The tractive force that a railroad locomotive can develop depends upon the frictional resistance between the drive wheels and the rails. When the potential exists for wheel slip to occur, such as when a train travels upgrade over wet rails, sand is deposited on top of the railhead to increase this friction.

CHAPTER 8

Friction

Chapter 8 Friction

8.1 INTRODUCTION

In the preceding chapters, it was assumed that surfaces in contact were either *frictionless* or *rough*. If they were frictionless, the force each surface exerted on the other was normal to the surfaces and the two surfaces could move freely with respect to each other. If they were rough, it was assumed that tangential forces could develop to prevent the motion of one surface with respect to the other.

This view was a simplified one. Actually, no perfectly frictionless surface exists. When two surfaces are in contact, tangential forces, called *friction forces,* will always develop if one attempts to move one surface with respect to the other. On the other hand, these friction forces are limited in magnitude and will not prevent motion if sufficiently large forces are applied. The distinction between frictionless and rough surfaces is thus a matter of degree. This will be seen more clearly in the present chapter, which is devoted to the study of friction and of its applications to common engineering situations.

There are two types of friction: *dry friction,* sometimes called *Coulomb friction,* and *fluid friction*. Fluid friction develops between layers of fluid moving at different velocities. Fluid friction is of great importance in problems involving the flow of fluids through pipes and orifices or dealing with bodies immersed in moving fluids. It is also basic in the analysis of the motion of *lubricated mechanisms*. Such problems are considered in texts on fluid mechanics. The present study is limited to dry friction, i.e., to problems involving rigid bodies which are in contact along *nonlubricated* surfaces.

In the first part of this chapter, the equilibrium of various rigid bodies and structures, assuming dry friction at the surfaces of contact, is analyzed. Later a number of specific engineering applications where dry friction plays an important role are considered: wedges, square-threaded screws, journal bearings, thrust bearings, rolling resistance, and belt friction.

8.2 THE LAWS OF DRY FRICTION. COEFFICIENTS OF FRICTION

The laws of dry friction are exemplified by the following experiment. A block of weight **W** is placed on a horizontal plane surface (Fig. 8.1*a*). The forces acting on the block are its weight **W** and the reaction of the surface. Since the weight has no horizontal component, the reaction of the surface also has no horizontal component; the reaction is therefore *normal* to the surface and is represented by **N** in Fig. 8.1*a*. Suppose, now, that a horizontal force **P** is applied to the block (Fig. 8.1*b*). If **P** is small, the block will not move; some other horizontal force must therefore exist, which balances **P**. This other force is the *static-friction force* **F**, which is actually the resultant of a great number of forces acting over the entire surface of contact between the block and the plane. The nature of these forces is not known exactly, but it is generally assumed that these forces are due to the irregularities of the surfaces in contact and, to a certain extent, to molecular attraction.

If the force **P** is increased, the friction force **F** also increases, continuing to oppose **P**, until its magnitude reaches a certain *maximum value* F_m (Fig. 8.1*c*). If **P** is further increased, the friction force

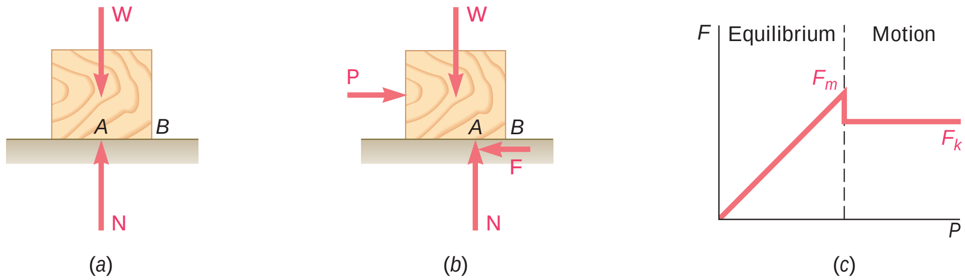


Fig. 8.1

cannot balance it any more and the block starts sliding.† As soon as the block has been set in motion, the magnitude of **F** drops from F_m to a lower value F_k. This is because there is less interpenetration between the irregularities of the surfaces in contact when these surfaces move with respect to each other. From then on, the block keeps sliding with increasing velocity while the friction force, denoted by $\mathbf{F}_k$ and called the *kinetic-friction force,* remains approximately constant.

Experimental evidence shows that the maximum value F_m of the static-friction force is proportional to the normal component N of the reaction of the surface. We have

$$F_m = \mu_s N \tag{8.1}$$

where μ_s is a constant called the *coefficient of static friction*. Similarly, the magnitude F_k of the kinetic-friction force may be put in the form

$$F_k = \mu_k N \tag{8.2}$$

where μ_k is a constant called the *coefficient of kinetic friction*. The coefficients of friction μ_s and μ_k do not depend upon the area of

†It should be noted that, as the magnitude F of the friction force increases from 0 to F_m, the point of application A of the resultant **N** of the normal forces of contact moves to the right, so that the couples formed, respectively, by **P** and **F** and by **W** and **N** remain balanced. If **N** reaches B before F reaches its maximum value F_m, the block will tip about B before it can start sliding (see Probs. 8.15 through 8.18).

the surfaces in contact. Both coefficients, however, depend strongly on the *nature* of the surfaces in contact. Since they also depend upon the exact condition of the surfaces, their value is seldom known with an accuracy greater than 5 percent. Approximate values of coefficients of static friction for various dry surfaces are given in Table 8.1. The corresponding values of the coefficient of kinetic friction would be about 25 percent smaller. Since coefficients of friction are dimensionless quantities, the values given in Table 8.1 can be used with both SI units and U.S. customary units.

TABLE 8.1 Approximate Values of Coefficient of Static Friction for Dry Surfaces

Metal on metal	0.15–0.60
Metal on wood	0.20–0.60
Metal on stone	0.30–0.70
Metal on leather	0.30–0.60
Wood on wood	0.25–0.50
Wood on leather	0.25–0.50
Stone on stone	0.40–0.70
Earth on earth	0.20–1.00
Rubber on concrete	0.60–0.90

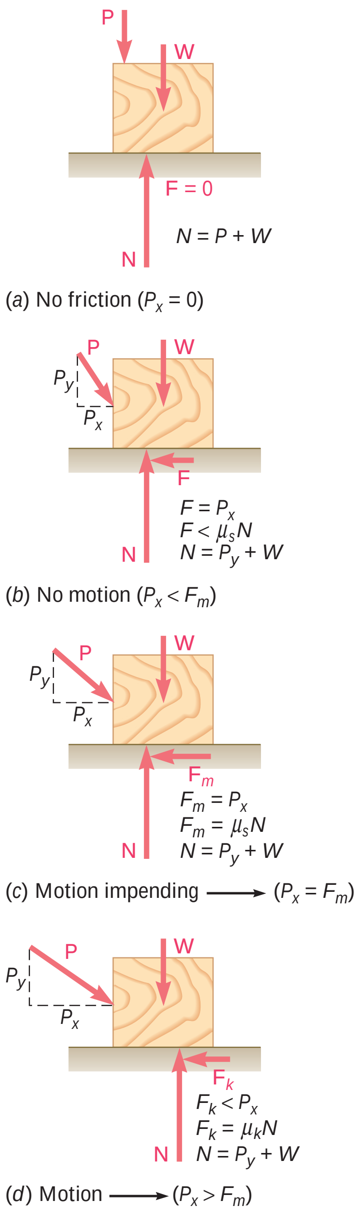


Fig. 8.2

From the description given above, it appears that four different situations can occur when a rigid body is in contact with a horizontal surface:

1. The forces applied to the body do not tend to move it along the surface of contact; there is no friction force (Fig. 8.2*a*).
2. The applied forces tend to move the body along the surface of contact but are not large enough to set it in motion. The friction force **F** which has developed can be found by solving the equations of equilibrium for the body. Since there is no evidence that **F** has reached its maximum value, the equation $F_m = \mu_s N$ *cannot be used* to determine the friction force (Fig. 8.2*b*).
3. The applied forces are such that the body is just about to slide. We say that *motion is impending*. The friction force **F** has reached its maximum value F_m and, together with the normal force **N**, balances the applied forces. Both the equations of equilibrium and the equation $F_m = \mu_s N$ *can be used*. We also note that the friction force has a sense opposite to the sense of impending motion (Fig. 8.2*c*).
4. The body is sliding under the action of the applied forces, and the equations of equilibrium do not apply any more. However, **F** is now equal to $\mathbf{F}_k$ and the equation $F_k = \mu_k N$ may be used. The sense of $\mathbf{F}_k$ is opposite to the sense of motion (Fig. 8.2*d*).

8.3 ANGLES OF FRICTION

It is sometimes convenient to replace the normal force **N** and the friction force **F** by their resultant **R**. Let us consider again a block of weight **W** resting on a horizontal plane surface. If no horizontal force is applied to the block, the resultant **R** reduces to the normal force **N** (Fig. 8.3*a*). However, if the applied force **P** has a horizontal component $\mathbf{P}_x$ which tends to move the block, the force **R** will have a horizontal component **F** and, thus, will form an angle ϕ with the normal to the surface (Fig. 8.3*b*). If $\mathbf{P}_x$ is increased until motion becomes impending, the angle between **R** and the vertical grows and reaches a maximum value (Fig. 8.3*c*). This value is called the *angle of static friction* and is denoted by ϕ_s. From the geometry of Fig. 8.3*c*, we note that

$$\tan \phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\boxed{\tan \phi_s = \mu_s} \tag{8.3}$$

If motion actually takes place, the magnitude of the friction force drops to F_k; similarly, the angle ϕ between **R** and **N** drops to a lower value ϕ_k, called the *angle of kinetic friction* (Fig. 8.3*d*). From the geometry of Fig. 8.3*d*, we write

$$\tan \phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\boxed{\tan \phi_k = \mu_k} \tag{8.4}$$

Another example will show how the angle of friction can be used to advantage in the analysis of certain types of problems. Consider a block resting on a board and subjected to no other force than its weight **W** and the reaction **R** of the board. The board can be given any desired inclination. If the board is horizontal, the force **R** exerted by the board on the block is perpendicular to the board and balances the weight **W** (Fig. 8.4*a*). If the board is given a small angle of inclination θ, the force **R** will deviate from the perpendicular to the board by the angle θ and will keep balancing **W** (Fig. 8.4*b*); it will then have a normal component **N** of magnitude $N = W \cos \theta$ and a tangential component **F** of magnitude $F = W \sin \theta$.

If we keep increasing the angle of inclination, motion will soon become impending. At that time, the angle between **R** and the normal will have reached its maximum value ϕ_s (Fig. 8.4*c*). The value of the angle of inclination corresponding to impending motion is called the *angle of repose*. Clearly, the angle of repose is equal to the angle of static friction ϕ_s. If the angle of inclination θ is further increased, motion starts and the angle between **R** and the normal drops to the lower value ϕ_k (Fig. 8.4*d*). The reaction **R** is not vertical any more, and the forces acting on the block are unbalanced.

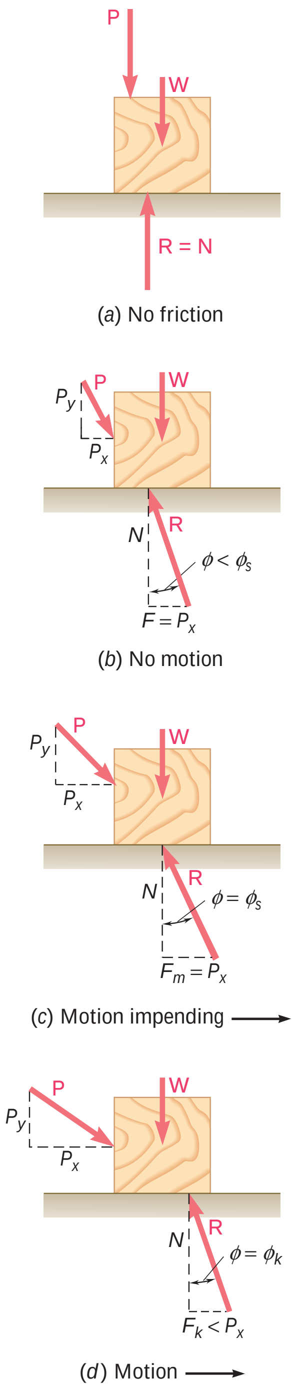


Fig. 8.3

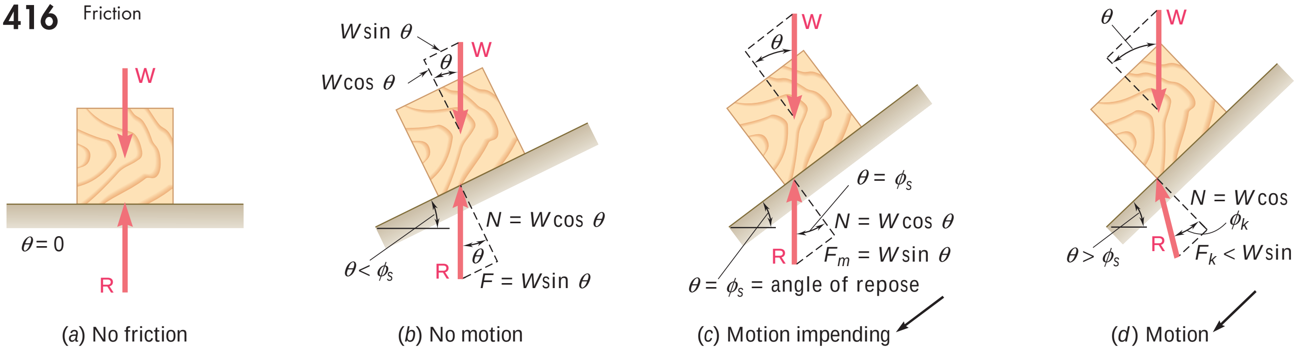


Fig. 8.4

Photo 8.1 The coefficient of static friction between a package and the inclined conveyer belt must be sufficiently large to enable the package to be transported without slipping.

8.4 PROBLEMS INVOLVING DRY FRICTION

Problems involving dry friction are found in many engineering applications. Some deal with simple situations such as the block sliding on a plane described in the preceding sections. Others involve more complicated situations as in Sample Prob. 8.3; many deal with the stability of rigid bodies in accelerated motion and will be studied in dynamics. Also, a number of common machines and mechanisms can be analyzed by applying the laws of dry friction. These include wedges, screws, journal and thrust bearings, and belt transmissions. They will be studied in the following sections.

The *methods* which should be used to solve problems involving dry friction are the same that were used in the preceding chapters. If a problem involves only a motion of translation, with no possible rotation, the body under consideration can usually be treated as a particle, and the methods of Chap. 2 used. If the problem involves a possible rotation, the body must be considered as a rigid body, and the methods of Chap. 4 should be used. If the structure considered is made of several parts, the principle of action and reaction must be used as was done in Chap. 6.

If the body considered is acted upon by more than three forces (including the reactions at the surfaces of contact), the reaction at each surface will be represented by its components **N** and **F** and the problem will be solved from the equations of equilibrium. If only three forces act on the body under consideration, it may be more convenient to represent each reaction by the single force **R** and to solve the problem by drawing a force triangle.

Most problems involving friction fall into one of the following *three groups:* In the *first group* of problems, all applied forces are given and the coefficients of friction are known; we are to determine whether the body considered will remain at rest or slide. The friction force **F** *required to maintain equilibrium* is unknown (its magnitude is *not* equal to $\mu_s N$) and should be determined, together with the normal force **N**, by drawing a free-body diagram and *solving the equations of equilibrium* (Fig. 8.5*a*). The value found for the magnitude F of the friction force is then compared with the maximum value $F_m = \mu_s N$. If F is smaller than or equal to F_m, the body remains at rest. If the value found for F is larger than F_m, equilibrium cannot be maintained and motion takes place; the actual magnitude of the friction force is then $F_k = \mu_k N$.

In problems of the *second group*, all applied forces are given and the motion is known to be impending; we are to determine the value of the coefficient of static friction. Here again, we determine the friction force and the normal force by drawing a free-body diagram and solving the equations of equilibrium (Fig. 8.5*b*). Since we know that the value found for F is the maximum value F_m, the coefficient of friction may be found by writing and solving the equation $F_m = \mu_s N$.

In problems of the *third group*, the coefficient of static friction is given, and it is known that the motion is impending in a given direction; we are to determine the magnitude or the direction of one of the applied forces. The friction force should be shown in the free-body diagram with a *sense opposite to that of the impending motion* and with a magnitude $F_m = \mu_s N$ (Fig. 8.5*c*). The equations of equilibrium can then be written, and the desired force determined.

As noted above, when only three forces are involved it may be more convenient to represent the reaction of the surface by a single force **R** and to solve the problem by drawing a force triangle. Such a solution is used in Sample Prob. 8.2.

When two bodies A and B are in contact (Fig. 8.6*a*), the forces of friction exerted, respectively, by A on B and by B on A are equal and opposite (Newton's third law). In drawing the free-body diagram of one of the bodies, it is important to include the appropriate friction force with its correct sense. The following rule should then be observed: *The sense of the friction force acting on A is opposite to that of the motion (or impending motion) of A as observed from B* (Fig. 8.6*b*).† The sense of the friction force acting on B is determined in a similar way (Fig. 8.6*c*). Note that the motion of A as observed from B is a *relative motion*. For example, if body A is fixed and body B moves, body A will have a relative motion with respect to B. Also, if both B and A are moving down but B is moving faster than A, body A will be observed, from B, to be moving up.

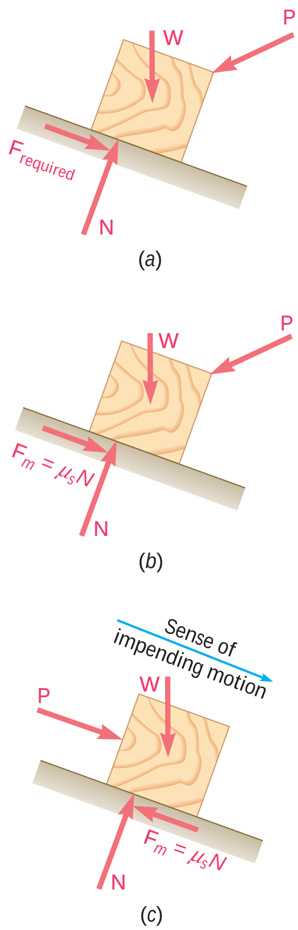


Fig. 8.5

†It is therefore *the same as that of the motion of B as observed from A*.

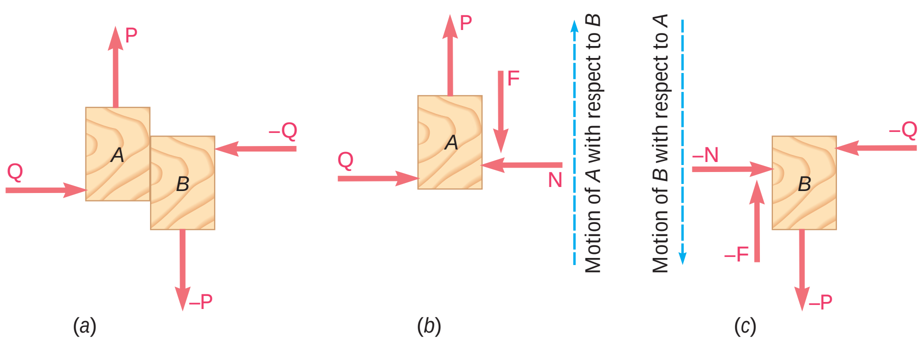
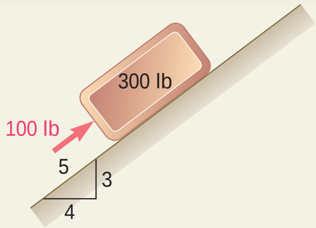


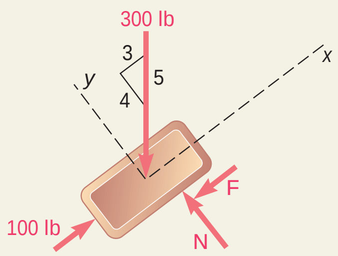
Fig. 8.6

SAMPLE PROBLEM 8.1



A 100-lb force acts as shown on a 300-lb block placed on an inclined plane. The coefficients of friction between the block and the plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the block is in equilibrium, and find the value of the friction force.

