

Many engineering problems can be solved by considering the equilibrium of a “particle.” In the case of this excavator, which is being loaded onto a ship, a relation between the tensions in the various cables involved can be obtained by considering the equilibrium of the hook to which the cables are attached.



# CHAPTER 2

## Statics of Particles



## Chapter 2 Statics of Particles

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## 2.1 INTRODUCTION

In this chapter you will study the effect of forces acting on particles. First you will learn how to replace two or more forces acting on a given particle by a single force having the same effect as the original forces. This single equivalent force is the *resultant* of the original forces acting on the particle. Later the relations which exist among the various forces acting on a particle in a state of *equilibrium* will be derived and used to determine some of the forces acting on the particle.

The use of the word "particle" does not imply that our study will be limited to that of small corpuscles. What it means is that the size and shape of the bodies under consideration will not significantly affect the solution of the problems treated in this chapter and that all the forces acting on a given body will be assumed to be applied at the same point. Since such an assumption is verified in many practical applications, you will be able to solve a number of engineering problems in this chapter.

The first part of the chapter is devoted to the study of forces contained in a single plane, and the second part to the analysis of forces in three-dimensional space.

## FORCES IN A PLANE

### 2.2 FORCE ON A PARTICLE. RESULTANT OF TWO FORCES

A force represents the action of one body on another and is generally characterized by its *point of application*, its *magnitude*, and its *direction*. Forces acting on a given particle, however, have the same point of application. Each force considered in this chapter will thus be completely defined by its magnitude and direction.

The magnitude of a force is characterized by a certain number of units. As indicated in Chap. 1, the SI units used by engineers to measure the magnitude of a force are the newton (N) and its multiple the kilonewton (kN), equal to 1000 N, while the U.S. customary units used for the same purpose are the pound (lb) and its multiple the kilopound (kip), equal to 1000 lb. The direction of a force is defined by the *line of action* and the *sense* of the force. The line of action is the infinite straight line along which the force acts; it is characterized by the angle it forms with some fixed axis (Fig. 2.1). The force itself is represented by a segment of

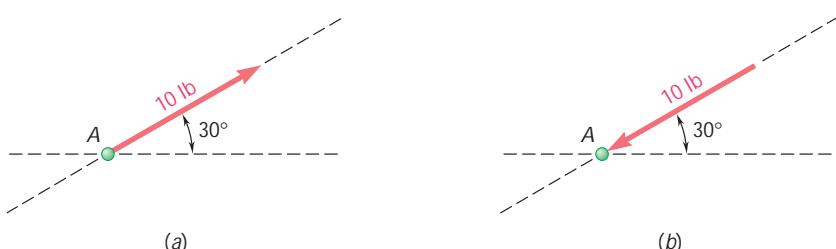


Fig. 2.1

that line; through the use of an appropriate scale, the length of this segment may be chosen to represent the magnitude of the force. Finally, the sense of the force should be indicated by an arrowhead. It is important in defining a force to indicate its sense. Two forces having the same magnitude and the same line of action but different sense, such as the forces shown in Fig. 2.1a and b, will have directly opposite effects on a particle.

Experimental evidence shows that two forces **P** and **Q** acting on a particle A (Fig. 2.2a) can be replaced by a single force **R** which has the same effect on the particle (Fig. 2.2c). This force is called the *resultant* of the forces **P** and **Q** and can be obtained, as shown in Fig. 2.2b, by constructing a parallelogram, using **P** and **Q** as two adjacent sides of the parallelogram. *The diagonal that passes through A represents the resultant.* This method for finding the resultant is known as the *parallelogram law* for the addition of two forces. This law is based on experimental evidence; it cannot be proved or derived mathematically.

## 2.3 VECTORS

It appears from the above that forces do not obey the rules of addition defined in ordinary arithmetic or algebra. For example, two forces acting at a right angle to each other, one of 4 lb and the other of 3 lb, add up to a force of 5 lb, *not* to a force of 7 lb. Forces are not the only quantities which follow the parallelogram law of addition. As you will see later, *displacements, velocities, accelerations, and momenta* are other examples of physical quantities possessing magnitude and direction that are added according to the parallelogram law. All these quantities can be represented mathematically by *vectors*, while those physical quantities which have magnitude but not direction, such as *volume, mass, or energy*, are represented by plain numbers or *scalars*.

Vectors are defined as *mathematical expressions possessing magnitude and direction, which add according to the parallelogram law*. Vectors are represented by arrows in the illustrations and will be distinguished from scalar quantities in this text through the use of boldface type (**P**). In longhand writing, a vector may be denoted by drawing a short arrow above the letter used to represent it ( $\vec{P}$ ) or by underlining the letter (P). The last method may be preferred since underlining can also be used on a typewriter or computer. The magnitude of a vector defines the length of the arrow used to represent the vector. In this text, italic type will be used to denote the magnitude of a vector. Thus, the magnitude of the vector **P** will be denoted by *P*.

A vector used to represent a force acting on a given particle has a well-defined point of application, namely, the particle itself. Such a vector is said to be a *fixed, or bound, vector* and cannot be moved without modifying the conditions of the problem. Other physical quantities, however, such as couples (see Chap. 3), are represented by vectors which may be freely moved in space; these

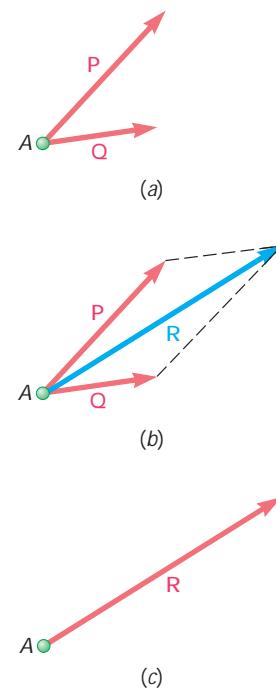


Fig. 2.2

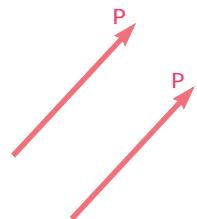


Fig. 2.4

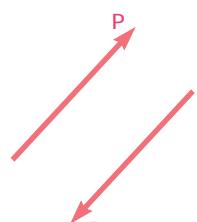


Fig. 2.5

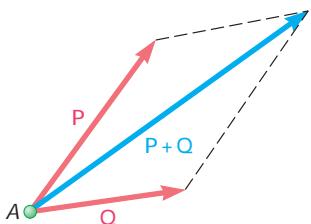


Fig. 2.6

vectors are called *free vectors*. Still other physical quantities, such as forces acting on a rigid body (see Chap. 3), are represented by vectors which can be moved, or slid, along their lines of action; they are known as *sliding vectors*.†

Two vectors which have the same magnitude and the same direction are said to be *equal*, whether or not they also have the same point of application (Fig. 2.4); equal vectors may be denoted by the same letter.

The *negative vector* of a given vector  $\mathbf{P}$  is defined as a vector having the same magnitude as  $\mathbf{P}$  and a direction opposite to that of  $\mathbf{P}$  (Fig. 2.5); the negative of the vector  $\mathbf{P}$  is denoted by  $-\mathbf{P}$ . The vectors  $\mathbf{P}$  and  $-\mathbf{P}$  are commonly referred to as *equal and opposite vectors*. Clearly, we have

$$\mathbf{P} + (-\mathbf{P}) = 0$$

## 2.4 ADDITION OF VECTORS

We saw in the preceding section that, by definition, vectors add according to the parallelogram law. Thus, the sum of two vectors  $\mathbf{P}$  and  $\mathbf{Q}$  is obtained by attaching the two vectors to the same point  $A$  and constructing a parallelogram, using  $\mathbf{P}$  and  $\mathbf{Q}$  as two sides of the parallelogram (Fig. 2.6). The diagonal that passes through  $A$  represents the sum of the vectors  $\mathbf{P}$  and  $\mathbf{Q}$ , and this sum is denoted by  $\mathbf{P} + \mathbf{Q}$ . The fact that the sign  $+$  is used to denote both vector and scalar addition should not cause any confusion if vector and scalar quantities are always carefully distinguished. Thus, we should note that the magnitude of the vector  $\mathbf{P} + \mathbf{Q}$  is *not*, in general, equal to the sum  $P + Q$  of the magnitudes of the vectors  $\mathbf{P}$  and  $\mathbf{Q}$ .

Since the parallelogram constructed on the vectors  $\mathbf{P}$  and  $\mathbf{Q}$  does not depend upon the order in which  $\mathbf{P}$  and  $\mathbf{Q}$  are selected, we conclude that the addition of two vectors is *commutative*, and we write

$$\mathbf{P} + \mathbf{Q} = \mathbf{Q} + \mathbf{P} \quad (2.1)$$

†Some expressions have magnitude and direction, but do not add according to the parallelogram law. While these expressions may be represented by arrows, they *cannot* be considered as vectors.

A group of such expressions is the finite rotations of a rigid body. Place a closed book on a table in front of you, so that it lies in the usual fashion, with its front cover up and its binding to the left. Now rotate it through  $180^\circ$  about an axis parallel to the binding (Fig. 2.3a); this rotation may be represented by an arrow of length equal to  $180$  units and oriented as shown. Picking up the book as it lies in its new position, rotate

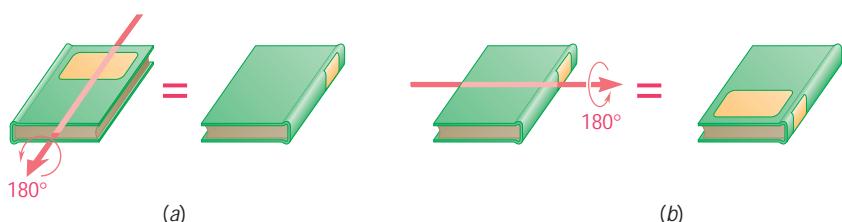


Fig. 2.3 Finite rotations of a rigid body

From the parallelogram law, we can derive an alternative method for determining the sum of two vectors. This method, known as the *triangle rule*, is derived as follows. Consider Fig. 2.6, where the sum of the vectors  $\mathbf{P}$  and  $\mathbf{Q}$  has been determined by the parallelogram law. Since the side of the parallelogram opposite  $\mathbf{Q}$  is equal to  $\mathbf{Q}$  in magnitude and direction, we could draw only half of the parallelogram (Fig. 2.7a). The sum of the two vectors can thus be found by *arranging  $\mathbf{P}$  and  $\mathbf{Q}$  in tip-to-tail fashion and then connecting the tail of  $\mathbf{P}$  with the tip of  $\mathbf{Q}$* . In Fig. 2.7b, the other half of the parallelogram is considered, and the same result is obtained. This confirms the fact that vector addition is commutative.

The *subtraction* of a vector is defined as the addition of the corresponding negative vector. Thus, the vector  $\mathbf{P} - \mathbf{Q}$  representing the difference between the vectors  $\mathbf{P}$  and  $\mathbf{Q}$  is obtained by adding to  $\mathbf{P}$  the negative vector  $-\mathbf{Q}$  (Fig. 2.8). We write

$$\mathbf{P} - \mathbf{Q} = \mathbf{P} + (-\mathbf{Q}) \quad (2.2)$$

Here again we should observe that, while the same sign is used to denote both vector and scalar subtraction, confusion will be avoided if care is taken to distinguish between vector and scalar quantities.

We will now consider the *sum of three or more vectors*. The sum of three vectors  $\mathbf{P}$ ,  $\mathbf{Q}$ , and  $\mathbf{S}$  will, *by definition*, be obtained by first adding the vectors  $\mathbf{P}$  and  $\mathbf{Q}$  and then adding the vector  $\mathbf{S}$  to the vector  $\mathbf{P} + \mathbf{Q}$ . We thus write

$$\mathbf{P} + \mathbf{Q} + \mathbf{S} = (\mathbf{P} + \mathbf{Q}) + \mathbf{S} \quad (2.3)$$

Similarly, the sum of four vectors will be obtained by adding the fourth vector to the sum of the first three. It follows that the sum of any number of vectors can be obtained by applying repeatedly the parallelogram law to successive pairs of vectors until all the given vectors are replaced by a single vector.

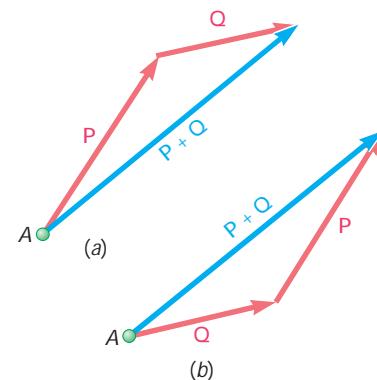


Fig. 2.7

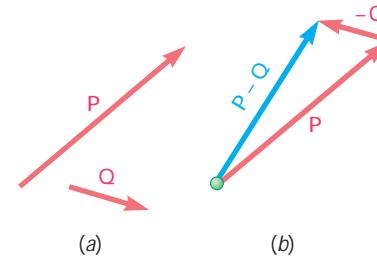
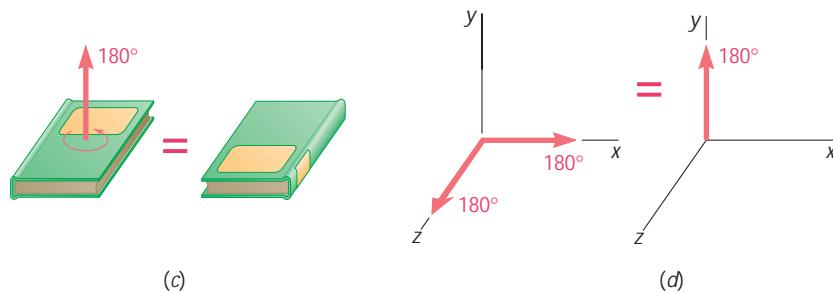


Fig. 2.8

it now through  $180^\circ$  about a horizontal axis perpendicular to the binding (Fig. 2.3b); this second rotation may be represented by an arrow  $180^\circ$  units long and oriented as shown. But the book could have been placed in this final position through a single  $180^\circ$  rotation about a vertical axis (Fig. 2.3c). We conclude that the sum of the two  $180^\circ$  rotations represented by arrows directed respectively along the  $z$  and  $x$  axes is a  $180^\circ$  rotation represented by an arrow directed along the  $y$  axis (Fig. 2.3d). Clearly, the finite rotations of a rigid body *do not* obey the parallelogram law of addition; therefore, they *cannot* be represented by vectors.



