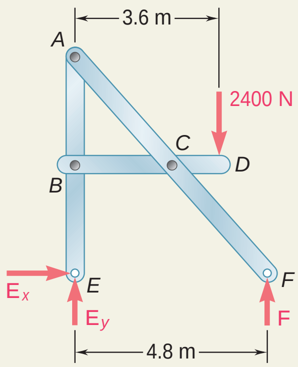


SAMPLE PROBLEM 6.5

Determine the components of the forces acting on each member of the frame shown.

SOLUTION

Free Body: Entire Frame. Since the external reactions involve only three unknowns, we compute the reactions by considering the free-body diagram of the entire frame.



$+\circlearrowleft\Sigma M_E = 0$: $\quad -(2400 \text{ N})(3.6 \text{ m}) + F(4.8 \text{ m}) = 0$

$F = +1800 \text{ N}$ $\qquad$ $\mathbf{F} = 1800 \text{ N}\uparrow$ ◀

$+\uparrow\Sigma F_y = 0$: $\quad -2400 \text{ N} + 1800 \text{ N} + E_y = 0$

$E_y = +600 \text{ N}$ $\qquad$ $\mathbf{E}_y = 600 \text{ N}\uparrow$ ◀

$\xrightarrow{+}\Sigma F_x = 0$: $\qquad$ $\mathbf{E}_x = 0$ ◀

Members. The frame is now dismembered; since only two members are connected at each joint, equal and opposite components are shown on each member at each joint.

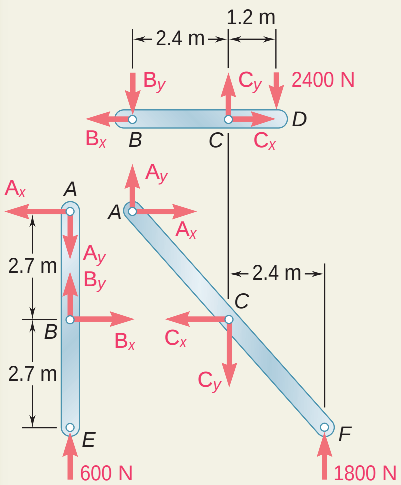
Free Body: Member BCD

$+\circlearrowleft\Sigma M_B = 0$: $\quad -(2400 \text{ N})(3.6 \text{ m}) + C_y(2.4 \text{ m}) = 0$ $\qquad$ $C_y = +3600 \text{ N}$ ◀

$+\circlearrowleft\Sigma M_C = 0$: $\quad -(2400 \text{ N})(1.2 \text{ m}) + B_y(2.4 \text{ m}) = 0$ $\qquad$ $B_y = +1200 \text{ N}$ ◀

$\xrightarrow{+}\Sigma F_x = 0$: $\quad -B_x + C_x = 0$

We note that neither B_x nor C_x can be obtained by considering only member *BCD*. The positive values obtained for B_y and C_y indicate that the force components $\mathbf{B}_y$ and $\mathbf{C}_y$ are directed as assumed.



Free Body: Member ABE

$+\circlearrowleft\Sigma M_A = 0$: $\quad B_x(2.7 \text{ m}) = 0$ $\qquad$ $B_x = 0$ ◀

$\xrightarrow{+}\Sigma F_x = 0$: $\quad +B_x - A_x = 0$ $\qquad$ $A_x = 0$ ◀

$+\uparrow\Sigma F_y = 0$: $\quad -A_y + B_y + 600 \text{ N} = 0$

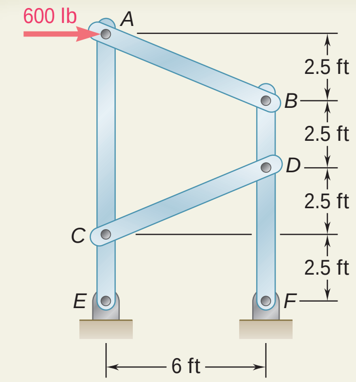
$-A_y + 1200 \text{ N} + 600 \text{ N} = 0$ $\qquad$ $A_y = +1800 \text{ N}$ ◀

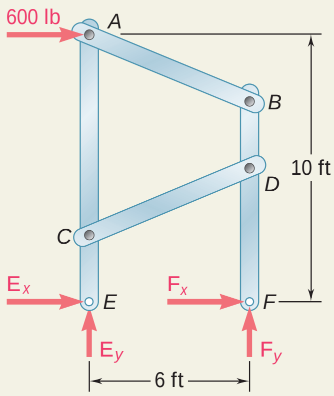
Free Body: Member BCD. Returning now to member *BCD*, we write

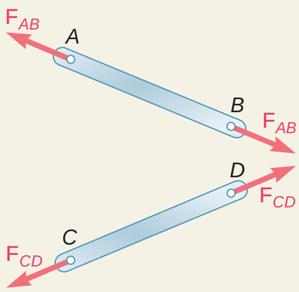
$\xrightarrow{+}\Sigma F_x = 0$: $\quad -B_x + C_x = 0 \qquad 0 + C_x = 0$ $\qquad$ $C_x = 0$ ◀

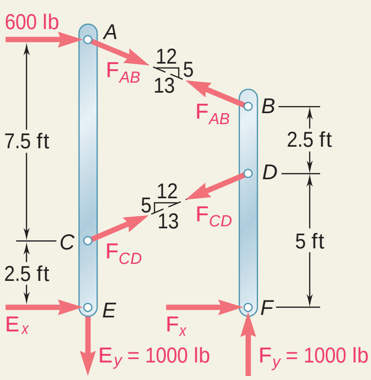
Free Body: Member ACF (Check). All unknown components have now been found; to check the results, we verify that member *ACF* is in equilibrium.

$$+\circlearrowleft\Sigma M_C = (1800 \text{ N})(2.4 \text{ m}) - A_y(2.4 \text{ m}) - A_x(2.7 \text{ m})$$
$$= (1800 \text{ N})(2.4 \text{ m}) - (1800 \text{ N})(2.4 \text{ m}) - 0 = 0 \qquad \text{(checks)}$$









SAMPLE PROBLEM 6.6

A 600-lb horizontal force is applied to pin A of the frame shown. Determine the forces acting on the two vertical members of the frame.

SOLUTION

Free Body: Entire Frame. The entire frame is chosen as a free body; although the reactions involve four unknowns, $\mathbf{E}_y$ and $\mathbf{F}_y$ may be determined by writing

$$+\circlearrowleft\Sigma M_E = 0: \quad -(600 \text{ lb})(10 \text{ ft}) + F_y(6 \text{ ft}) = 0$$
$$F_y = +1000 \text{ lb} \qquad \mathbf{F}_y = 1000 \text{ lb}\uparrow \blacktriangleleft$$
$$+\uparrow\Sigma F_y = 0: \quad E_y + F_y = 0$$
$$E_y = -1000 \text{ lb} \qquad \mathbf{E}_y = 1000 \text{ lb}\downarrow \blacktriangleleft$$

Members. The equations of equilibrium of the entire frame are not sufficient to determine $\mathbf{E}_x$ and $\mathbf{F}_x$. The free-body diagrams of the various members must now be considered in order to proceed with the solution. In dismembering the frame we will assume that pin A is attached to the multiforce member ACE and, thus, that the 600-lb force is applied to that member. We also note that AB and CD are two-force members.

Free Body: Member ACE

$$+\uparrow\Sigma F_y = 0: \quad -\tfrac{5}{13}F_{AB} + \tfrac{5}{13}F_{CD} - 1000 \text{ lb} = 0$$
$$+\circlearrowleft\Sigma M_E = 0: \quad -(600 \text{ lb})(10 \text{ ft}) - (\tfrac{12}{13}F_{AB})(10 \text{ ft}) - (\tfrac{12}{13}F_{CD})(2.5 \text{ ft}) = 0$$

Solving these equations simultaneously, we find

$$F_{AB} = -1040 \text{ lb} \qquad F_{CD} = +1560 \text{ lb} \blacktriangleleft$$

The signs obtained indicate that the sense assumed for F_{CD} was correct and the sense for F_{AB} incorrect. Summing now x components,

$$\xrightarrow{+}\Sigma F_x = 0: \quad 600 \text{ lb} + \tfrac{12}{13}(-1040 \text{ lb}) + \tfrac{12}{13}(+1560 \text{ lb}) + E_x = 0$$
$$E_x = -1080 \text{ lb} \qquad \mathbf{E}_x = 1080 \text{ lb} \leftarrow \blacktriangleleft$$

Free Body: Entire Frame. Since $\mathbf{E}_x$ has been determined, we can return to the free-body diagram of the entire frame and write

$$\xrightarrow{+}\Sigma F_x = 0: \quad 600 \text{ lb} - 1080 \text{ lb} + F_x = 0$$
$$F_x = +480 \text{ lb} \qquad \mathbf{F}_x = 480 \text{ lb} \rightarrow \blacktriangleleft$$

Free Body: Member BDF (Check). We can check our computations by verifying that the equation $\Sigma M_B = 0$ is satisfied by the forces acting on member BDF.

$$+\circlearrowleft\Sigma M_B = -\left(\tfrac{12}{13}F_{CD}\right)(2.5 \text{ ft}) + (F_x)(7.5 \text{ ft})$$
$$= -\tfrac{12}{13}(1560 \text{ lb})(2.5 \text{ ft}) + (480 \text{ lb})(7.5 \text{ ft})$$
$$= -3600 \text{ lb} \cdot \text{ft} + 3600 \text{ lb} \cdot \text{ft} = 0 \qquad \text{(checks)}$$

SOLVING PROBLEMS ON YOUR OWN

In this lesson you learned to analyze *frames containing one or more multiforce members*. In the problems that follow you will be asked to determine the external reactions exerted on the frame and the internal forces that hold together the members of the frame.

In solving problems involving frames containing one or more multiforce members, follow these steps:

1. Draw a free-body diagram of the entire frame. Use this free-body diagram to calculate, to the extent possible, the reactions at the supports. (In Sample Prob. 6.6 only two of the four reaction components could be found from the free body of the entire frame.)

2. Dismember the frame, and draw a free-body diagram of each member.

3. Considering first the two-force members, apply equal and opposite forces to each two-force member at the points where it is connected to another member. If the two-force member is a straight member, these forces will be directed along the axis of the member. If you cannot tell at this point whether the member is in tension or compression, just *assume* that the member is in tension and *direct both of the forces away from the member*. Since these forces have the same unknown magnitude, give them both the *same name* and, to avoid any confusion later, *do not use a plus sign or a minus sign*.

4. Next, consider the multiforce members. For each of these members, show all the forces acting on the member, including *applied loads, reactions, and internal forces at connections*. The magnitude and direction of any reaction or reaction component found earlier from the free-body diagram of the entire frame should be clearly indicated.

a. Where a multiforce member is connected to a two-force member, apply to the multiforce member a force *equal and opposite* to the force drawn on the free-body diagram of the two-force member, *giving it the same name*.

b. Where a multiforce member is connected to another multiforce member, use *horizontal and vertical components* to represent the internal forces at that point, since neither the direction nor the magnitude of these forces is known. The direction you choose for each of the two force components exerted on the first multiforce member is arbitary, but *you must apply equal and opposite force components of the same name* to the other multiforce member. Again, *do not use a plus sign or a minus sign*.

(continued)

5. The internal forces may now be determined, as well as any *reactions* that you have not already found.

a. The free-body diagram of each of the multiforce members can provide you with *three equilibrium equations*.

b. To simplify your solution, you should seek a way to write an equation involving a single unknown. If you can locate *a point where all but one of the unknown force components intersect,* you will obtain an equation in a single unknown by summing moments about that point. *If all unknown forces except one are parallel,* you will obtain an equation in a single unknown by summing force components in a direction perpendicular to the parallel forces.

c. Since you arbitrarily chose the direction of each of the unknown forces, you cannot determine until the solution is completed whether your guess was correct. To do that, consider the *sign* of the value found for each of the unknowns: a *positive* sign means that the direction you selected was *correct;* a *negative* sign means that the direction is *opposite* to the direction you assumed.

6. To be more effective and efficient as you proceed through your solution, observe the following rules:

a. If an equation involving only one unknown can be found, write that equation and *solve it for that unknown*. Immediately *replace* that unknown wherever it appears on other free-body diagrams *by the value you have found*. Repeat this process by seeking equilibrium equations involving only one unknown until you have found all of the internal forces and unknown reactions.

b. If an equation involving only one unknown cannot be found, you may have to *solve a pair of simultaneous equations*. Before doing so, check that you have shown the values of all of the reactions that were obtained from the free-body diagram of the entire frame.

c. The total number of equations of equilibrium for the entire frame and for the individual members *will be larger than the number of unknown forces and reactions*. After you have found all the reactions and all the internal forces, you can use the remaining equations to check the accuracy of your computations.

PROBLEMS

FREE BODY PRACTICE PROBLEMS

6.F1 For the frame and loading shown, draw the free-body diagram(s) needed to determine the forces acting on member *ABC* at *B* and *C*.

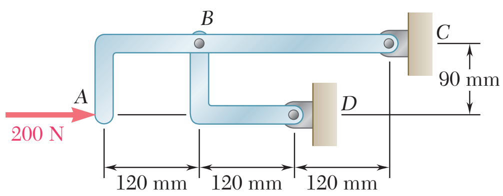


Fig. P6.F1

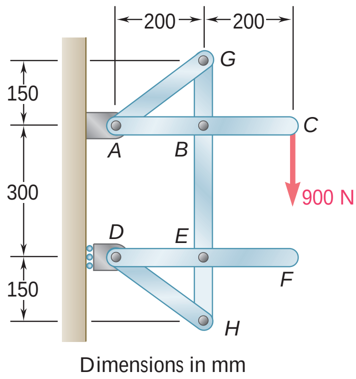


Fig. P6.F2

6.F2 For the frame and loading shown, draw the free-body diagram(s) needed to determine all forces acting on member *GBEH*.