SOLUTION



Force Required for Equilibrium. We first determine the value of the friction force *required to maintain equilibrium*. Assuming that **F** is directed down and to the left, we draw the free-body diagram of the block and write

$+\nearrow \Sigma F_x = 0$: $\quad 100 \text{ lb} - \frac{3}{5}(300 \text{ lb}) - F = 0$

$F = -80 \text{ lb} \qquad \mathbf{F} = 80 \text{ lb} \nearrow$

$+\nwarrow \Sigma F_y = 0$: $\quad N - \frac{4}{5}(300 \text{ lb}) = 0$

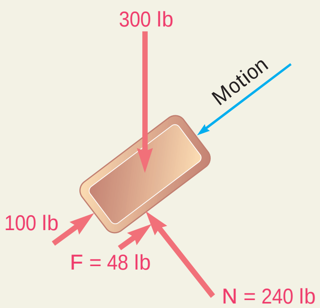
$N = +240 \text{ lb} \qquad \mathbf{N} = 240 \text{ lb} \nwarrow$

The force **F** required to maintain equilibrium is an 80-lb force directed up and to the right; the tendency of the block is thus to move down the plane.

Maximum Friction Force. The magnitude of the maximum friction force which may be developed is

$$F_m = \mu_s N \qquad F_m = 0.25(240 \text{ lb}) = 60 \text{ lb}$$

Since the value of the force required to maintain equilibrium (80 lb) is larger than the maximum value which may be obtained (60 lb), equilibrium will not be maintained and *the block will slide down the plane.*



Actual Value of Friction Force. The magnitude of the actual friction force is obtained as follows:

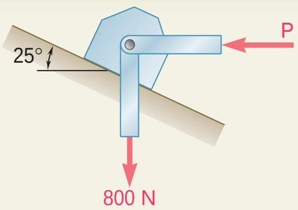
$$F_{\text{actual}} = F_k = \mu_k N = 0.20(240 \text{ lb}) = 48 \text{ lb}$$

The sense of this force is opposite to the sense of motion; the force is thus directed up and to the right:

$$\mathbf{F}_{\text{actual}} = 48 \text{ lb} \nearrow \quad ◀$$

It should be noted that the forces acting on the block are not balanced; the resultant is

$$\frac{3}{5}(300 \text{ lb}) - 100 \text{ lb} - 48 \text{ lb} = 32 \text{ lb} \swarrow$$



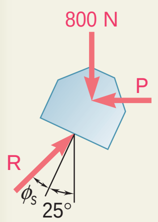
SAMPLE PROBLEM 8.2

A support block is acted upon by two forces as shown. Knowing that the coefficients of friction between the block and the incline are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the force **P** required (*a*) to start the block moving up the incline, (*b*) to keep it moving up, (*c*) to prevent it from sliding down.

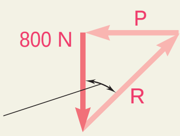
SOLUTION

Free-Body Diagram. For each part of the problem we draw a free-body diagram of the block and a force triangle including the 800-N vertical force, the horizontal force **P**, and the force **R** exerted on the block by the incline. The direction of **R** must be determined in each separate case. We note that since **P** is perpendicular to the 800-N force, the force triangle is a right triangle, which can easily be solved for **P**. In most other problems, however, the force triangle will be an oblique triangle and should be solved by applying the law of sines.

a. Force *P* to Start Block Moving Up

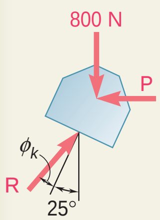


$\tan \phi_s = \mu_s$
$= 0.35$
$\phi_s = 19.29°$
$25° + 19.29° = 44.29°$

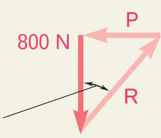


$P = (800\ \text{N}) \tan 44.29°$ $\mathbf{P} = 780\ \text{N}$ z ◀

b. Force *P* to Keep Block Moving Up

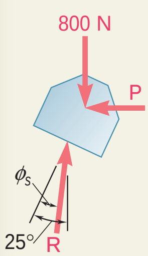


$\tan \phi_k = \mu_k$
$= 0.25$
$\phi_k = 14.04°$
$25° + 14.04° = 39.04°$

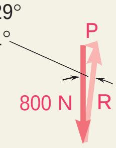


$P = (800\ \text{N}) \tan 39.04°$ $\mathbf{P} = 649\ \text{N}$ z ◀

c. Force *P* to Prevent Block from Sliding Down

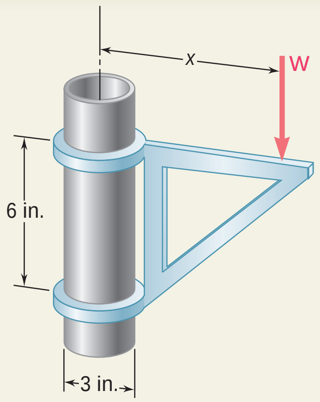


$\phi_s = 19.29°$
$25° - 19.29° = 5.71°$



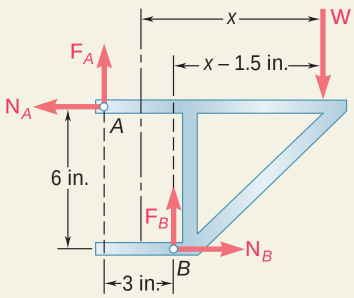
$P = (800\ \text{N}) \tan 5.71°$ $\mathbf{P} = 80.0\ \text{N}$ z ◀

SAMPLE PROBLEM 8.3



The movable bracket shown may be placed at any height on the 3-in.-diameter pipe. If the coefficient of static friction between the pipe and bracket is 0.25, determine the minimum distance x at which the load **W** can be supported. Neglect the weight of the bracket.

SOLUTION



Free-Body Diagram. We draw the free-body diagram of the bracket. When **W** is placed at the minimum distance x from the axis of the pipe, the bracket is just about to slip, and the forces of friction at A and B have reached their maximum values:

$$F_A = \mu_s N_A = 0.25\ N_A$$
$$F_B = \mu_s N_B = 0.25\ N_B$$

Equilibrium Equations

$\xrightarrow{+}\Sigma F_x = 0$: $\quad N_B - N_A = 0$
$\quad N_B = N_A$

$+\uparrow\Sigma F_y = 0$: $\quad F_A + F_B - W = 0$
$\quad 0.25N_A + 0.25N_B = W$

And, since N_B has been found equal to N_A,

$$0.50N_A = W$$
$$N_A = 2W$$

$+\circlearrowleft\Sigma M_B = 0$: $\quad N_A(6\text{ in.}) - F_A(3\text{ in.}) - W(x - 1.5\text{ in.}) = 0$
$\quad 6N_A - 3(0.25N_A) - Wx + 1.5W = 0$
$\quad 6(2W) - 0.75(2W) - Wx + 1.5W = 0$

Dividing through by W and solving for x,

$$x = 12\text{ in.}$$

SOLVING PROBLEMS ON YOUR OWN

In this lesson you studied and applied the *laws of dry friction*. Previously you had encountered only (*a*) frictionless surfaces that could move freely with respect to each other, (*b*) rough surfaces that allowed no motion relative to each other.

A. In solving problems involving dry friction, you should keep the following in mind.

1. The reaction R exerted by a surface on a free body can be resolved into a component **N** and a tangential component **F**. The tangential component is known as the *friction force*. When a body is in contact with a fixed surface the direction of the friction force **F** is opposite to that of the actual or impending motion of the body.

a. No motion will occur as long as F does not exceed the maximum value $F_m = \mu_s N$, where μ_s is the *coefficient of static friction*.

b. Motion will occur if a value of F larger than F_m is required to maintain equilibrium. As motion takes place, the actual value of F drops to $F_k = \mu_k N$, where μ_k is the *coefficient of kinetic friction* [Sample Prob. 8.1].

2. When only three forces are involved an alternative approach to the analysis of friction may be preferred [Sample Prob. 8.2]. The reaction **R** is defined by its magnitude R and the angle ϕ it forms with the normal to the surface. No motion will occur as long as ϕ does not exceed the maximum value ϕ_s, where $\tan \phi_s = \mu_s$. Motion will occur if a value of ϕ larger than ϕ_s is required to maintain equilibrium, and the actual value of ϕ will drop to ϕ_k, where $\tan \phi_k = \mu_k$.

3. When two bodies are in contact the sense of the actual or impending relative motion at the point of contact must be determined. On each of the two bodies a friction force **F** should be shown in a direction opposite to that of the actual or impending motion of the body as seen from the other body.

(*continued*)

B. Methods of solution. The first step in your solution is to *draw a free-body diagram* of the body under consideration, resolving the force exerted on each surface where friction exists into a normal component **N** and a friction force **F**. If several bodies are involved, draw a free-body diagram of each of them, labeling and directing the forces at each surface of contact as you learned to do when analyzing frames in Chap. 6.

The problem you have to solve may fall in one of the following three categories:

1. All the applied forces and the coefficients of friction are known, and you must determine whether equilibrium is maintained. Note that in this situation the friction force is unknown and *cannot be assumed to be equal* to $\mu_s N$.

a. Write the equations of equilibrium to determine N and F.

b. Calculate the maximum allowable friction force, $F_m = \mu_s N$. If $F \leq F_m$, equilibrium is maintained. If $F > F_m$, motion occurs, and the magnitude of the friction force is $F_k = \mu_k N$ [Sample Prob. 8.1].

2. All the applied forces are known, and you must find the smallest allowable value of μ_s for which equilibrium is maintained. You will assume that motion is impending and determine the corresponding value of μ_s.

a. Write the equations of equilibrium to determine N and F.

b. Since motion is impending, $F = F_m$. Substitute the values found for N and F into the equation $F_m = \mu_s N$ and solve for μ_s.

3. The motion of the body is impending and μ_s is known; you must find some unknown quantity, such as a distance, an angle, the magnitude of a force, or the direction of a force.

a. Assume a possible motion of the body and, on the free-body diagram, draw the friction force in a direction opposite to that of the assumed motion.

b. Since motion is impending, $F = F_m = \mu_s N$. Substituting for μ_s its known value, you can express F in terms of N on the free-body diagram, thus eliminating one unknown.

c. Write and solve the equilibrium equations for the unknown you seek [Sample Prob. 8.3].

PROBLEMS

FREE BODY PRACTICE PROBLEMS

8.F1 Draw the free-body diagram needed to determine the smallest force **P** for which equilibrium of the 7.5-kg block is maintained.

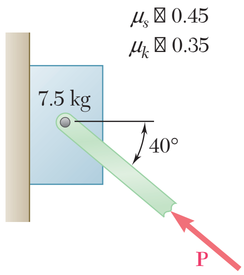


Fig. P8.F1

8.F2 Two blocks A and B are connected by a cable as shown. Knowing that the coefficient of static friction at all surfaces of contact is 0.30 and neglecting the friction of the pulleys, draw the free-body diagrams needed to determine the smallest force **P** required to move the blocks.

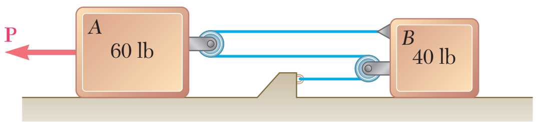


Fig. P8.F2

8.F3 The cylinder shown is of weight W and radius r, and the coefficient of static friction m_s is the same at A and B. Draw the free-body diagram needed to determine the largest couple **M** that can be applied to the cylinder if it is not to rotate.

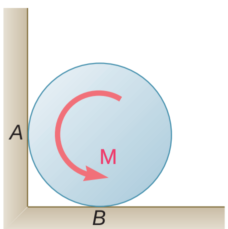


Fig. P8.F3

8.F4 A uniform crate of mass 30 kg must be moved up along the 15° incline without tipping. Knowing that the force **P** is horizontal, draw the free-body diagram needed to determine the largest allowable coefficient of static friction between the crate and the incline, and the corresponding force **P**.

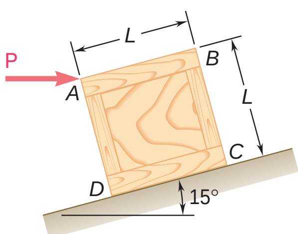


Fig. P8.F4

END-OF-SECTION PROBLEMS

8.1 Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 25°$ and $P = 750$ N.

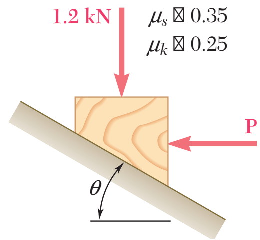


Fig. P8.1 and P8.2

8.2 Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 30°$ and $P = 150$ N.

8.3 Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 100$ lb.

8.4 Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 60$ lb.

8.5 Determine the smallest value of P required to (*a*) start the block up the incline, (*b*) keep it moving up, (*c*) prevent it from moving down.

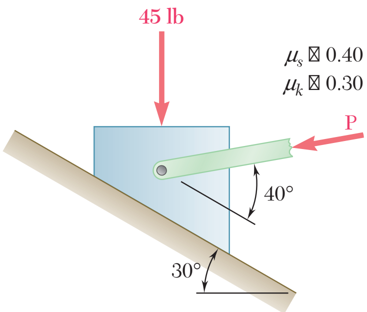


Fig. P8.3, P8.4, and P8.5

8.6 Knowing that the coefficient of friction between the 25-kg block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of P required to start the block moving up the incline, (*b*) the corresponding value of β.

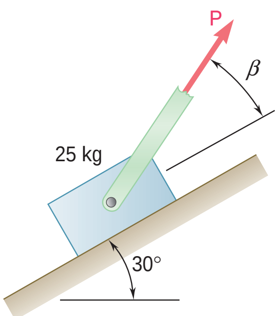


Fig. P8.6

8.7 The 80-lb block is attached to link AB and rests on a moving belt. Knowing that $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the magnitude of the horizontal force **P** that should be applied to the belt to maintain its motion (*a*) to the right, (*b*) to the left.

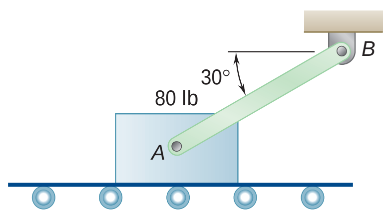


Fig. *P8.7*

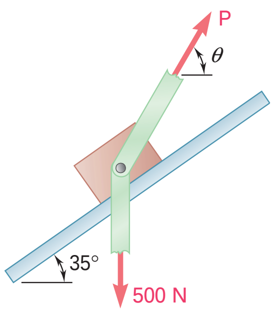


Fig. P8.8

8.8 The coefficients of friction between the block and the rail are $\mu_s = 0.30$ and $\mu_k = 0.25$. Knowing that $\theta = 65°$, determine the smallest value of P required (*a*) to start the block moving up the rail, (*b*) to keep it from moving down.

8.9 Considering only values of u less than 90°, determine the smallest value of u required to start the block moving to the right when (*a*) $W = 75$ lb, (*b*) $W = 100$ lb.

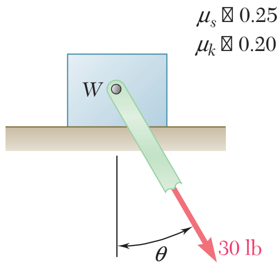


Fig. *P8.9*

8.10 Determine the range of values of P for which equilibrium of the block shown is maintained.

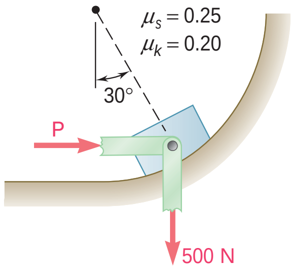


Fig. P8.10

8.11 The 20-lb block A and the 30-lb block B are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between the two blocks and zero between block B and the incline, determine the value of u for which motion is impending.

8.12 The 20-lb block A and the 30-lb block B are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between all surfaces of contact, determine the value of u for which motion is impending.

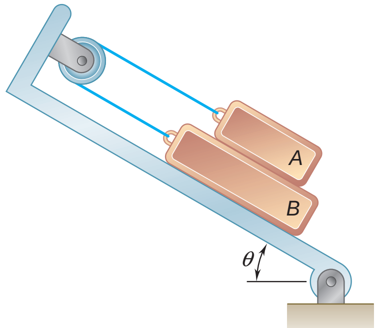


Fig. P8.11 and P8.12

8.13 and 8.14 The coefficients of friction are $m_s = 0.40$ and $m_k = 0.30$ between all surfaces of contact. Determine the smallest force **P** required to start the 30-kg block moving if cable AB (*a*) is attached as shown, (*b*) is removed.

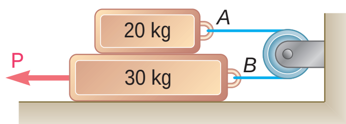


Fig. P8.13

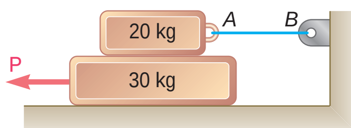


Fig. P8.14

8.15 A 40-kg packing crate must be moved to the left along the floor without tipping. Knowing that the coefficient of static friction between the crate and the floor is 0.35, determine (*a*) the largest allowable value of a, (*b*) the corresponding magnitude of the force **P**.

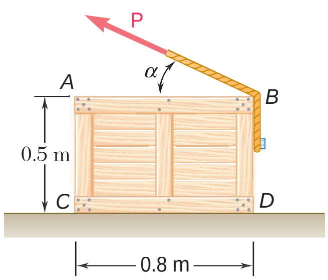


Fig. *P8.15* and *P8.16*

8.16 A 40-kg packing crate is pulled by a rope as shown. The coefficient of static friction between the crate and the floor is 0.35. If a = 40°, determine (*a*) the magnitude of the force **P** required to move the crate, (*b*) whether the crate will slide or tip.

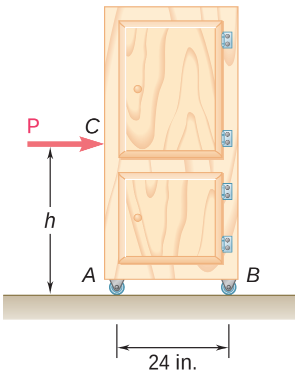


Fig. P8.17 and P8.18

8.17 A 120-lb cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. If $h = 32$ in., determine the magnitude of the force **P** required to move the cabinet to the right (*a*) if all casters are locked, (*b*) if the casters at *B* are locked and the casters at *A* are free to rotate, (*c*) if the casters at *A* are locked and the casters at *B* are free to rotate.

8.18 A 120-lb cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Assuming that the casters at both *A* and *B* are locked, determine (*a*) the force **P** required to move the cabinet to the right, (*b*) the largest allowable value of *h* if the cabinet is not to tip over.

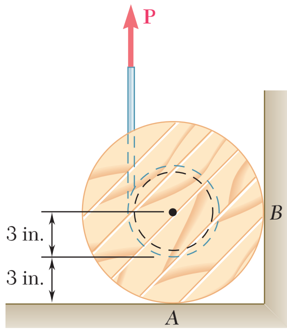


Fig. P8.19

8.19 Wire is being drawn at a constant rate from a spool by applying a vertical force **P** to the wire as shown. The spool and the wire wrapped on the spool have a combined weight of 20 lb. Knowing that the coefficients of friction at both *A* and *B* are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine the required magnitude of the force **P**.

8.20 Solve Prob. 8.19 assuming that the coefficients of friction at *B* are zero.

8.21 The hydraulic cylinder shown exerts a force of 3 kN directed to the right on point *B* and to the left on point *E*. Determine the magnitude of the couple **M** required to rotate the drum clockwise at a constant speed.

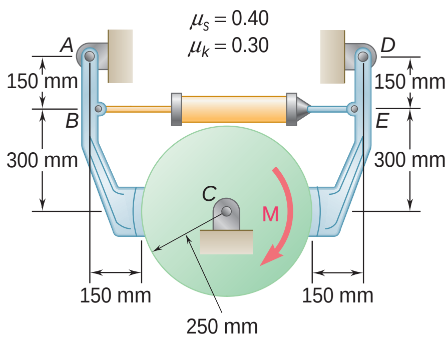


Fig. P8.21 and P8.22

8.22 A couple **M** of magnitude 100 N · m is applied to the drum as shown. Determine the smallest force that must be exerted by the hydraulic cylinder on joints *B* and *E* if the drum is not to rotate.