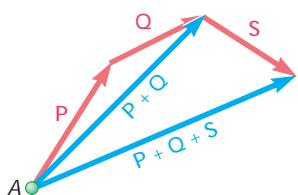


Fig. 2.9

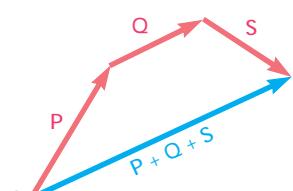


Fig. 2.10

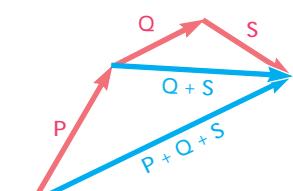


Fig. 2.11

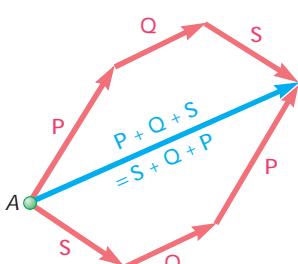


Fig. 2.12

If the given vectors are *coplanar*, i.e., if they are contained in the same plane, their sum can be easily obtained graphically. For this case, the repeated application of the triangle rule is preferred to the application of the parallelogram law. In Fig. 2.9 the sum of three vectors  $\mathbf{P}$ ,  $\mathbf{Q}$ , and  $\mathbf{S}$  was obtained in that manner. The triangle rule was first applied to obtain the sum  $\mathbf{P} + \mathbf{Q}$  of the vectors  $\mathbf{P}$  and  $\mathbf{Q}$ ; it was applied again to obtain the sum of the vectors  $\mathbf{P} + \mathbf{Q}$  and  $\mathbf{S}$ . The determination of the vector  $\mathbf{P} + \mathbf{Q}$ , however, could have been omitted and the sum of the three vectors could have been obtained directly, as shown in Fig. 2.10, by *arranging the given vectors in tip-to-tail fashion and connecting the tail of the first vector with the tip of the last one*. This is known as the *polygon rule* for the addition of vectors.

We observe that the result obtained would have been unchanged if, as shown in Fig. 2.11, the vectors  $\mathbf{Q}$  and  $\mathbf{S}$  had been replaced by their sum  $\mathbf{Q} + \mathbf{S}$ . We may thus write

$$\mathbf{P} + \mathbf{Q} + \mathbf{S} = (\mathbf{P} + \mathbf{Q}) + \mathbf{S} = \mathbf{P} + (\mathbf{Q} + \mathbf{S}) \quad (2.4)$$

which expresses the fact that vector addition is *associative*. Recalling that vector addition has also been shown, in the case of two vectors, to be commutative, we write

$$\begin{aligned} \mathbf{P} + \mathbf{Q} + \mathbf{S} &= (\mathbf{P} + \mathbf{Q}) + \mathbf{S} = \mathbf{S} + (\mathbf{P} + \mathbf{Q}) \\ &= \mathbf{S} + (\mathbf{Q} + \mathbf{P}) = \mathbf{S} + \mathbf{Q} + \mathbf{P} \end{aligned} \quad (2.5)$$

This expression, as well as others which may be obtained in the same way, shows that the order in which several vectors are added together is immaterial (Fig. 2.12).

**Product of a Scalar and a Vector.** Since it is convenient to denote the sum  $\mathbf{P} + \mathbf{P}$  by  $2\mathbf{P}$ , the sum  $\mathbf{P} + \mathbf{P} + \mathbf{P}$  by  $3\mathbf{P}$ , and, in general, the sum of  $n$  equal vectors  $\mathbf{P}$  by the product  $n\mathbf{P}$ , we will define the product  $n\mathbf{P}$  of a positive integer  $n$  and a vector  $\mathbf{P}$  as a vector having the same direction as  $\mathbf{P}$  and the magnitude  $nP$ . Extending this definition to include all scalars, and recalling the definition of a negative vector given in Sec. 2.3, we define the product  $k\mathbf{P}$  of a scalar  $k$  and a vector  $\mathbf{P}$  as a vector having the same direction as  $\mathbf{P}$  (if  $k$  is positive), or a direction opposite to that of  $\mathbf{P}$  (if  $k$  is negative), and a magnitude equal to the product of  $P$  and of the absolute value of  $k$  (Fig. 2.13).

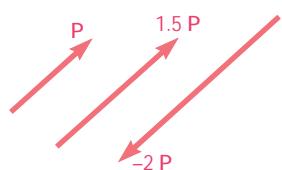


Fig. 2.13

## 2.5 RESULTANT OF SEVERAL CONCURRENT FORCES

Consider a particle A acted upon by several coplanar forces, i.e., by several forces contained in the same plane (Fig. 2.14a). Since the forces considered here all pass through A, they are also said to be *concurrent*. The vectors representing the forces acting on A may be added by the polygon rule (Fig. 2.14b). Since the use of the polygon rule is equivalent to the repeated application of the parallelogram law, the vector  $\mathbf{R}$  thus obtained represents the resultant of the given concurrent forces, i.e., the single force which has the same effect on

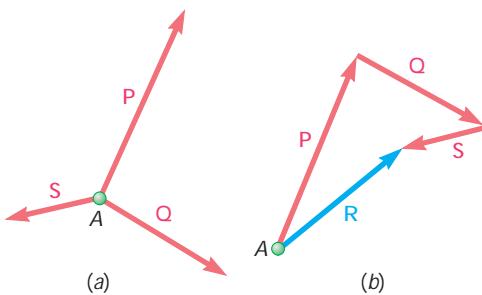


Fig. 2.14

the particle  $A$  as the given forces. As indicated in the previous section, the order in which the vectors  $\mathbf{P}$ ,  $\mathbf{Q}$ , and  $\mathbf{S}$  representing the given forces are added together is immaterial.

## 2.6 RESOLUTION OF A FORCE INTO COMPONENTS

We have seen that two or more forces acting on a particle may be replaced by a single force which has the same effect on the particle. Conversely, a single force  $\mathbf{F}$  acting on a particle may be replaced by two or more forces which, together, have the same effect on the particle. These forces are called the *components* of the original force  $\mathbf{F}$ , and the process of substituting them for  $\mathbf{F}$  is called *resolving the force  $\mathbf{F}$  into components*.

Clearly, for each force  $\mathbf{F}$  there exist an infinite number of possible sets of components. Sets of *two components*  $\mathbf{P}$  and  $\mathbf{Q}$  are the most important as far as practical applications are concerned. But, even then, the number of ways in which a given force  $\mathbf{F}$  may be resolved into two components is unlimited (Fig. 2.15). Two cases are of particular interest:

1. *One of the Two Components,  $\mathbf{P}$ , Is Known.* The second component,  $\mathbf{Q}$ , is obtained by applying the triangle rule and joining the tip of  $\mathbf{P}$  to the tip of  $\mathbf{F}$  (Fig. 2.16); the magnitude and direction of  $\mathbf{Q}$  are determined graphically or by trigonometry. Once  $\mathbf{Q}$  has been determined, both components  $\mathbf{P}$  and  $\mathbf{Q}$  should be applied at  $A$ .
2. *The Line of Action of Each Component Is Known.* The magnitude and sense of the components are obtained by applying the parallelogram law and drawing lines, through the tip of  $\mathbf{F}$ , parallel to the given lines of action (Fig. 2.17). This process leads to two well-defined components,  $\mathbf{P}$  and  $\mathbf{Q}$ , which can be determined graphically or computed trigonometrically by applying the law of sines.

Many other cases can be encountered; for example, the direction of one component may be known, while the magnitude of the other component is to be as small as possible (see Sample Prob. 2.2). In all cases the appropriate triangle or parallelogram which satisfies the given conditions is drawn.

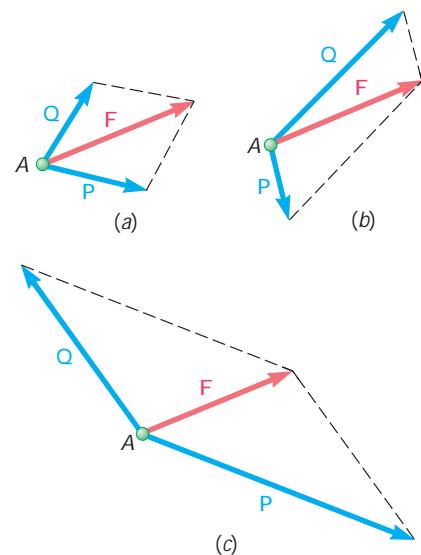


Fig. 2.15

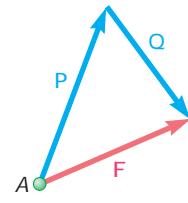


Fig. 2.16

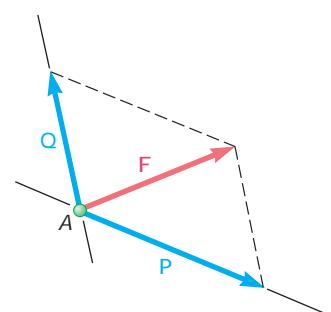
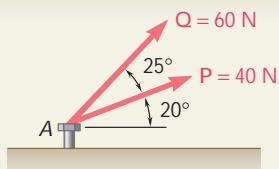


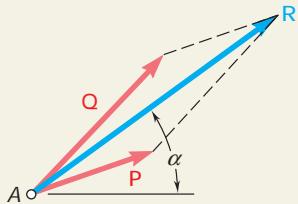
Fig. 2.17



## SAMPLE PROBLEM 2.1

The two forces **P** and **Q** act on a bolt **A**. Determine their resultant.

### SOLUTION

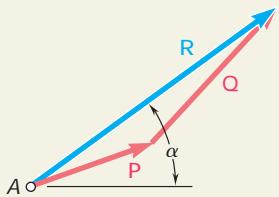


**Graphical Solution.** A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant are measured and found to be

$$R = 98 \text{ N} \quad a = 35^\circ \quad \mathbf{R} = 98 \text{ N a } 35^\circ \quad \blacktriangleleft$$

The triangle rule may also be used. Forces **P** and **Q** are drawn in tip-to-tail fashion. Again the magnitude and direction of the resultant are measured.

$$R = 98 \text{ N} \quad a = 35^\circ \quad \mathbf{R} = 98 \text{ N a } 35^\circ \quad \blacktriangleleft$$



**Trigonometric Solution.** The triangle rule is again used; two sides and the included angle are known. We apply the law of cosines.

$$R^2 = P^2 + Q^2 - 2PQ \cos B$$

$$R^2 = (40 \text{ N})^2 + (60 \text{ N})^2 - 2(40 \text{ N})(60 \text{ N}) \cos 155^\circ$$

$$R = 97.73 \text{ N}$$

Now, applying the law of sines, we write

$$\frac{\sin A}{Q} = \frac{\sin B}{R} \quad \frac{\sin A}{60 \text{ N}} = \frac{\sin 155^\circ}{97.73 \text{ N}} \quad (1)$$

Solving Eq. (1) for  $\sin A$ , we have

$$\sin A = \frac{(60 \text{ N}) \sin 155^\circ}{97.73 \text{ N}}$$

Using a calculator, we first compute the quotient, then its arc sine, and obtain

$$A = 15.04^\circ \quad a = 20^\circ + A = 35.04^\circ$$

We use 3 significant figures to record the answer (cf. Sec. 1.6):

$$\mathbf{R} = 97.7 \text{ N a } 35.0^\circ \quad \blacktriangleleft$$

**Alternative Trigonometric Solution.** We construct the right triangle **BCD** and compute

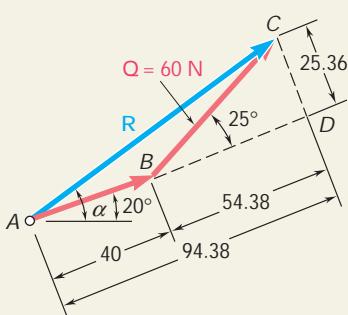
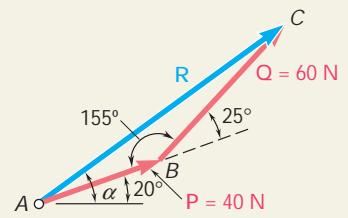
$$CD = (60 \text{ N}) \sin 25^\circ = 25.36 \text{ N}$$

$$BD = (60 \text{ N}) \cos 25^\circ = 54.38 \text{ N}$$

Then, using triangle **ACD**, we obtain

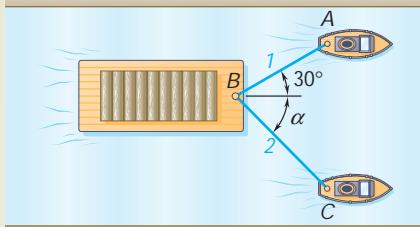
$$\tan A = \frac{25.36 \text{ N}}{94.38 \text{ N}} \quad A = 15.04^\circ$$

$$R = \frac{25.36}{\sin A} \quad R = 97.73 \text{ N}$$



Again,

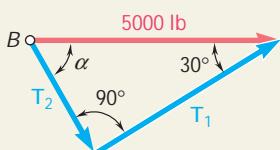
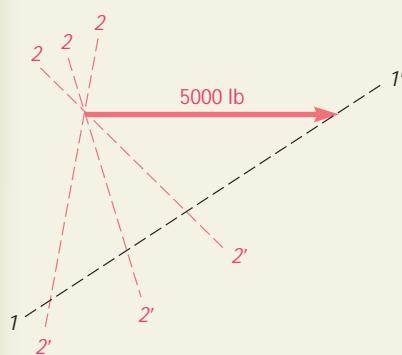
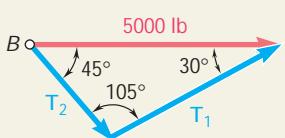
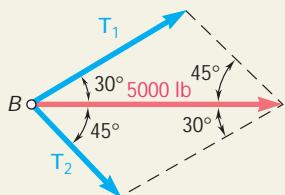
$$a = 20^\circ + A = 35.04^\circ \quad \mathbf{R} = 97.7 \text{ N a } 35.0^\circ \quad \blacktriangleleft$$



## SAMPLE PROBLEM 2.2

A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 5000-lb force directed along the axis of the barge, determine (a) the tension in each of the ropes knowing that  $\alpha = 45^\circ$ , (b) the value of  $\alpha$  for which the tension in rope 2 is minimum.