6.F3 For the frame and loading shown, draw the free-body diagram(s) needed to determine the reactions at *B* and *F*.

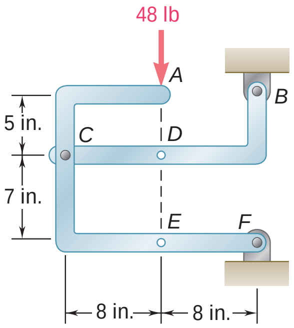


Fig. P6.F3

6.F4 Knowing that the surfaces at *A* and *D* are frictionless, draw the free-body diagram(s) needed to determine the forces exerted at *B* and *C* on member *BCE*.

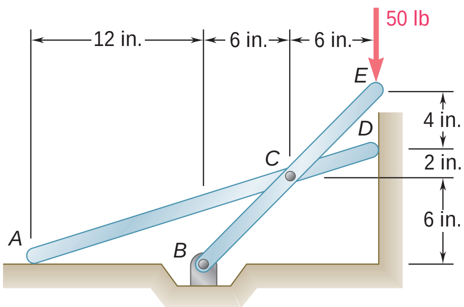


Fig. P6.F4

END-OF-SECTION PROBLEMS

6.75 and 6.76 Determine the force in member *BD* and the component of the reaction at *C*.

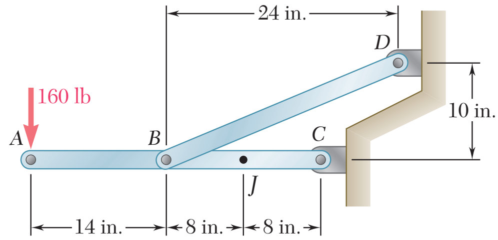


Fig. P6.75

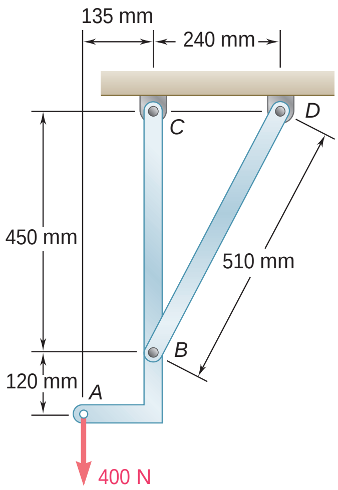


Fig. P6.76

6.77 Determine the components of all forces acting on member *ABCD* of the assembly shown.

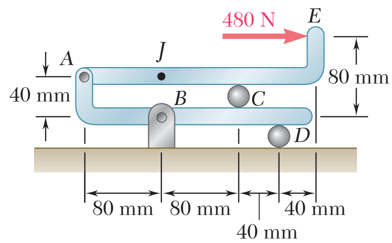


Fig. P6.77

6.78 Determine the components of all forces acting on member *ABD* of the frame shown.

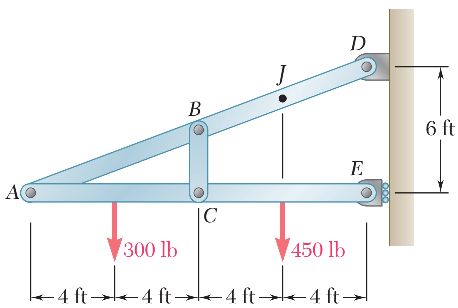


Fig. P6.78

6.79 For the frame and loading shown, determine the components of all forces acting on member *ABC*.

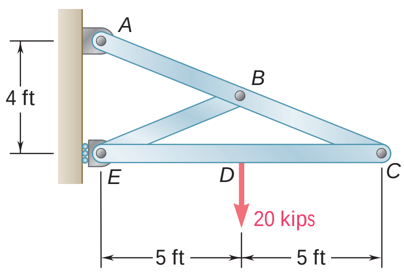


Fig. P6.79

6.80 Solve Prob. 6.79 assuming that the 20-kip load is replaced by a clockwise couple of magnitude 100 kip · ft applied to member *EDC* at point *D*.

6.81 Determine the components of all forces acting on member *ABCD* when $\theta = 0$.

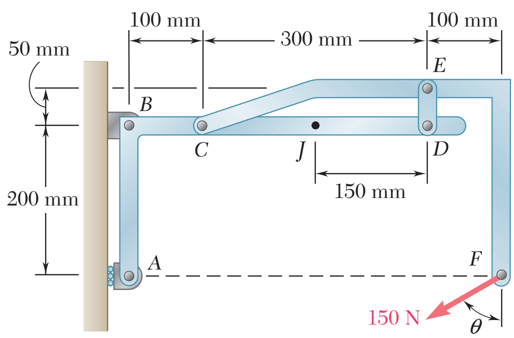


Fig. P6.81 and P6.82

6.82 Determine the components of all forces acting on member *ABCD* when $\theta = 90°$.

6.83 and 6.84 Determine the components of the reactions at A and E if a 750-N force directed vertically downward is applied (*a*) at B, (*b*) at D.

6.85 and 6.86 Determine the components of the reactions at A and E if the frame is loaded by a clockwise couple of magnitude 36 N · m applied (*a*) at B, (*b*) at D.

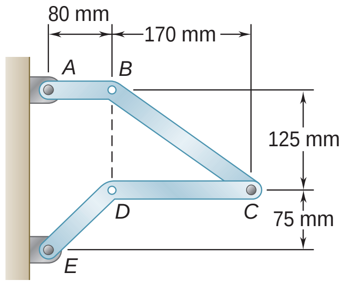


Fig. P6.83 and *P6.85*

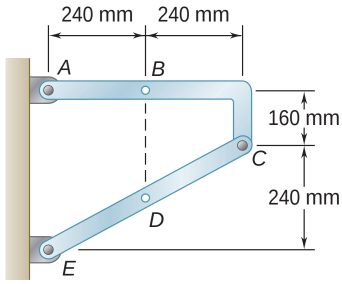


Fig. P6.84 and *P6.86*

6.87 Determine all the forces exerted on member AI if the frame is loaded by a clockwise couple of magnitude 1200 lb · in. applied (*a*) at point D, (*b*) at point E.

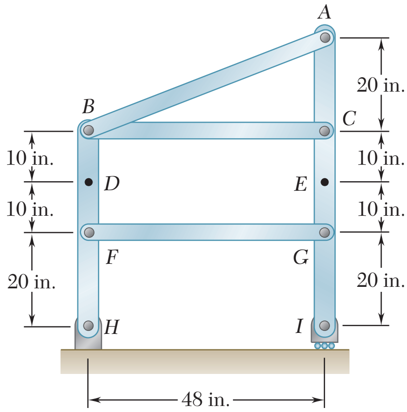


Fig. P6.87 and P6.88

6.88 Determine all the forces exerted on member AI if the frame is loaded by a 40-lb force directed horizontally to the right and applied (*a*) at point D, (*b*) at point E.

6.89 Determine the components of the reactions at A and B, (*a*) if the 100-lb load is applied as shown, (*b*) if the 100-lb load is moved along its line of action and is applied at point F.

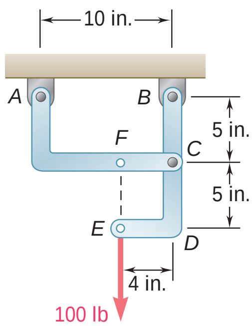


Fig. P6.89

6.90 (*a*) Show that when a frame supports a pulley at A, an equivalent loading of the frame and of each of its component parts can be obtained by removing the pulley and applying at A two forces equal and parallel to the forces that the cable exerted on the pulley. (*b*) Show that if one end of the cable is attached to the frame at a point B, a force of magnitude equal to the tension in the cable should also be applied at B.

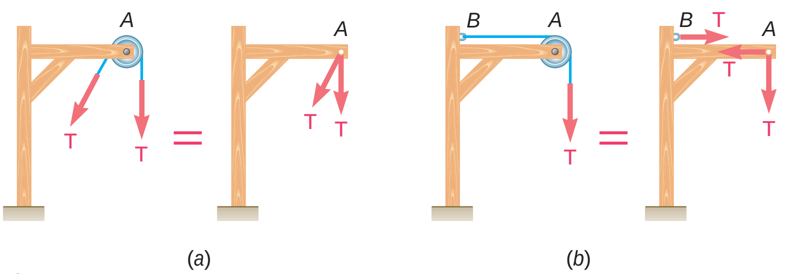


Fig. P6.90

6.91 A 3-ft-diameter pipe is supported every 16 ft by a small frame like that shown. Knowing that the combined weight of the pipe and its contents is 500 lb/ft and assuming frictionless surfaces, determine the components (*a*) of the reaction at E, (*b*) of the force exerted at C on member CDE.

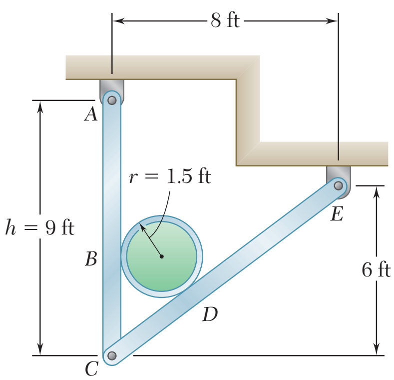


Fig. P6.91

6.92 Solve Prob. 6.91 for a frame where $h = 6$ ft.

6.93 Knowing that the pulley has a radius of 0.5 m, determine the components of the reactions at A and E.

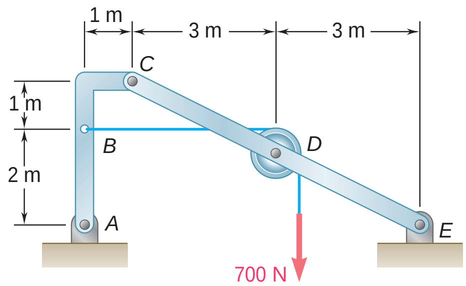


Fig. P6.93

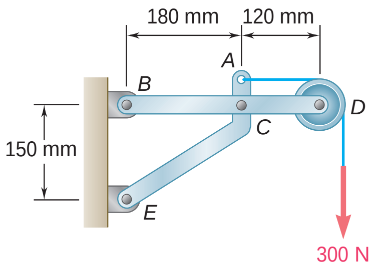


Fig. P6.94

6.94 Knowing that the pulley has a radius of 50 mm, determine the components of the reactions at B and E.

6.95 A trailer weighing 2400 lb is attached to a 2900-lb pickup truck by a ball-and-socket truck hitch at D. Determine (*a*) the reactions at each of the six wheels when the truck and trailer are at rest, (*b*) the additional load on each of the truck wheels due to the trailer.

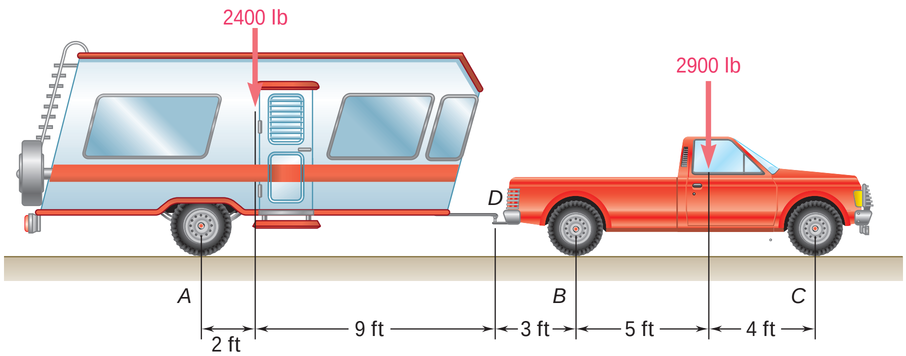


Fig. P6.95

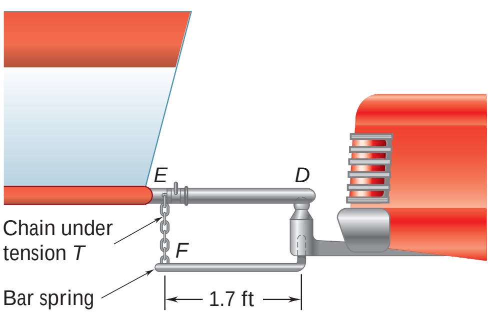


Fig. P6.96

6.96 In order to obtain a better weight distribution over the four wheels of the pickup truck of Prob. 6.95, a compensating hitch of the type shown is used to attach the trailer to the truck. The hitch consists of two bar springs (only one is shown in the figure) that fit into bearings inside a support rigidly attached to the truck. The springs are also connected by chains to the trailer frame, and specially designed hooks make it possible to place both chains in tension. (*a*) Determine the tension T required in each of the two chains if the additional load due to the trailer is to be evenly distributed over the four wheels of the truck. (*b*) What are the resulting reactions at each of the six wheels of the trailer-truck combination?

6.97 The cab and motor units of the front-end loader shown are connected by a vertical pin located 2 m behind the cab wheels. The distance from C to D is 1 m. The center of gravity of the 300-kN motor unit is located at G_m, while the centers of gravity of the 100-kN cab and 75-kN load are located, respectively, at G_c and G_l. Knowing that the machine is at rest with its brakes released, determine (*a*) the reactions at each of the four wheels, (*b*) the forces exerted on the motor unit at C and D.