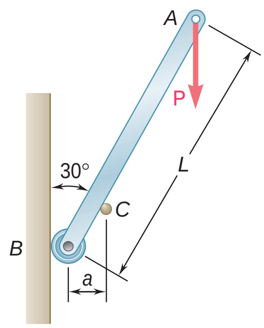


Fig. P8.23

8.23 A slender rod of length *L* is lodged between peg *C* and the vertical wall, and supports a load **P** at end *A*. Knowing that the coefficient of static friction between the peg and the rod is 0.15 and neglecting friction at the roller, determine the range of values of the ratio *L*/*a* for which equilibrium is maintained.

8.24 Solve Prob. 8.23 assuming that the coefficient of static friction between the peg and the rod is 0.60.

8.25 A 6.5-m ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is zero at B, determine the smallest value of μ_s at A for which equilibrium is maintained.

8.26 A 6.5-m ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at A and B, determine the smallest value of μ_s for which equilibrium is maintained.

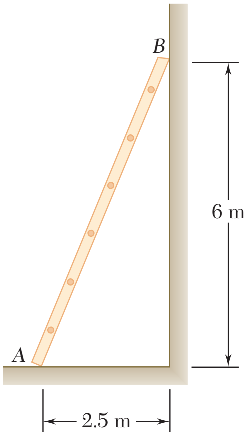


Fig. P8.25 and *P8.26*

8.27 The press shown is used to emboss a small seal at E. Knowing that the coefficient of static friction between the vertical guide and the embossing die D is 0.30, determine the force exerted by the die on the seal.

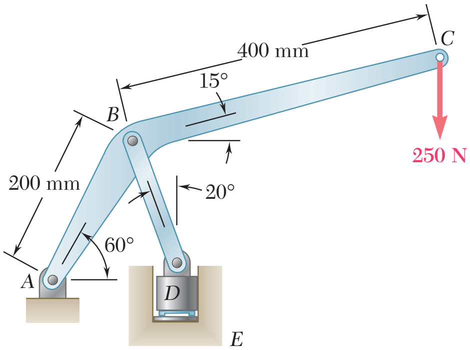


Fig. P8.27

8.28 The machine base shown has a mass of 75 kg and is fitted with skids at A and B. The coefficient of static friction between the skids and the floor is 0.30. If a force $\mathbf{P}$ of magnitude 500 N is applied at corner C, determine the range of values of θ for which the base will not move.

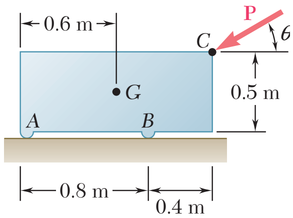


Fig. *P8.28*

8.29 The 50-lb plate $ABCD$ is attached at A and D to collars that can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at E is (*a*) $P = 0$, (*b*) $P = 20$ lb.

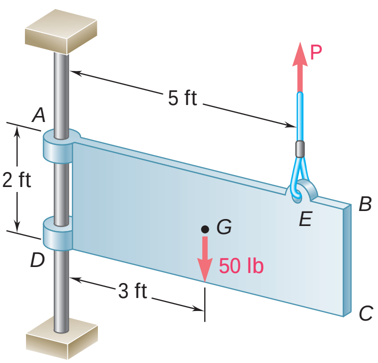


Fig. P8.29

8.30 In Prob. 8.29, determine the range of values of the magnitude P of the vertical force applied at E for which the plate will move downward.

8.31 A rod DE and a small cylinder are placed between two guides as shown. The rod is not to slip downward, however large the force **P** may be; i.e., the arrangement is said to be self-locking. Neglecting the weight of the cylinder, determine the minimum allowable coefficients of static friction at A, B, and C.

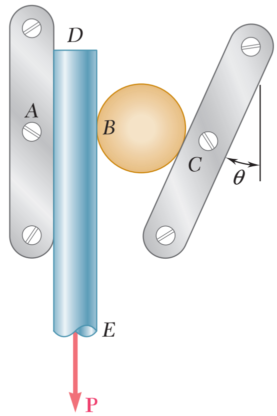


Fig. *P8.31*

8.32 A 500-N concrete block is to be lifted by the pair of tongs shown. Determine the smallest allowable value of the coefficient of static friction between the block and the tongs at F and G.

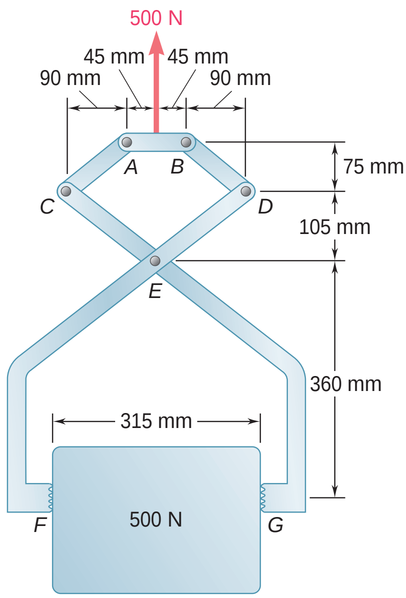


Fig. P8.32

8.33 The 100-mm-radius cam shown is used to control the motion of the plate CD. Knowing that the coefficient of static friction between the cam and the plate is 0.45 and neglecting friction at the roller supports, determine (*a*) the force **P** required to maintain the motion of the plate, knowing that the plate is 20 mm thick, (*b*) the largest thickness of the plate for which the mechanism is self-locking (i.e., for which the plate cannot be moved however large the force **P** may be).

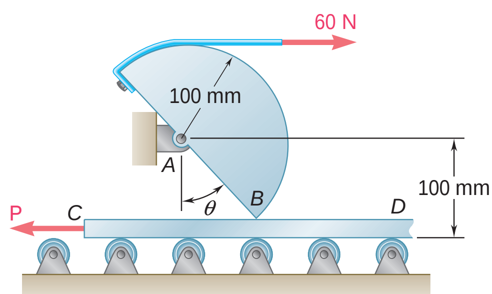


Fig. *P8.33*

8.34 A safety device used by workers climbing ladders fixed to high structures consists of a rail attached to the ladder and a sleeve that can slide on the flange of the rail. A chain connects the worker's belt to the end of an eccentric cam that can be rotated about an axle attached to the sleeve at C. Determine the smallest allowable common value of the coefficient of static friction between the flange of the rail, the pins at A and B, and the eccentric cam if the sleeve is not to slide down when the chain is pulled vertically downward.

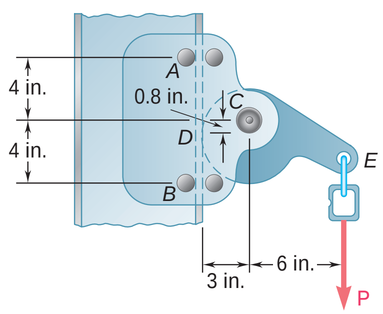


Fig. P8.34

8.35 To be of practical use, the safety sleeve described in Prob. 8.34 must be free to slide along the rail when pulled upward. Determine the largest allowable value of the coefficient of static friction between the flange of the rail and the pins at A and B if the sleeve is to be free to slide when pulled as shown in the figure, assuming (a) $\theta = 60°$, (b) $\theta = 50°$, (c) $\theta = 40°$.

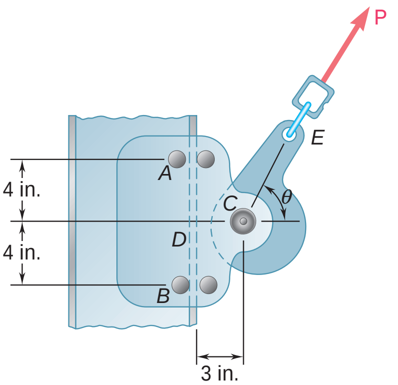


Fig. P8.35

8.36 Two 10-lb blocks A and B are connected by a slender rod of negligible weight. The coefficient of static friction is 0.30 between all surfaces of contact, and the rod forms an angle $\theta = 30°$ with the vertical. (a) Show that the system is in equilibrium when $P = 0$. (b) Determine the largest value of P for which equilibrium is maintained.

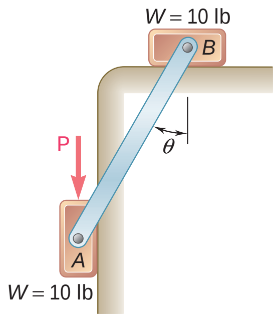


Fig. P8.36

8.37 Bar AB is attached to collars that can slide on the inclined rods shown. A force **P** is applied at point D located at a distance a from end A. Knowing that the coefficient of static friction μ_s between each collar and the rod upon which it slides is 0.30 and neglecting the weights of the bar and of the collars, determine the smallest value of the ratio a/L for which equilibrium is maintained.

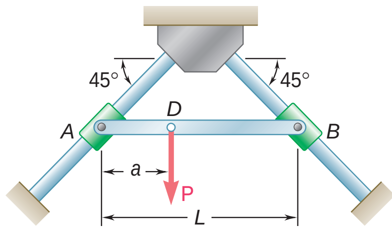


Fig. P8.37

8.38 Two identical uniform boards, each of weight 40 lb, are temporarily leaned against each other as shown. Knowing that the coefficient of static friction between all surfaces is 0.40, determine (a) the largest magnitude of the force **P** for which equilibrium will be maintained, (b) the surface at which motion will impend.

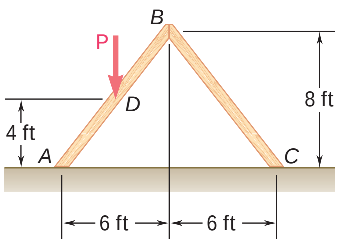


Fig. *P8.38*

8.39 Knowing that the coefficient of static friction between the collar and the rod is 0.35, determine the range of values of P for which equilibrium is maintained when $\theta = 50°$ and $M = 20$ N · m.

8.40 Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of M for which equilibrium is maintained when $\theta = 60°$ and $P = 200$ N.

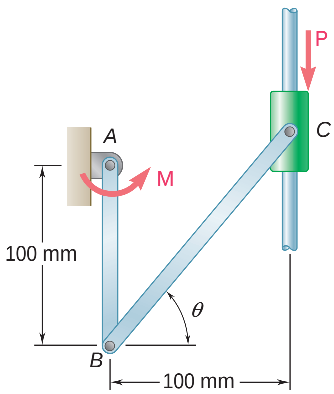


Fig. P8.39 and P8.40

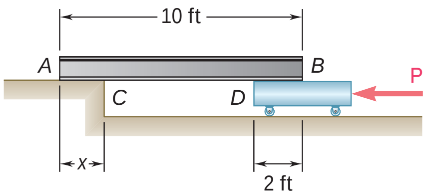


Fig. P8.41

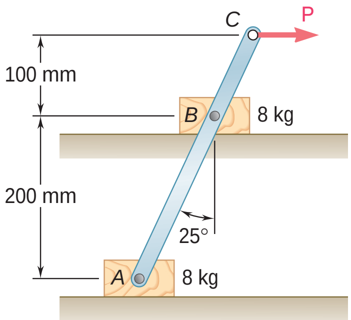


Fig. P8.43

8.41 A 10-ft beam, weighing 1200 lb, is to be moved to the left onto the platform. A horizontal force **P** is applied to the dolly, which is mounted on frictionless wheels. The coefficients of friction between all surfaces are $\mu_s = 0.30$ and $\mu_k = 0.25$, and initially $x = 2$ ft. Knowing that the top surface of the dolly is slightly higher than the platform, determine the force **P** required to start moving the beam. (*Hint:* The beam is supported at *A* and *D*.)

8.42 (*a*) Show that the beam of Prob. 8.41 *cannot* be moved if the top surface of the dolly is slightly *lower* than the platform. (*b*) Show that the beam *can* be moved if two 175-lb workers stand on the beam at *B* and determine how far to the left the beam can be moved.

8.43 Two 8-kg blocks *A* and *B* resting on shelves are connected by a rod of negligible mass. Knowing that the magnitude of a horizontal force **P** applied at *C* is slowly increased from zero, determine the value of *P* for which motion occurs, and what that motion is, when the coefficient of static friction between all surfaces is (*a*) $\mu_s = 0.40$, (*b*) $\mu_s = 0.50$.

8.44 A slender steel rod of length 225 mm is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

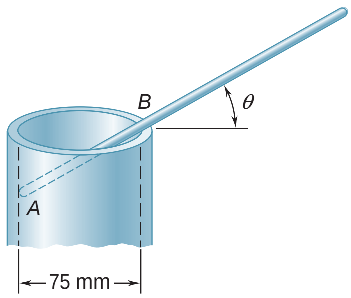


Fig. P8.44

8.45 In Prob. 8.44, determine the smallest value of θ for which the rod will not fall out of the pipe.

8.46 Two slender rods of negligible weight are pin-connected at *C* and attached to blocks *A* and *B*, each of weight *W*. Knowing that $\theta = 80°$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of *P* for which equilibrium is maintained.

8.47 Two slender rods of negligible weight are pin-connected at *C* and attached to blocks *A* and *B*, each of weight *W*. Knowing that $P = 1.260W$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the range of values of θ, between 0 and 180°, for which equilibrium is maintained.

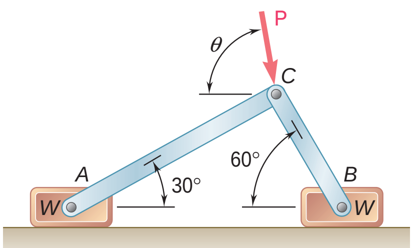


Fig. P8.46 and P8.47

8.5 WEDGES

Wedges are simple machines used to raise large stone blocks and other heavy loads. These loads can be raised by applying to the wedge a force usually considerably smaller than the weight of the load. In addition, because of the friction between the surfaces in contact, a properly shaped wedge will remain in place after being forced under the load. Wedges can thus be used advantageously to make small adjustments in the position of heavy pieces of machinery.

Consider the block *A* shown in Fig. 8.7*a*. This block rests against a vertical wall *B* and is to be raised slightly by forcing a wedge *C* between block *A* and a second wedge *D*. We want to find the minimum value of the force **P** which must be applied to the wedge *C* to move the block. It will be assumed that the weight **W** of the block is known, either given in pounds or determined in newtons from the mass of the block expressed in kilograms.

The free-body diagrams of block *A* and of wedge *C* have been drawn in Fig. 8.7*b* and *c*. The forces acting on the block include its weight and the normal and friction forces at the surfaces of contact with wall *B* and wedge *C*. The magnitudes of the friction forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal, respectively, to $\mu_s N_1$ and $\mu_s N_2$ since the motion of the block must be started. It is important to show the friction forces with their correct sense. Since the block will move upward, the force $\mathbf{F}_1$ exerted by the wall on the block must be directed downward. On the other hand, since the wedge *C* moves to the right, the relative motion of *A* with respect to *C* is to the left and the force $\mathbf{F}_2$ exerted by *C* on *A* must be directed to the right.

Considering now the free body *C* in Fig. 8.7*c*, we note that the forces acting on *C* include the applied force **P** and the normal and friction forces at the surfaces of contact with *A* and *D*. The weight of the wedge is small compared with the other forces involved and can be neglected. The forces exerted by *A* on *C* are equal and opposite to the forces $\mathbf{N}_2$ and $\mathbf{F}_2$ exerted by *C* on *A* and are denoted, respectively, by $-\mathbf{N}_2$ and $-\mathbf{F}_2$; the friction force $-\mathbf{F}_2$ must therefore be directed to the left. We check that the force $\mathbf{F}_3$ exerted by *D* is also directed to the left.

The total number of unknowns involved in the two free-body diagrams can be reduced to four if the friction forces are expressed in terms of the normal forces. Expressing that block *A* and wedge *C* are in equilibrium will provide four equations which can be solved to obtain the magnitude of **P**. It should be noted that in the example considered here, it will be more convenient to replace each pair of normal and friction forces by their resultant. Each free body is then subjected to only three forces, and the problem can be solved by drawing the corresponding force triangles (see Sample Prob. 8.4).

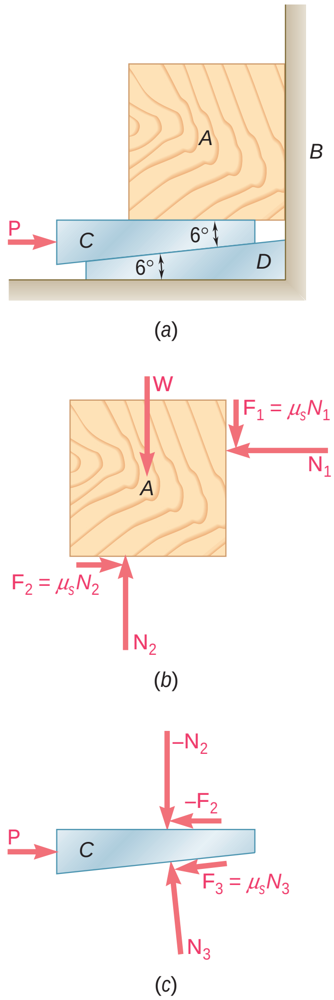


Fig. 8.7

8.6 SQUARE-THREADED SCREWS

Square-threaded screws are frequently used in jacks, presses, and other mechanisms. Their analysis is similar to the analysis of a block sliding along an inclined plane.

Consider the jack shown in Fig. 8.8. The screw carries a load **W** and is supported by the base of the jack. Contact between screw and base takes place along a portion of their threads. By applying a force **P** on the handle, the screw can be made to turn and to raise the load **W**.

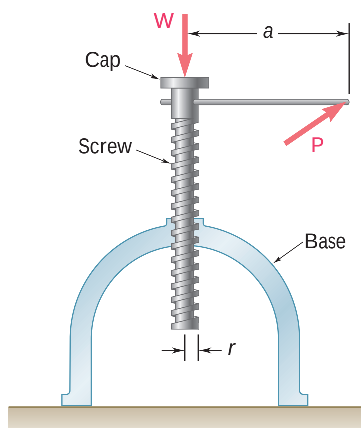
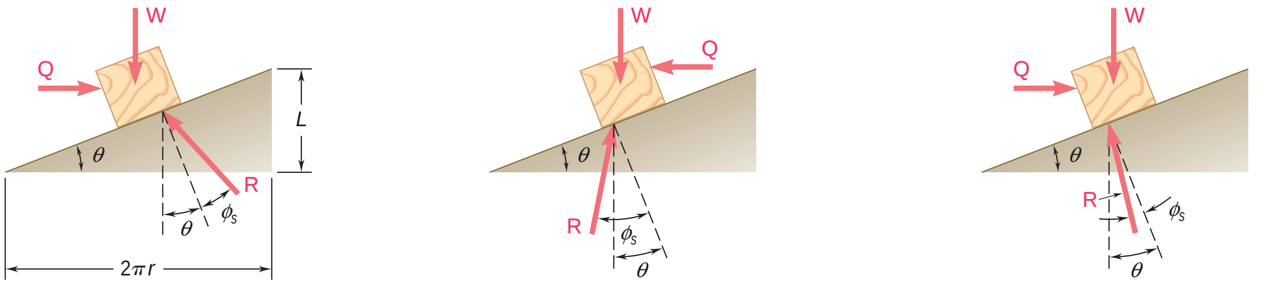


Fig. 8.8

Photo 8.2 Wedges are used as shown to split tree trunks because the normal forces exerted by the wedges on the wood are much larger than the forces required to insert the wedges.

The thread of the base has been unwrapped and shown as a straight line in Fig. 8.9*a*. The correct slope was obtained by plotting horizontally the product $2\pi r$, where r is the mean radius of the thread, and vertically the *lead* L of the screw, i.e., the distance through which the screw advances in one turn. The angle θ this line forms with the horizontal is the *lead angle*. Since the force of friction between two surfaces in contact does not depend upon the area of contact, a much smaller than actual area of contact between the two threads can be assumed, and the screw can be represented by the block shown in Fig. 8.9*a*. It should be noted, however, that in this analysis of the jack, the friction between cap and screw is neglected.