### SOLUTION



**a. Tension for  $\alpha = 45^\circ$ .** *Graphical Solution.* The parallelogram law is used; the diagonal (resultant) is known to be equal to 5000 lb and to be directed to the right. The sides are drawn parallel to the ropes. If the drawing is done to scale, we measure

$$T_1 = 3700 \text{ lb} \quad T_2 = 2600 \text{ lb}$$

**Trigonometric Solution.** The triangle rule can be used. We note that the triangle shown represents half of the parallelogram shown above. Using the law of sines, we write

$$\frac{T_1}{\sin 45^\circ} = \frac{T_2}{\sin 30^\circ} = \frac{5000 \text{ lb}}{\sin 105^\circ}$$

With a calculator, we first compute and store the value of the last quotient. Multiplying this value successively by  $\sin 45^\circ$  and  $\sin 30^\circ$ , we obtain

$$T_1 = 3660 \text{ lb} \quad T_2 = 2590 \text{ lb}$$

**b. Value of  $\alpha$  for Minimum  $T_2$ .** To determine the value of  $\alpha$  for which the tension in rope 2 is minimum, the triangle rule is again used. In the sketch shown, line 1-1' is the known direction of  $\mathbf{T}_1$ . Several possible directions of  $\mathbf{T}_2$  are shown by the lines 2-2'. We note that the minimum value of  $T_2$  occurs when  $\mathbf{T}_1$  and  $\mathbf{T}_2$  are perpendicular. The minimum value of  $T_2$  is

$$T_2 = (5000 \text{ lb}) \sin 30^\circ = 2500 \text{ lb}$$

Corresponding values of  $T_1$  and  $\alpha$  are

$$T_1 = (5000 \text{ lb}) \cos 30^\circ = 4330 \text{ lb}$$

$$\alpha = 90^\circ - 30^\circ$$

$$\alpha = 60^\circ$$

# SOLVING PROBLEMS ON YOUR OWN

The preceding sections were devoted to the *parallelogram law* for the addition of vectors and to its applications.

Two sample problems were presented. In Sample Prob. 2.1, the parallelogram law was used to determine the resultant of two forces of known magnitude and direction. In Sample Prob. 2.2, it was used to resolve a given force into two components of known direction.

You will now be asked to solve problems on your own. Some may resemble one of the sample problems; others may not. What all problems and sample problems in this section have in common is that they can be solved by the direct application of the parallelogram law.

Your solution of a given problem should consist of the following steps:

**1. Identify which of the forces are the applied forces and which is the resultant.** It is often helpful to write the vector equation which shows how the forces are related. For example, in Sample Prob. 2.1 we would have

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$

You may want to keep that relation in mind as you formulate the next part of your solution.

**2. Draw a parallelogram with the applied forces as two adjacent sides and the resultant as the included diagonal (Fig. 2.2).** Alternatively, you can *use the triangle rule*, with the applied forces drawn in tip-to-tail fashion and the resultant extending from the tail of the first vector to the tip of the second (Fig. 2.7).

**3. Indicate all dimensions.** Using one of the triangles of the parallelogram, or the triangle constructed according to the triangle rule, indicate all dimensions—whether sides or angles—and determine the unknown dimensions either graphically or by trigonometry. If you use trigonometry, remember that the law of cosines should be applied first if two sides and the included angle are known [Sample Prob. 2.1], and the law of sines should be applied first if one side and all angles are known [Sample Prob. 2.2].

If you have had prior exposure to mechanics, you might be tempted to ignore the solution techniques of this lesson in favor of resolving the forces into rectangular components. While this latter method is important and will be considered in the next section, use of the parallelogram law simplifies the solution of many problems and should be mastered at this time.

# PROBLEMS<sup>†</sup>

- 2.1** Two forces are applied at point *B* of beam *AB*. Determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

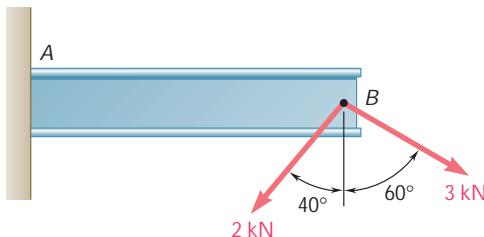


Fig. P2.1

- 2.2** The cable stays *AB* and *AD* help support pole *AC*. Knowing that the tension is 120 lb in *AB* and 40 lb in *AD*, determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at *A* using (a) the parallelogram law, (b) the triangle rule.

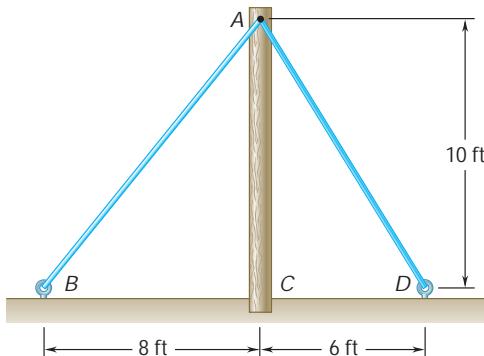


Fig. P2.2

- 2.3** Two structural members *B* and *C* are bolted to bracket *A*. Knowing that both members are in tension and that  $P = 10$  kN and  $Q = 15$  kN, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.

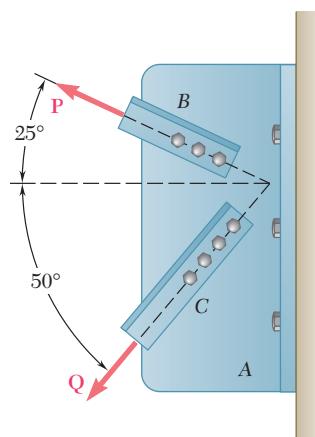


Fig. P2.3 and P2.4

- 2.4** Two structural members *B* and *C* are bolted to bracket *A*. Knowing that both members are in tension and that  $P = 6$  kips and  $Q = 4$  kips, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.

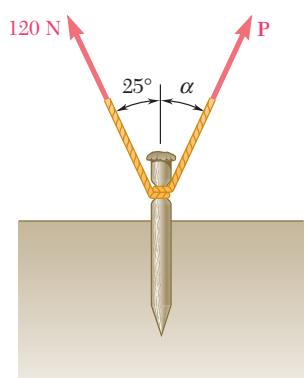


Fig. P2.5

<sup>†</sup>Answers to all problems set in straight type (such as 2.1) are given at the end of the book. Answers to problems with a number set in italic type (such as 2.3) are not given.

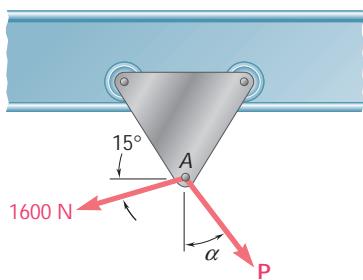


Fig. P2.6 and P2.7

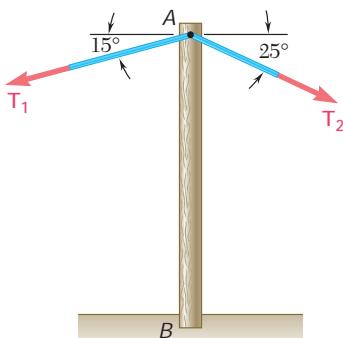


Fig. P2.8 and P2.9

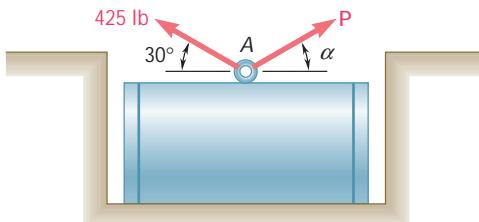


Fig. P2.11, P2.12, and P2.13

- 2.6** A trolley that moves along a horizontal beam is acted upon by two forces as shown. (a) Knowing that  $\alpha = 25^\circ$ , determine by trigonometry the magnitude of the force  $\mathbf{P}$  so that the resultant force exerted on the trolley is vertical. (b) What is the corresponding magnitude of the resultant?

- 2.7** A trolley that moves along a horizontal beam is acted upon by two forces as shown. Determine by trigonometry the magnitude and direction of the force  $\mathbf{P}$  so that the resultant is a vertical force of 2500 N.

- 2.8** A telephone cable is clamped at A to the pole AB. Knowing that the tension in the left-hand portion of the cable is  $T_1 = 800$  lb, determine by trigonometry (a) the required tension  $T_2$  in the right-hand portion if the resultant  $\mathbf{R}$  of the forces exerted by the cable at A is to be vertical, (b) the corresponding magnitude of  $\mathbf{R}$ .

- 2.9** A telephone cable is clamped at A to the pole AB. Knowing that the tension in the right-hand portion of the cable is  $T_2 = 1000$  lb, determine by trigonometry (a) the required tension  $T_1$  in the left-hand portion if the resultant  $\mathbf{R}$  of the forces exerted by the cable at A is to be vertical, (b) the corresponding magnitude of  $\mathbf{R}$ .

- 2.10** Two forces are applied as shown to a hook support. Knowing that the magnitude of  $\mathbf{P}$  is 35 N, determine by trigonometry (a) the required angle  $\alpha$  if the resultant  $\mathbf{R}$  of the two forces applied to the support is to be horizontal, (b) the corresponding magnitude of  $\mathbf{R}$ .

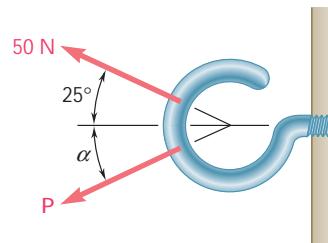


Fig. P2.10

- 2.11** A steel tank is to be positioned in an excavation. Knowing that  $\alpha = 20^\circ$ , determine by trigonometry (a) the required magnitude of the force  $\mathbf{P}$  if the resultant  $\mathbf{R}$  of the two forces applied at A is to be vertical, (b) the corresponding magnitude of  $\mathbf{R}$ .

- 2.12** A steel tank is to be positioned in an excavation. Knowing that the magnitude of  $\mathbf{P}$  is 500 lb, determine by trigonometry (a) the required angle  $\alpha$  if the resultant  $\mathbf{R}$  of the two forces applied at A is to be vertical, (b) the corresponding magnitude of  $\mathbf{R}$ .

- 2.13** A steel tank is to be positioned in an excavation. Determine by trigonometry (a) the magnitude and direction of the smallest force  $\mathbf{P}$  for which the resultant  $\mathbf{R}$  of the two forces applied at A is vertical, (b) the corresponding magnitude of  $\mathbf{R}$ .

- 2.14** For the hook support of Prob. 2.10, determine by trigonometry (a) the magnitude and direction of the smallest force  $\mathbf{P}$  for which the resultant  $\mathbf{R}$  of the two forces applied to the support is horizontal, (b) the corresponding magnitude of  $\mathbf{R}$ .

**2.15** Solve Prob. 2.2 by trigonometry.

**2.16** Solve Prob. 2.4 by trigonometry.

**2.17** For the stake of Prob. 2.5, knowing that the tension in one rope is 120 N, determine by trigonometry the magnitude and direction of the force  $\mathbf{P}$  so that the resultant is a vertical force of 160 N.

**2.18** For the hook support of Prob. 2.10, knowing that  $P = 75$  N and  $\alpha = 50^\circ$ , determine by trigonometry the magnitude and direction of the resultant of the two forces applied to the support.

**2.19** Two forces  $\mathbf{P}$  and  $\mathbf{Q}$  are applied to the lid of a storage bin as shown. Knowing that  $P = 48$  N and  $Q = 60$  N, determine by trigonometry the magnitude and direction of the resultant of the two forces.

**2.20** Two forces  $\mathbf{P}$  and  $\mathbf{Q}$  are applied to the lid of a storage bin as shown. Knowing that  $P = 60$  N and  $Q = 48$  N, determine by trigonometry the magnitude and direction of the resultant of the two forces.

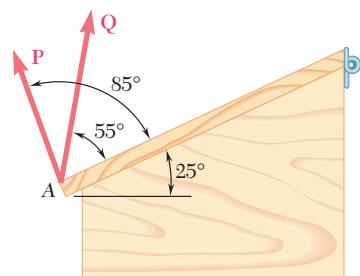


Fig. P2.19 and P2.20

## 2.7 RECTANGULAR COMPONENTS OF A FORCE. UNIT VECTORS†

In many problems it will be found desirable to resolve a force into two components which are perpendicular to each other. In Fig. 2.18, the force  $\mathbf{F}$  has been resolved into a component  $\mathbf{F}_x$  along the  $x$  axis and a component  $\mathbf{F}_y$  along the  $y$  axis. The parallelogram drawn to obtain the two components is a *rectangle*, and  $\mathbf{F}_x$  and  $\mathbf{F}_y$  are called *rectangular components*.

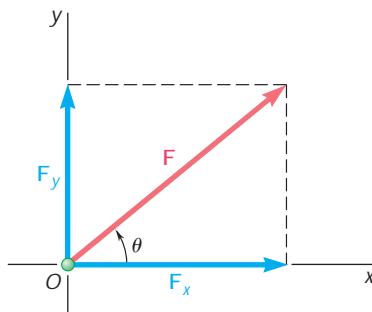


Fig. 2.18

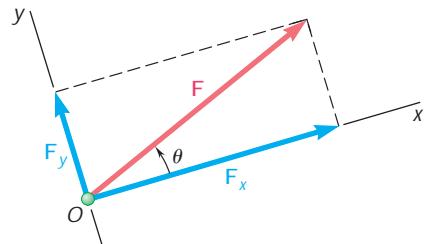


Fig. 2.19

The  $x$  and  $y$  axes are usually chosen horizontal and vertical, respectively, as in Fig. 2.18; they may, however, be chosen in any two perpendicular directions, as shown in Fig. 2.19. In determining the rectangular components of a force, the student should think of the construction lines shown in Figs. 2.18 and 2.19 as being *parallel* to the  $x$  and  $y$  axes, rather than *perpendicular* to these axes. This practice will help avoid mistakes in determining *oblique* components as in Sec. 2.6.

†The properties established in Secs. 2.7 and 2.8 may be readily extended to the rectangular components of any vector quantity.