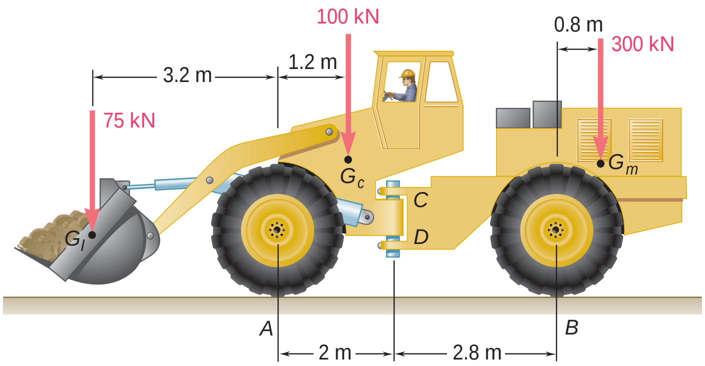


Fig. *P6.97*

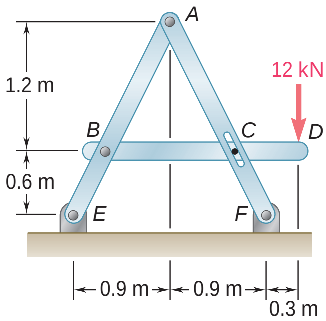


Fig. P6.99

6.98 Solve Prob. 6.97 assuming that the 75-kN load has been removed.

6.99 and 6.100 For the frame and loading shown, determine the components of all forces acting on member ABE.

6.101 For the frame and loading shown, determine the components of all forces acting on member ABD.

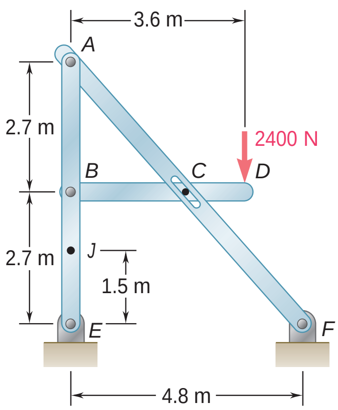


Fig. P6.100

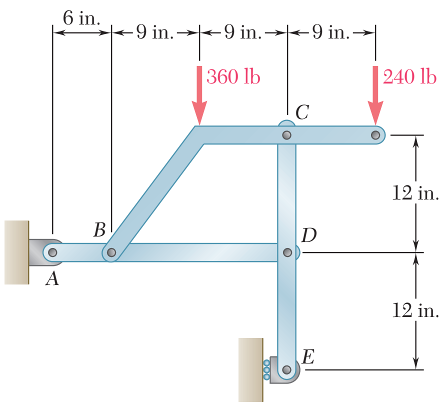


Fig. P6.101

6.102 Solve Prob. 6.101 assuming that the 360-lb load has been removed.

6.103 For the frame and loading shown, determine the components of the forces acting on member CDE at C and D.

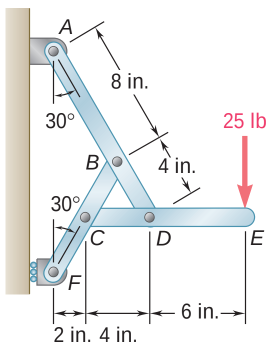


Fig. P6.103

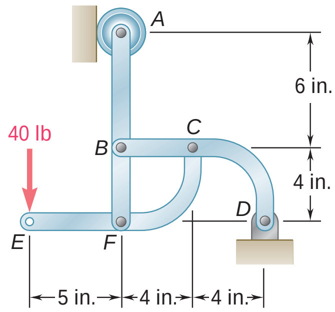


Fig. P6.104

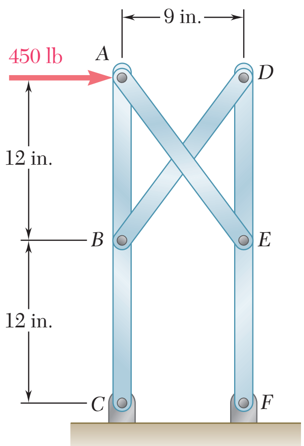


Fig. P6.107

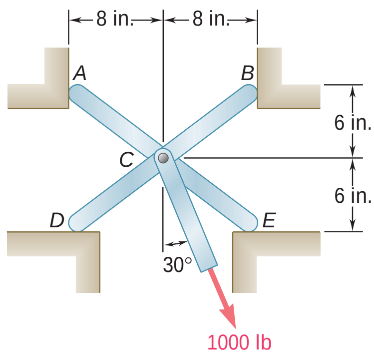


Fig. P6.108

6.104 For the frame and loading shown, determine the components of the forces acting on member *CFE* at *C* and *F*.

6.105 For the frame and loading shown, determine the components of all forces acting on member *ABD*.

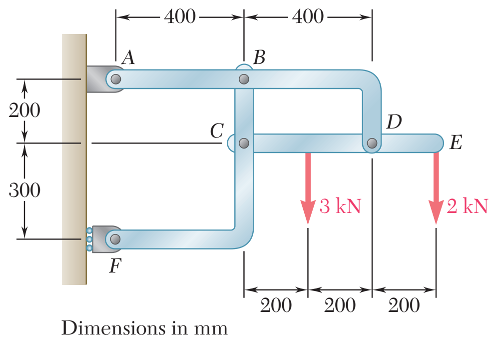


Fig. *P6.105*

6.106 Solve Prob. 6.105 assuming that the 3-kN load has been removed.

6.107 Determine the reaction at *F* and the force in members *AE* and *BD*.

6.108 For the frame and loading shown, determine the reactions at *A*, *B*, *D*, and *E*. Assume that the surface at each support is frictionless.

6.109 The axis of the three-hinge arch *ABC* is a parabola with vertex at *B*. Knowing that $P = 112$ kN and $Q = 140$ kN, determine (*a*) the components of the reaction at *A*, (*b*) the components of the force exerted at *B* on segment *AB*.

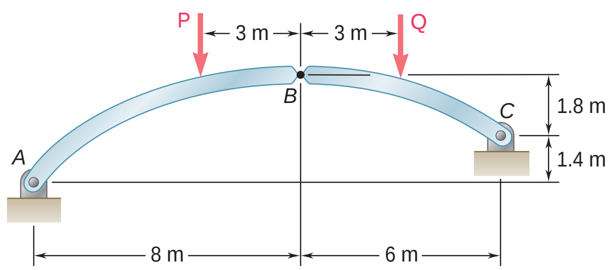


Fig. P6.109 and P6.110

6.110 The axis of the three-hinge arch *ABC* is a parabola with vertex at *B*. Knowing that $P = 140$ kN and $Q = 112$ kN, determine (*a*) the components of the reaction at *A*, (*b*) the components of the force exerted at *B* on segment *AB*.

6.111, 6.112, and 6.113 Members ABC and CDE are pin-connected at C and supported by four links. For the loading shown, determine the force in each link.

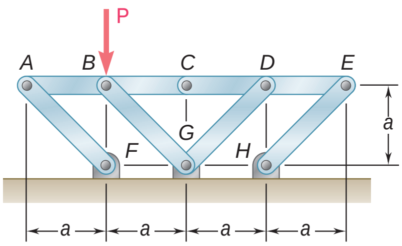


Fig. P6.111

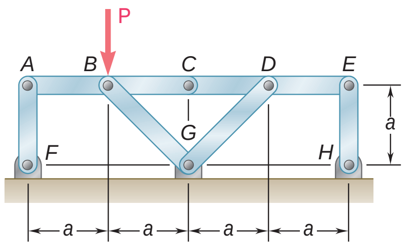


Fig. P6.112

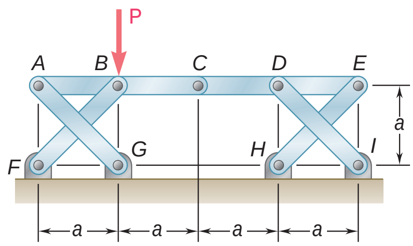


Fig. P6.113

6.114 Members ABC and CDE are pin-connected at C and supported by the four links AF, BG, DG, and EH. For the loading shown, determine the force in each link.

6.115 Solve Prob. 6.112 assuming that the force **P** is replaced by a clockwise couple of moment $\mathbf{M}_0$ applied to member CDE at D.

6.116 Solve Prob. 6.114 assuming that the force **P** is replaced by a clockwise couple of moment $\mathbf{M}_0$ applied at the same point.

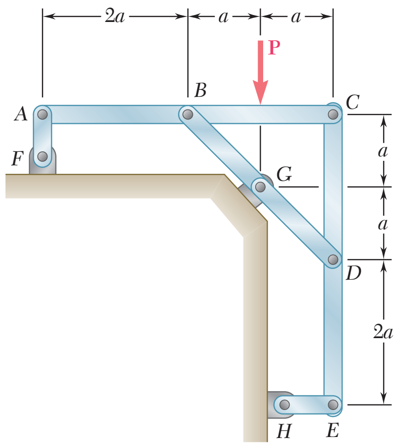


Fig. P6.114

6.117 Four beams, each of length $3a$, are held together by single nails at A, B, C, and D. Each beam is attached to a support located at a distance a from an end of the beam as shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at E, F, G, and H.

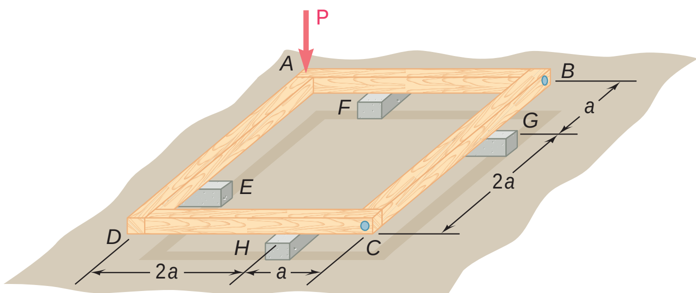


Fig. P6.117

6.118 Four beams, each of length $2a$, are nailed together at their midpoints to form the support system shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at A, D, E, and H.

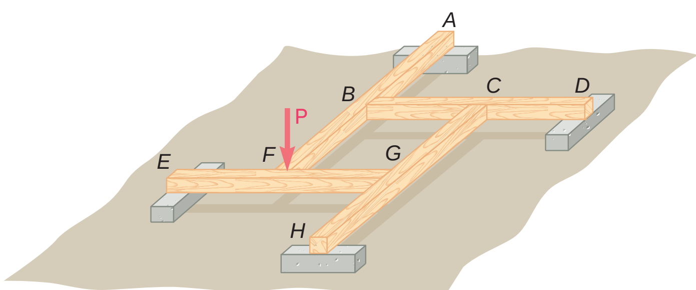


Fig. P6.118

6.119 through 6.121 Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.

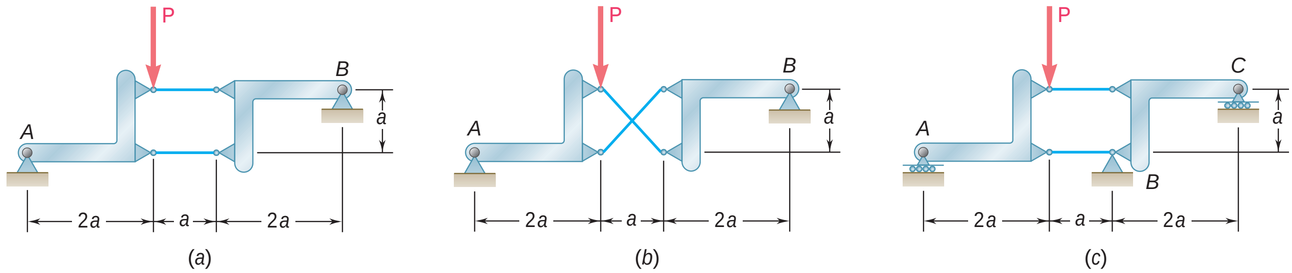


Fig. *P6.119*

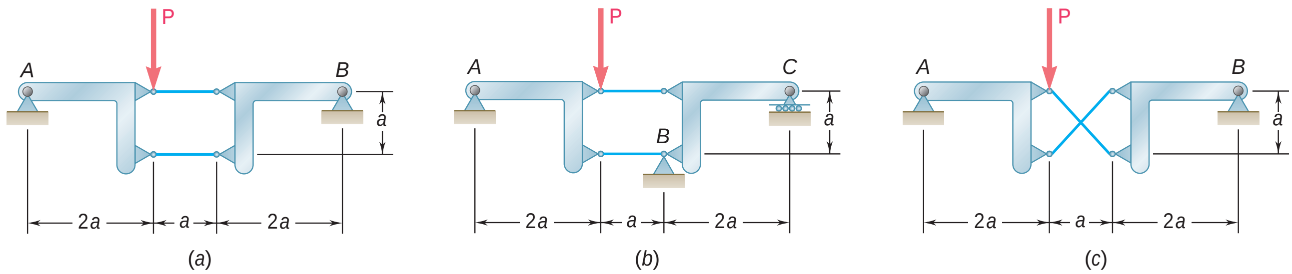


Fig. *P6.120*

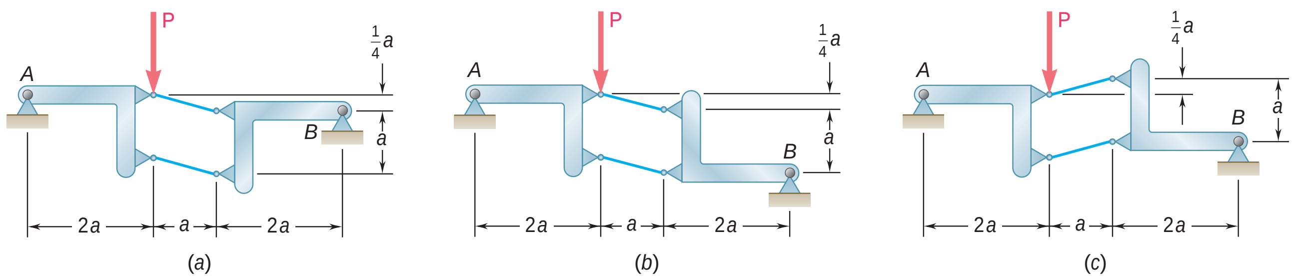


Fig. P6.121

6.12 MACHINES

Machines are structures designed to transmit and modify forces. Whether they are simple tools or include complicated mechanisms, their main purpose is to transform *input forces* into *output forces*. Consider, for example, a pair of cutting pliers used to cut a wire (Fig. 6.22*a*). If we apply two equal and opposite forces **P** and −**P** on their handles, they will exert two equal and opposite forces **Q** and −**Q** on the wire (Fig. 6.22*b*).