The free-body diagram of the block should include the load **W**, the reaction **R** of the base thread, and a horizontal force **Q** having the same effect as the force **P** exerted on the handle. The force **Q** should have the same moment as **P** about the axis of the screw and its magnitude should thus be $Q = Pa/r$. The force **Q**, and thus the force **P** required to raise the load **W**, can be obtained from the free-body diagram shown in Fig. 8.9*a*. The friction angle is taken equal to ϕ_s since the load will presumably be raised through a succession of short strokes. In mechanisms providing for the continuous rotation of a screw, it may be desirable to distinguish between the force required to start motion (using ϕ_s) and that required to maintain motion (using ϕ_k).

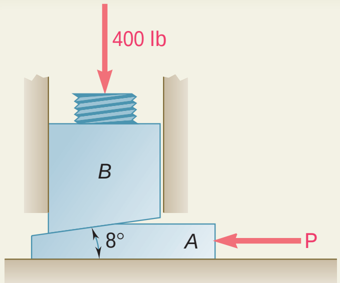


(*a*) Impending motion upward (*b*) Impending motion downward with $\phi_s > \theta$ (*c*) Impending motion downward with $\phi_s < \theta$

Fig. 8.9 Block-and-incline analysis of a screw.

If the friction angle ϕ_s is larger than the lead angle θ, the screw is said to be *self-locking;* it will remain in place under the load. To lower the load, we must then apply the force shown in Fig. 8.9*b*. If ϕ_s is smaller than θ, the screw will unwind under the load; it is then necessary to apply the force shown in Fig. 8.9*c* to maintain equilibrium.

The lead of a screw should not be confused with its *pitch*. The lead was defined as the distance through which the screw advances in one turn; the pitch is the distance measured between two consecutive threads. While lead and pitch are equal in the case of *single-threaded* screws, they are different in the case of *multiple-threaded* screws, i.e., screws having several independent threads. It is easily verified that for double-threaded screws, the lead is twice as large as the pitch; for triple-threaded screws, it is three times as large as the pitch; etc.



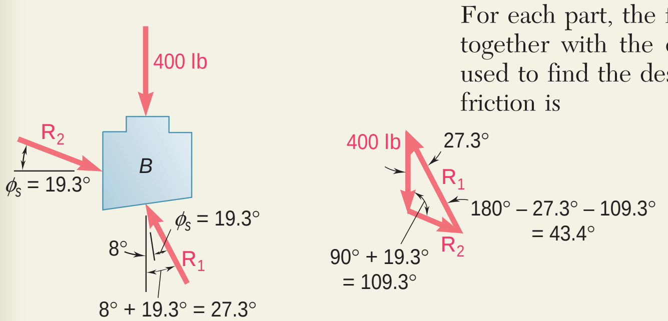
SAMPLE PROBLEM 8.4

The position of the machine block B is adjusted by moving the wedge A. Knowing that the coefficient of static friction is 0.35 between all surfaces of contact, determine the force **P** required (*a*) to raise block B, (*b*) to lower block B.

SOLUTION

For each part, the free-body diagrams of block B and wedge A are drawn, together with the corresponding force triangles, and the law of sines is used to find the desired forces. We note that since $m_s = 0.35$, the angle of friction is

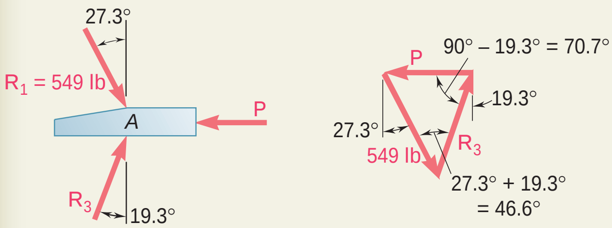
$$f_s = \tan^{-1} 0.35 = 19.3°$$



a. Force P to Raise Block

Free Body: Block B

$$\frac{R_1}{\sin 109.3°} = \frac{400 \text{ lb}}{\sin 43.4°}$$
$$R_1 = 549 \text{ lb}$$

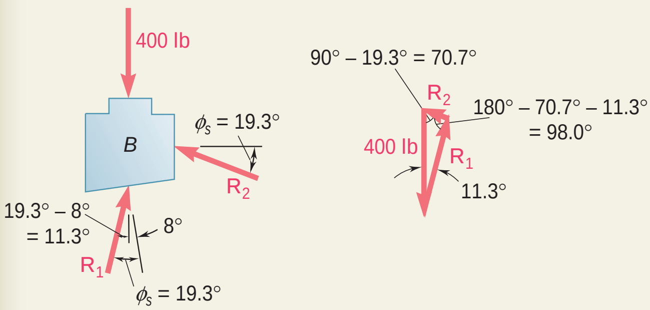


Free Body: Wedge A

$$\frac{P}{\sin 46.6°} = \frac{549 \text{ lb}}{\sin 70.7°}$$
$$P = 423 \text{ lb} \qquad \mathbf{P} = 423 \text{ lb z} \quad ◀$$

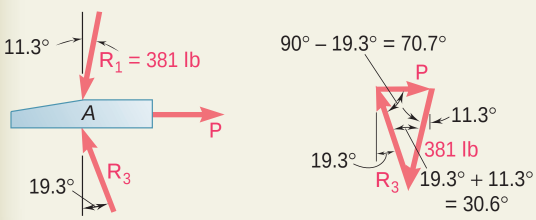
b. Force P to Lower Block

Free Body: Block B

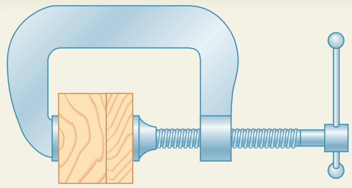


$$\frac{R_1}{\sin 70.7°} = \frac{400 \text{ lb}}{\sin 98.0°}$$
$$R_1 = 381 \text{ lb}$$

Free Body: Wedge A

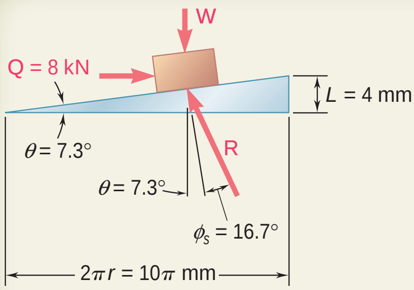


$$\frac{P}{\sin 30.6°} = \frac{381 \text{ lb}}{\sin 70.7°}$$
$$P = 206 \text{ lb} \qquad \mathbf{P} = 206 \text{ lb y} \quad ◀$$



SAMPLE PROBLEM 8.5

A clamp is used to hold two pieces of wood together as shown. The clamp has a double square thread of mean diameter equal to 10 mm with a pitch of 2 mm. The coefficient of friction between threads is $\mu_s = 0.30$. If a maximum couple of 40 N · m is applied in tightening the clamp, determine (*a*) the force exerted on the pieces of wood, (*b*) the couple required to loosen the clamp.

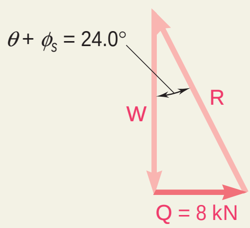


SOLUTION

a. Force Exerted by Clamp. The mean radius of the screw is $r = 5$ mm. Since the screw is double-threaded, the lead L is equal to twice the pitch: $L = 2(2\text{ mm}) = 4$ mm. The lead angle θ and the friction angle ϕ_s are obtained by writing

$$\tan\theta = \frac{L}{2\pi r} = \frac{4\text{ mm}}{10\pi\text{ mm}} = 0.1273 \qquad \theta = 7.3°$$

$$\tan\phi_s = \mu_s = 0.30 \qquad \phi_s = 16.7°$$



The force **Q** which should be applied to the block representing the screw is obtained by expressing that its moment Qr about the axis of the screw is equal to the applied couple.

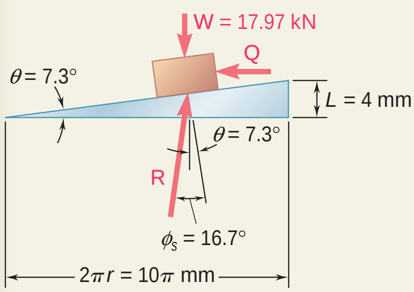
$$Q(5\text{ mm}) = 40\text{ N}\cdot\text{m}$$

$$Q = \frac{40\text{ N}\cdot\text{m}}{5\text{ mm}} = \frac{40\text{ N}\cdot\text{m}}{5\times10^{-3}\text{ m}} = 8000\text{ N} = 8\text{ kN}$$

The free-body diagram and the corresponding force triangle can now be drawn for the block; the magnitude of the force **W** exerted on the pieces of wood is obtained by solving the triangle.

$$W = \frac{Q}{\tan(\theta+\phi_s)} = \frac{8\text{ kN}}{\tan 24.0°}$$

$$W = 17.97\text{ kN} \quad \blacktriangleleft$$

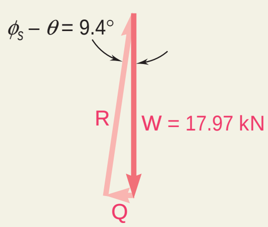


b. Couple Required to Loosen Clamp. The force **Q** required to loosen the clamp and the corresponding couple are obtained from the free-body diagram and force triangle shown.

$$\begin{aligned} Q &= W\tan(\phi_s - \theta) = (17.97\text{ kN})\tan 9.4° \\ &= 2.975\text{ kN} \end{aligned}$$

$$\begin{aligned} \text{Couple} &= Qr = (2.975\text{ kN})(5\text{ mm}) \\ &= (2.975\times10^{3}\text{ N})(5\times10^{-3}\text{ m}) = 14.87\text{ N}\cdot\text{m} \end{aligned}$$

$$\text{Couple} = 14.87\text{ N}\cdot\text{m} \quad \blacktriangleleft$$



SOLVING PROBLEMS ON YOUR OWN

In this lesson you learned to apply the laws of friction to the solution of problems involving *wedges* and *square-threaded screws*.

1. Wedges. Keep the following in mind when solving a problem involving a wedge:

a. First draw a free-body diagram of the wedge and of all the other bodies involved. Carefully note the sense of the relative motion of all surfaces of contact and show each friction force acting in *a direction opposite* to the direction of that relative motion.

b. Show the maximum static friction force $\mathbf{F}_m$ at each surface if the wedge is to be inserted or removed, *since motion will be impending in each of these cases*.

c. The reaction R and the angle of friction, rather than the normal force and the friction force, can be used in many applications. You can then draw one or more force triangles and determine the unknown quantities either graphically or by trigonometry [Sample Prob. 8.4].

2. Square-Threaded Screws. The analysis of a square-threaded screw is equivalent to the analysis of a block sliding on an incline. To draw the appropriate incline, you should unwrap the thread of the screw and represent it by a straight line [Sample Prob. 8.5]. When solving a problem involving a square-threaded screw, keep the following in mind:

a. Do not confuse the pitch of a screw with the lead of a screw. The *pitch* of a screw is the distance between two consecutive threads, while the *lead* of a screw is the distance the screw advances in one full turn. The lead and the pitch are equal only in single-threaded screws. In a double-threaded screw, the lead is twice the pitch.

b. The couple required to tighten a screw is different from the couple required to loosen it. Also, screws used in jacks and clamps are usually *self-locking;* that is, the screw will remain stationary as long as no couple is applied to it, and a couple must be applied to the screw to loosen it [Sample Prob. 8.5].

PROBLEMS

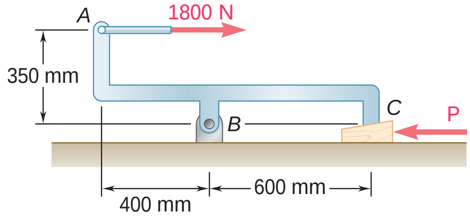


Fig. P8.48

8.48 The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine (*a*) the force **P** required to move the wedge, (*b*) the components of the corresponding reaction at *B*.

8.49 Solve Prob. 8.48 assuming that the force **P** is directed to the right.

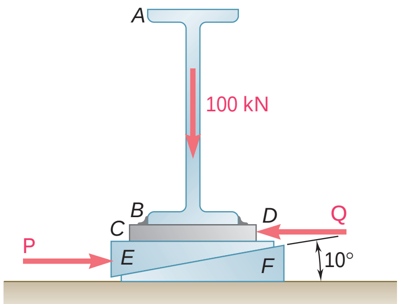


Fig. P8.50

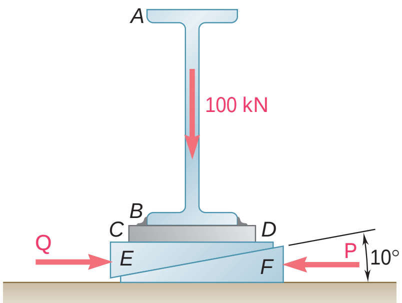


Fig. P8.51

8.50* and *8.51 The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 100 kN. The coefficient of static friction is 0.30 between two steel surfaces and 0.60 between steel and concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

8.52 and 8.53 Two 10° wedges of negligible weight are used to move and position the 400-lb block. Knowing that the coefficient of static friction is 0.25 at all surfaces of contact, determine the smallest force **P** that should be applied as shown to one of the wedges.

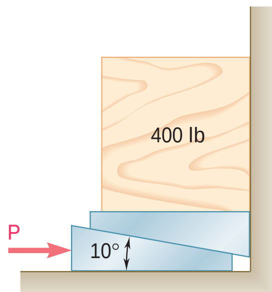


Fig. P8.52

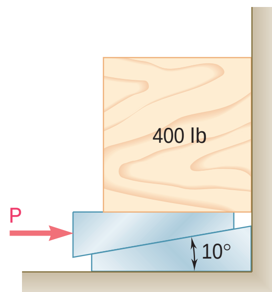


Fig. P8.53

8.54 Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that u = 45°, determine the smallest force **P** required to raise block *A*.

8.55 Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that u = 45°, determine the smallest force **P** for which equilibrium is maintained.

8.56 Block *A* supports a pipe column and rests as shown on wedge *B*. The coefficient of static friction at all surfaces of contact is 0.25. If **P** = 0, determine (*a*) the angle u for which sliding is impending, (*b*) the corresponding force exerted on the block by the vertical wall.

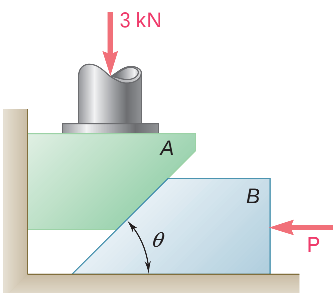


Fig. P8.54, P8.55, and P8.56

8.57 A wedge A of negligible weight is to be driven between two 100-lb plates B and C. The coefficient of static friction between all surfaces of contact is 0.35. Determine the magnitude of the force **P** required to start moving the wedge (a) if the plates are equally free to move, (b) if plate C is securely bolted to the surface.

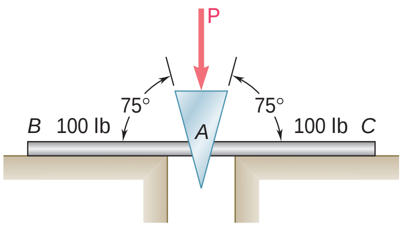


Fig. P8.57

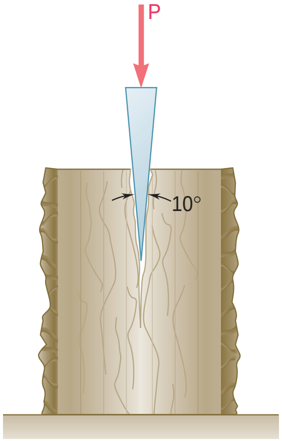


Fig. *P8.58*

8.58 A 10° wedge is used to split a section of a log. The coefficient of static friction between the wedge and the log is 0.35. Knowing that a force **P** of magnitude 600 lb was required to insert the wedge, determine the magnitude of the forces exerted on the wood by the wedge after insertion.

8.59 A 10° wedge is to be forced under end B of the 5-kg rod AB. Knowing that the coefficient of static friction is 0.40 between the wedge and the rod and 0.20 between the wedge and the floor, determine the smallest force **P** required to raise end B of the rod.

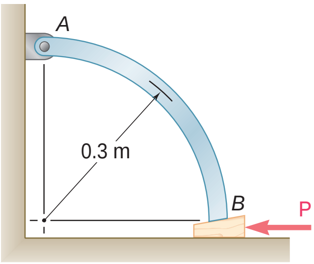


Fig. P8.59

8.60 The spring of the door latch has a constant of 1.8 lb/in. and in the position shown exerts a 0.6-lb force on the bolt. The coefficient of static friction between the bolt and the strike plate is 0.40; all other surfaces are well lubricated and may be assumed frictionless. Determine the magnitude of the force **P** required to start closing the door.

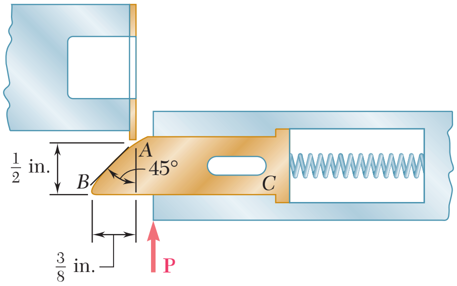


Fig. P8.60

8.61 In Prob. 8.60, determine the angle that the face of the bolt should form with the line BC if the force **P** required to close the door is to be the same for both the position shown and the position when B is almost at the strike plate.

8.62 A 5° wedge is to be forced under a 1400-lb machine base at A. Knowing that the coefficient of static friction at all surfaces is 0.20, (a) determine the force **P** required to move the wedge, (b) indicate whether the machine base will move.

8.63 Solve Prob. 8.62 assuming that the wedge is to be forced under the machine base at B instead of A.

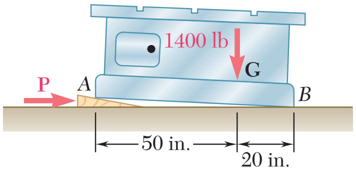


Fig. P8.62

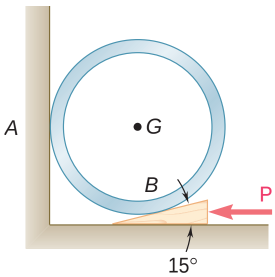


Fig. P8.64 and P8.65

8.64 A 15° wedge is forced under a 50-kg pipe as shown. The coefficient of static friction at all surfaces is 0.20. (*a*) Show that slipping will occur between the pipe and the vertical wall. (*b*) Determine the force **P** required to move the wedge.

8.65 A 15° wedge is forced under a 50-kg pipe as shown. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine the largest coefficient of static friction between the pipe and the vertical wall for which slipping will occur at *A*.

***8.66** A 200-N block rests as shown on a wedge of negligible weight. The coefficient of static friction μ_s is the same at both surfaces of the wedge, and friction between the block and the vertical wall may be neglected. For $P = 100$ N, determine the value of μ_s for which motion is impending. (*Hint:* Solve the equation obtained by trial and error.)

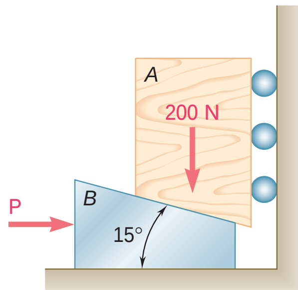


Fig. P8.66

***8.67** Solve Prob. 8.66 assuming that the rollers are removed and that μ_s is the coefficient of friction at all surfaces of contact.

8.68 Derive the following formulas relating the load **W** and the force **P** exerted on the handle of the jack discussed in Sec. 8.6. (*a*) $P = (Wr/a)\tan(\theta + \phi_s)$, to raise the load; (*b*) $P = (Wr/a)\tan(\phi_s - \theta)$, to lower the load if the screw is self-locking; (*c*) $P = (Wr/a)\tan(\theta - \phi_s)$, to hold the load if the screw is not self-locking.