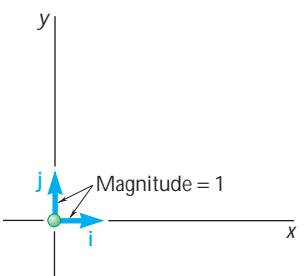


Fig. 2.20

Two vectors of unit magnitude, directed respectively along the positive  $x$  and  $y$  axes, will be introduced at this point. These vectors are called *unit vectors* and are denoted by  $\mathbf{i}$  and  $\mathbf{j}$ , respectively (Fig. 2.20). Recalling the definition of the product of a scalar and a vector given in Sec. 2.4, we note that the rectangular components  $\mathbf{F}_x$  and  $\mathbf{F}_y$  of a force  $\mathbf{F}$  may be obtained by multiplying respectively the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  by appropriate scalars (Fig. 2.21). We write

$$\mathbf{F}_x = F_x \mathbf{i} \quad \mathbf{F}_y = F_y \mathbf{j} \quad (2.6)$$

and

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} \quad (2.7)$$

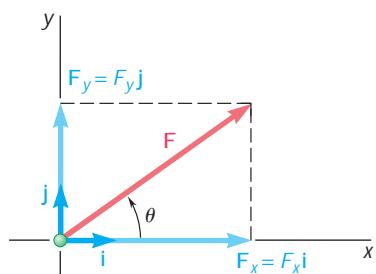


Fig. 2.21

While the scalars  $F_x$  and  $F_y$  may be positive or negative, depending upon the sense of  $\mathbf{F}_x$  and of  $\mathbf{F}_y$ , their absolute values are respectively equal to the magnitudes of the component forces  $\mathbf{F}_x$  and  $\mathbf{F}_y$ . The scalars  $F_x$  and  $F_y$  are called the *scalar components* of the force  $\mathbf{F}$ , while the actual component forces  $\mathbf{F}_x$  and  $\mathbf{F}_y$  should be referred to as the *vector components* of  $\mathbf{F}$ . However, when there exists no possibility of confusion, the vector as well as the scalar components of  $\mathbf{F}$  may be referred to simply as the *components* of  $\mathbf{F}$ . We note that the scalar component  $F_x$  is positive when the vector component  $\mathbf{F}_x$  has the same sense as the unit vector  $\mathbf{i}$  (i.e., the same sense as the positive  $x$  axis) and is negative when  $\mathbf{F}_x$  has the opposite sense. A similar conclusion may be drawn regarding the sign of the scalar component  $F_y$ .

Denoting by  $F$  the magnitude of the force  $\mathbf{F}$  and by  $\alpha$  the angle between  $\mathbf{F}$  and the  $x$  axis, measured counterclockwise from the positive  $x$  axis (Fig. 2.21), we may express the scalar components of  $\mathbf{F}$  as follows:

$$F_x = F \cos \alpha \quad F_y = F \sin \alpha \quad (2.8)$$

We note that the relations obtained hold for any value of the angle  $\alpha$  from  $0^\circ$  to  $360^\circ$  and that they define the signs as well as the absolute values of the scalar components  $F_x$  and  $F_y$ .

**EXAMPLE 1.** A force of 800 N is exerted on a bolt  $A$  as shown in Fig. 2.22a. Determine the horizontal and vertical components of the force.

In order to obtain the correct sign for the scalar components  $F_x$  and  $F_y$ , the value  $180^\circ - 35^\circ = 145^\circ$  should be substituted for  $\alpha$  in Eqs. (2.8). However, it will be found more practical to determine by inspection the signs of  $F_x$  and  $F_y$  (Fig. 2.22b) and to use the trigonometric functions of the angle  $\alpha = 35^\circ$ . We write, therefore,

$$F_x = -F \cos \alpha = -(800 \text{ N}) \cos 35^\circ = -655 \text{ N}$$

$$F_y = +F \sin \alpha = +(800 \text{ N}) \sin 35^\circ = +459 \text{ N}$$

The vector components of  $\mathbf{F}$  are thus

$$\mathbf{F}_x = -(655 \text{ N})\mathbf{i} \quad \mathbf{F}_y = +(459 \text{ N})\mathbf{j}$$

and we may write  $\mathbf{F}$  in the form

$$\mathbf{F} = -(655 \text{ N})\mathbf{i} + (459 \text{ N})\mathbf{j} \blacksquare$$

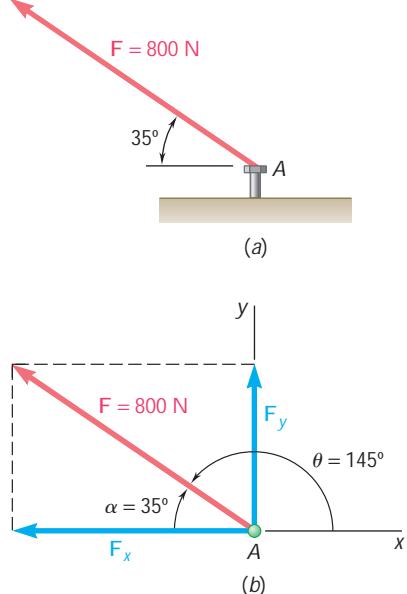


Fig. 2.22

**EXAMPLE 2.** A man pulls with a force of 300 N on a rope attached to a building, as shown in Fig. 2.23a. What are the horizontal and vertical components of the force exerted by the rope at point A?

It is seen from Fig. 2.23b that

$$F_x = +(300 \text{ N}) \cos \alpha \quad F_y = -(300 \text{ N}) \sin \alpha$$

Observing that  $AB = 10 \text{ m}$ , we find from Fig. 2.23a

$$\cos \alpha = \frac{8 \text{ m}}{10 \text{ m}} = \frac{8 \text{ m}}{10 \text{ m}} = \frac{4}{5} \quad \sin \alpha = \frac{6 \text{ m}}{10 \text{ m}} = \frac{6 \text{ m}}{10 \text{ m}} = \frac{3}{5}$$

We thus obtain

$$F_x = +(300 \text{ N}) \frac{4}{5} = +240 \text{ N} \quad F_y = -(300 \text{ N}) \frac{3}{5} = -180 \text{ N}$$

and write

$$\mathbf{F} = (240 \text{ N})\mathbf{i} - (180 \text{ N})\mathbf{j} \blacksquare$$

When a force  $\mathbf{F}$  is defined by its rectangular components  $F_x$  and  $F_y$  (see Fig. 2.21), the angle  $\alpha$  defining its direction can be obtained by writing

$$\tan \alpha = \frac{F_y}{F_x} \quad (2.9)$$

The magnitude  $F$  of the force can be obtained by applying the Pythagorean theorem and writing

$$F = \sqrt{F_x^2 + F_y^2} \quad (2.10)$$

or by solving for  $F$  one of the Eqs. (2.8).

**EXAMPLE 3.** A force  $\mathbf{F} = (700 \text{ lb})\mathbf{i} + (1500 \text{ lb})\mathbf{j}$  is applied to a bolt A. Determine the magnitude of the force and the angle  $\alpha$  it forms with the horizontal.

First we draw a diagram showing the two rectangular components of the force and the angle  $\alpha$  (Fig. 2.24). From Eq. (2.9), we write

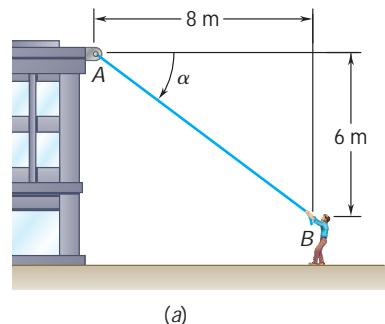
$$\tan \alpha = \frac{F_y}{F_x} = \frac{1500 \text{ lb}}{700 \text{ lb}}$$

Using a calculator,<sup>†</sup> we enter 1500 lb and divide by 700 lb; computing the arc tangent of the quotient, we obtain  $\alpha = 65.0^\circ$ . Solving the second of Eqs. (2.8) for  $F$ , we have

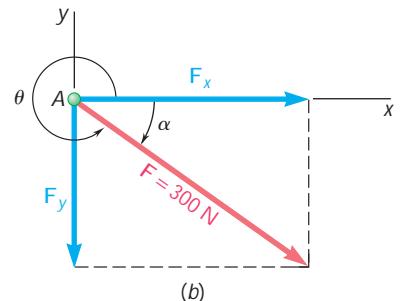
$$F = \frac{F_y}{\sin \alpha} = \frac{1500 \text{ lb}}{\sin 65.0^\circ} = 1655 \text{ lb}$$

The last calculation is facilitated if the value of  $F_y$  is stored when originally entered; it may then be recalled to be divided by  $\sin \alpha$ .  $\blacksquare$

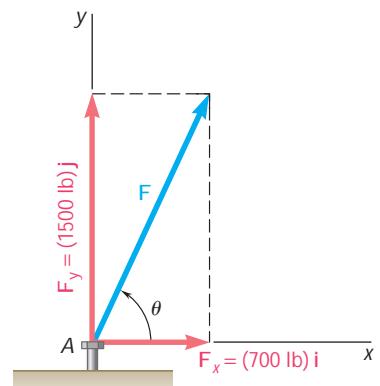
## 2.7 Rectangular Components of a Force. Unit Vectors



(a)



(b)

**Fig. 2.23****Fig. 2.24**

<sup>†</sup>It is assumed that the calculator used has keys for the computation of trigonometric and inverse trigonometric functions. Some calculators also have keys for the direct conversion of rectangular coordinates into polar coordinates, and vice versa. Such calculators eliminate the need for the computation of trigonometric functions in Examples 1, 2, and 3 and in problems of the same type.

## 2.8 ADDITION OF FORCES BY SUMMING X AND Y COMPONENTS

It was seen in Sec. 2.2 that forces should be added according to the parallelogram law. From this law, two other methods, more readily applicable to the *graphical* solution of problems, were derived in Secs. 2.4 and 2.5: the triangle rule for the addition of two forces and the polygon rule for the addition of three or more forces. It was also seen that the force triangle used to define the resultant of two forces could be used to obtain a *trigonometric* solution.

When three or more forces are to be added, no practical trigonometric solution can be obtained from the force polygon which defines the resultant of the forces. In this case, an *analytic* solution of the problem can be obtained by resolving each force into two rectangular components. Consider, for instance, three forces  $\mathbf{P}$ ,  $\mathbf{Q}$ , and  $\mathbf{S}$  acting on a particle A (Fig. 2.25a). Their resultant  $\mathbf{R}$  is defined by the relation

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{S} \quad (2.11)$$

Resolving each force into its rectangular components, we write

$$\begin{aligned} R_x \mathbf{i} + R_y \mathbf{j} &= P_x \mathbf{i} + P_y \mathbf{j} + Q_x \mathbf{i} + Q_y \mathbf{j} + S_x \mathbf{i} + S_y \mathbf{j} \\ &= (P_x + Q_x + S_x) \mathbf{i} + (P_y + Q_y + S_y) \mathbf{j} \end{aligned}$$

from which it follows that

$$R_x = P_x + Q_x + S_x \quad R_y = P_y + Q_y + S_y \quad (2.12)$$

or, for short,

$$R_x = \sum F_x \quad R_y = \sum F_y \quad (2.13)$$

We thus conclude that the *scalar components*  $R_x$  and  $R_y$  of the resultant  $\mathbf{R}$  of several forces acting on a particle are obtained by adding algebraically the corresponding scalar components of the given forces.†

In practice, the determination of the resultant  $\mathbf{R}$  is carried out in three steps as illustrated in Fig. 2.25. First the given forces shown in Fig. 2.25a are resolved into their  $x$  and  $y$  components (Fig. 2.25b). Adding these components, we obtain the  $x$  and  $y$  components of  $\mathbf{R}$  (Fig. 2.25c). Finally, the resultant  $\mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j}$  is determined by applying the parallelogram law (Fig. 2.25d). The procedure just described will be carried out most efficiently if the computations are arranged in a table. While it is the only practical analytic method for adding three or more forces, it is also often preferred to the trigonometric solution in the case of the addition of two forces.

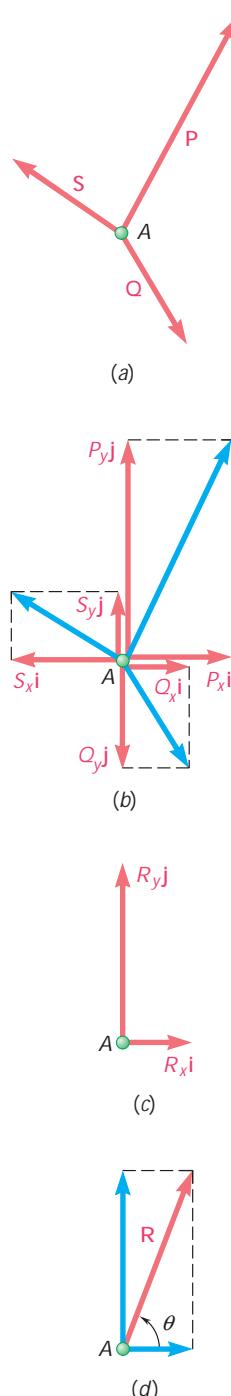
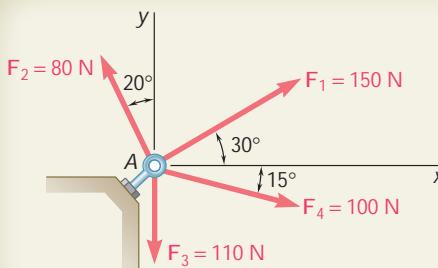


Fig. 2.25

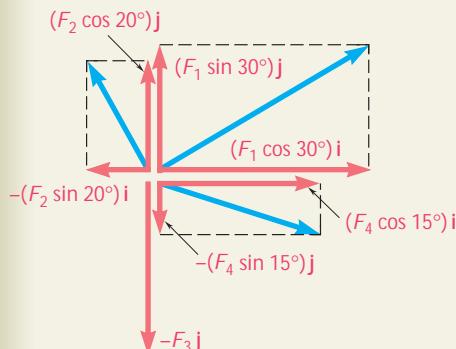
†Clearly, this result also applies to the addition of other vector quantities, such as velocities, accelerations, or momenta.



### SAMPLE PROBLEM 2.3

Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt.

### SOLUTION



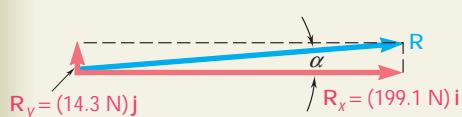
The  $x$  and  $y$  components of each force are determined by trigonometry as shown and are entered in the table below. According to the convention adopted in Sec. 2.7, the scalar number representing a force component is positive if the force component has the same sense as the corresponding coordinate axis. Thus,  $x$  components acting to the right and  $y$  components acting upward are represented by positive numbers.