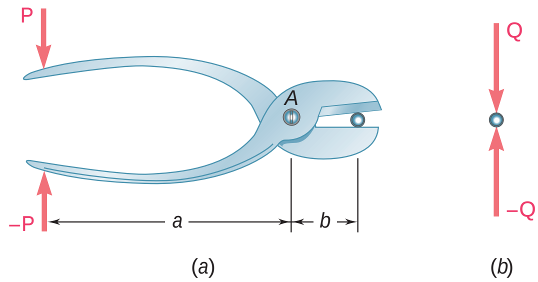


Fig. 6.22

Photo 6.5 The lamp shown can be placed in many positions. By considering various free bodies, the force in the springs and the internal forces at the joints can be determined.

To determine the magnitude Q of the output forces when the magnitude P of the input forces is known (or, conversely, to determine P when Q is known), we draw a free-body diagram of the pliers *alone,* showing the input forces **P** and −**P** and the *reactions* −**Q** and **Q** that the wire exerts on the pliers (Fig. 6.23). However, since a pair of pliers forms a nonrigid structure, we must use one of the component parts as a free body in order to determine the unknown forces. Considering Fig. 6.24*a*, for example, and taking moments about A, we obtain the relation $Pa = Qb$, which defines the magnitude Q in terms of P or P in terms of Q. The same free-body diagram can be used to determine the components of the internal force at A; we find $A_x = 0$ and $A_y = P + Q$.

In the case of more complicated machines, it generally will be necessary to use several free-body diagrams and, possibly, to solve simultaneous equations involving various internal forces. The free bodies should be chosen to include the input forces and the reactions to the output forces, and the total number of unknown force components involved should not exceed the number of available independent equations. It is advisable, before attempting to solve a problem, to determine whether the structure considered is determinate. There is no point, however, in discussing the rigidity of a machine, since a machine includes moving parts and thus *must* be nonrigid.

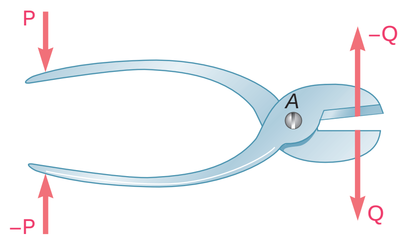


Fig. 6.23

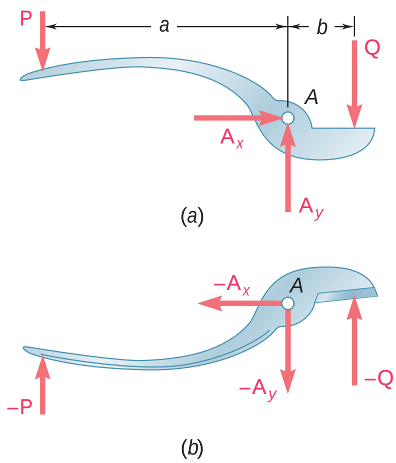
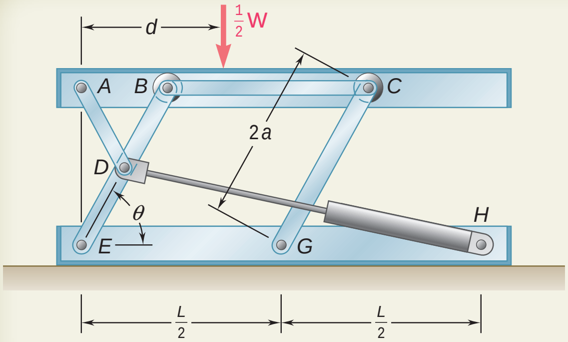
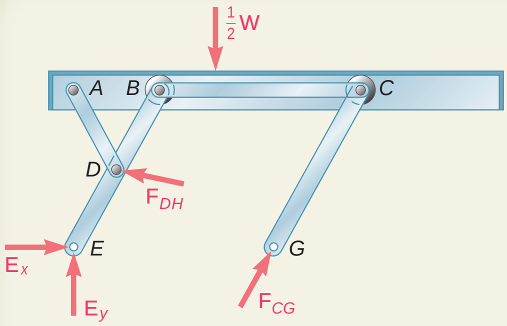


Fig. 6.24



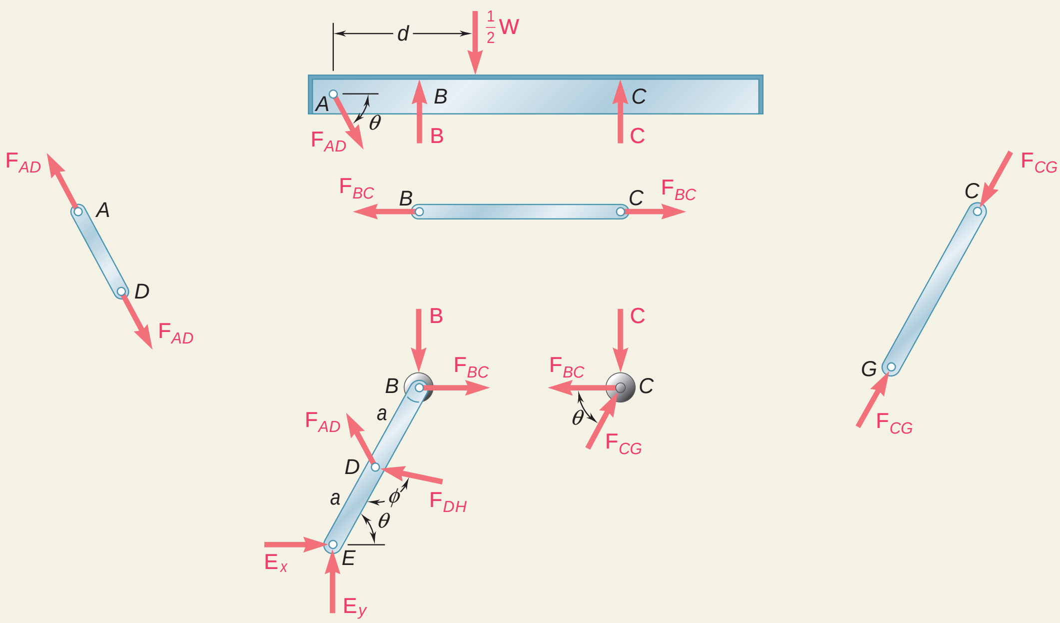
SAMPLE PROBLEM 6.7

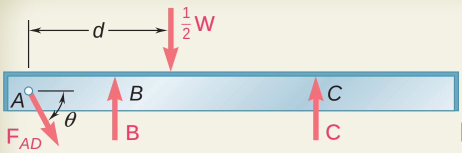
A hydraulic-lift table is used to raise a 1000-kg crate. It consists of a platform and two identical linkages on which hydraulic cylinders exert equal forces. (Only one linkage and one cylinder are shown.) Members *EDB* and *CG* are each of length $2a$, and member *AD* is pinned to the midpoint of *EDB*. If the crate is placed on the table, so that half of its weight is supported by the system shown, determine the force exerted by each cylinder in raising the crate for u = 60°, a = 0.70 m, and L = 3.20 m. Show that the result obtained is independent of the distance d.



SOLUTION

The machine considered consists of the platform and of the linkage. Its free-body diagram includes an input force $\mathbf{F}_{DH}$ exerted by the cylinder, the weight $\frac{1}{2}\mathbf{W}$, equal and opposite to the output force, and reactions at E and G that we assume to be directed as shown. Since more than three unknowns are involved, this diagram will not be used. The mechanism is dismembered and a free-body diagram is drawn for each of its component parts. We note that *AD*, *BC*, and *CG* are two-force members. We already assumed member *CG* to be in compression; we now assume that *AD* and *BC* are in tension and direct as shown the forces exerted on them. Equal and opposite vectors will be used to represent the forces exerted by the two-force members on the platform, on member *BDE*, and on roller *C*.



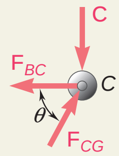


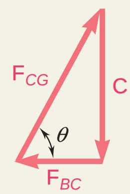
Free Body: Platform *ABC*.

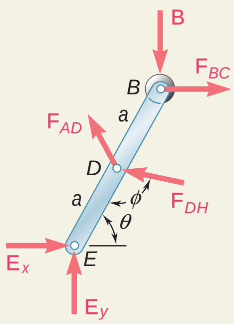
$$\overset{+}{\rightarrow}\Sigma F_x = 0: \qquad F_{AD}\cos\theta = 0 \qquad F_{AD} = 0$$

$$+\uparrow\Sigma F_y = 0: \qquad B + C - \tfrac{1}{2}W = 0 \qquad B + C = \tfrac{1}{2}W \tag{1}$$

Free Body: Roller *C*. We draw a force triangle and obtain $F_{BC} = C\cot\theta$.







Free Body: Member *BDE*. Recalling that $F_{AD} = 0$,

$$+\circlearrowleft\Sigma M_E = 0: \qquad F_{DH}\cos(\phi - 90°)a - B(2a\cos\theta) - F_{BC}(2a\sin\theta) = 0$$

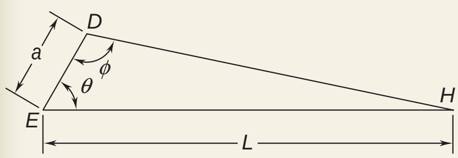
$$F_{DH}a\sin\phi - B(2a\cos\theta) - (C\cot\theta)(2a\sin\theta) = 0$$

$$F_{DH}\sin\phi - 2(B + C)\cos\theta = 0$$

Recalling Eq. (1), we have

$$F_{DH} = W\frac{\cos\theta}{\sin\phi} \tag{2}$$

and we observe that *the result obtained is independent of d*. ◀



Applying first the law of sines to triangle EDH, we write

$$\frac{\sin\phi}{EH} = \frac{\sin\theta}{DH} \qquad \sin\phi = \frac{EH}{DH}\sin\theta \tag{3}$$

Using now the law of cosines, we have

$$(DH)^2 = a^2 + L^2 - 2aL\cos\theta$$

$$= (0.70)^2 + (3.20)^2 - 2(0.70)(3.20)\cos 60°$$

$$(DH)^2 = 8.49 \qquad DH = 2.91\text{ m}$$

We also note that

$$W = mg = (1000\text{ kg})(9.81\text{ m/s}^2) = 9810\text{ N} = 9.81\text{ kN}$$

Substituting for $\sin\phi$ from (3) into (2) and using the numerical data, we write

$$F_{DH} = W\frac{DH}{EH}\cot\theta = (9.81\text{ kN})\frac{2.91\text{ m}}{3.20\text{ m}}\cot 60°$$

$$F_{DH} = 5.15\text{ kN}$$ ◀

SOLVING PROBLEMS ON YOUR OWN

This lesson was devoted to the analysis of *machines*. Since machines are designed to transmit or modify forces, they always contain moving parts. However, the machines considered here will always be at rest, and you will be working with the set of *forces required to maintain the equilibrium of the machine*.

Known forces that act on a machine are called *input forces. A machine transforms the input forces into output forces,* such as the cutting forces applied by the pliers of Fig. 6.22. You will determine the output forces by finding the forces equal and opposite to the output forces that should be applied to the machine to maintain its equilibrium.

In the preceding lesson you analyzed frames; you will now use almost the same procedure to analyze machines:

1. Draw a free-body diagram of the whole machine, and use it to determine as many as possible of the unknown forces exerted on the machine.

2. Dismember the machine, and draw a free-body diagram of each member.

3. Considering first the two-force members, apply equal and opposite forces to each two-force member at the points where it is connected to another member. If you cannot tell at this point whether the member is in tension or in compression just *assume* that the member is in tension and *direct both of the forces away from the member*. Since these forces have the same unknown magnitude, *give them both the same name*.

4. Next consider the multiforce members. For each of these members, show all the forces acting on the member, including applied loads and forces, reactions, and internal forces at connections.

a. Where a multiforce member is connected to a two-force member, apply to the multiforce member a force *equal and opposite* to the force drawn on the free-body diagram of the two-force member, *giving it the same name*.

b. Where a multiforce member is connected to another multiforce member, use *horizontal and vertical components* to represent the internal forces at that point. The directions you choose for each of the two force components exerted on the first multiforce member are arbitrary, but *you must apply equal and opposite force components of the same name* to the other multiforce member.

5. Equilibrium equations can be written after you have completed the various free-body diagrams.

a. To simplify your solution, you should, whenever possible, write and solve equilibrium equations involving single unknowns.

b. Since you arbitrarily chose the direction of each of the unknown forces, you must determine at the end of the solution whether your guess was correct. To that effect, *consider the sign* of the value found for each of the unknowns. A *positive* sign indicates that your guess was correct, and a *negative* sign indicates that it was not.