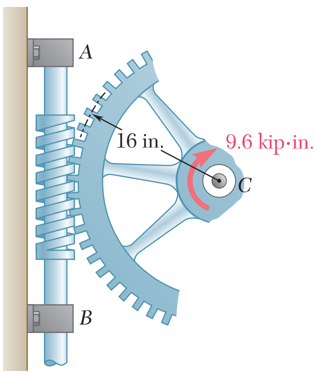


Fig. *P8.69*

8.69 The square-threaded worm gear shown has a mean radius of 2 in. and a lead of 0.5 in. The large gear is subjected to a constant clockwise couple of 9.6 kip · in. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

8.70 In Prob. 8.69, determine the couple that must be applied to shaft *AB* in order to rotate the large gear clockwise.

8.71 High-strength bolts are used in the construction of many steel structures. For a 24-mm-nominal-diameter bolt, the required minimum bolt tension is 210 kN. Assuming the coefficient of friction to be 0.40, determine the required couple that should be applied to the bolt and nut. The mean diameter of the thread is 22.6 mm, and the lead is 3 mm. Neglect friction between the nut and washer, and assume the bolt to be square-threaded.

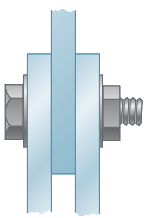


Fig. P8.71

8.72 The position of the automobile jack shown is controlled by a screw *ABC* that is single-threaded at each end (right-handed thread at *A*, left-handed thread at *C*). Each thread has a pitch of 0.1 in. and a mean diameter of 0.375 in. If the coefficient of static friction is 0.15, determine the magnitude of the couple **M** that must be applied to raise the automobile.

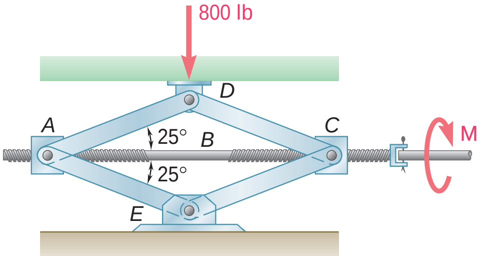


Fig. P8.72

8.73 For the jack of Prob. 8.72, determine the magnitude of the couple **M** that must be applied to lower the automobile.

8.74 In the gear-pulling assembly shown, the square-threaded screw *AB* has a mean radius of 15 mm and a lead of 4 mm. Knowing that the coefficient of static friction is 0.10, determine the couple that must be applied to the screw in order to produce a force of 3 kN on the gear. Neglect friction at end *A* of the screw.

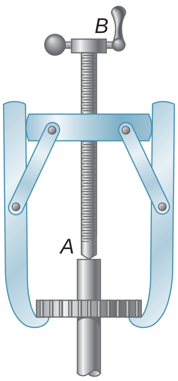


Fig. *P8.74*

8.75 The ends of two fixed rods *A* and *B* are each made in the form of a single-threaded screw of mean radius 6 mm and pitch 2 mm. Rod *A* has a right-handed thread and rod *B* has a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.

Fig. P8.75

8.76 Assuming that in Prob. 8.75 a right-handed thread is used on *both* rods *A* and *B*, determine the magnitude of the couple that must be applied to the sleeve in order to rotate it.

*8.7 JOURNAL BEARINGS. AXLE FRICTION

Journal bearings are used to provide lateral support to rotating shafts and axles. Thrust bearings, which will be studied in the next section, are used to provide axial support to shafts and axles. If the journal bearing is fully lubricated, the frictional resistance depends upon the speed of rotation, the clearance between axle and bearing, and the viscosity of the lubricant. As indicated in Sec. 8.1, such problems are studied in fluid mechanics. The methods of this chapter, however, can be applied to the study of axle friction when the bearing is not lubricated or only partially lubricated. It can then be assumed that the axle and the bearing are in direct contact along a single straight line.

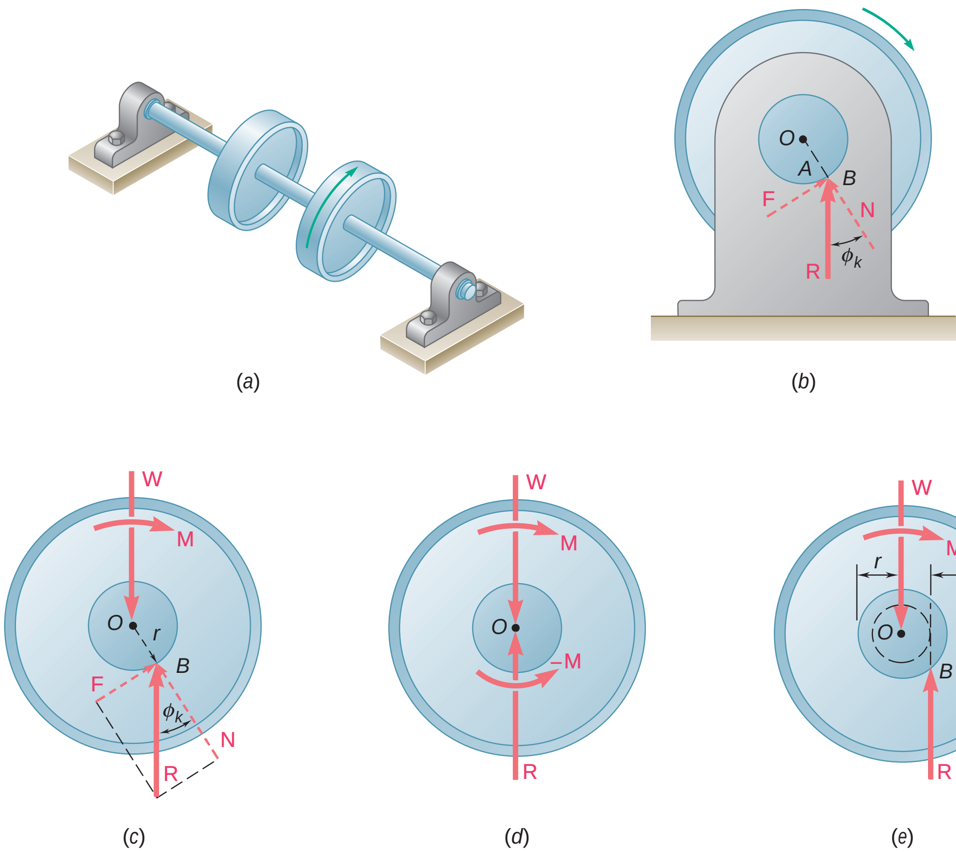


Fig. 8.10

Consider two wheels, each of weight **W**, rigidly mounted on an axle supported symmetrically by two journal bearings (Fig. 8.10*a*). If the wheels rotate, we find that to keep them rotating at constant speed, it is necessary to apply to each of them a couple **M**. The free-body diagram in Fig. 8.10*c* represents one of the wheels and the corresponding half axle in projection on a plane perpendicular to the axle. The forces acting on the free body include the weight **W** of the wheel, the couple **M** required to maintain its motion, and a force **R** representing the reaction of the bearing. This force is vertical, equal, and opposite to **W** but does not pass through the center O of the axle; **R** is located to the right of O at a distance such that its moment about O balances the moment **M** of the couple. Therefore, contact between the axle and bearing does not take place at the lowest point A when the axle rotates. It takes place at point B (Fig. 8.10*b*) or, rather, along a straight line intersecting the plane of the figure at B. Physically, this is explained by the fact that when the wheels are set in motion, the axle "climbs" in the bearings until slippage occurs. After sliding back slightly, the axle settles more or less in the position shown. This position is such that the angle between the reaction **R** and the normal to the surface of the bearing is equal to the angle of kinetic friction ϕ_k. The distance from O to the line of action of **R** is thus $r \sin \phi_k$, where r is the radius of the axle. Writing that $\Sigma M_O = 0$ for the forces acting on the free body considered, we obtain the magnitude of the couple **M** required to overcome the frictional resistance of one of the bearings:

$$M = Rr \sin \phi_k \tag{8.5}$$

Observing that, for small values of the angle of friction, sin ϕ_k can be replaced by tan ϕ_k, that is, by μ_k, we write the approximate formula

$$M \approx Rr\mu_k \tag{8.6}$$

In the solution of certain problems, it may be more convenient to let the line of action of **R** pass through O, as it does when the axle does not rotate. A couple $-$**M** of the same magnitude as the couple **M** but of opposite sense must then be added to the reaction **R** (Fig. 8.10*d*). This couple represents the frictional resistance of the bearing.

In case a graphical solution is preferred, the line of action of **R** can be readily drawn (Fig. 8.10*e*) if we note that it must be tangent to a circle centered at O and of radius

$$r_f = r \sin \phi_k \approx r\mu_k \tag{8.7}$$

This circle is called the *circle of friction* of the axle and bearing and is independent of the loading conditions of the axle.

*8.8 THRUST BEARINGS. DISK FRICTION

Two types of thrust bearings are used to provide axial support to rotating shafts and axles: (1) *end bearings* and (2) *collar bearings* (Fig. 8.11). In the case of collar bearings, friction forces develop between the two ring-shaped areas which are in contact. In the case of end bearings, friction takes place over full circular areas, or over ring-shaped areas when the end of the shaft is hollow. Friction between circular areas, called *disk friction*, also occurs in other mechanisms, such as *disk clutches*.

(*a*) End bearing (*b*) Collar bearing

Fig. 8.11 Thrust bearings.

To obtain a formula which is valid in the most general case of disk friction, let us consider a rotating hollow shaft. A couple **M** keeps the shaft rotating at constant speed while a force **P** maintains it in contact with a fixed bearing (Fig. 8.12). Contact between the shaft and

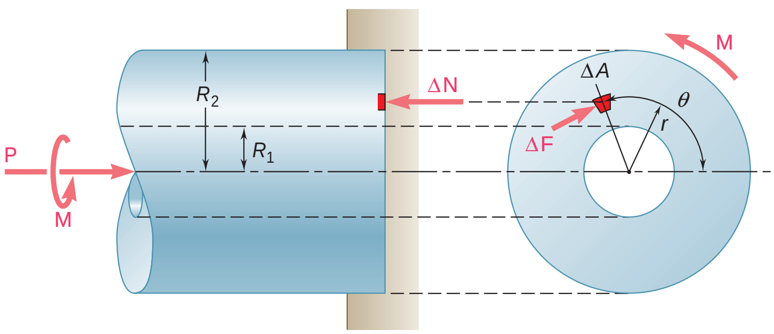


Fig. 8.12

the bearing takes place over a ring-shaped area of inner radius R_1 and outer radius R_2. Assuming that the pressure between the two surfaces in contact is uniform, we find that the magnitude of the normal force $\Delta\mathbf{N}$ exerted on an element of area ΔA is $\Delta N = P\,\Delta A/A$, where $A = \pi(R_2^2 - R_1^2)$, and that the magnitude of the friction force $\Delta\mathbf{F}$ acting on ΔA is $\Delta F = \mu_k\,\Delta N$. Denoting by r the distance from the axis of the shaft to the element of area ΔA, we express the magnitude ΔM of the moment of $\Delta\mathbf{F}$ about the axis of the shaft as follows:

$$\Delta M = r\,\Delta F = \frac{r\mu_k P\,\Delta A}{\pi(R_2^2 - R_1^2)}$$

The equilibrium of the shaft requires that the moment $\mathbf{M}$ of the couple applied to the shaft be equal in magnitude to the sum of the moments of the friction forces $\Delta\mathbf{F}$. Replacing ΔA by the infinitesimal element $dA = r\,d\theta\,dr$ used with polar coordinates, and integrating over the area of contact, we thus obtain the following expression for the magnitude of the couple $\mathbf{M}$ required to overcome the frictional resistance of the bearing:

$$\begin{aligned} M &= \frac{\mu_k P}{\pi(R_2^2 - R_1^2)}\int_0^{2\pi}\int_{R_1}^{R_2} r^2\,dr\,d\theta \\ &= \frac{\mu_k P}{\pi(R_2^2 - R_1^2)}\int_0^{2\pi} \tfrac{1}{3}(R_2^3 - R_1^3)\,d\theta \end{aligned}$$

$$M = \tfrac{2}{3}\mu_k P\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \tag{8.8}$$

When contact takes place over a full circle of radius R, formula (8.8) reduces to

$$M = \tfrac{2}{3}\mu_k PR \tag{8.9}$$

The value of M is then the same as would be obtained if contact between shaft and bearing took place at a single point located at a distance $2R/3$ from the axis of the shaft.

The largest couple which can be transmitted by a disk clutch without causing slippage is given by a formula similar to (8.9), where μ_k has been replaced by the coefficient of static friction μ_s.

*8.9 WHEEL FRICTION. ROLLING RESISTANCE

The wheel is one of the most important inventions of our civilization. Its use makes it possible to move heavy loads with relatively little effort. Because the point of the wheel in contact with the ground at any given instant has no relative motion with respect to the ground, the wheel eliminates the large friction forces which would arise if the load were in direct contact with the ground. However, some resistance to the wheel's motion exists. This resistance has two distinct causes. It is due (1) to a combined effect of axle friction and friction at the rim and (2) to the fact that the wheel and the ground deform, with the result that contact between wheel and ground takes place over a certain area, rather than at a single point.

To understand better the first cause of resistance to the motion of a wheel, let us consider a railroad car supported by eight wheels mounted on axles and bearings. The car is assumed to be moving to the right at constant speed along a straight horizontal track. The free-body diagram of one of the wheels is shown in Fig. 8.13*a*. The forces acting on the free body include the load **W** supported by the wheel and the normal reaction **N** of the track. Since **W** is drawn through the center *O* of the axle, the frictional resistance of the bearing should be represented by a counterclockwise couple **M** (see Sec. 8.7). To keep the free body in equilibrium, we must add two equal and opposite forces **P** and **F**, forming a clockwise couple of moment $-\mathbf{M}$. The force **F** is the friction force exerted by the track on the wheel, and **P** represents the force which should be applied to the wheel to keep it rolling at constant speed. Note that the forces **P** and **F** would not exist if there were no friction between wheel and track. The couple **M** representing the axle friction would then be zero; the wheel would slide on the track without turning in its bearing.

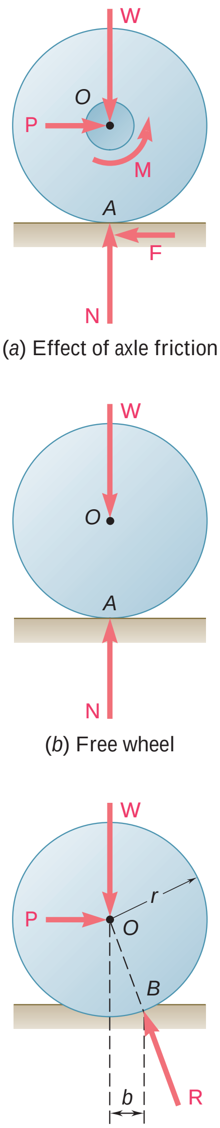
The couple **M** and the forces **P** and **F** also reduce to zero when there is no axle friction. For example, a wheel which is not held in bearings and rolls freely and at constant speed on horizontal ground (Fig. 8.13*b*) will be subjected to only two forces: its own weight **W** and the normal reaction **N** of the ground. Regardless of the value of the coefficient of friction between wheel and ground no friction force will act on the wheel. A wheel rolling freely on horizontal ground should thus keep rolling indefinitely.

Experience, however, indicates that the wheel will slow down and eventually come to rest. This is due to the second type of resistance mentioned at the beginning of this section, known as the *rolling resistance*. Under the load **W**, both the wheel and the ground deform slightly, causing the contact between wheel and ground to take place over a certain area. Experimental evidence shows that the resultant of the forces exerted by the ground on the wheel over this area is a force **R** applied at a point *B*, which is not located directly under the center *O* of the wheel, but slightly in front of it (Fig. 8.13*c*). To balance the moment of **W** about *B* and to keep the wheel rolling at constant speed, it is necessary to apply a horizontal force **P** at the center of the wheel. Writing $\Sigma M_B = 0$, we obtain

$$Pr = Wb \tag{8.10}$$

where r = radius of wheel
b = horizontal distance between O and B

The distance b is commonly called the *coefficient of rolling resistance*. It should be noted that b is not a dimensionless coefficient since it represents a length; b is usually expressed in inches or in millimeters. The value of b depends upon several parameters in a manner which has not yet been clearly established. Values of the coefficient of rolling resistance vary from about 0.01 in. or 0.25 mm for a steel wheel on a steel rail to 5.0 in. or 125 mm for the same wheel on soft ground.

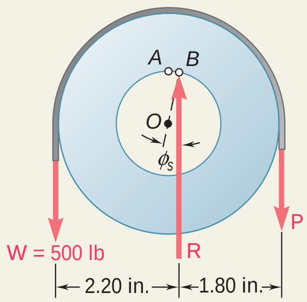


(*a*) Effect of axle friction
(*b*) Free wheel
(*c*) Rolling resistance
Fig. 8.13

SAMPLE PROBLEM 8.6

A pulley of diameter 4 in. can rotate about a fixed shaft of diameter 2 in. The coefficient of static friction between the pulley and shaft is 0.20. Determine (*a*) the smallest vertical force **P** required to start raising a 500-lb load, (*b*) the smallest vertical force **P** required to hold the load, (*c*) the smallest horizontal force **P** required to start raising the same load.

SOLUTION

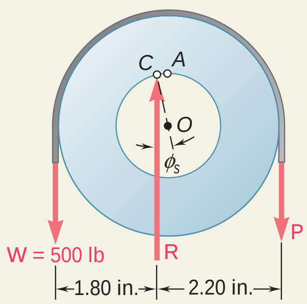


a. Vertical Force P Required to Start Raising the Load. When the forces in both parts of the rope are equal, contact between the pulley and shaft takes place at *A*. When **P** is increased, the pulley rolls around the shaft slightly and contact takes place at *B*. The free-body diagram of the pulley when motion is impending is drawn. The perpendicular distance from the center *O* of the pulley to the line of action of **R** is

$$r_f = r \sin \phi_s \approx r\mu_s \qquad r_f \approx (1 \text{ in.})0.20 = 0.20 \text{ in.}$$

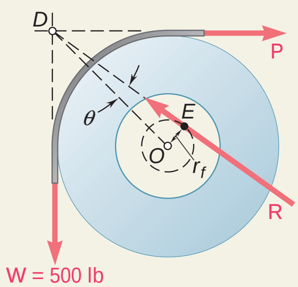
Summing moments about *B*, we write

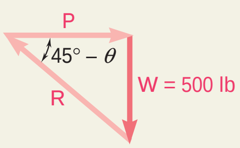
$$+\circlearrowleft \Sigma M_B = 0: \qquad (2.20 \text{ in.})(500 \text{ lb}) - (1.80 \text{ in.})P = 0$$
$$P = 611 \text{ lb} \qquad\qquad \mathbf{P} = 611 \text{ lb} \downarrow \blacktriangleleft$$



b. Vertical Force P to Hold the Load. As the force **P** is decreased, the pulley rolls around the shaft and contact takes place at *C*. Considering the pulley as a free body and summing moments about *C*, we write

$$+\circlearrowleft \Sigma M_C = 0: \qquad (1.80 \text{ in.})(500 \text{ lb}) - (2.20 \text{ in.})P = 0$$
$$P = 409 \text{ lb} \qquad\qquad \mathbf{P} = 409 \text{ lb} \downarrow \blacktriangleleft$$





c. Horizontal Force P to Start Raising the Load. Since the three forces **W**, **P**, and **R** are not parallel, they must be concurrent. The direction of **R** is thus determined from the fact that its line of action must pass through the point of intersection *D* of **W** and **P**, and must be tangent to the circle of friction. Recalling that the radius of the circle of friction is $r_f = 0.20$ in., we write

$$\sin\theta = \frac{OE}{OD} = \frac{0.20 \text{ in.}}{(2 \text{ in.})\sqrt{2}} = 0.0707 \qquad \theta = 4.1°$$

From the force triangle, we obtain

$$P = W \cot (45° - \theta) = (500 \text{ lb}) \cot 40.9°$$
$$= 577 \text{ lb} \qquad\qquad \mathbf{P} = 577 \text{ lb} \rightarrow \blacktriangleleft$$

SOLVING PROBLEMS ON YOUR OWN

In this lesson you learned about several additional engineering applications of the laws of friction.