Force	Magnitude, N	$x$ Component, N	$y$ Component, N
$\mathbf{F}_1$	150	+129.9	+75.0
$\mathbf{F}_2$	80	-27.4	+75.2
$\mathbf{F}_3$	110	0	-110.0
$\mathbf{F}_4$	100	+96.6	-25.9
		$R_x = +199.1$	$R_y = +14.3$

Thus, the resultant  $\mathbf{R}$  of the four forces is

$$\mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j} \quad \mathbf{R} = (199.1 \text{ N}) \mathbf{i} + (14.3 \text{ N}) \mathbf{j}$$

The magnitude and direction of the resultant may now be determined. From the triangle shown, we have



$$\tan \alpha = \frac{R_y}{R_x} = \frac{14.3 \text{ N}}{199.1 \text{ N}} \quad \alpha = 4.1^\circ$$

$$R = \frac{14.3 \text{ N}}{\sin \alpha} = 199.6 \text{ N} \quad \mathbf{R} = 199.6 \text{ N} \text{ at } 4.1^\circ$$

With a calculator, the last computation may be facilitated if the value of  $R_y$  is stored when originally entered; it may then be recalled to be divided by  $\sin \alpha$ . (Also see the footnote on p. 29.)

# SOLVING PROBLEMS ON YOUR OWN

You saw in the preceding lesson that the resultant of two forces may be determined either graphically or from the trigonometry of an oblique triangle.

**A. When three or more forces are involved**, the determination of their resultant  $\mathbf{R}$  is best carried out by first resolving each force into *rectangular components*. Two cases may be encountered, depending upon the way in which each of the given forces is defined:

**Case 1. The force  $\mathbf{F}$  is defined by its magnitude  $F$  and the angle  $\alpha$  it forms with the  $x$  axis.** The  $x$  and  $y$  components of the force can be obtained by multiplying  $F$  by  $\cos \alpha$  and  $\sin \alpha$ , respectively [Example 1].

**Case 2. The force  $\mathbf{F}$  is defined by its magnitude  $F$  and the coordinates of two points  $A$  and  $B$  on its line of action** (Fig. 2.23). The angle  $\alpha$  that  $\mathbf{F}$  forms with the  $x$  axis may first be determined by trigonometry. However, the components of  $\mathbf{F}$  may also be obtained directly from proportions among the various dimensions involved, without actually determining  $\alpha$  [Example 2].

**B. Rectangular components of the resultant.** The components  $R_x$  and  $R_y$  of the resultant can be obtained by adding algebraically the corresponding components of the given forces [Sample Prob. 2.3].

You can express the resultant in *vectorial form* using the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ , which are directed along the  $x$  and  $y$  axes, respectively:

$$\mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j}$$

Alternatively, you can determine the *magnitude and direction* of the resultant by solving the right triangle of sides  $R_x$  and  $R_y$  for  $R$  and for the angle that  $\mathbf{R}$  forms with the  $x$  axis.

# PROBLEMS

- 2.21 and 2.22** Determine the  $x$  and  $y$  components of each of the forces shown.

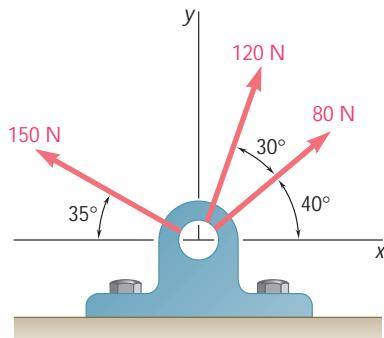


Fig. P2.21

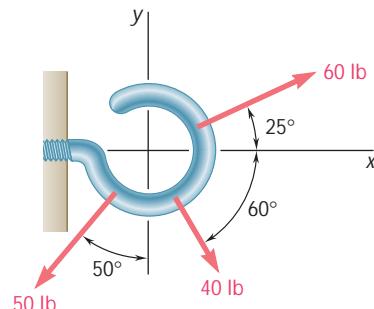


Fig. P2.22

- 2.23 and 2.24** Determine the  $x$  and  $y$  components of each of the forces shown.

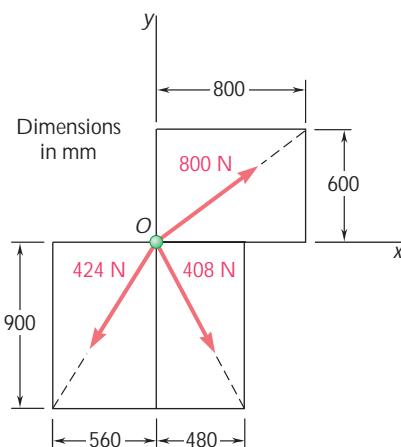


Fig. P2.23

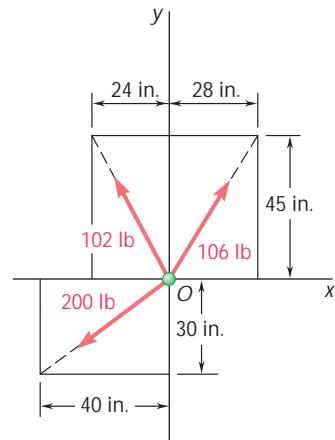


Fig. P2.24

- 2.25** The hydraulic cylinder  $BD$  exerts on member  $ABC$  a force  $\mathbf{P}$  directed along line  $BD$ . Knowing that  $\mathbf{P}$  must have a 750-N component perpendicular to member  $ABC$ , determine (a) the magnitude of the force  $\mathbf{P}$ , (b) its component parallel to  $ABC$ .

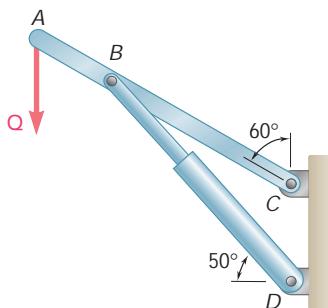


Fig. P2.25

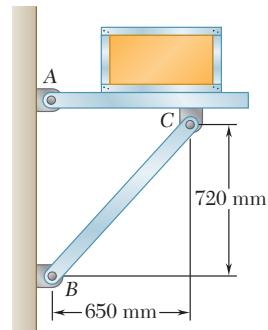


Fig. P2.27

- 2.26** Cable  $AC$  exerts on beam  $AB$  a force  $\mathbf{P}$  directed along line  $AC$ . Knowing that  $\mathbf{P}$  must have a 350-lb vertical component, determine (a) the magnitude of the force  $\mathbf{P}$ , (b) its horizontal component.

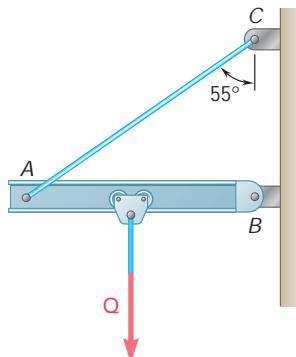


Fig. P2.26

- 2.27** Member  $BC$  exerts on member  $AC$  a force  $\mathbf{P}$  directed along line  $BC$ . Knowing that  $\mathbf{P}$  must have a 325-N horizontal component, determine (a) the magnitude of the force  $\mathbf{P}$ , (b) its vertical component.

- 2.28** Member  $BD$  exerts on member  $ABC$  a force  $\mathbf{P}$  directed along line  $BD$ . Knowing that  $\mathbf{P}$  must have a 240-lb vertical component, determine (a) the magnitude of the force  $\mathbf{P}$ , (b) its horizontal component.

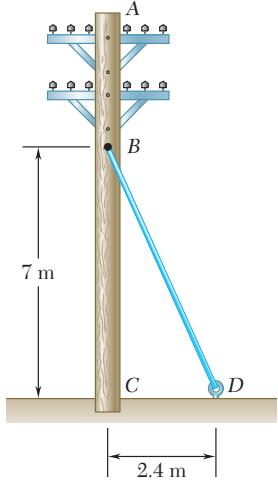


Fig. P2.29

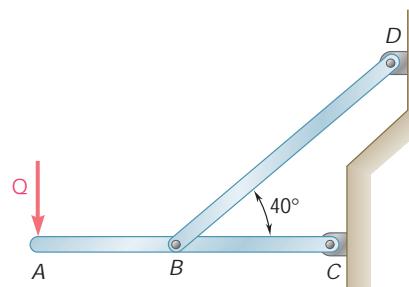


Fig. P2.28

- 2.29** The guy wire  $BD$  exerts on the telephone pole  $AC$  a force  $\mathbf{P}$  directed along  $BD$ . Knowing that  $\mathbf{P}$  must have a 720-N component perpendicular to the pole  $AC$ , determine (a) the magnitude of the force  $\mathbf{P}$ , (b) its component along line  $AC$ .

- 2.30** The hydraulic cylinder  $BC$  exerts on member  $AB$  a force  $\mathbf{P}$  directed along line  $BC$ . Knowing that  $\mathbf{P}$  must have a 600-N component perpendicular to member  $AB$ , determine (a) the magnitude of the force  $\mathbf{P}$ , (b) its component along line  $AB$ .

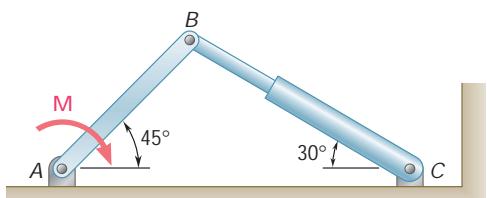


Fig. P2.30

- 2.31** Determine the resultant of the three forces of Prob. 2.23.

- 2.32** Determine the resultant of the three forces of Prob. 2.21.

- 2.33** Determine the resultant of the three forces of Prob. 2.22.

- 2.34** Determine the resultant of the three forces of Prob. 2.24.

- 2.35** Knowing that  $\alpha = 35^\circ$ , determine the resultant of the three forces shown.

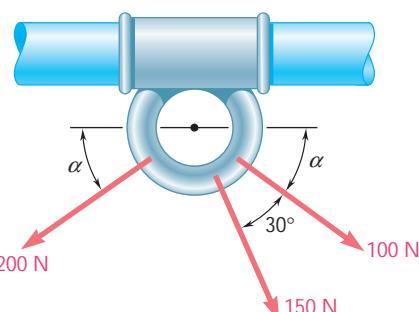


Fig. P2.35

- 2.36** Knowing that the tension in rope  $AC$  is 365 N, determine the resultant of the three forces exerted at point  $C$  of post  $BC$ .

- 2.37** Knowing that  $\alpha = 40^\circ$ , determine the resultant of the three forces shown.

- 2.38** Knowing that  $\alpha = 75^\circ$ , determine the resultant of the three forces shown.

- 2.39** For the collar of Prob. 2.35, determine (a) the required value of  $\alpha$  if the resultant of the three forces shown is to be vertical, (b) the corresponding magnitude of the resultant.

- 2.40** For the post of Prob. 2.36, determine (a) the required tension in rope  $AC$  if the resultant of the three forces exerted at point  $C$  is to be horizontal, (b) the corresponding magnitude of the resultant.

- 2.41** A hoist trolley is subjected to the three forces shown. Knowing that  $\alpha = 40^\circ$ , determine (a) the required magnitude of the force  $\mathbf{P}$  if the resultant of the three forces is to be vertical, (b) the corresponding magnitude of the resultant.

- 2.42** A hoist trolley is subjected to the three forces shown. Knowing that  $P = 250$  lb, determine (a) the required value of  $\alpha$  if the resultant of the three forces is to be vertical, (b) the corresponding magnitude of the resultant.

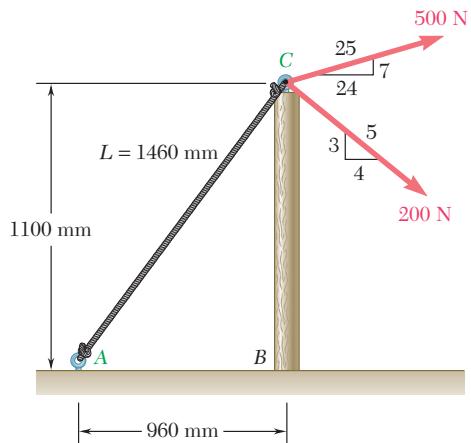


Fig. P2.36

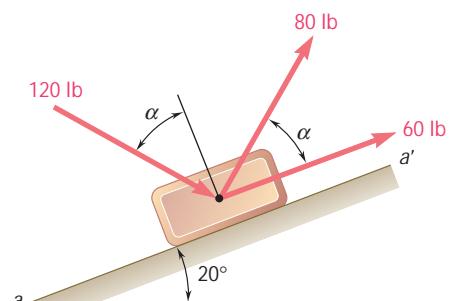


Fig. P2.37 and P2.38

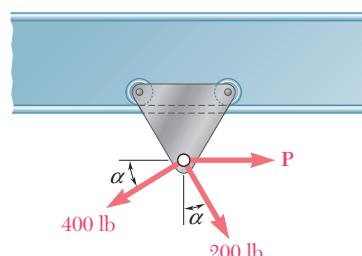


Fig. P2.41 and P2.42

## 2.9 EQUILIBRIUM OF A PARTICLE

In the preceding sections, we discussed the methods for determining the resultant of several forces acting on a particle. Although it has not occurred in any of the problems considered so far, it is quite possible for the resultant to be zero. In such a case, the net effect of the given forces is zero, and the particle is said to be in equilibrium. We thus have the following definition: *When the resultant of all the forces acting on a particle is zero, the particle is in equilibrium.*

A particle which is acted upon by two forces will be in equilibrium if the two forces have the same magnitude and the same line of action but opposite sense. The resultant of the two forces is then zero. Such a case is shown in Fig. 2.26.

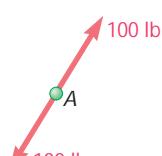


Fig. 2.26

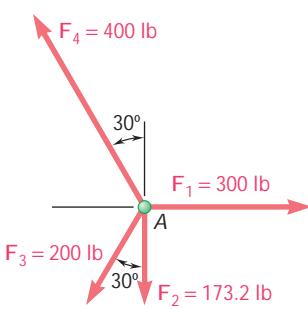


Fig. 2.27

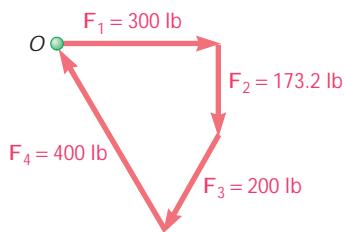


Fig. 2.28

Another case of equilibrium of a particle is represented in Fig. 2.27, where four forces are shown acting on A. In Fig. 2.28, the resultant of the given forces is determined by the polygon rule. Starting from point  $O$  with  $\mathbf{F}_1$  and arranging the forces in tip-to-tail fashion, we find that the tip of  $\mathbf{F}_4$  coincides with the starting point  $O$ . Thus the resultant  $\mathbf{R}$  of the given system of forces is zero, and the particle is in equilibrium.