6. Finally, you should check your solution by substituting the results obtained into an equilibrium equation that you have not previously used.

PROBLEMS

FREE BODY PRACTICE PROBLEMS

6.F5 The position of member *ABC* is controlled by the hydraulic cylinder *CD*. Knowing that u = 30°, draw the free-body diagram(s) needed to determine the force exerted by the hydraulic cylinder on pin *C*, and the reaction at *B*.

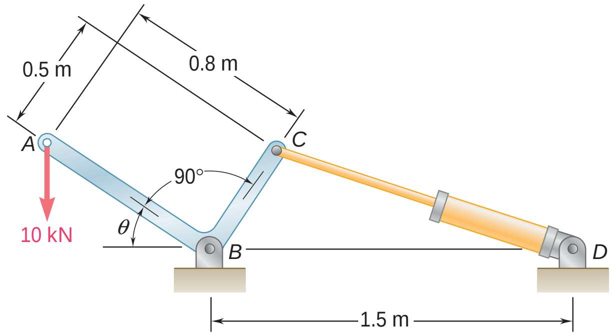


Fig. P6.F5

6.F6 Arm *ABC* is connected by pins to a collar at *B* and to crank *CD* at *C*. Neglecting the effect of friction, draw the free-body diagram(s) needed to determine the couple **M** to hold the system in equilibrium when u = 30°.

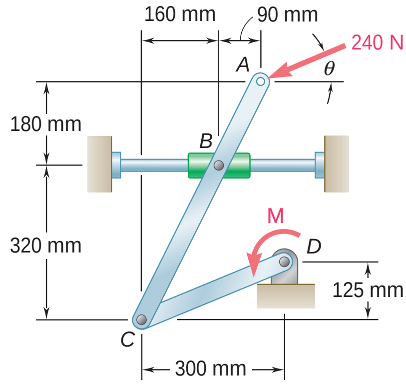


Fig. P6.F6

6.F7 Since the brace shown must remain in position even when the magnitude of **P** is very small, a single safety spring is attached at *D* and *E*. The spring *DE* has a constant of 50 lb/in. and an unstretched length of 7 in. Knowing that l = 10 in. and that the magnitude of **P** is 800 lb, draw the free-body diagram(s) needed to determine the force **Q** required to release the brace.

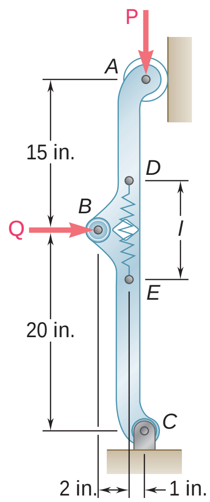


Fig. P6.F7

6.F8 A log weighing 800 lb is lifted by a pair of tongs as shown. Draw the free-body diagram(s) needed to determine the forces exerted at *E* and *F* on tong *DEF*.

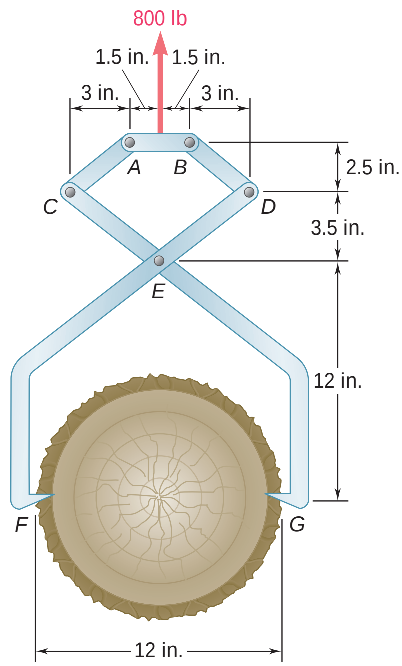


Fig. P6.F8

END-OF-SECTION PROBLEMS

6.122 The shear shown is used to cut and trim electronic-circuit-board laminates. For the position shown, determine (*a*) the vertical component of the force exerted on the shearing blade at *D*, (*b*) the reaction at *C*.

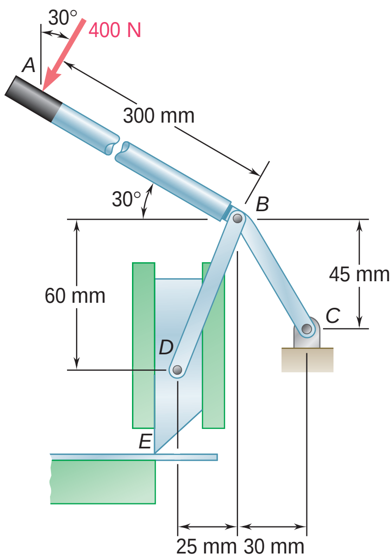


Fig. P6.122

6.123 The press shown is used to emboss a small seal at *E*. Knowing that $P = 250$ N, determine (*a*) the vertical component of the force exerted on the seal, (*b*) the reaction at *A*.

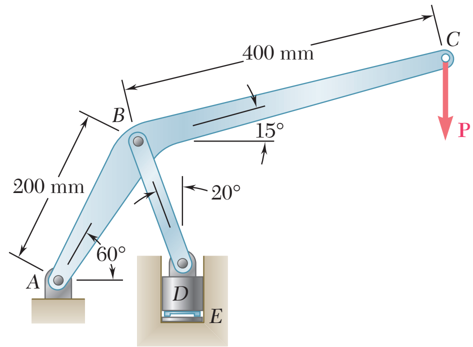


Fig. P6.123 and *P6.124*

6.124 The press shown is used to emboss a small seal at *E*. Knowing that the vertical component of the force exerted on the seal must be 900 N, determine (*a*) the required vertical force **P**, (*b*) the corresponding reaction at *A*.

6.125 Water pressure in the supply system exerts a downward force of 135 N on the vertical plug at *A*. Determine the tension in the fusible link *DE* and the force exerted on member *BCE* at *B*.

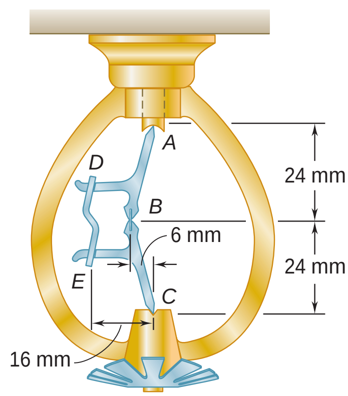


Fig. *P6.125*

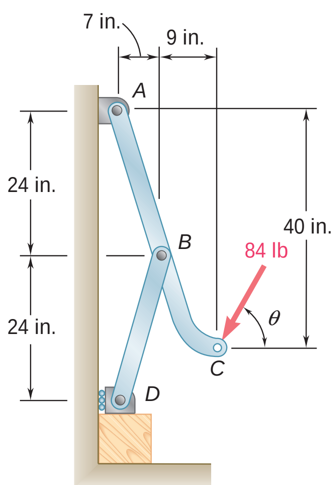


Fig. P6.126

6.126 An 84-lb force is applied to the toggle vise at *C*. Knowing that $\theta = 90°$, determine (*a*) the vertical force exerted on the block at *D*, (*b*) the force exerted on member *ABC* at *B*.

6.127 Solve Prob. 6.126 when $\theta = 0$.

6.128 For the system and loading shown, determine (*a*) the force **P** required for equilibrium, (*b*) the corresponding force in member *BD*, (*c*) the corresponding reaction at *C*.

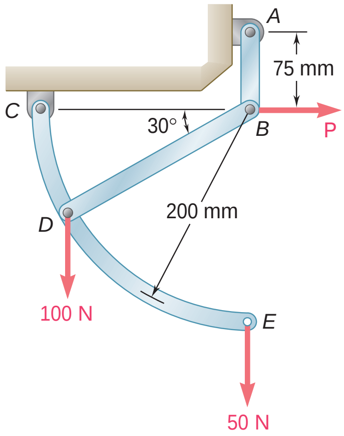


Fig. P6.128

6.129 The Whitworth mechanism shown is used to produce a quick-return motion of point *D*. The block at *B* is pinned to the crank *AB* and is free to slide in a slot cut in member *CD*. Determine the couple **M** that must be applied to the crank *AB* to hold the mechanism in equilibrium when (*a*) a = 0, (*b*) a = 30°.

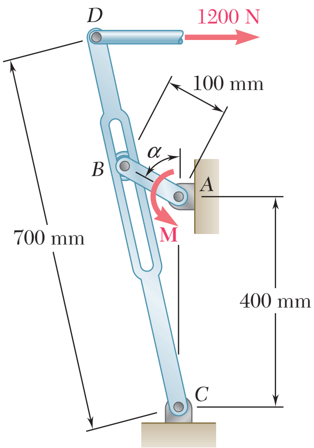


Fig. P6.129

6.130 Solve Prob. 6.129 when (*a*) a = 60°, (*b*) a = 90°.

6.131 A couple **M** of magnitude 1.5 kN · m is applied to the crank of the engine system shown. For each of the two positions shown, determine the force **P** required to hold the system in equilibrium.

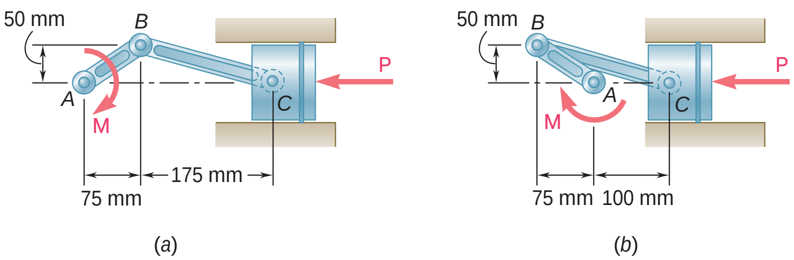


Fig. P6.131 and P6.132

6.132 A force **P** of magnitude 16 kN is applied to the piston of the engine system shown. For each of the two positions shown, determine the couple **M** required to hold the system in equilibrium.

6.133 The pin at *B* is attached to member *ABC* and can slide freely along the slot cut in the fixed plate. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when u = 30°.

6.134 The pin at *B* is attached to member *ABC* and can slide freely along the slot cut in the fixed plate. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when u = 60°.

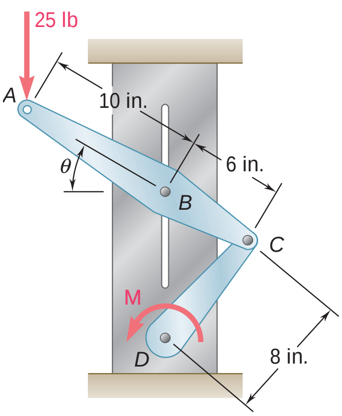


Fig. P6.133 and P6.134

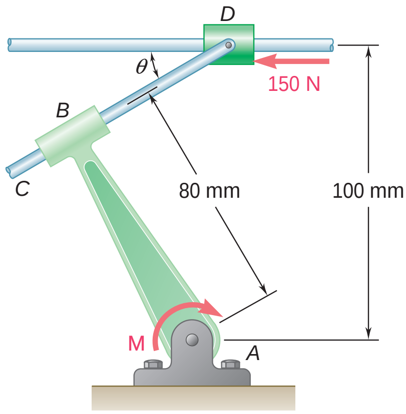


Fig. P6.135

6.135 and 6.136 Rod *CD* is attached to the collar *D* and passes through a collar welded to end *B* of lever *AB*. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when u = 30°.

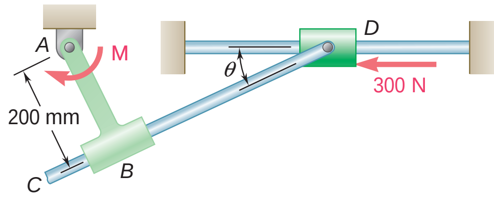
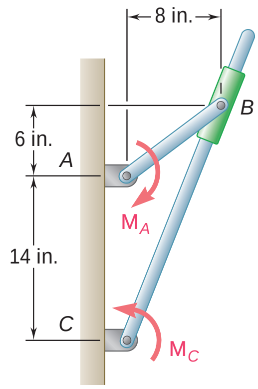


Fig. P6.136

6.137* and *6.138 Two rods are connected by a frictionless collar *B*. Knowing that the magnitude of the couple $\mathbf{M}_A$ is 500 lb · in., determine (*a*) the couple $\mathbf{M}_C$ required for equilibrium, (*b*) the corresponding components of the reaction at *C*.



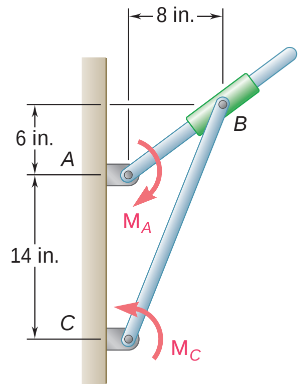


Fig. *P6.137* Fig. *P6.138*

6.139 Two hydraulic cylinders control the position of the robotic arm *ABC*. Knowing that in the position shown the cylinders are parallel, determine the force exerted by each cylinder when $P = 160$ N and $Q = 80$ N.