1. Journal bearings and axle friction. In journal bearings, the *reaction does not pass through the center of the shaft or axle* which is being supported. The distance from the center of the shaft or axle to the line of action of the reaction (Fig. 8.10) is defined by the equation.

$$r_f = r \sin \phi_k \approx r\mu_k$$

if motion is actually taking place, and by the equation

$$r_f = r \sin \phi_s \approx r\mu_s$$

if the motion is impending.

Once you have determined the line of action of the reaction, you can draw a *free-body diagram* and use the corresponding equations of equilibrium to complete your solution [Sample Prob. 8.6]. In some problems, it is useful to observe that the line of action of the reaction must be tangent to a circle of radius $r_f \approx r\mu_k$, or $r_f \approx r\mu_s$, known as the *circle of friction* [Sample Prob. 8.6, part *c*].

2. Thrust bearings and disk friction. In a *thrust bearing* the magnitude of the couple required to overcome frictional resistance is equal to the sum of the moments of the *kinetic friction forces* exerted on the elements of the end of the shaft [Eqs. (8.8) and (8.9)].

An example of disk friction is the *disk clutch*. It is analyzed in the same way as a thrust bearing, except that to determine the largest couple that can be transmitted, you must compute the sum of the moments of the *maximum static friction forces* exerted on the disk.

3. Wheel friction and rolling resistance. You saw that the rolling resistance of a wheel is caused by deformations of both the wheel and the ground. The line of action of the reaction **R** of the ground on the wheel intersects the ground at a horizontal distance b from the center of the wheel. The distance b is known as the *coefficient of rolling resistance* and is expressed in inches or millimeters.

4. In problems involving both rolling resistance and axle friction, your free-body diagram should show that the line of action of the reaction **R** of the ground on the wheel is tangent to the friction circle of the axle and intersects the ground at a horizontal distance from the center of the wheel equal to the coefficient of rolling resistance.

PROBLEMS

8.77 A lever of negligible weight is loosely fitted onto a 30-mm-radius fixed shaft as shown. Knowing that a force **P** of magnitude 275 N will just start the lever rotating clockwise, determine (*a*) the coefficient of static friction between the shaft and the lever, (*b*) the smallest force **P** for which the lever does not start rotating counterclockwise.

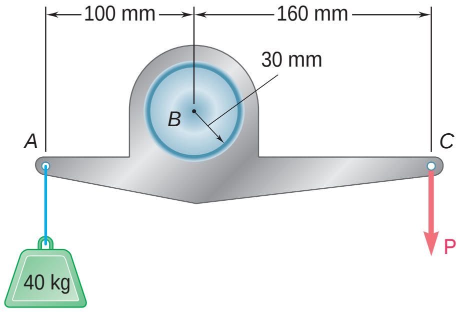


Fig. P8.77

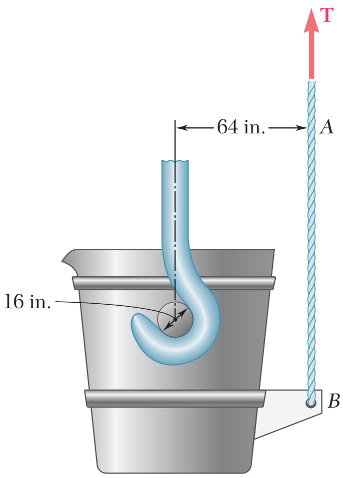


Fig. P8.78

8.78 A hot-metal ladle and its contents weigh 130 kips. Knowing that the coefficient of static friction between the hooks and the pinion is 0.30, determine the tension in cable *AB* required to start tipping the ladle.

8.79 and 8.80 The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

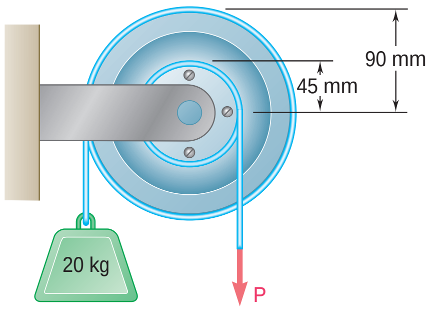


Fig. P8.79 and P8.81

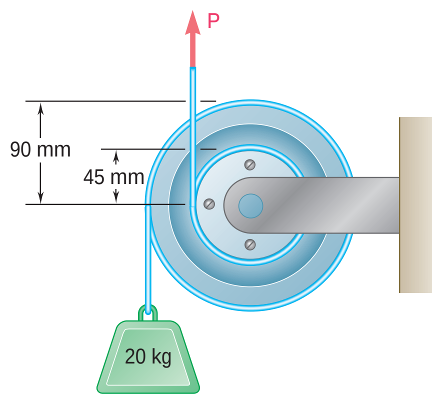


Fig. P8.80 and P8.82

8.81 and 8.82 The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the smallest force **P** required to maintain equilibrium.

8.83 The block and tackle shown are used to raise a 150-lb load. Each of the 3-in.-diameter pulleys rotates on a 0.5-in.-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly raised.

8.84 The block and tackle shown are used to lower a 150-lb load. Each of the 3-in.-diameter pulleys rotates on a 0.5-in.-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly lowered.

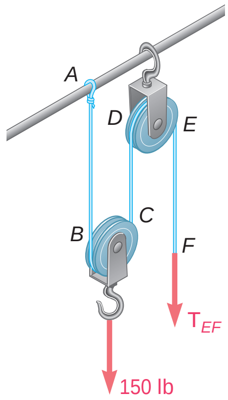


Fig. *P8.83* and P8.84

8.85 A scooter is to be designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 25-mm-diameter axles and the bearings is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

8.86 The link arrangement shown is frequently used in highway bridge construction to allow for expansion due to changes in temperature. At each of the 60-mm-diameter pins *A* and *B* the coefficient of static friction is 0.20. Knowing that the vertical component of the force exerted by *BC* on the link is 200 kN, determine (*a*) the horizontal force that should be exerted on beam *BC* to just move the link, (*b*) the angle that the resulting force exerted by beam *BC* on the link will form with the vertical.

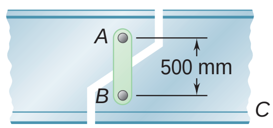


Fig. P8.86

***8.87* and 8.88** A lever *AB* of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating counterclockwise.

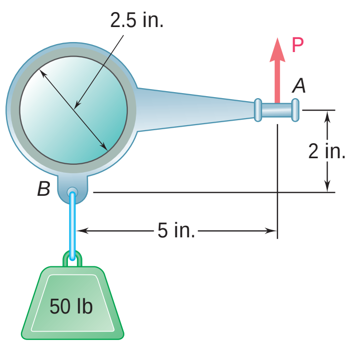


Fig. *P8.87* and P8.89

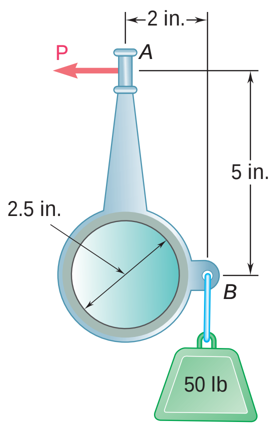


Fig. P8.88 and P8.90

8.89 and 8.90 A lever *AB* of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating clockwise.

8.91 A loaded railroad car has a mass of 30 Mg and is supported by eight 800-mm-diameter wheels with 125-mm-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (*a*) to start the car moving, (*b*) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the rails.

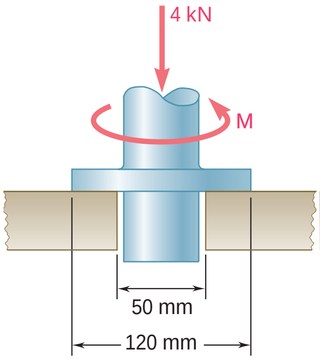


Fig. P8.92

8.92 Knowing that a couple of magnitude 30 N · m is required to start the vertical shaft rotating, determine the coefficient of static friction between the annular surfaces of contact.

8.93 A 50-lb electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

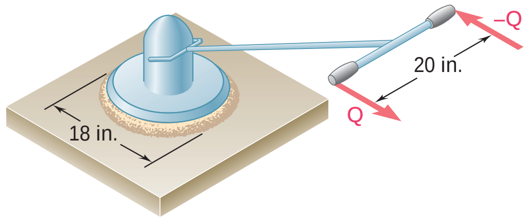


Fig. P8.93

***8.94** The frictional resistance of a thrust bearing decreases as the shaft and bearing surfaces wear out. It is generally assumed that the wear is directly proportional to the distance traveled by any given point of the shaft and thus to the distance r from the point to the axis of the shaft. Assuming, then, that the normal force per unit area is inversely proportional to r, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out end bearing (with contact over the full circular area) is equal to 75 percent of the value given by Eq. (8.9) for a new bearing.

***8.95** Assuming that bearings wear out as indicated in Prob. 8.94, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out collar bearing is

$$M = \tfrac{1}{2}\mathrm{m}_k P(R_1 + R_2)$$

where P = magnitude of the total axial force
R_1, R_2 = inner and outer radii of the collar

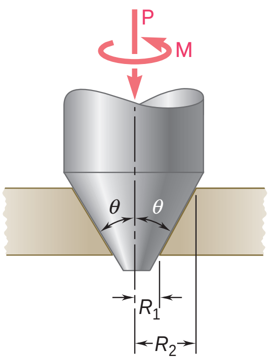


Fig. *P8.96*

***8.96** Assuming that the pressure between the surfaces of contact is uniform, show that the magnitude M of the couple required to overcome frictional resistance for the conical bearing shown is

$$M = \frac{2}{3}\frac{\mathrm{m}_k P}{\sin \mathrm{u}}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

8.97 Solve Prob. 8.93 assuming that the normal force per unit area between the disk and the floor varies linearly from a maximum at the center to zero at the circumference of the disk.

8.98 Determine the horizontal force required to move a 2500-lb automobile with 23-in.-diameter tires along a horizontal road at a constant speed. Neglect all forms of friction except rolling resistance, and assume the coefficient of rolling resistance to be 0.05 in.

8.99 Knowing that a 6-in.-diameter disk rolls at a constant velocity down a 2 percent incline, determine the coefficient of rolling resistance between the disk and the incline.

8.100 A 900-kg machine base is rolled along a concrete floor using a series of steel pipes with outside diameters of 100 mm. Knowing that the coefficient of rolling resistance is 0.5 mm between the pipes and the base and 1.25 mm between the pipes and the concrete floor, determine the magnitude of the force **P** required to slowly move the base along the floor.

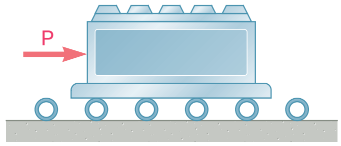


Fig. P8.100

8.101 Solve Prob. 8.85 including the effect of a coefficient of rolling resistance of 1.75 mm.

8.102 Solve Prob. 8.91 including the effect of a coefficient of rolling resistance of 0.5 mm.

8.10 BELT FRICTION

Consider a flat belt passing over a fixed cylindrical drum (Fig. 8.14*a*). We propose to determine the relation existing between the values T_1 and T_2 of the tension in the two parts of the belt when the belt is just about to slide toward the right.

Let us detach from the belt a small element PP' subtending an angle Δu. Denoting by T the tension at P and by $T + \Delta T$ the tension at P', we draw the free-body diagram of the element of the belt (Fig. 8.14*b*). Besides the two forces of tension, the forces acting on the free body are the normal component $\Delta \mathbf{N}$ of the reaction of the drum and the friction force $\Delta \mathbf{F}$. Since motion is assumed to be impending, we have $\Delta F = m_s\, \Delta N$. It should be noted that if Δu is made to approach zero, the magnitudes ΔN and ΔF, and the *difference* ΔT between the tension at P and the tension at P', will also approach zero; the value T of the tension at P, however, will remain unchanged. This observation helps in understanding our choice of notations.

Choosing the coordinate axes shown in Fig. 8.14*b*, we write the equations of equilibrium for the element PP':

$$\Sigma F_x = 0: \qquad (T + \Delta T)\cos\frac{\Delta u}{2} - T\cos\frac{\Delta u}{2} - m_s \Delta N = 0 \tag{8.11}$$

$$\Sigma F_y = 0: \qquad \Delta N - (T + \Delta T)\sin\frac{\Delta u}{2} - T\sin\frac{\Delta u}{2} = 0 \tag{8.12}$$

Solving Eq. (8.12) for ΔN and substituting into (8.11), we obtain after reductions

$$\Delta T\cos\frac{\Delta u}{2} - m_s(2T + \Delta T)\sin\frac{\Delta u}{2} = 0$$

Both terms are now divided by Δu. For the first term, this is done simply by dividing ΔT by Δu. The division of the second term is

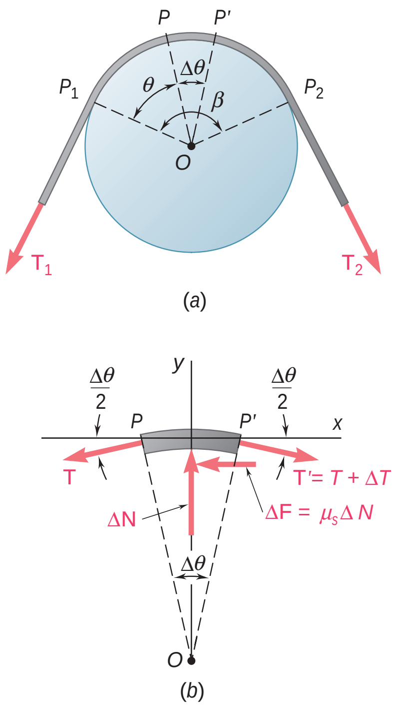


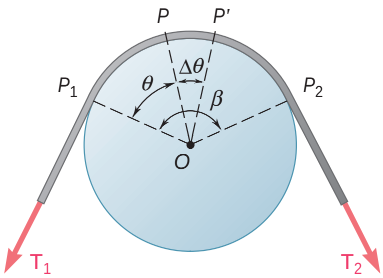
Fig. 8.14

carried out by dividing the terms in the parentheses by 2 and the sine by $\Delta\theta/2$. We write

$$\frac{\Delta T}{\Delta\theta}\cos\frac{\Delta\theta}{2} - \mu_s\left(T + \frac{\Delta T}{2}\right)\frac{\sin(\Delta\theta/2)}{\Delta\theta/2} = 0$$

If we now let $\Delta\theta$ approach 0, the cosine approaches 1 and $\Delta T/2$ approaches zero, as noted above. The quotient of sin $(\Delta\theta/2)$ over $\Delta\theta/2$ approaches 1, according to a lemma derived in all calculus textbooks. Since the limit of $\Delta T/\Delta\theta$ is by definition equal to the derivative $dT/d\theta$, we write

$$\frac{dT}{d\theta} - \mu_s T = 0 \qquad\qquad \frac{dT}{T} = \mu_s d\theta$$



(a)

Fig. 8.14a (*repeated*)

Both members of the last equation (Fig. 8.14*a*) will now be integrated from P_1 to P_2. At P_1, we have $\theta = 0$ and $T = T_1$; at P_2, we have $\theta = \beta$ and $T = T_2$. Integrating between these limits, we write

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} \mu_s\, d\theta$$

$$\ln T_2 - \ln T_1 = \mu_s\beta$$

or, noting that the left-hand member is equal to the natural logarithm of the quotient of T_2 and T_1,

$$\boxed{\ln \frac{T_2}{T_1} = \mu_s\beta} \qquad (8.13)$$

This relation can also be written in the form

$$\boxed{\frac{T_2}{T_1} = e^{\mu_s\beta}} \qquad (8.14)$$

Photo 8.3 By wrapping the rope around the bollard, the force exerted by the worker to control the rope is much smaller than the tension in the taut portion of the rope.

The formulas we have derived apply equally well to problems involving flat belts passing over fixed cylindrical drums and to problems involving ropes wrapped around a post or capstan. They can also be used to solve problems involving band brakes. In such problems, it is the drum which is about to rotate, while the band remains fixed. The formulas can also be applied to problems involving belt drives. In these problems, both the pulley and the belt rotate; our concern is then to find whether the belt will slip, i.e., whether it will move *with respect* to the pulley.

Formulas (8.13) and (8.14) should be used only if the belt, rope, or brake is *about to slip*. Formula (8.14) will be used if T_1 or T_2 is desired; formula (8.13) will be preferred if either μ_s or the angle of contact β is desired. We should note that T_2 is always larger than T_1; T_2 therefore represents the tension in that part of the belt or rope which *pulls*, while T_1 is the tension in the part which *resists*. We should also observe that the angle of contact β must be expressed in *radians*. The angle β may be larger than 2π; for example, if a rope is wrapped n times around a post, β is equal to $2\pi n$.

If the belt, rope, or brake is actually slipping, formulas similar to (8.13) and (8.14), but involving the coefficient of kinetic friction μ_k, should be used. If the belt, rope, or brake is not slipping and is not about to slip, none of these formulas can be used.

The belts used in belt drives are often V-shaped. In the V belt shown in Fig. 8.15*a* contact between belt and pulley takes place

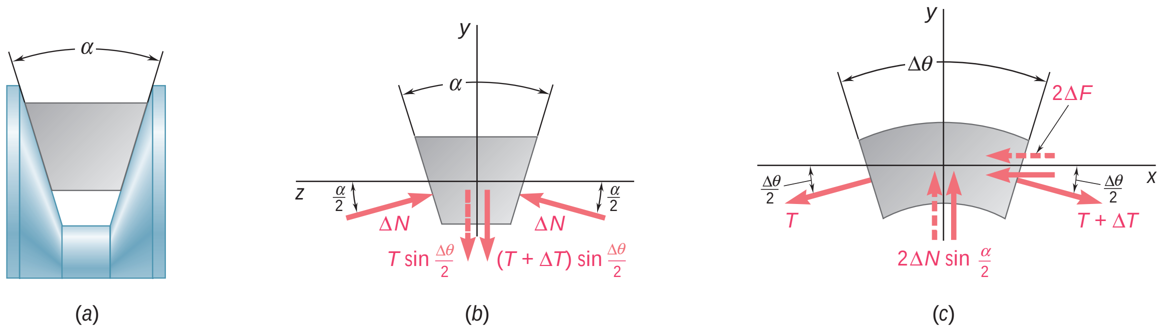


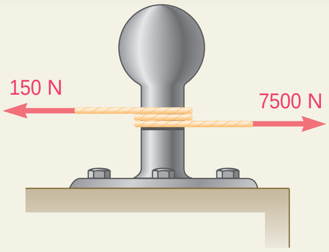
Fig. 8.15

along the sides of the groove. The relation existing between the values T_1 and T_2 of the tension in the two parts of the belt when the belt is just about to slip can again be obtained by drawing the free-body diagram of an element of belt (Fig. 8.15*b* and *c*). Equations similar to (8.11) and (8.12) are derived, but the magnitude of the total friction force acting on the element is now $2\,\Delta F$, and the sum of the y components of the normal forces is $2\,\Delta N \sin(\alpha/2)$. Proceeding as above, we obtain

$$\ln \frac{T_2}{T_1} = \frac{\mu_s \beta}{\sin(\alpha/2)} \tag{8.15}$$

or,

$$\frac{T_2}{T_1} = e^{\mu_s \beta/\sin(\alpha/2)} \tag{8.16}$$



SAMPLE PROBLEM 8.7

A hawser thrown from a ship to a pier is wrapped two full turns around a bollard. The tension in the hawser is 7500 N; by exerting a force of 150 N on its free end, a dockworker can just keep the hawser from slipping. (*a*) Determine the coefficient of friction between the hawser and the bollard. (*b*) Determine the tension in the hawser that could be resisted by the 150-N force if the hawser were wrapped three full turns around the bollard.