The closed polygon drawn in Fig. 2.28 provides a *graphical* expression of the equilibrium of A. To express *algebraically* the conditions for the equilibrium of a particle, we write

$$\mathbf{R} = \sum \mathbf{F} = 0 \quad (2.14)$$

Resolving each force  $\mathbf{F}$  into rectangular components, we have

$$\Sigma(F_x \mathbf{i} + F_y \mathbf{j}) = 0 \quad \text{or} \quad (\Sigma F_x) \mathbf{i} + (\Sigma F_y) \mathbf{j} = 0$$

We conclude that the necessary and sufficient conditions for the equilibrium of a particle are

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad (2.15)$$

Returning to the particle shown in Fig. 2.27, we check that the equilibrium conditions are satisfied. We write

$$\begin{aligned} \Sigma F_x &= 300 \text{ lb} - (200 \text{ lb}) \sin 30^\circ - (400 \text{ lb}) \sin 30^\circ \\ &= 300 \text{ lb} - 100 \text{ lb} - 200 \text{ lb} = 0 \\ \Sigma F_y &= -173.2 \text{ lb} - (200 \text{ lb}) \cos 30^\circ + (400 \text{ lb}) \cos 30^\circ \\ &= -173.2 \text{ lb} - 173.2 \text{ lb} + 346.4 \text{ lb} = 0 \end{aligned}$$

## 2.10 NEWTON'S FIRST LAW OF MOTION

In the latter part of the seventeenth century, Sir Isaac Newton formulated three fundamental laws upon which the science of mechanics is based. The first of these laws can be stated as follows:

*If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).*

From this law and from the definition of equilibrium given in Sec. 2.9, it is seen that a particle in equilibrium either is at rest or is moving in a straight line with constant speed. In the following section, various problems concerning the equilibrium of a particle will be considered.

## 2.11 PROBLEMS INVOLVING THE EQUILIBRIUM OF A PARTICLE. FREE-BODY DIAGRAMS

In practice, a problem in engineering mechanics is derived from an actual physical situation. A sketch showing the physical conditions of the problem is known as a *space diagram*.

The methods of analysis discussed in the preceding sections apply to a system of forces acting on a particle. A large number of problems involving actual structures, however, can be reduced to problems concerning the equilibrium of a particle. This is done by

choosing a significant particle and drawing a separate diagram showing this particle and all the forces acting on it. Such a diagram is called a *free-body diagram*.

As an example, consider the 75-kg crate shown in the space diagram of Fig. 2.29a. This crate was lying between two buildings, and it is now being lifted onto a truck, which will remove it. The crate is supported by a vertical cable, which is joined at A to two ropes which pass over pulleys attached to the buildings at B and C. It is desired to determine the tension in each of the ropes AB and AC.

In order to solve this problem, a free-body diagram showing a particle in equilibrium must be drawn. Since we are interested in the rope tensions, the free-body diagram should include at least one of these tensions or, if possible, both tensions. Point A is seen to be a good free body for this problem. The free-body diagram of point A is shown in Fig. 2.29b. It shows point A and the forces exerted on A by the vertical cable and the two ropes. The force exerted by the cable is directed downward, and its magnitude is equal to the weight  $W$  of the crate. Recalling Eq. (1.4), we write

$$W = mg = (75 \text{ kg})(9.81 \text{ m/s}^2) = 736 \text{ N}$$

and indicate this value in the free-body diagram. The forces exerted by the two ropes are not known. Since they are respectively equal in magnitude to the tensions in rope AB and rope AC, we denote them by  $T_{AB}$  and  $T_{AC}$  and draw them away from A in the directions shown in the space diagram. No other detail is included in the free-body diagram.

Since point A is in equilibrium, the three forces acting on it must form a closed triangle when drawn in tip-to-tail fashion. This *force triangle* has been drawn in Fig. 2.29c. The values  $T_{AB}$  and  $T_{AC}$  of the tension in the ropes may be found graphically if the triangle is drawn to scale, or they may be found by trigonometry. If the latter method of solution is chosen, we use the law of sines and write

$$\frac{T_{AB}}{\sin 60^\circ} = \frac{T_{AC}}{\sin 40^\circ} = \frac{736 \text{ N}}{\sin 80^\circ}$$

$$T_{AB} = 647 \text{ N} \quad T_{AC} = 480 \text{ N}$$

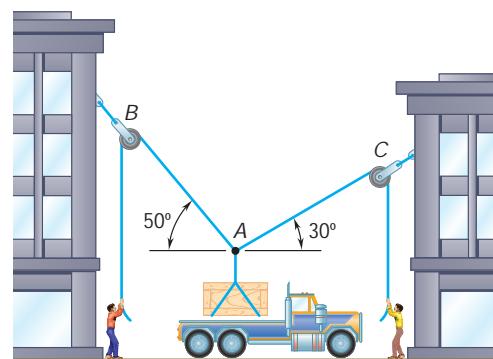
When a particle is in *equilibrium under three forces*, the problem can be solved by drawing a force triangle. When a particle is in *equilibrium under more than three forces*, the problem can be solved graphically by drawing a force polygon. If an analytic solution is desired, the *equations of equilibrium* given in Sec. 2.9 should be solved:

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad (2.15)$$

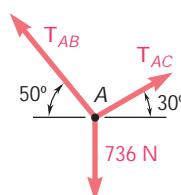
These equations can be solved for no more than *two unknowns*; similarly, the force triangle used in the case of equilibrium under three forces can be solved for two unknowns.

The more common types of problems are those in which the two unknowns represent (1) the two components (or the magnitude and direction) of a single force, (2) the magnitudes of two forces, each of known direction. Problems involving the determination of the maximum or minimum value of the magnitude of a force are also encountered (see Probs. 2.57 through 2.62).

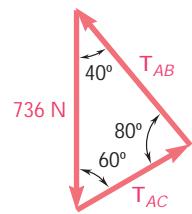
## 2.11 Problems Involving the Equilibrium of a Particle. Free-Body Diagrams



(a) Space diagram

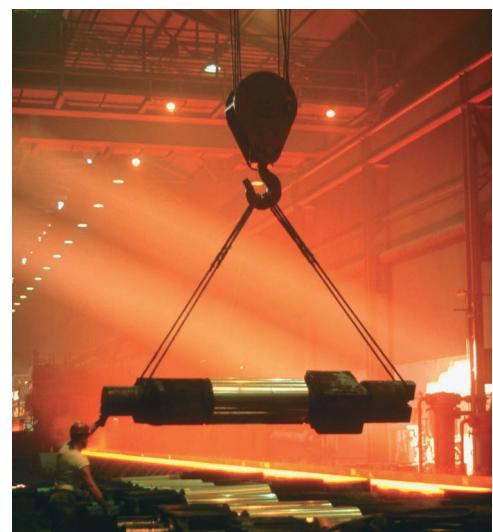


(b) Free-body diagram

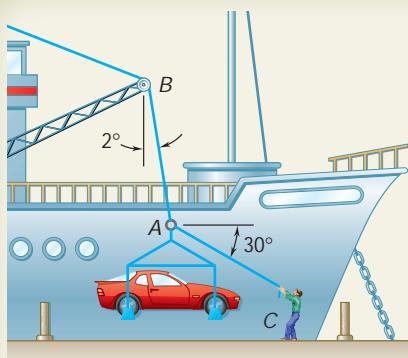


(c) Force triangle

Fig. 2.29



**Photo 2.1** As illustrated in the above example, it is possible to determine the tensions in the cables supporting the shaft shown by treating the hook as a particle and then applying the equations of equilibrium to the forces acting on the hook.



## SAMPLE PROBLEM 2.4

In a ship-unloading operation, a 3500-lb automobile is supported by a cable. A rope is tied to the cable at A and pulled in order to center the automobile over its intended position. The angle between the cable and the vertical is  $2^\circ$ , while the angle between the rope and the horizontal is  $30^\circ$ . What is the tension in the rope?

## SOLUTION

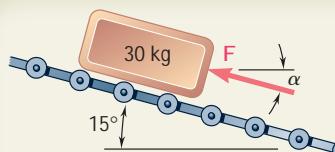
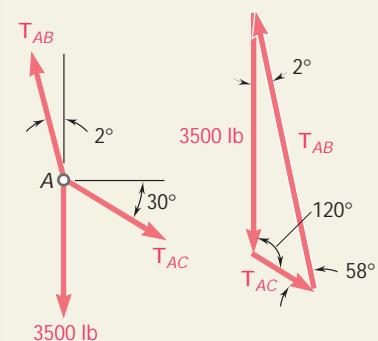
**Free-Body Diagram.** Point A is chosen as a free body, and the complete free-body diagram is drawn.  $T_{AB}$  is the tension in the cable AB, and  $T_{AC}$  is the tension in the rope.

**Equilibrium Condition.** Since only three forces act on the free body, we draw a force triangle to express that it is in equilibrium. Using the law of sines, we write

$$\frac{T_{AB}}{\sin 120^\circ} = \frac{T_{AC}}{\sin 2^\circ} = \frac{3500 \text{ lb}}{\sin 58^\circ}$$

With a calculator, we first compute and store the value of the last quotient. Multiplying this value successively by  $\sin 120^\circ$  and  $\sin 2^\circ$ , we obtain

$$T_{AB} = 3570 \text{ lb} \quad T_{AC} = 144 \text{ lb} \quad \blacktriangleleft$$



## SAMPLE PROBLEM 2.5

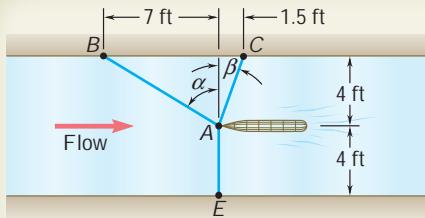
Determine the magnitude and direction of the smallest force  $\mathbf{F}$  which will maintain the package shown in equilibrium. Note that the force exerted by the rollers on the package is perpendicular to the incline.

## SOLUTION

**Free-Body Diagram.** We choose the package as a free body, assuming that it can be treated as a particle. We draw the corresponding free-body diagram.

**Equilibrium Condition.** Since only three forces act on the free body, we draw a force triangle to express that it is in equilibrium. Line 1-1' represents the known direction of  $\mathbf{P}$ . In order to obtain the minimum value of the force  $\mathbf{F}$ , we choose the direction of  $\mathbf{F}$  perpendicular to that of  $\mathbf{P}$ . From the geometry of the triangle obtained, we find

$$F = (294 \text{ N}) \sin 15^\circ = 76.1 \text{ N} \quad a = 15^\circ \quad \mathbf{F} = 76.1 \text{ N} \text{ b } 15^\circ \quad \blacktriangleleft$$



## SAMPLE PROBLEM 2.6

As part of the design of a new sailboat, it is desired to determine the drag force which may be expected at a given speed. To do so, a model of the proposed hull is placed in a test channel and three cables are used to keep its bow on the centerline of the channel. Dynamometer readings indicate that for a given speed, the tension is 40 lb in cable AB and 60 lb in cable AE. Determine the drag force exerted on the hull and the tension in cable AC.

## SOLUTION

**Determination of the Angles.** First, the angles  $\alpha$  and  $\beta$  defining the direction of cables AB and AC are determined. We write

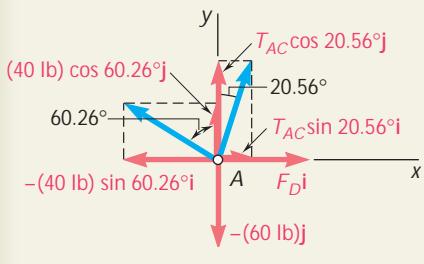
$$\begin{aligned}\tan \alpha &= \frac{7 \text{ ft}}{4 \text{ ft}} = 1.75 & \tan \beta &= \frac{1.5 \text{ ft}}{4 \text{ ft}} = 0.375 \\ \alpha &= 60.26^\circ & \beta &= 20.56^\circ\end{aligned}$$

**Free-Body Diagram.** Choosing the hull as a free body, we draw the free-body diagram shown. It includes the forces exerted by the three cables on the hull, as well as the drag force  $\mathbf{F}_D$  exerted by the flow.

**Equilibrium Condition.** We express that the hull is in equilibrium by writing that the resultant of all forces is zero:

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AE} + \mathbf{F}_D = 0 \quad (1)$$

Since more than three forces are involved, we resolve the forces into  $x$  and  $y$  components:



$$\begin{aligned}\mathbf{T}_{AB} &= -(40 \text{ lb}) \sin 60.26^\circ \mathbf{i} + (40 \text{ lb}) \cos 60.26^\circ \mathbf{j} \\ &= -(34.73 \text{ lb}) \mathbf{i} + (19.84 \text{ lb}) \mathbf{j} \\ \mathbf{T}_{AC} &= T_{AC} \sin 20.56^\circ \mathbf{i} + T_{AC} \cos 20.56^\circ \mathbf{j} \\ &= 0.3512 T_{AC} \mathbf{i} + 0.9363 T_{AC} \mathbf{j} \\ \mathbf{T}_{AE} &= -(60 \text{ lb}) \mathbf{j} \\ \mathbf{F}_D &= F_D \mathbf{i}\end{aligned}$$

Substituting the expressions obtained into Eq. (1) and factoring the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ , we have

$$(-34.73 \text{ lb} + 0.3512 T_{AC} + F_D) \mathbf{i} + (19.84 \text{ lb} + 0.9363 T_{AC} - 60 \text{ lb}) \mathbf{j} = 0$$

This equation will be satisfied if, and only if, the coefficients of  $\mathbf{i}$  and  $\mathbf{j}$  are equal to zero. We thus obtain the following two equilibrium equations, which express, respectively, that the sum of the  $x$  components and the sum of the  $y$  components of the given forces must be zero.

$$(\sum F_x = 0: \quad -34.73 \text{ lb} + 0.3512 T_{AC} + F_D = 0 \quad (2)$$

$$(\sum F_y = 0: \quad 19.84 \text{ lb} + 0.9363 T_{AC} - 60 \text{ lb} = 0 \quad (3)$$

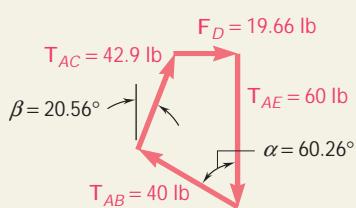
From Eq. (3) we find

$$T_{AC} = +42.9 \text{ lb} \quad \blacksquare$$

and, substituting this value into Eq. (2),

$$F_D = +19.66 \text{ lb} \quad \blacksquare$$

In drawing the free-body diagram, we assumed a sense for each unknown force. A positive sign in the answer indicates that the assumed sense is correct. The complete force polygon may be drawn to check the results.