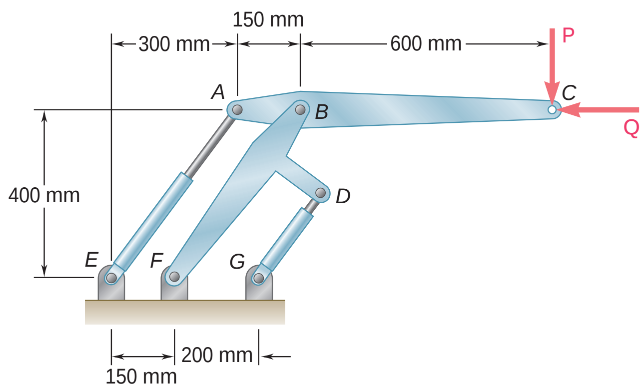


Fig. P6.139 and P6.140

6.140 Two hydraulic cylinders control the position of the robotic arm *ABC*. In the position shown, the cylinders are parallel and both are in tension. Knowing that $F_{AE} = 600$ N and $F_{DG} = 50$ N, determine the forces **P** and **Q** applied at *C* to arm *ABC*.

6.141 The tongs shown are used to apply a total upward force of 45 kN on a pipe cap. Determine the forces exerted at D and F on tong ADF.

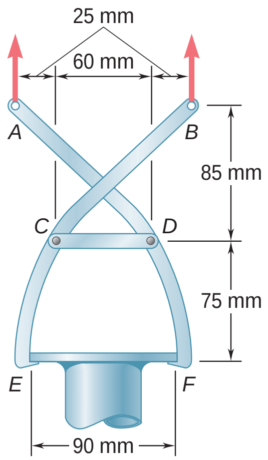


Fig. P6.141

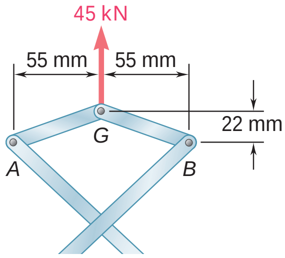


Fig. P6.142

6.142 If the toggle shown is added to the tongs of Prob. 6.141 and a single vertical force is applied at G, determine the forces exerted at D and F on tong ADF.

6.143 A small barrel weighing 60 lb is lifted by a pair of tongs as shown. Knowing that $a = 5$ in., determine the forces exerted at B and D on tong ABD.

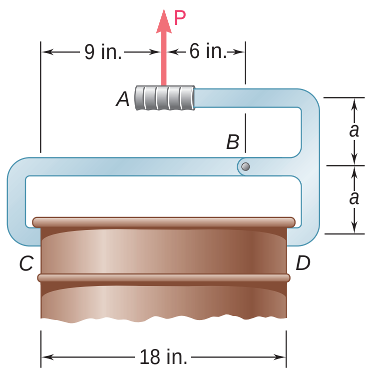


Fig. *P6.143*

6.144 A 39-ft length of railroad rail of weight 44 lb/ft is lifted by the tongs shown. Determine the forces exerted at D and F on tong BDF.

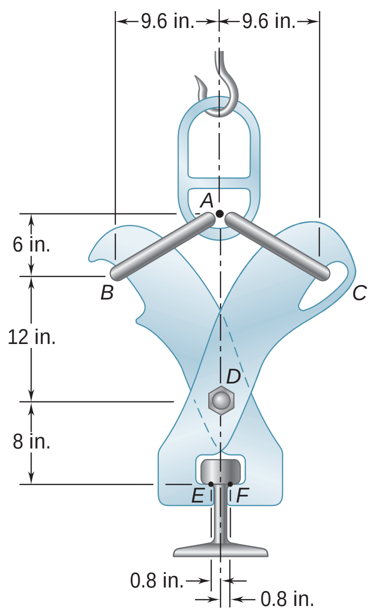


Fig. P6.144

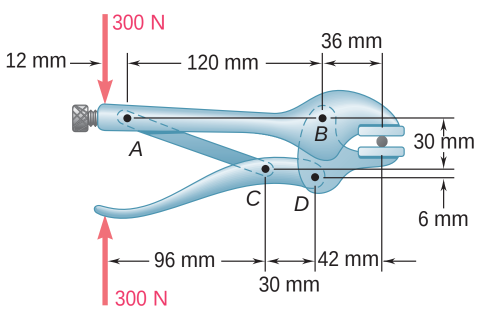


Fig. P6.145

6.145 Determine the magnitude of the gripping forces produced when two 300-N forces are applied as shown.

6.146 The compound-lever pruning shears shown can be adjusted by placing pin A at various ratchet positions on blade ACE. Knowing that 300-lb vertical forces are required to complete the pruning of a small branch, determine the magnitude P of the forces that must be applied to the handles when the shears are adjusted as shown.

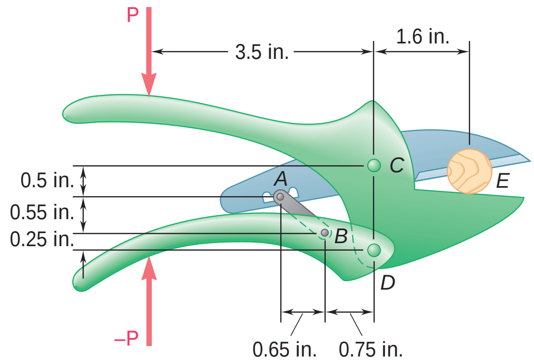


Fig. P6.146

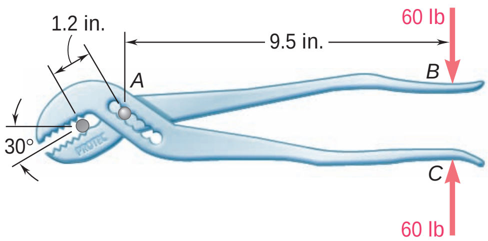


Fig. P6.147

6.147 The pliers shown are used to grip a 0.3-in.-diameter rod. Knowing that two 60-lb forces are applied to the handles, determine (*a*) the magnitude of the forces exerted on the rod, (*b*) the force exerted by the pin at A on portion AB of the pliers.

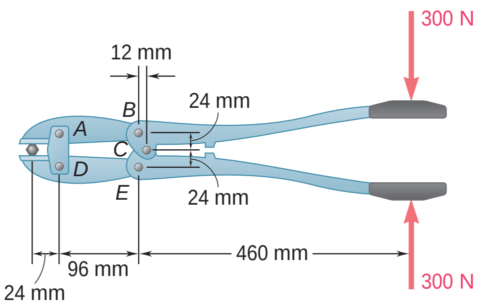


Fig. P6.148

6.148 In using the bolt cutter shown, a worker applies two 300-N forces to the handles. Determine the magnitude of the forces exerted by the cutter on the bolt.

6.149 The specialized plumbing wrench shown is used in confined areas (e.g., under a basin or sink). It consists essentially of a jaw BC pinned at B to a long rod. Knowing that the forces exerted on the nut are equivalent to a clockwise (when viewed from above) couple of magnitude 135 lb · in., determine (*a*) the magnitude of the force exerted by pin B on jaw BC, (*b*) the couple $\mathbf{M}_0$ that is applied to the wrench.

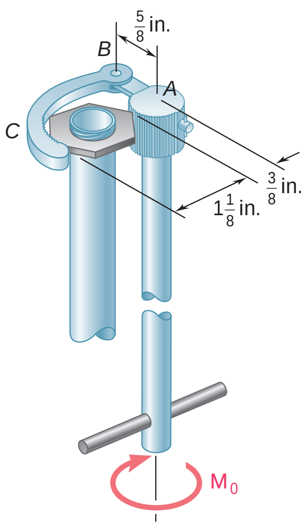


Fig. P6.149

6.150 and *6.151* Determine the force **P** that must be applied to the toggle CDE to maintain bracket ABC in the position shown.

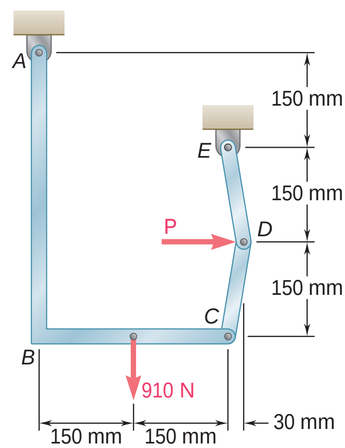


Fig. P6.150

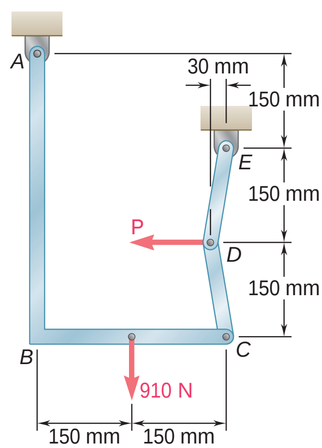


Fig. P6.151

6.152 A 45-lb shelf is held horizontally by a self-locking brace that consists of two parts *EDC* and *CDB* hinged at *C* and bearing against each other at *D*. Determine the force **P** required to release the brace.

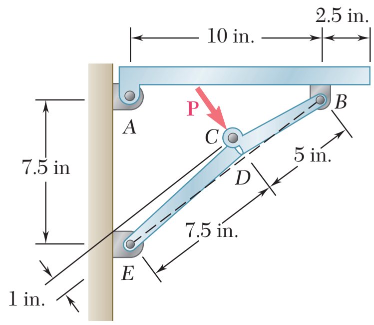


Fig. P6.152

6.153 The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together have a mass of 200 kg and have a combined center of gravity located directly above *C*. For the position when u = 20°, determine (*a*) the force exerted at *B* by the single hydraulic cylinder *BD*, (*b*) the force exerted on the supporting carriage at *A*.

6.154 The telescoping arm *ABC* of Prob. 6.153 can be lowered until end *C* is close to the ground, so that workers can easily board the platform. For the position when u = −20°, determine (*a*) the force exerted at *B* by the single hydraulic cylinder *BD*, (*b*) the force exerted on the supporting carriage at *A*.

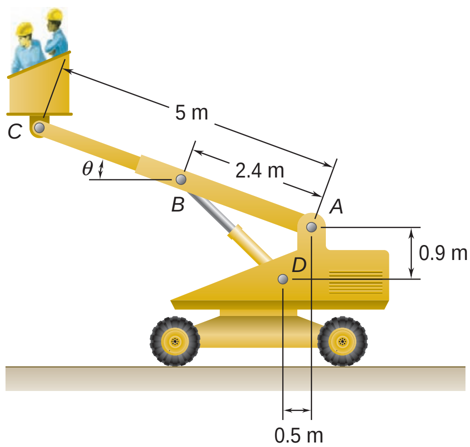


Fig. P6.153

6.155 The bucket of the front-end loader shown carries a 3200-lb load. The motion of the bucket is controlled by two identical mechanisms, only one of which is shown. Knowing that the mechanism shown supports one-half of the 3200-lb load, determine the force exerted (*a*) by cylinder *CD*, (*b*) by cylinder *FH*.

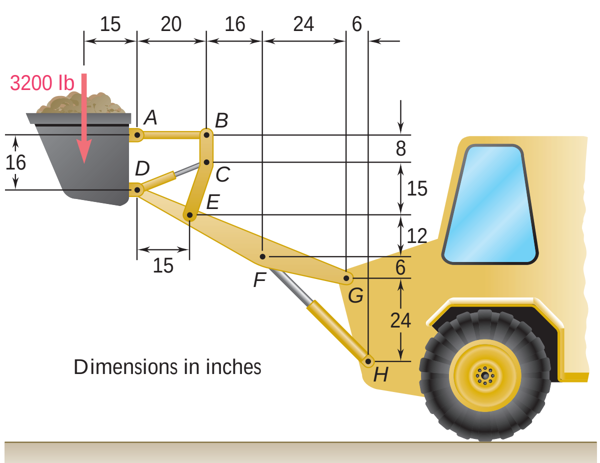


Fig. P6.155

6.156 The motion of the bucket of the front-end loader shown is controlled by two arms and a linkage that are pin-connected at *D*. The arms are located symmetrically with respect to the central, vertical, and longitudinal plane of the loader; one arm *AFJ* and its control cylinder *EF* are shown. The single linkage *GHDB* and its control cylinder *BC* are located in the plane of symmetry. For the position and loading shown, determine the force exerted (*a*) by cylinder *BC*, (*b*) by cylinder *EF*.

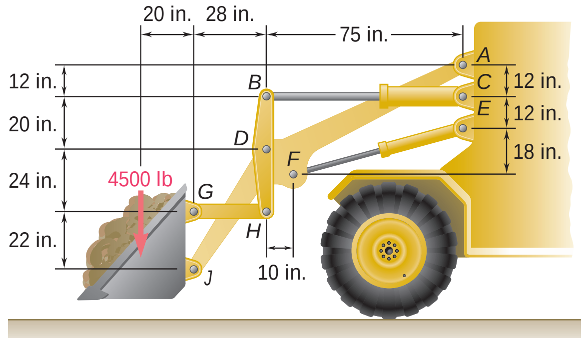


Fig. P6.156

6.157 The motion of the backhoe bucket shown is controlled by the hydraulic cylinders *AD*, *CG*, and *EF*. As a result of an attempt to dislodge a portion of a slab, a 2-kip force **P** is exerted on the bucket teeth at *J*. Knowing that u = 45°, determine the force exerted by each cylinder.