SOLUTION

a. Coefficient of Friction. Since slipping of the hawser is impending, we use Eq. (8.13):

$$\ln \frac{T_2}{T_1} = \mu_s \beta$$

Since the hawser is wrapped two full turns around the bollard, we have

$$\beta = 2(2\pi \text{ rad}) = 12.57 \text{ rad}$$
$$T_1 = 150 \text{ N} \qquad T_2 = 7500 \text{ N}$$

Therefore,

$$\mu_s \beta = \ln \frac{T_2}{T_1}$$
$$\mu_s(12.57 \text{ rad}) = \ln \frac{7500 \text{ N}}{150 \text{ N}} = \ln 50 = 3.91$$
$$\mu_s = 0.311 \qquad\qquad \mu_s = 0.311 \blacktriangleleft$$

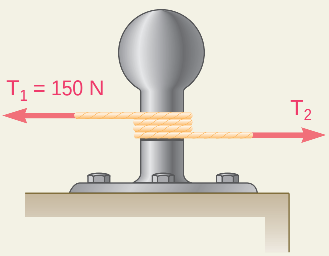
b. Hawser Wrapped Three Turns Around Bollard. Using the value of μ_s obtained in part *a*, we now have

$$\beta = 3(2\pi \text{ rad}) = 18.85 \text{ rad}$$
$$T_1 = 150 \text{ N} \qquad \mu_s = 0.311$$

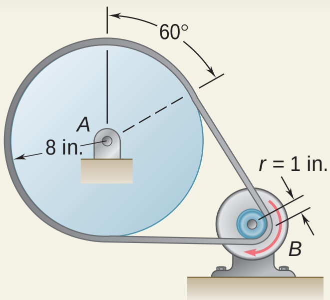
Substituting these values into Eq. (8.14), we obtain

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$
$$\frac{T_2}{150 \text{ N}} = e^{(0.311)(18.85)} = e^{5.862} = 351.5$$
$$T_2 = 52\,725 \text{ N}$$

$$T_2 = 52.7 \text{ kN} \blacktriangleleft$$

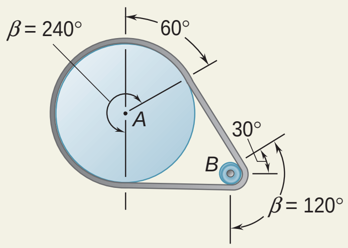


SAMPLE PROBLEM 8.8

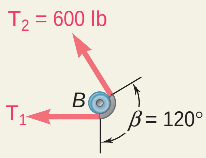


A flat belt connects pulley A, which drives a machine tool, to pulley B, which is attached to the shaft of an electric motor. The coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$ between both pulleys and the belt. Knowing that the maximum allowable tension in the belt is 600 lb, determine the largest torque which can be exerted by the belt on pulley A.

SOLUTION



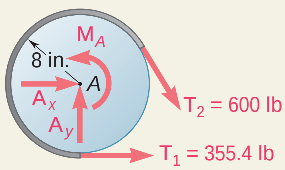
Since the resistance to slippage depends upon the angle of contact β between pulley and belt, as well as upon the coefficient of static friction μ_s, and since μ_s is the same for both pulleys, slippage will occur first on pulley B, for which β is smaller.



Pulley *B*. Using Eq. (8.14) with $T_2 = 600$ lb, $\mu_s = 0.25$, and $\beta = 120° = 2\pi/3$ rad, we write

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \qquad \frac{600\text{ lb}}{T_1} = e^{0.25(2\pi/3)} = 1.688$$

$$T_1 = \frac{600\text{ lb}}{1.688} = 355.4\text{ lb}$$



Pulley *A*. We draw the free-body diagram of pulley A. The couple $\mathbf{M}_A$ is applied to the pulley by the machine tool to which it is attached and is equal and opposite to the torque exerted by the belt. We write

$+\circlearrowleft \Sigma M_A = 0$: $\quad M_A - (600\text{ lb})(8\text{ in.}) + (355.4\text{ lb})(8\text{ in.}) = 0$

$M_A = 1957\text{ lb}\cdot\text{in.} \qquad M_A = 163.1\text{ lb}\cdot\text{ft}$ ◀

Note. We may check that the belt does not slip on pulley A by computing the value of μ_s required to prevent slipping at A and verifying that it is smaller than the actual value of μ_s. From Eq. (8.13) we have

$$\mu_s \beta = \ln\frac{T_2}{T_1} = \ln\frac{600\text{ lb}}{355.4\text{ lb}} = 0.524$$

and, since $\beta = 240° = 4\pi/3$ rad,

$$\frac{4\pi}{3}\mu_s = 0.524 \qquad \mu_s = 0.125 < 0.25$$

SOLVING PROBLEMS ON YOUR OWN

In the preceding section you learned about *belt friction*. The problems you will solve include belts passing over fixed drums, band brakes in which the drum rotates while the band remains fixed, and belt drives.

1. Problems involving belt friction fall into one of the following two categories:

a. Problems in which slipping is impending. One of the following formulas, involving the *coefficient of static friction* μ_s, may then be used,

$$\ln \frac{T_2}{T_1} = \mu_s \beta \qquad (8.13)$$

or

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \qquad (8.14)$$

b. Problems in which slipping is occurring. The formulas to be used can be obtained from Eqs. (8.13) and (8.14) by replacing μ_s with the *coefficient of kinetic friction* μ_k.

2. As you start solving a belt-friction problem, be sure to remember the following:

a. The angle β must be expressed in radians. In a belt-and-drum problem, this is the angle subtending the arc of the drum on which the belt is wrapped.

b. The larger tension is always denoted by T_2 and the *smaller tension is denoted by* T_1.

c. The larger tension occurs at the end of the belt which is in the direction of the motion, or impending motion, of the belt relative to the drum.

3. In each of the problems you will be asked to solve, three of the four quantities T_1, T_2, β, and μ_s (or μ_k) will either be given or readily found, and you will then solve the appropriate equation for the fourth quantity. Here are two kinds of problems that you will encounter:

a. Find μ_s between belt and drum, knowing that slipping is impending. From the given data, determine T_1, T_2, and β; substitute these values into Eq. (8.13) and solve for μ_s [Sample Prob. 8.7, part *a*]. Follow the same procedure to find the *smallest value* of μ_s for which slipping will not occur.

b. Find the magnitude of a force or couple applied to the belt or drum, knowing that slipping is impending. The given data should include μ_s and β. If it also includes T_1 or T_2, use Eq. (8.14) to find the other tension. If neither T_1 nor T_2 is known but some other data is given, use the free-body diagram of the belt-drum system to write an equilibrium equation that you will solve simultaneously with Eq. (8.14) for T_1 and T_2. You will then be able to find the magnitude of the specified force or couple from the free-body diagram of the system. Follow the same procedure to determine the *largest value* of a force or couple which can be applied to the belt or drum if no slipping is to occur [Sample Prob. 8.8].

PROBLEMS

8.103 A 300-lb block is supported by a rope that is wrapped $1\frac{1}{2}$ times around a horizontal rod. Knowing that the coefficient of static friction between the rope and the rod is 0.15, determine the range of values of P for which equilibrium is maintained.

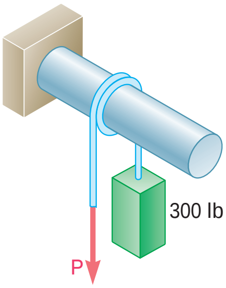


Fig. P8.103

8.104 A hawser is wrapped two full turns around a bollard. By exerting an 80-lb force on the free end of the hawser, a dockworker can resist a force of 5000 lb on the other end of the hawser. Determine (*a*) the coefficient of static friction between the hawser and the bollard, (*b*) the number of times the hawser should be wrapped around the bollard if a 20,000-lb force is to be resisted by the same 80-lb force.

8.105 A rope *ABCD* is looped over two pipes as shown. Knowing that the coefficient of static friction is 0.25, determine (*a*) the smallest value of the mass m for which equilibrium is possible, (*b*) the corresponding tension in portion *BC* of the rope.

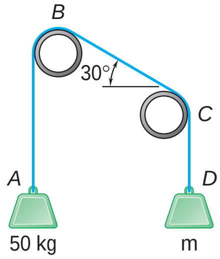


Fig. P8.105 and P8.106

8.106 A rope *ABCD* is looped over two pipes as shown. Knowing that the coefficient of static friction is 0.25, determine (*a*) the largest value of the mass m for which equilibrium is possible, (*b*) the corresponding tension in portion *BC* of the rope.

8.107 Knowing that the coefficient of static friction is 0.25 between the rope and the horizontal pipe and 0.20 between the rope and the vertical pipe, determine the range of values of P for which equilibrium is maintained.

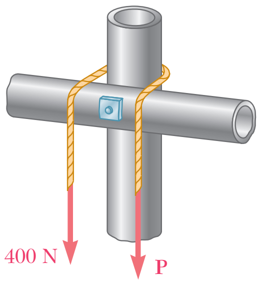


Fig. *P8.107* and *P8.108*

8.108 Knowing that the coefficient of static friction is 0.30 between the rope and the horizontal pipe and that the smallest value of P for which equilibrium is maintained is 80 N, determine (*a*) the largest value of P for which equilibrium is maintained, (*b*) the coefficient of static friction between the rope and the vertical pipe.

8.109 A band brake is used to control the speed of a flywheel as shown. The coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.25$. Determine the magnitude of the couple being applied to the flywheel, knowing that $P = 45$ N and that the flywheel is rotating counterclockwise at a constant speed.

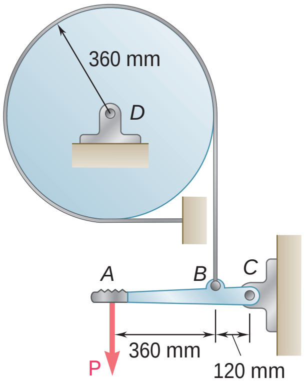


Fig. P8.109

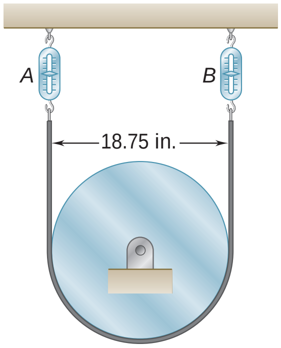


Fig. P8.110 and P8.111

8.110 The setup shown is used to measure the output of a small turbine. When the flywheel is at rest, the reading of each spring scale is 14 lb. If a 105-lb · in. couple must be applied to the flywheel to keep it rotating clockwise at a constant speed, determine (*a*) the reading of each scale at that time, (*b*) the coefficient of kinetic friction. Assume that the length of the belt does not change.

8.111 The setup shown is used to measure the output of a small turbine. The coefficient of kinetic friction is 0.20 and the reading of each spring scale is 16 lb when the flywheel is at rest. Determine (*a*) the reading of each scale when the flywheel is rotating clockwise at a constant speed, (*b*) the couple that must be applied to the flywheel. Assume that the length of the belt does not change.

8.112 A flat belt is used to transmit a couple from drum *B* to drum *A*. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum *A*.

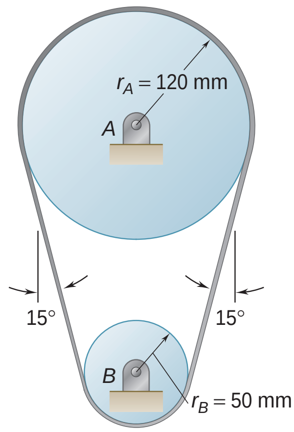


Fig. P8.112

8.113 A flat belt is used to transmit a couple from pulley *A* to pulley *B*. The radius of each pulley is 60 mm, and a force of magnitude *P* = 900 N is applied as shown to the axle of pulley *A*. Knowing that the coefficient of static friction is 0.35, determine (*a*) the largest couple that can be transmitted, (*b*) the corresponding maximum value of the tension in the belt.

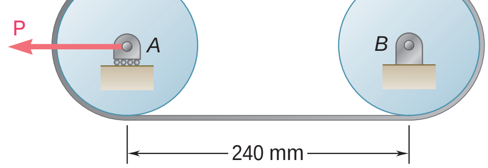


Fig. P8.113

8.114 Solve Prob. 8.113 assuming that the belt is looped around the pulleys in a figure eight.

8.115 The speed of the brake drum shown is controlled by a belt attached to the control bar AD. A force $\mathbf{P}$ of magnitude 25 lb is applied to the control bar at A. Determine the magnitude of the couple being applied to the drum, knowing that the coefficient of kinetic friction between the belt and the drum is 0.25, that $a = 4$ in., and that the drum is rotating at a constant speed (a) counterclockwise, (b) clockwise.

8.116 The speed of the brake drum shown is controlled by a belt attached to the control bar AD. Knowing that $a = 4$ in., determine the maximum value of the coefficient of static friction for which the brake is not self-locking when the drum rotates counterclockwise.

8.117 The speed of the brake drum shown is controlled by a belt attached to the control bar AD. Knowing that the coefficient of static friction is 0.30 and that the brake drum is rotating counterclockwise, determine the minimum value of a for which the brake is not self-locking.

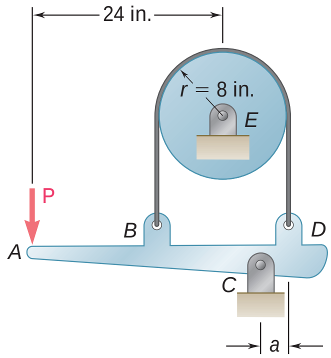


Fig. *P8.115, P8.116,* and P8.117

8.118 Bucket A and block C are connected by a cable that passes over drum B. Knowing that drum B rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined mass m of the bucket and its contents for which block C will (a) remain at rest, (b) start moving up the incline, (c) continue moving up the incline at a constant speed.

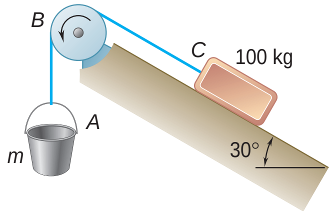


Fig. P8.118

8.119 Solve Prob. 8.118 assuming that drum B is frozen and cannot rotate.

8.120 and *8.122* A cable is placed around three parallel pipes. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (a) the smallest weight W for which equilibrium is maintained, (b) the largest weight W that can be raised if pipe B is slowly rotated counterclockwise while pipes A and C remain fixed.

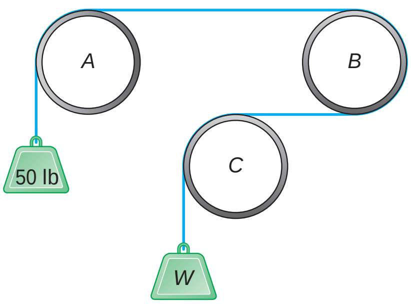


Fig. P8.120 and P8.121

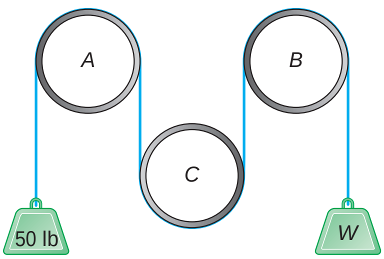


Fig. *P8.122* and *P8.123*

8.121 and *8.123* A cable is placed around three parallel pipes. Two of the pipes are fixed and do not rotate; the third pipe is slowly rotated. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the largest weight W that can be raised (a) if only pipe A is rotated counterclockwise, (b) if only pipe C is rotated clockwise.

8.124 A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

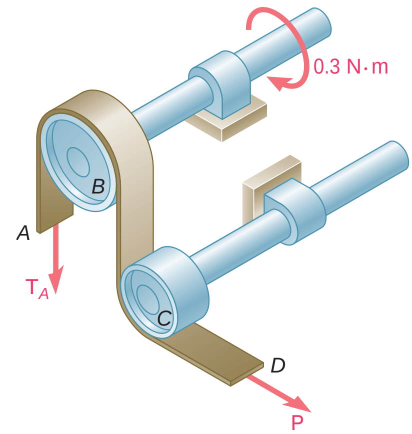


Fig. P8.124

8.125 Solve Prob. 8.124 assuming that the idler drum C is frozen and cannot rotate.

8.126 The strap wrench shown is used to grip the pipe firmly without marring the external surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 200$ mm, $r = 30$ mm, and $\theta = 65°$.

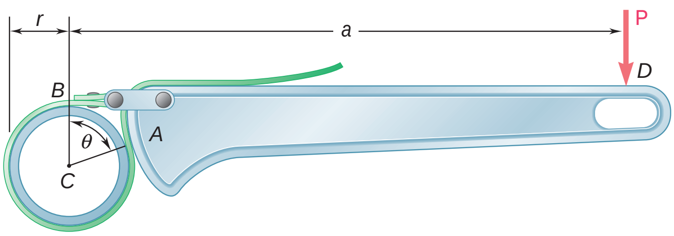


Fig. P8.126

8.127 Solve Prob. 8.126 assuming that $\theta = 75°$.

8.128 The 10-lb bar AE is suspended by a cable that passes over a 5-in.-radius drum. Vertical motion of end E of the bar is prevented by the two stops shown. Knowing that $\mu_s = 0.30$ between the cable and the drum, determine (a) the largest counterclockwise couple $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (b) the corresponding force exerted on end E of the bar.

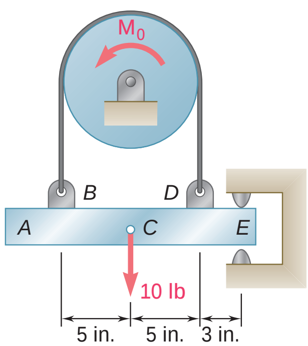


Fig. P8.128

8.129 Solve Prob. 8.128 assuming that a clockwise couple $\mathbf{M}_0$ is applied to the drum.

8.130 Prove that Eqs. (8.13) and (8.14) are valid for any shape of surface provided that the coefficient of friction is the same at all points of contact.

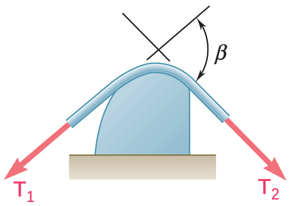


Fig. P8.130

8.131 Complete the derivation of Eq. (8.15), which relates the tension in both parts of a V belt.

8.132 Solve Prob. 8.112 assuming that the flat belt and drums are replaced by a V belt and V pulleys with a = 36°. (The angle a is as shown in Fig. 8.15*a*.)

8.133 Solve Prob. 8.113 assuming that the flat belt and pulleys are replaced by a V belt and V pulleys with a = 36°. (The angle a is as shown in Fig. 8.15*a*.)

REVIEW AND SUMMARY

This chapter was devoted to the study of *dry friction,* i.e., to problems involving rigid bodies which are in contact along *nonlubricated surfaces*.

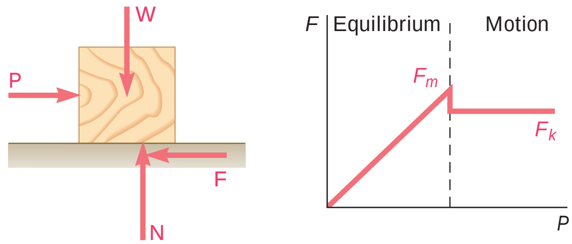


Fig. 8.16

Static and kinetic friction

Applying a horizontal force **P** to a block resting on a horizontal surface [Sec. 8.2], we note that the block at first does not move. This shows that a *friction force* **F** must have developed to balance **P** (Fig. 8.16). As the magnitude of **P** is increased, the magnitude of **F** also increases until it reaches a maximum value F_m. If **P** is further increased, the block starts sliding and the magnitude of **F** drops from F_m to a lower value F_k. Experimental evidence shows that F_m and F_k are proportional to the normal component N of the reaction of the surface. We have

$$F_m = \mu_s N \qquad F_k = \mu_k N \qquad (8.1,\ 8.2)$$

where μ_s and μ_k are called, respectively, the *coefficient of static friction* and the *coefficient of kinetic friction*. These coefficients depend on the nature and the condition of the surfaces in contact. Approximate values of the coefficients of static friction were given in Table 8.1.