# SOLVING PROBLEMS ON YOUR OWN

When a particle is in *equilibrium*, the resultant of the forces acting on the particle must be zero. Expressing this fact in the case of a particle under *coplanar forces* will provide you with two relations among these forces. As you saw in the preceding sample problems, these relations may be used to determine two unknowns—such as the magnitude and direction of one force or the magnitudes of two forces.

**Drawing a free-body diagram is the first step** in the solution of a problem involving the equilibrium of a particle. This diagram shows the particle and all the forces acting on it. Indicate in your free-body diagram the magnitudes of known forces, as well as any angle or dimensions that define the direction of a force. Any unknown magnitude or angle should be denoted by an appropriate symbol. Nothing else should be included in the free-body diagram.

*Drawing a clear and accurate free-body diagram is a must in the solution of any equilibrium problem.* Skipping this step might save you pencil and paper, but is very likely to lead you to a wrong solution.

**Case 1. If only three forces are involved** in the free-body diagram, the rest of the solution is best carried out by drawing these forces in tip-to-tail fashion to form a *force triangle*. This triangle can be solved graphically or by trigonometry for no more than two unknowns [Sample Probs. 2.4 and 2.5].

**Case 2. If more than three forces are involved**, it is to your advantage to use an *analytic solution*. You select  $x$  and  $y$  axes and resolve each of the forces shown in the free-body diagram into  $x$  and  $y$  components. Expressing that the sum of the  $x$  components and the sum of the  $y$  components of all the forces are both zero, you will obtain two equations which you can solve for no more than two unknowns [Sample Prob. 2.6].

It is strongly recommended that when using an analytic solution the equations of equilibrium be written in the same form as Eqs. (2) and (3) of Sample Prob. 2.6. The practice adopted by some students of initially placing the unknowns on the left side of the equation and the known quantities on the right side may lead to confusion in assigning the appropriate sign to each term.

We have noted that regardless of the method used to solve a two-dimensional equilibrium problem we can determine at most two unknowns. If a two-dimensional problem involves more than two unknowns, one or more additional relations must be obtained from the information contained in the statement of the problem.

# PROBLEMS

## FREE BODY PRACTICE PROBLEMS

- 2.F1** Two cables are tied together at *C* and loaded as shown. Draw the free-body diagram needed to determine the tension in *AC* and *BC*.
- 2.F2** A chairlift has been stopped in the position shown. Knowing that each chair weighs 250 N and that the skier in chair *E* weighs 765 N, draw the free-body diagrams needed to determine the weight of the skier in chair *F*.

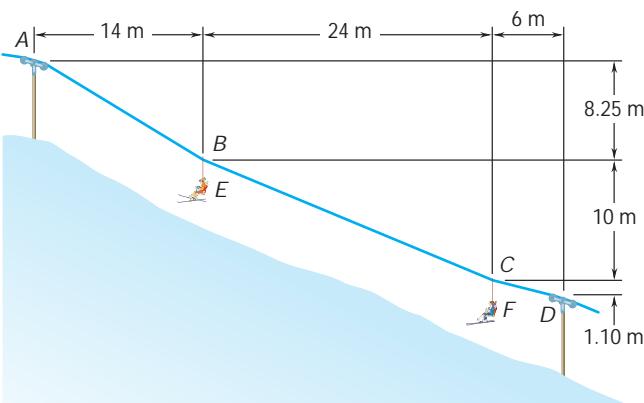


Fig. P2.F2

- 2.F3** Two cables are tied together at *A* and loaded as shown. Draw the free-body diagram needed to determine the tension in each cable.
- 2.F4** The 60-lb collar *A* can slide on a frictionless vertical rod and is connected as shown to a 65-lb counterweight *C*. Draw the free-body diagram needed to determine the value of *h* for which the system is in equilibrium.

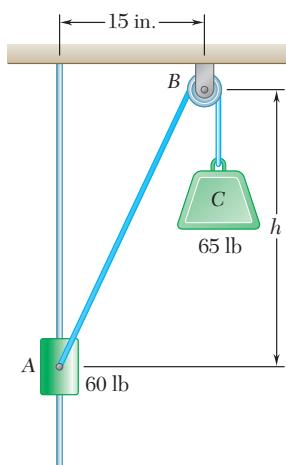


Fig. P2.F4

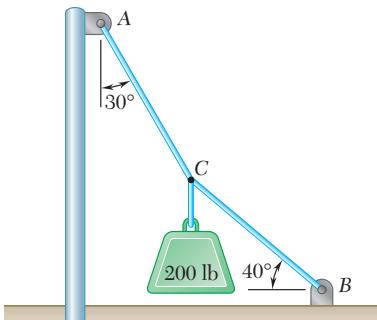


Fig. P2.F1

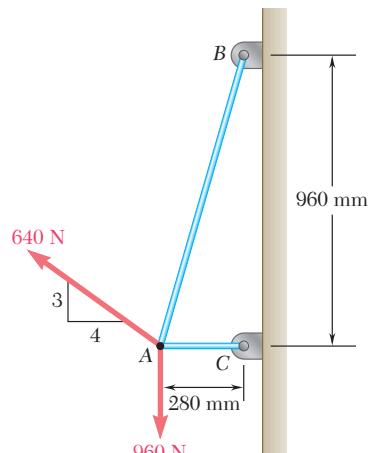


Fig. P2.F3

## END-OF-SECTION PROBLEMS

**2.43 and 2.44** Two cables are tied together at *C* and are loaded as shown. Determine the tension (a) in cable *AC*, (b) in cable *BC*.

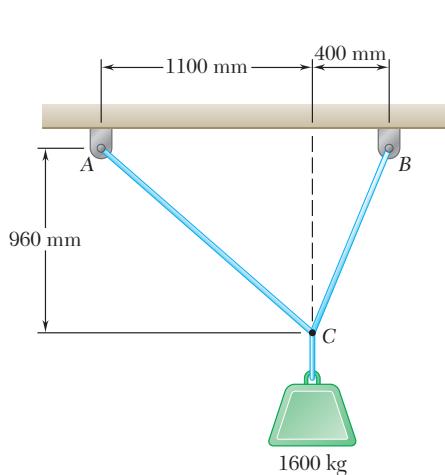


Fig. P2.43

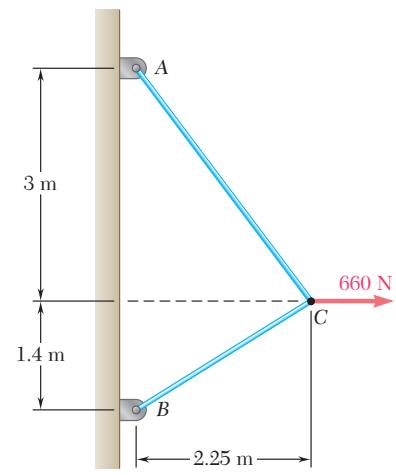


Fig. P2.44

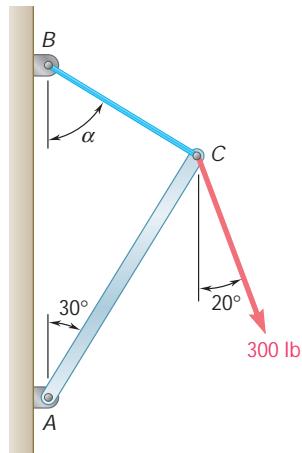


Fig. P2.45

**2.45** Knowing that  $\alpha = 20^\circ$ , determine the tension (a) in cable *AC*, (b) in rope *BC*.

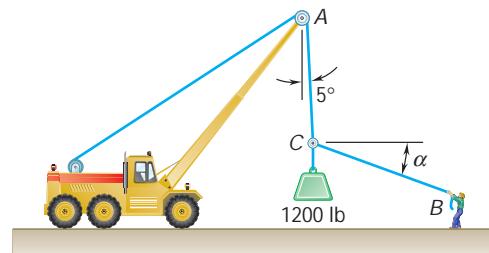


Fig. P2.46

**2.46** Knowing that  $\alpha = 55^\circ$  and that boom *AC* exerts on pin *C* a force directed along line *AC*, determine (a) the magnitude of that force, (b) the tension in cable *BC*.

**2.47** Two cables are tied together at *C* and loaded as shown. Determine the tension (a) in cable *AC*, (b) in cable *BC*.

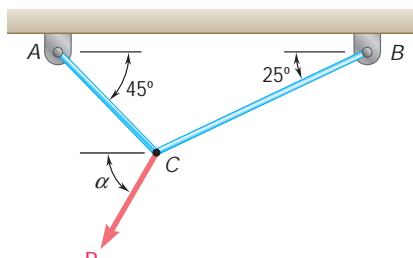


Fig. P2.48

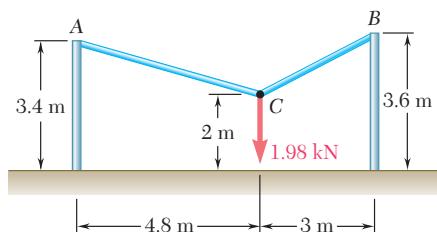


Fig. P2.47

**2.48** Two cables are tied together at *C* and are loaded as shown. Knowing that  $\mathbf{P} = 500 \text{ N}$  and  $\alpha = 60^\circ$ , determine the tension (a) in cable *AC*, (b) in cable *BC*.

- 2.49** Two forces of magnitude  $T_A = 8$  kips and  $T_B = 15$  kips are applied as shown to a welded connection. Knowing that the connection is in equilibrium, determine the magnitudes of the forces  $T_C$  and  $T_D$ .

- 2.50** Two forces of magnitude  $T_A = 6$  kips and  $T_C = 9$  kips are applied as shown to a welded connection. Knowing that the connection is in equilibrium, determine the magnitudes of the forces  $T_B$  and  $T_D$ .

- 2.51** Two cables are tied together at  $C$  and loaded as shown. Knowing that  $P = 360$  N, determine the tension (a) in cable  $AC$ , (b) in cable  $BC$ .

- 2.52** Two cables are tied together at  $C$  and loaded as shown. Determine the range of values of  $P$  for which both cables remain taut.

- 2.53** A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable  $ACB$  and is pulled at a constant speed by cable  $CD$ . Knowing that  $\alpha = 30^\circ$  and  $\beta = 10^\circ$  and that the combined weight of the boatswain's chair and the sailor is 900 N, determine the tension (a) in the support cable  $ACB$ , (b) in the traction cable  $CD$ .

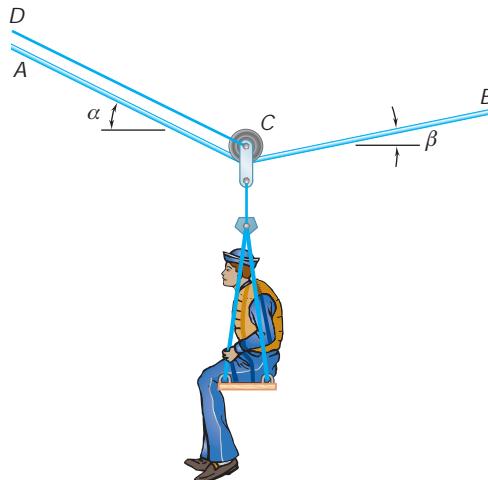


Fig. P2.53 and P2.54

- 2.54** A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable  $ACB$  and is pulled at a constant speed by cable  $CD$ . Knowing that  $\alpha = 25^\circ$  and  $\beta = 15^\circ$  and that the tension in cable  $CD$  is 80 N, determine (a) the combined weight of the boatswain's chair and the sailor, (b) the tension in the support cable  $ACB$ .

- 2.55** Two forces  $\mathbf{P}$  and  $\mathbf{Q}$  are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that  $P = 500$  lb and  $Q = 650$  lb, determine the magnitudes of the forces exerted on the rods  $A$  and  $B$ .

- 2.56** Two forces  $\mathbf{P}$  and  $\mathbf{Q}$  are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that the magnitudes of the forces exerted on rods  $A$  and  $B$  are  $F_A = 750$  lb and  $F_B = 400$  lb, determine the magnitudes of  $\mathbf{P}$  and  $\mathbf{Q}$ .

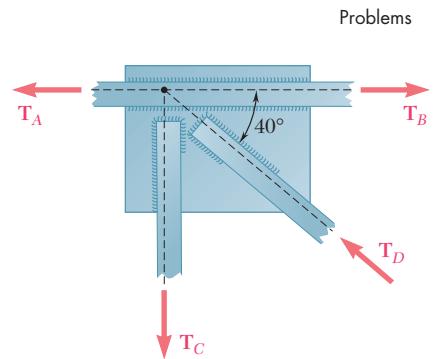


Fig. P2.49 and P2.50

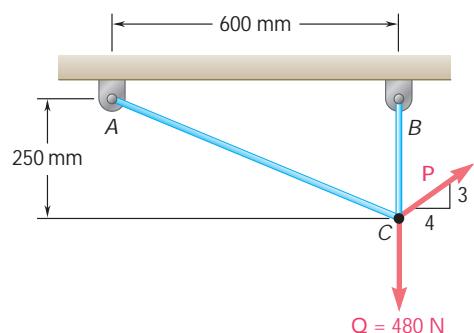


Fig. P2.51 and P2.52

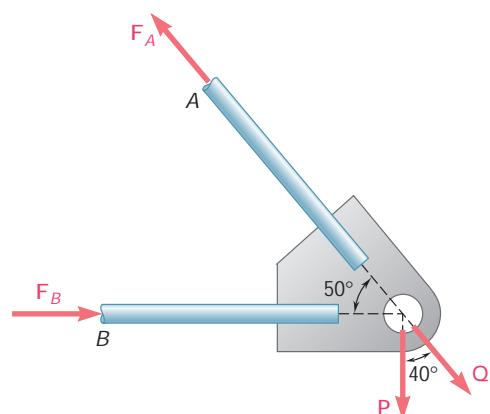


Fig. P2.55 and P2.56

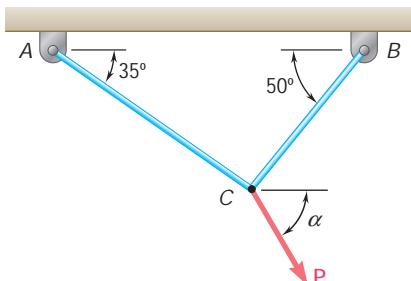


Fig. P2.57 and P2.58

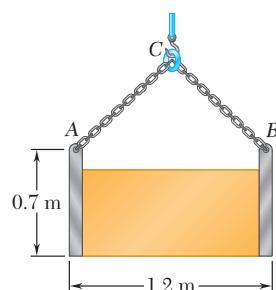


Fig. P2.62

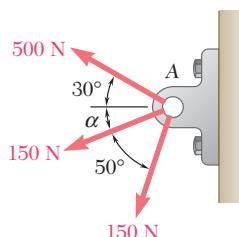


Fig. P2.65

- 2.57** Two cables tied together at *C* are loaded as shown. Knowing that the maximum allowable tension in each cable is 800 N, determine (a) the magnitude of the largest force **P** that can be applied at *C*, (b) the corresponding value of *a*.