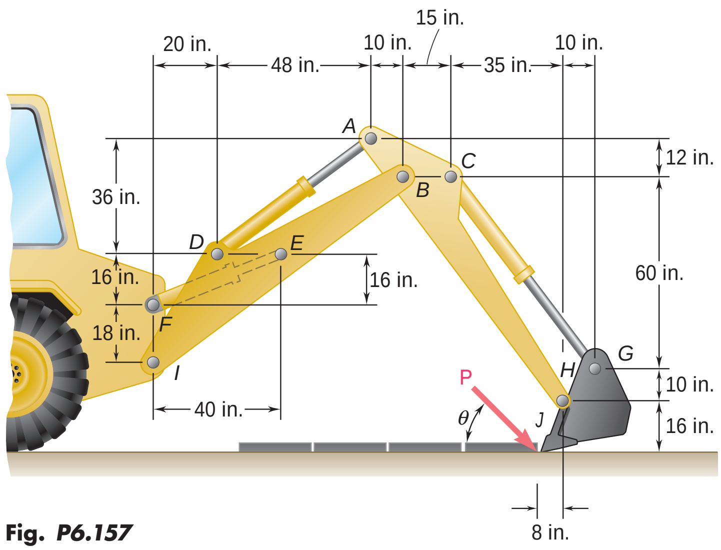


Fig. P6.157

6.158 Solve Prob. 6.157 assuming that the 2-kip force **P** acts horizontally to the right (u = 0).

6.159 In the planetary gear system shown, the radius of the central gear *A* is $a = 18$ mm, the radius of each planetary gear is b, and the radius of the outer gear *E* is $(a + 2b)$. A clockwise couple of magnitude $M_A = 10$ N · m is applied to the central gear *A* and a counterclockwise couple of magnitude $M_S = 50$ N · m is applied to the spider *BCD*. If the system is to be in equilibrium, determine (*a*) the required radius b of the planetary gears, (*b*) the magnitude M_E of the couple that must be applied to the outer gear *E*.

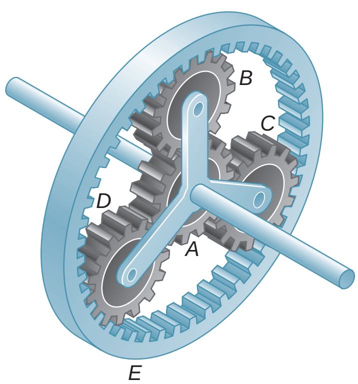


Fig. P6.159

6.160 The gears D and G are rigidly attached to shafts that are held by frictionless bearings. If $r_D = 90$ mm and $r_G = 30$ mm, determine (*a*) the couple $\mathbf{M}_0$ that must be applied for equilibrium, (*b*) the reactions at A and B.

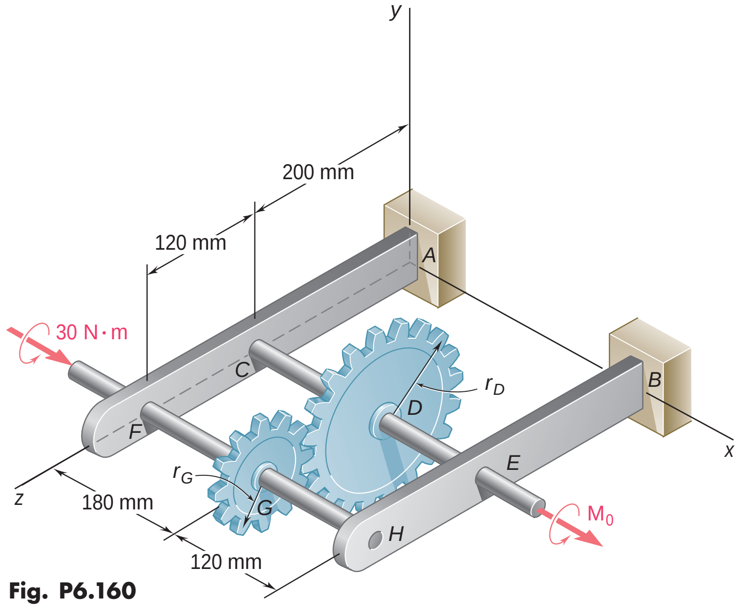


Fig. P6.160

***6.161** Two shafts AC and CF, which lie in the vertical xy plane, are connected by a universal joint at C. The bearings at B and D do not exert any axial force. A couple of magnitude 500 lb · in. (clockwise when viewed from the positive x axis) is applied to shaft CF at F. At a time when the arm of the crosspiece attached to shaft CF is horizontal, determine (*a*) the magnitude of the couple that must be applied to shaft AC at A to maintain equilibrium, (*b*) the reactions at B, D, and E. (*Hint:* The sum of the couples exerted on the crosspiece must be zero.)

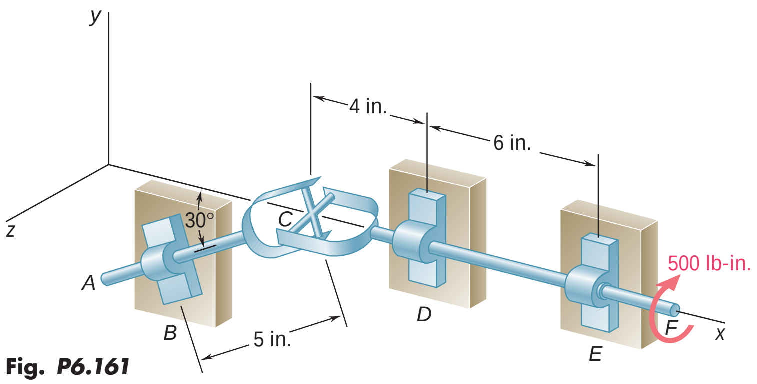


Fig. P6.161

***6.162** Solve Prob. 6.161 assuming that the arm of the crosspiece attached to shaft CF is vertical.

***6.163** The large mechanical tongs shown are used to grab and lift a thick 7500-kg steel slab HJ. Knowing that slipping does not occur between the tong grips and the slab at H and J, determine the components of all forces acting on member EFH. (*Hint:* Consider the symmetry of the tongs to establish relationships between the components of the force acting at E on EFH and the components of the force acting at D on DGJ.)

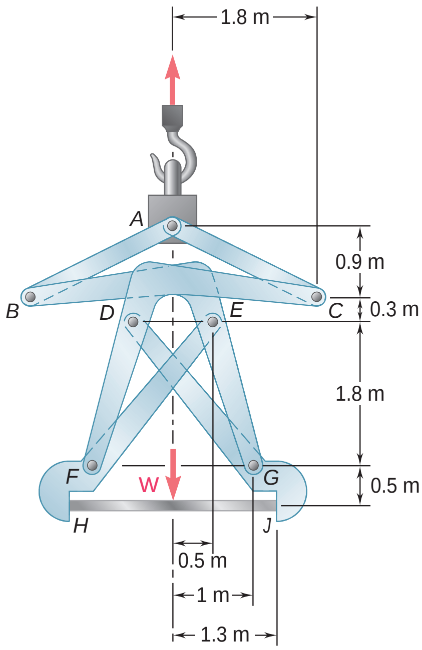


Fig. P6.163

REVIEW AND SUMMARY

In this chapter you learned to determine the *internal forces* holding together the various parts of a structure.

Analysis of trusses

The first half of the chapter was devoted to the analysis of *trusses*, i.e., to the analysis of structures consisting of *straight members connected at their extremities only*. The members being slender and unable to support lateral loads, all the loads must be applied at the joints; a truss may thus be assumed to consist of *pins and two-force members* [Sec. 6.2].

Simple trusses

A truss is said to be *rigid* if it is designed in such a way that it will not greatly deform or collapse under a small load. A triangular truss consisting of three members connected at three joints is clearly a rigid truss (Fig. 6.25*a*) and so will be the truss obtained by adding two new members to the first one and connecting them at a new joint (Fig. 6.25*b*). Trusses obtained by repeating this procedure are called *simple trusses*. We may check that in a simple truss the total number of members is $m = 2n - 3$, where n is the total number of joints [Sec. 6.3].

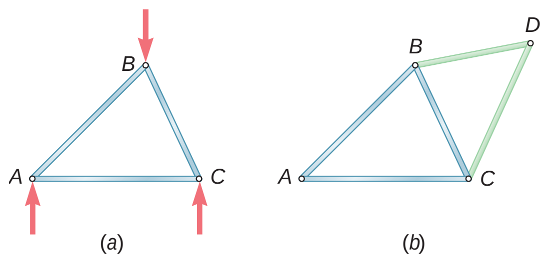


Fig. 6.25

Method of joints

The forces in the various members of a simple truss can be determined by the *method of joints* [Sec. 6.4]. First, the reactions at the supports can be obtained by considering the entire truss as a free body. The free-body diagram of each pin is then drawn, showing the forces exerted on the pin by the members or supports it connects. Since the members are straight two-force members, the force exerted by a member on the pin is directed along that member, and only the magnitude of the force is unknown. It is always possible in the case of a simple truss to draw the free-body diagrams of the pins in such an order that only two unknown forces are included in each diagram. These forces can be obtained from the corresponding two equilibrium equations or—if only three forces are involved—from the corresponding force triangle. If the force exerted by a member on a pin is directed toward that pin, the member is in *compression;*

if it is directed away from the pin, the member is in *tension* [Sample Prob. 6.1]. The analysis of a truss is sometimes expedited by first recognizing *joints under special loading conditions* [Sec. 6.5]. The method of joints can also be extended to the analysis of three-dimensional or *space trusses* [Sec. 6.6].

Method of sections

The *method of sections* is usually preferred to the method of joints when the force in only one member—or very few members—of a truss is desired [Sec. 6.7]. To determine the force in member BD of the truss of Fig. 6.26*a*, for example, we *pass a section* through members BD, BE, and CE, remove these members, and use the portion ABC of the truss as a free body (Fig. 6.26*b*). Writing $\Sigma M_E = 0$, we determine the magnitude of the force $\mathbf{F}_{BD}$, which represents the force in member BD. A positive sign indicates that the member is in *tension;* a negative sign indicates that it is in *compression* [Sample Probs. 6.2 and 6.3].

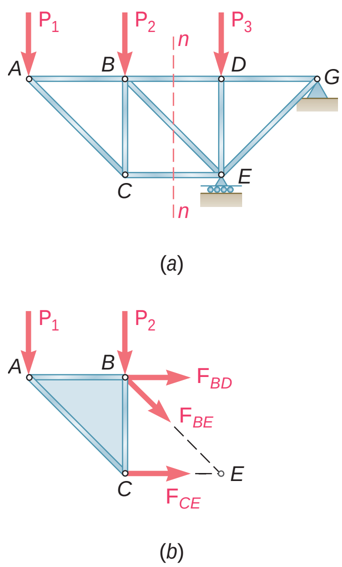


Fig. 6.26

Compound trusses

The method of sections is particularly useful in the analysis of *compound trusses,* i.e., trusses which cannot be constructed from the basic triangular truss of Fig. 6.25*a* but which can be obtained by rigidly connecting several simple trusses [Sec. 6.8]. If the component trusses have been properly connected (e.g., one pin and one link, or three nonconcurrent and nonparallel links) and if the resulting structure is properly supported (e.g., one pin and one roller), the compound truss is *statically determinate, rigid, and completely constrained*. The following necessary—but not sufficient—condition is then satisfied: $m + r = 2n$, where m is the number of members, r is the number of unknowns representing the reactions at the supports, and n is the number of joints.

Frames and machines

The second part of the chapter was devoted to the analysis of *frames and machines*. Frames and machines are structures which contain *multiforce members,* i.e., members acted upon by three or more forces. Frames are designed to support loads and are usually stationary, fully constrained structures. Machines are designed to transmit or modify forces and always contain moving parts [Sec. 6.9].

Analysis of a frame

To *analyze a frame,* we first consider the *entire frame as a free body* and write three equilibrium equations [Sec. 6.10]. If the frame remains rigid when detached from its supports, the reactions involve only three unknowns and may be determined from these equations [Sample Probs. 6.4 and 6.5]. On the other hand, if the frame ceases to be rigid when detached from its supports, the reactions involve more than three unknowns and cannot be completely determined from the equilibrium equations of the frame [Sec. 6.11; Sample Prob. 6.6].

Multiforce members

We then *dismember the frame* and identify the various members as either two-force members or multiforce members; pins are assumed to form an integral part of one of the members they connect. We draw the free-body diagram of each of the multiforce members, noting that when two multiforce members are connected to the same two-force member, they are acted upon by that member with *equal and opposite forces of unknown magnitude but known direction*. When two multiforce members are connected by a pin, they exert on each other *equal and opposite forces of unknown direction,* which should be represented by *two unknown components*. The equilibrium equations obtained from the free-body diagrams of the multiforce members can then be solved for the various internal forces [Sample Probs. 6.4 and 6.5]. The equilibrium equations can also be used to complete the determination of the reactions at the supports [Sample Prob. 6.6]. Actually, if the frame is *statically determinate and rigid,* the free-body diagrams of the multiforce members could provide as many equations as there are unknown forces (including the reactions) [Sec. 6.11]. However, as suggested above, it is advisable to first consider the free-body diagram of the entire frame to minimize the number of equations that must be solved simultaneously.

Analysis of a machine

To *analyze a machine,* we dismember it and, following the same procedure as for a frame, draw the free-body diagram of each of the multiforce members. The corresponding equilibrium equations yield the *output forces* exerted by the machine in terms of the *input forces* applied to it, as well as the *internal forces* at the various connections [Sec. 6.12; Sample Prob. 6.7].

REVIEW PROBLEMS

6.164 Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

6.165 Using the method of joints, determine the force in each member of the roof truss shown. State whether each member is in tension or compression.