Angles of friction

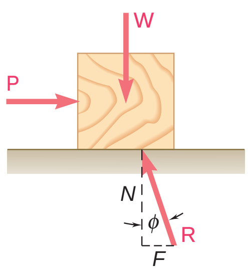


Fig. 8.17

It is sometimes convenient to replace the normal force **N** and the friction force **F** by their resultant **R** (Fig. 8.17). As the friction force increases and reaches its maximum value $F_m = \mu_s N$, the angle ϕ that **R** forms with the normal to the surface increases and reaches a maximum value ϕ_s, called the *angle of static friction*. If motion actually takes place, the magnitude of **F** drops to F_k; similarly the angle ϕ drops to a lower value ϕ_k, called the *angle of kinetic friction*. As shown in Sec. 8.3, we have

$$\tan \phi_s = \mu_s \qquad \tan \phi_k = \mu_k \qquad (8.3,\ 8.4)$$

Problems involving friction

When solving equilibrium problems involving friction, we should keep in mind that the magnitude F of the friction force is equal to $F_m = \mu_s N$ *only if the body is about to slide* [Sec. 8.4]. *If motion is not impending,* F and N should be considered as *independent unknowns* to be determined from the equilibrium equations (Fig. 8.18*a*). We

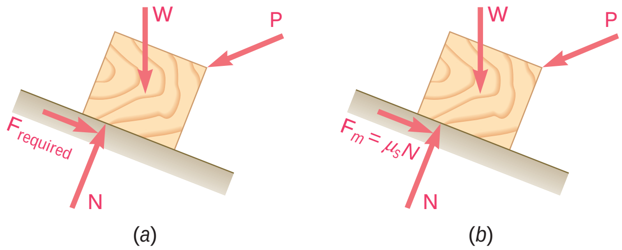


Fig. 8.18

should also check that the value of F required to maintain equilibrium is not larger than F_m; if it were, the body would move and the magnitude of the friction force would be $F_k = \mu_k N$ [Sample Prob. 8.1]. On the other hand, *if motion is known to be impending,* F has reached its maximum value $F_m = \mu_s N$ (Fig. 8.18*b*), and this expression may be substituted for F in the equilibrium equations [Sample Prob. 8.3]. When only three forces are involved in a free-body diagram, including the reaction **R** of the surface in contact with the body, it is usually more convenient to solve the problem by drawing a force triangle [Sample Prob. 8.2].

When a problem involves the analysis of the forces exerted on each other by *two bodies A and B*, it is important to show the friction forces with their correct sense. The correct sense for the friction force exerted by B on A, for instance, is opposite to that of the *relative motion* (or impending motion) of A with respect to B [Fig. 8.6].

Wedges and screws

In the second part of the chapter we considered a number of specific engineering applications where dry friction plays an important role. In the case of *wedges,* which are simple machines used to raise heavy loads [Sec. 8.5], two or more free-body diagrams were drawn and care was taken to show each friction force with its correct sense [Sample Prob. 8.4]. The analysis of *square-threaded screws,* which are frequently used in jacks, presses, and other mechanisms, was reduced to the analysis of a block sliding on an incline by unwrapping the thread of the screw and showing it as a straight line [Sec. 8.6]. This is done again in Fig. 8.19, where r denotes the *mean radius* of the thread, L is the *lead* of the screw, i.e., the distance through which the screw advances in one turn, **W** is the load, and Qr is equal to the couple exerted on the screw. It was noted that in the case of multiple-threaded screws the lead L of the screw is *not* equal to its pitch, which is the distance measured between two consecutive threads.

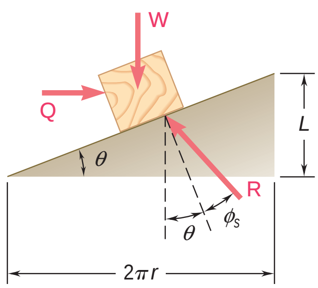


Fig. 8.19

Other engineering applications considered in this chapter were *journal bearings* and *axle friction* [Sec. 8.7], *thrust bearings* and *disk friction* [Sec. 8.8], *wheel friction* and *rolling resistance* [Sec. 8.9], and *belt friction* [Sec. 8.10].

Belt friction

In solving a problem involving a *flat belt* passing over a fixed cylinder, it is important to first determine the direction in which the belt slips or is about to slip. If the drum is rotating, the motion or impending motion of the belt should be determined *relative* to the rotating drum. For instance, if the belt shown in Fig. 8.20 is about to slip to

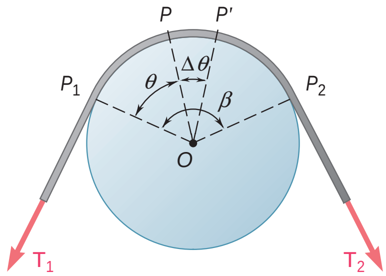


Fig. 8.20

the right relative to the drum, the friction forces exerted by the drum on the belt will be directed to the left and the tension will be larger in the right-hand portion of the belt than in the left-hand portion. Denoting the larger tension by T_2, the smaller tension by T_1, the coefficient of static friction by μ_s, and the angle (in radians) subtended by the belt by β, we derived in Sec. 8.10 the formulas

$$\ln\frac{T_2}{T_1} = \mu_s\beta \tag{8.13}$$

$$\frac{T_2}{T_1} = e^{\mu_s\beta} \tag{8.14}$$

which were used in solving Sample Probs. 8.7 and 8.8. If the belt actually slips on the drum, the coefficient of static friction μ_s should be replaced by the coefficient of kinetic friction μ_k in both of these formulas.

REVIEW PROBLEMS

8.134 Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 35°$ and $P = 200$ N.

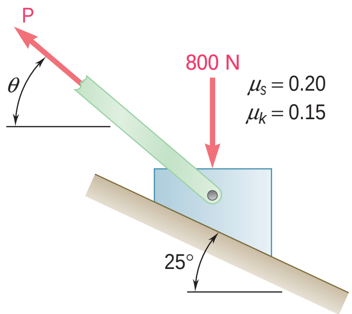


Fig. P8.134

8.135 Three 4-kg packages A, B, and C are placed on a conveyor belt that is at rest. Between the belt and both packages A and C the coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.20$; between package B and the belt the coefficients are $\mu_s = 0.10$ and $\mu_k = 0.08$. The packages are placed on the belt so that they are in contact with each other and at rest. Determine which, if any, of the packages will move and the friction force acting on each package.

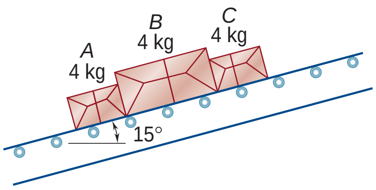


Fig. *P8.135*

8.136 The cylinder shown is of weight W and radius r. Express in terms W and r the magnitude of the largest couple **M** that can be applied to the cylinder if it is not to rotate, assuming the coefficient of static friction to be (*a*) zero at A and 0.30 at B, (*b*) 0.25 at A and 0.30 at B.

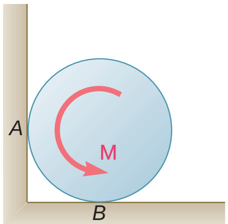


Fig. P8.136

8.137 End A of a slender, uniform rod of length L and weight W bears on a surface as shown, while end B is supported by a cord BC. Knowing that the coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine (*a*) the largest value of θ for which motion is impending, (*b*) the corresponding value of the tension in the cord.

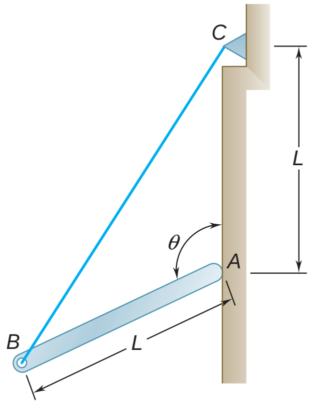


Fig. P8.137

8.138 A worker slowly moves a 50-kg crate to the left along a loading dock by applying a force **P** at corner B as shown. Knowing that the crate starts to tip about the edge E of the loading dock when $a = 200$ mm, determine (a) the coefficient of kinetic friction between the crate and the loading dock, (b) the corresponding magnitude P of the force.

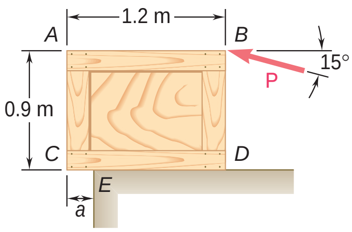


Fig. *P8.138*

8.139 A window sash weighing 10 lb is normally supported by two 5-lb sash weights. Knowing that the window remains open after one sash cord has broken, determine the smallest possible value of the coefficient of static friction. (Assume that the sash is slightly smaller than the frame and will bind only at points A and D.)

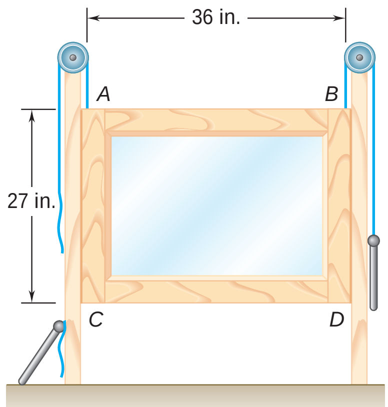


Fig. P8.139

8.140 The slender rod AB of length $l = 600$ mm is attached to a collar at B and rests on a small wheel located at a horizontal distance $a = 80$ mm from the vertical rod on which the collar slides. Knowing that the coefficient of static friction between the collar and the vertical rod is 0.25 and neglecting the radius of the wheel, determine the range of values of P for which equilibrium is maintained when $Q = 100$ N and u = 30°.

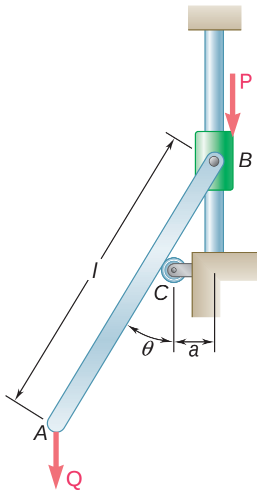


Fig. P8.140

8.141 The machine part ABC is supported by a frictionless hinge at B and a 10° wedge at C. Knowing that the coefficient of static friction is 0.20 at both surfaces of the wedge, determine (a) the force **P** required to move the wedge to the left, (b) the components of the corresponding reaction at B.

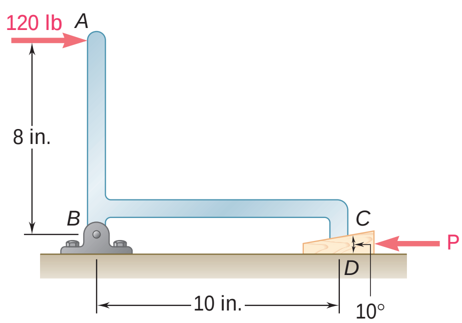


Fig. P8.141

8.142 A conical wedge is placed between two horizontal plates that are then slowly moved toward each other. Indicate what will happen to the wedge (*a*) if $\mu_s = 0.20$, (*b*) if $\mu_s = 0.30$.

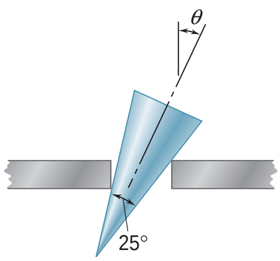


Fig. *P8.142*

8.143 In the machinist's vise shown, the movable jaw *D* is rigidly attached to the tongue *AB* that fits loosely into the fixed body of the vise. The screw is single-threaded into the fixed base and has a mean diameter of 0.75 in. and a pitch of 0.25 in. The coefficient of static friction is 0.25 between the threads and also between the tongue and the body. Neglecting bearing friction between the screw and the movable head, determine the couple that must be applied to the handle in order to produce a clamping force of 1 kip.

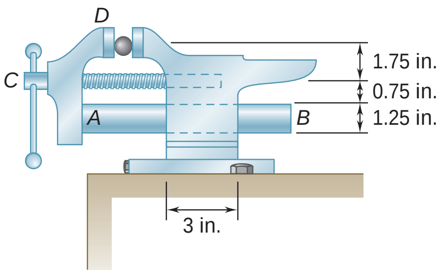


Fig. P8.143

8.144 A lever of negligible weight is loosely fitted onto a 75-mm-diameter fixed shaft. It is observed that the lever will just start rotating if a 3-kg mass is added at *C*. Determine the coefficient of static friction between the shaft and the lever.

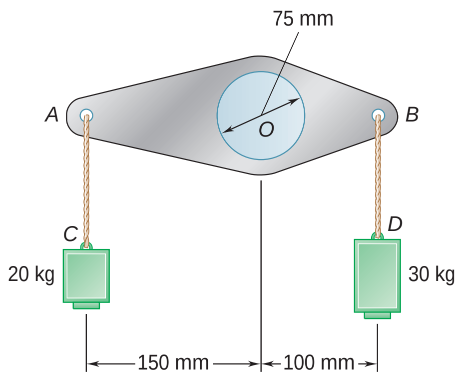


Fig. P8.144

8.145 In the pivoted motor mount shown, the weight **W** of the 175-lb motor is used to maintain tension in the drive belt. Knowing that the coefficient of static friction between the flat belt and drums *A* and *B* is 0.40, and neglecting the weight of platform *CD*, determine the largest couple that can be transmitted to drum *B* when the drive drum *A* is rotating clockwise.

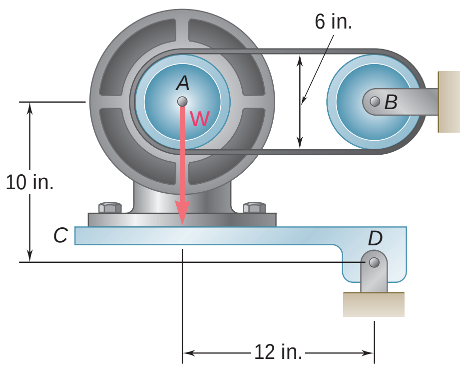


Fig. *P8.145*

COMPUTER PROBLEMS

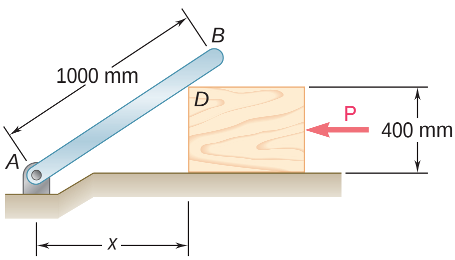


Fig. P8.C1

8.C1 The position of the 10-kg rod *AB* is controlled by the 2-kg block shown, which is slowly moved to the left by the force **P**. Knowing that the coefficient of kinetic friction between all surfaces of contact is 0.25, write a computer program and use it to calculate the magnitude *P* of the force for values of *x* from 900 to 100 mm, using 50-mm decrements. Using appropriate smaller decrements, determine the maximum value of *P* and the corresponding value of *x*.

8.C2 Blocks *A* and *B* are supported by an incline that is held in the position shown. Knowing that block *A* weighs 20 lb and that the coefficient of static friction between all surfaces of contact is 0.15, write a computer program and use it to calculate the value of u for which motion is impending for weights of block *B* from 0 to 100 lb, using 10-lb increments.

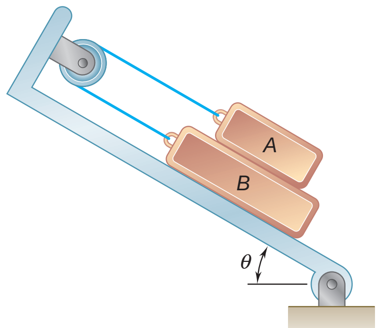


Fig. P8.C2

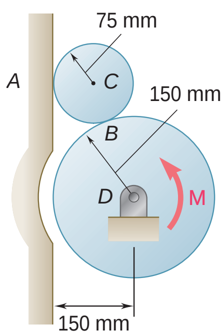


Fig. P8.C3

8.C3 A 300-g cylinder *C* rests on cylinder *D* as shown. Knowing that the coefficient of static friction m_s is the same at *A* and *B*, write a computer program and use it to determine, for values of m_s from 0 to 0.40 and using 0.05 increments, the largest counterclockwise couple **M** that can be applied to cylinder *D* if it is not to rotate.

8.C4 Two rods are connected by a slider block *D* and are held in equilibrium by the couple $\mathbf{M}_A$ as shown. Knowing that the coefficient of static friction between rod *AC* and the slider block is 0.40, write a computer program and use it to determine, for values of u from 0 to 120° and using 10° increments, the range of values of M_A for which equilibrium is maintained.

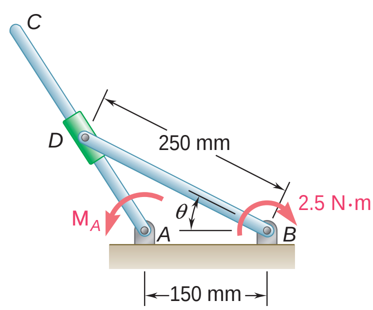


Fig. P8.C4

8.C5 The 10-lb block A is slowly moved up the circular cylindrical surface by a cable that passes over a small fixed cylindrical drum at B. The coefficient of kinetic friction is known to be 0.30 between the block and the surface and between the cable and the drum. Write a computer program and use it to calculate the force **P** required to maintain the motion for values of θ from 0 to 90°, using 10° increments. For the same values of θ calculate the magnitude of the reaction between the block and the surface. [Note that the angle of contact between the cable and the fixed drum is $\beta = \pi - (\theta/2)$.]

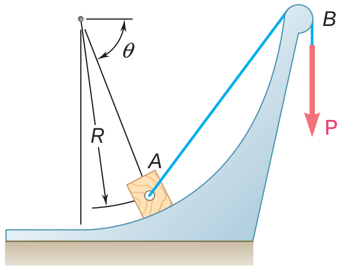


Fig. P8.C5

8.C6 A flat belt is used to transmit a couple from drum A to drum B. The radius of each drum is 80 mm, and the system is fitted with an idler wheel C that is used to increase the contact between the belt and the drums. The allowable belt tension is 200 N, and the coefficient of static friction between the belt and the drums is 0.30. Write a computer program and use it to calculate the largest couple that can be transmitted for values of θ from 0 to 30°, using 5° increments.

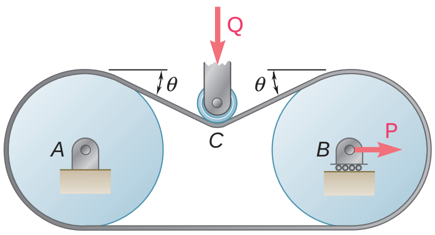


Fig. P8.C6

8.C7 Two collars A and B that slide on vertical rods with negligible friction are connected by a 30-in. cord that passes over a fixed shaft at C. The coefficient of static friction between the cord and the fixed shaft is 0.30. Knowing that the weight of collar B is 8 lb, write a computer program and use it to determine, for values of θ from 0 to 60° and using 10° increments, the largest and smallest weight of collar A for which equilibrium is maintained.

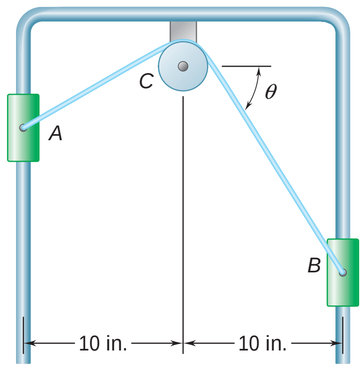


Fig. P8.C7

8.C8 The end B of a uniform beam of length L is being pulled by a stationary crane. Initially the beam lies on the ground with end A directly below pulley C. As the cable is slowly pulled in, the beam first slides to the left with $\theta = 0$ until it has moved through a distance x_0. In a second phase, end B is raised, while end A keeps sliding to the left until x reaches its maximum value x_m and θ the corresponding value θ_1. The beam then rotates about A' while θ keeps increasing. As θ reaches the value θ_2, end A starts sliding to the right and keeps sliding in an irregular manner until B reaches C. Knowing that the coefficients of friction between the beam and the ground are $\mu_s = 0.50$ and $\mu_k = 0.40$, (*a*) write a program to compute x for any value of θ while the beam is sliding to the left and use this program to determine x_0, x_m, and θ_1, (*b*) modify the program to compute for any θ the value of x for which sliding would be impending to the right and use this new program to determine the value θ_2 of θ corresponding to $x = x_m$.

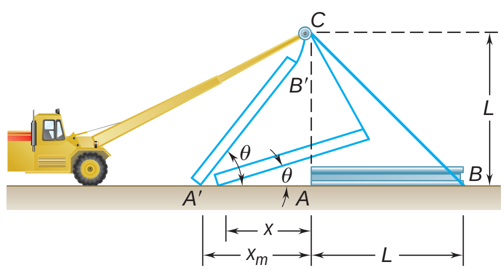


Fig. P8.C8