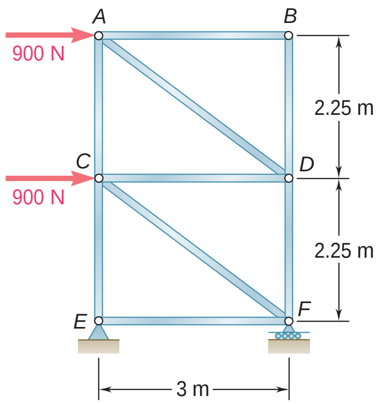


Fig. P6.164

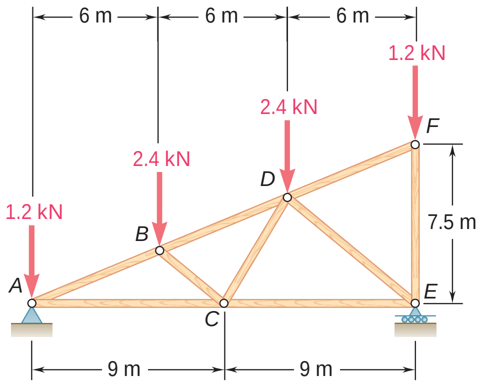


Fig. P6.165

6.166 A Howe scissors roof truss is loaded as shown. Determine the force in members *DF*, *DG*, and *EG*.

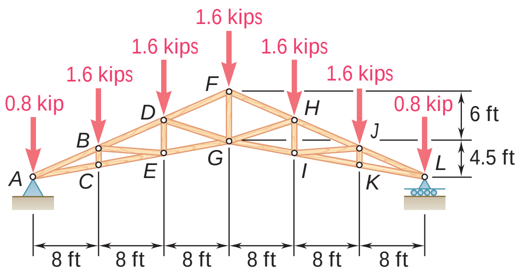


Fig. P6.166 and *P6.167*

6.167 A Howe scissors roof truss is loaded as shown. Determine the force in members *GI*, *HI*, and *HJ*.

6.168 Rod *CD* is fitted with a collar at *D* that can be moved along rod *AB*, which is bent in the shape of an arc of circle. For the position when u = 30°, determine (*a*) the force in rod *CD*, (*b*) the reaction at *B*.

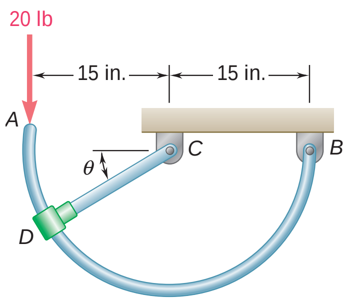


Fig. P6.168

6.169 For the frame and loading shown, determine the components of all forces acting on member ABC.

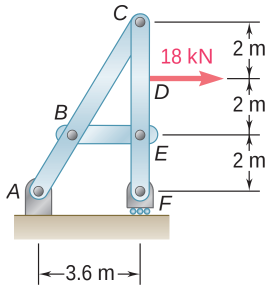


Fig. P6.169

6.170 Knowing that each pulley has a radius of 250 mm, determine the components of the reactions at D and E.

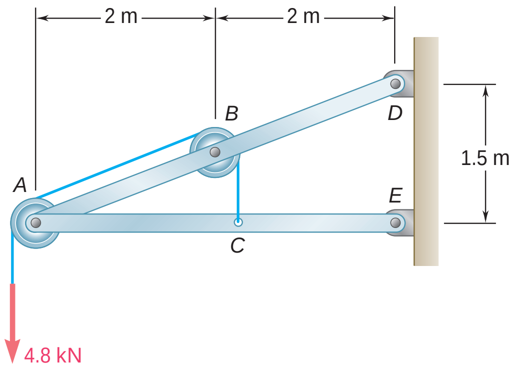


Fig. P6.170

6.171 For the frame and loading shown, determine the components of the forces acting on member $DABC$ at B and D.

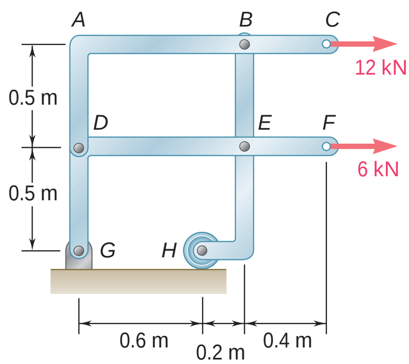


Fig. P6.171

6.172 For the frame and loading shown, determine (*a*) the reaction at C, (*b*) the force in member AD.

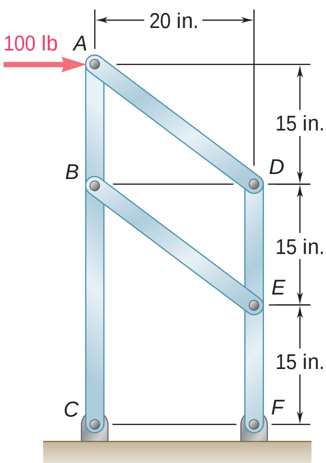


Fig. P6.172

6.173 The control rod CE passes through a horizontal hole in the body of the toggle system shown. Knowing that link BD is 250 mm long, determine the force **Q** required to hold the system in equilibrium when $b = 20°$.

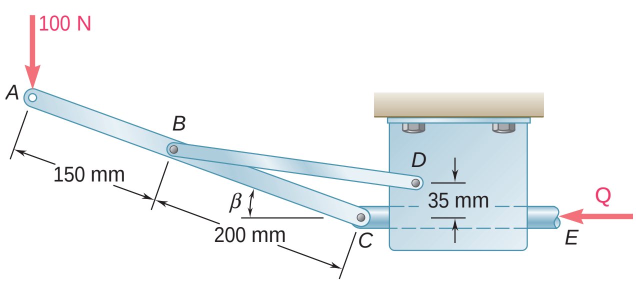


Fig. P6.173

6.174 Determine the magnitude of the gripping forces exerted along line aa on the nut when two 50-lb forces are applied to the handles as shown. Assume that pins A and D slide freely in slots cut in the jaws.

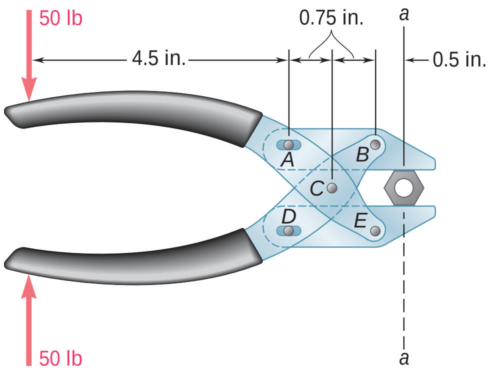


Fig. P6.174

6.175 Knowing that the frame shown has a sag at B of $a = 1$ in., determine the force $\mathbf{P}$ required to maintain equilibrium in the position shown.

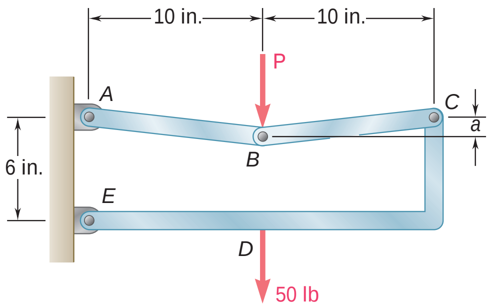


Fig. P6.175

COMPUTER PROBLEMS

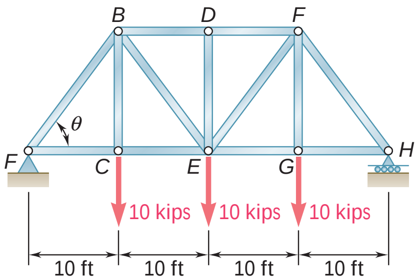


Fig. P6.C1

6.C1 A Pratt steel truss is to be designed to support three 10-kip loads as shown. The length of the truss is to be 40 ft. The height of the truss and thus the angle u, as well as the cross-sectional areas of the various members, are to be selected to obtain the most economical design. Specifically, the cross-sectional area of each member is to be chosen so that the stress (force divided by area) in that member is equal to 20 kips/in^2, the allowable stress for the steel used; the total weight of the steel, and thus its cost, must be as small as possible. (*a*) Knowing that the specific weight of the steel used is 0.284 lb/in^3, write a computer program that can be used to calculate the weight of the truss and the cross-sectional area of each load-bearing member located to the left of *DE* for values of u from 20° to 80° using 5° increments. (*b*) Using appropriate smaller increments, determine the optimum value of u and the corresponding values of the weight of the truss and of the cross-sectional areas of the various members. Ignore the weight of any zero-force member in your computations.

6.C2 The floor of a bridge will rest on stringers that will be simply supported by transverse floor beams, as in Fig. 6.3. The ends of the beams will be connected to the upper joints of two trusses, one of which is shown in Fig. P6.C2. As part of the design of the bridge, it is desired to simulate the effect on this truss of driving a 12-kN truck over the bridge. Knowing that the distance between the truck's axles is $b = 2.25$ m and assuming that the weight of the truck is equally distributed over its four wheels, write a computer program that can be used to calculate the forces created by the truck in members *BH* and *GH* for values of x from 0 to 17.25 m using 0.75-m increments. From the results obtained, determine (*a*) the maximum tensile force in *BH*, (*b*) the maximum compressive force in *BH*, (*c*) the maximum tensile force in *GH*. Indicate in each case the corresponding value of x. (*Note:* The increments have been selected so that the desired values are among those that will be tabulated.)

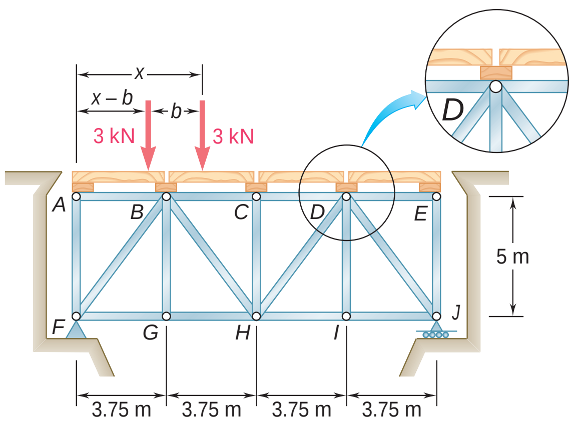


Fig. P6.C2

6.C3 In the mechanism shown the position of boom *AC* is controlled by arm *BD*. For the loading shown, write a computer program and use it to determine the couple **M** required to hold the system in equilibrium for values of u from −30° to 90° using 10° increments. Also, for the same values of u, determine the reaction at *A*. As a part of the design process of the mechanism, use appropriate smaller increments and determine (*a*) the value of u for which *M* is maximum and the corresponding value of *M*, (*b*) the value of u for which the reaction at *A* is maximum and the corresponding magnitude of this reaction.

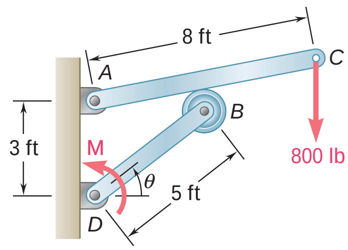


Fig. P6.C3

6.C4 The design of a robotic system calls for the two-rod mechanism shown. Rods *AC* and *BD* are connected by a slider block *D* as shown. Neglecting the effect of friction, write a computer program and use it to determine the couple $\mathbf{M}_A$ required to hold the rods in equilibrium for values of u from 0 to 120° using 10° increments. For the same values of u, determine the magnitude of the force **F** exerted by rod *AC* on the slider block.

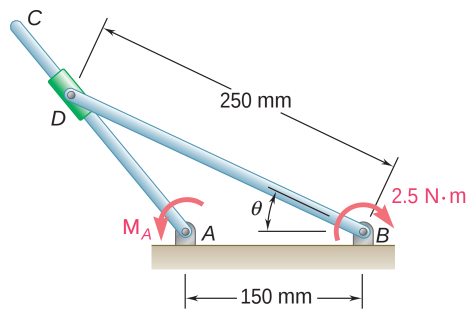


Fig. P6.C4

6.C5 The compound-lever pruning shears shown can be adjusted by placing pin *A* at various ratchet positions on blade *ACE*. Knowing that the length *AB* is 0.85 in., write a computer program and use it to determine the magnitude of the vertical forces applied to the small branch for values of *d* from 0.4 in. to 0.6 in. using 0.025-in. increments. As a part of the design of the shears, use appropriate smaller increments and determine the smallest allowable value of *d* if the force in link *AB* is not to exceed 500 lb.

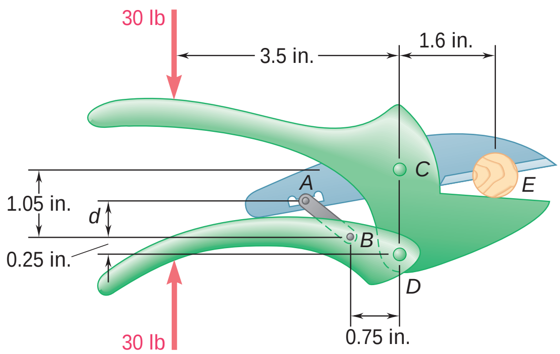


Fig. P6.C5

6.C6 Rod *CD* is attached to collar *D* and passes through a collar welded to end *B* of lever *AB*. As an initial step in the design of lever *AB*, write a computer program and use it to calculate the magnitude *M* of the couple required to hold the system in equilibrium for values of u from 15° to 90° using 5° increments. Using appropriate smaller increments, determine the value of u for which *M* is minimum and the corresponding value of *M*.

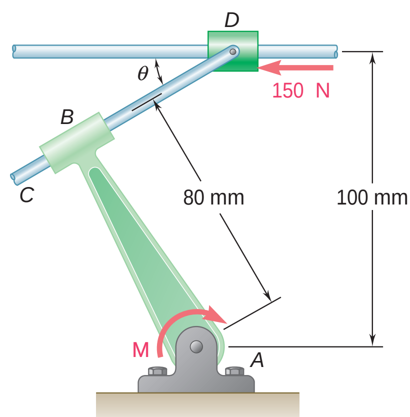


Fig. P6.C6