- 2.58** Two cables tied together at *C* are loaded as shown. Knowing that the maximum allowable tension is 1200 N in cable *AC* and 600 N in cable *BC*, determine (a) the magnitude of the largest force **P** that can be applied at *C*, (b) the corresponding value of *a*.

- 2.59** For the situation described in Fig. P2.45, determine (a) the value of *a* for which the tension in rope *BC* is as small as possible, (b) the corresponding value of the tension.

- 2.60** For the structure and loading of Prob. 2.46, determine (a) the value of *a* for which the tension in cable *BC* is as small as possible, (b) the corresponding value of the tension.

- 2.61** For the cables of Prob. 2.48, it is known that the maximum allowable tension is 600 N in cable *AC* and 750 N in cable *BC*. Determine (a) the maximum force **P** that can be applied at *C*, (b) the corresponding value of *a*.

- 2.62** A movable bin and its contents have a combined weight of 2.8 kN. Determine the shortest chain sling *ACB* that can be used to lift the loaded bin if the tension in the chain is not to exceed 5 kN.

- 2.63** Collar *A* is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the magnitude of the force **P** required to maintain the equilibrium of the collar when (a) *x* = 4.5 in., (b) *x* = 15 in.

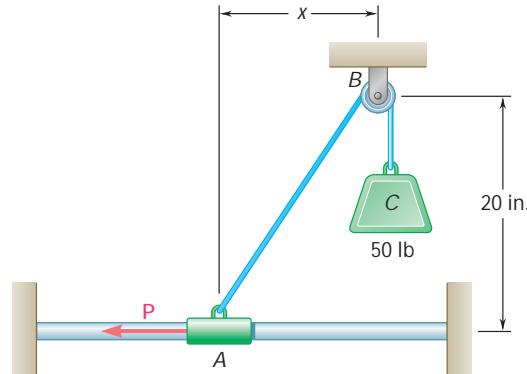


Fig. P2.63 and P2.64

- 2.64** Collar *A* is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the distance *x* for which the collar is in equilibrium when *P* = 48 lb.

- 2.65** Three forces are applied to a bracket as shown. The directions of the two 150-N forces may vary, but the angle between these forces is always 50°. Determine the range of values of *a* for which the magnitude of the resultant of the forces acting at *A* is less than 600 N.

- 2.66** A 200-kg crate is to be supported by the rope-and-pulley arrangement shown. Determine the magnitude and direction of the force  $\mathbf{P}$  that must be exerted on the free end of the rope to maintain equilibrium. (Hint: The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Chap. 4.)

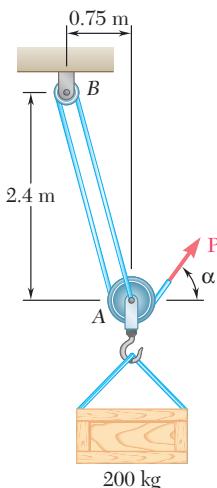


Fig. P2.66

- 2.67** A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Prob. 2.66.)

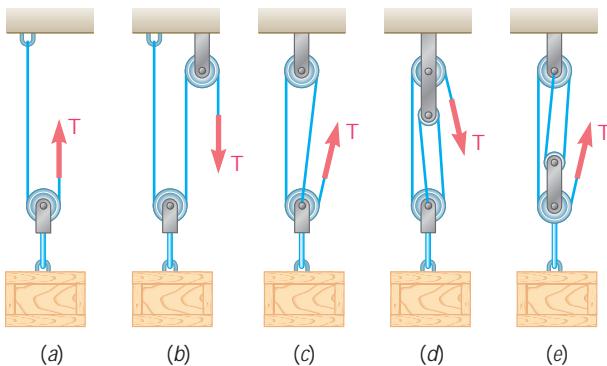


Fig. P2.67

- 2.68** Solve parts *b* and *d* of Prob. 2.67, assuming that the free end of the rope is attached to the crate.

- 2.69** A load  $\mathbf{Q}$  is applied to the pulley  $C$ , which can roll on the cable  $ACB$ . The pulley is held in the position shown by a second cable  $CAD$ , which passes over the pulley  $A$  and supports a load  $\mathbf{P}$ . Knowing that  $P = 750$  N, determine (a) the tension in cable  $ACB$ , (b) the magnitude of load  $\mathbf{Q}$ .

- 2.70** An 1800-N load  $\mathbf{Q}$  is applied to the pulley  $C$ , which can roll on the cable  $ACB$ . The pulley is held in the position shown by a second cable  $CAD$ , which passes over the pulley  $A$  and supports a load  $\mathbf{P}$ . Determine (a) the tension in cable  $ACB$ , (b) the magnitude of load  $\mathbf{P}$ .

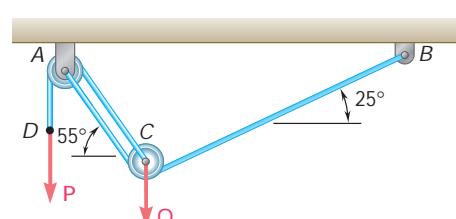


Fig. P2.69 and P2.70

## FORCES IN SPACE

### 2.12 RECTANGULAR COMPONENTS OF A FORCE IN SPACE

The problems considered in the first part of this chapter involved only two dimensions; they could be formulated and solved in a single plane. In this section and in the remaining sections of the chapter, we will discuss problems involving the three dimensions of space.

Consider a force  $\mathbf{F}$  acting at the origin  $O$  of the system of rectangular coordinates  $x, y, z$ . To define the direction of  $\mathbf{F}$ , we draw the vertical plane  $OBAC$  containing  $\mathbf{F}$  (Fig. 2.30a). This plane passes through the vertical  $y$  axis; its orientation is defined by the angle  $\phi$  it forms with the  $xy$  plane. The direction of  $\mathbf{F}$  within the plane is defined by the angle  $u_y$  that  $\mathbf{F}$  forms with the  $y$  axis. The force  $\mathbf{F}$  may be resolved into a vertical component  $\mathbf{F}_y$  and a horizontal component  $\mathbf{F}_h$ ; this operation, shown in Fig. 2.30b, is carried out in plane  $OBAC$  according to the rules developed in the first part of the chapter. The corresponding scalar components are

$$F_y = F \cos u_y \quad F_h = F \sin u_y \quad (2.16)$$

But  $\mathbf{F}_h$  may be resolved into two rectangular components  $\mathbf{F}_x$  and  $\mathbf{F}_z$  along the  $x$  and  $z$  axes, respectively. This operation, shown in Fig. 2.30c, is carried out in the  $xz$  plane. We obtain the following expressions for the corresponding scalar components:

$$\begin{aligned} F_x &= F_h \cos \phi = F \sin u_y \cos \phi \\ F_z &= F_h \sin \phi = F \sin u_y \sin \phi \end{aligned} \quad (2.17)$$

The given force  $\mathbf{F}$  has thus been resolved into three rectangular vector components  $\mathbf{F}_x, \mathbf{F}_y, \mathbf{F}_z$ , which are directed along the three coordinate axes.

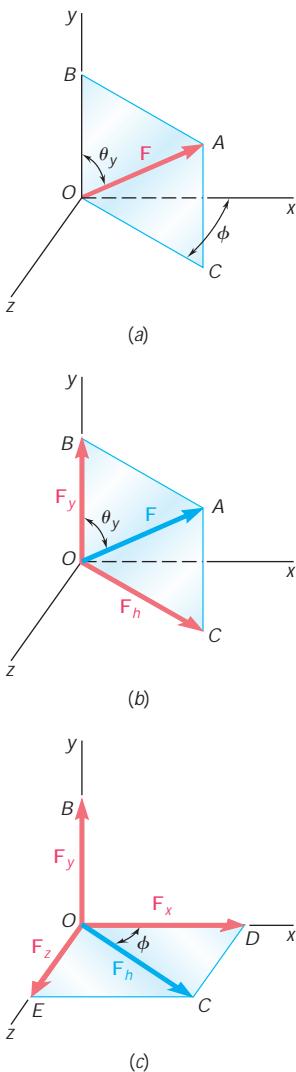
Applying the Pythagorean theorem to the triangles  $OAB$  and  $OCD$  of Fig. 2.30, we write

$$\begin{aligned} F^2 &= (OA)^2 = (OB)^2 + (BA)^2 = F_y^2 + F_h^2 \\ F_h^2 &= (OC)^2 = (OD)^2 + (DC)^2 = F_x^2 + F_z^2 \end{aligned}$$

Eliminating  $F_h^2$  from these two equations and solving for  $F$ , we obtain the following relation between the magnitude of  $\mathbf{F}$  and its rectangular scalar components:

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \quad (2.18)$$

The relationship existing between the force  $\mathbf{F}$  and its three components  $\mathbf{F}_x, \mathbf{F}_y, \mathbf{F}_z$  is more easily visualized if a “box” having  $\mathbf{F}_x, \mathbf{F}_y, \mathbf{F}_z$  for edges is drawn as shown in Fig. 2.31. The force  $\mathbf{F}$  is then represented by the diagonal  $OA$  of this box. Figure 2.31b shows the right triangle  $OAB$  used to derive the first of the formulas (2.16):  $F_y = F \cos u_y$ . In Fig. 2.31a and c, two other right triangles have also been drawn:  $OAD$  and  $OAE$ . These triangles are seen to occupy in the box positions comparable with that of triangle  $OAB$ . Denoting by  $u_x$  and  $u_z$ , respectively, the angles that  $\mathbf{F}$  forms



**Fig. 2.30**

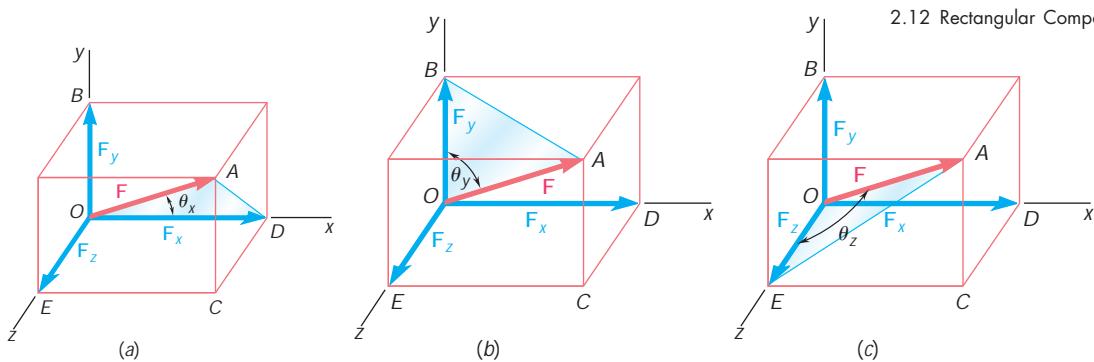


Fig. 2.31

with the  $x$  and  $z$  axes, we can derive two formulas similar to  $F_y = F \cos u_y$ . We thus write

$$F_x = F \cos u_x \quad F_y = F \cos u_y \quad F_z = F \cos u_z \quad (2.19)$$

The three angles  $u_x$ ,  $u_y$ ,  $u_z$  define the direction of the force  $\mathbf{F}$ ; they are more commonly used for this purpose than the angles  $\psi_y$  and  $\phi$  introduced at the beginning of this section. The cosines of  $u_x$ ,  $u_y$ ,  $u_z$  are known as the *direction cosines* of the force  $\mathbf{F}$ .

Introducing the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$ , directed respectively along the  $x$ ,  $y$ , and  $z$  axes (Fig. 2.32), we can express  $\mathbf{F}$  in the form

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k} \quad (2.20)$$

where the scalar components  $F_x$ ,  $F_y$ ,  $F_z$  are defined by the relations (2.19).

**EXAMPLE 1.** A force of 500 N forms angles of  $60^\circ$ ,  $45^\circ$ , and  $120^\circ$ , respectively, with the  $x$ ,  $y$ , and  $z$  axes. Find the components  $F_x$ ,  $F_y$ , and  $F_z$  of the force.

Substituting  $F = 500$  N,  $u_x = 60^\circ$ ,  $u_y = 45^\circ$ ,  $u_z = 120^\circ$  into formulas (2.19), we write

$$F_x = (500 \text{ N}) \cos 60^\circ = +250 \text{ N}$$

$$F_y = (500 \text{ N}) \cos 45^\circ = +354 \text{ N}$$

$$F_z = (500 \text{ N}) \cos 120^\circ = -250 \text{ N}$$

Carrying into Eq. (2.20) the values obtained for the scalar components of  $\mathbf{F}$ , we have

$$\mathbf{F} = (250 \text{ N}) \mathbf{i} + (354 \text{ N}) \mathbf{j} - (250 \text{ N}) \mathbf{k}$$

As in the case of two-dimensional problems, a plus sign indicates that the component has the same sense as the corresponding axis, and a minus sign indicates that it has the opposite sense. ■

The angle a force  $\mathbf{F}$  forms with an axis should be measured from the positive side of the axis and will always be between  $0$  and  $180^\circ$ . An angle  $u_x$  smaller than  $90^\circ$  (acute) indicates that  $\mathbf{F}$  (assumed attached to  $O$ ) is on the same side of the  $yz$  plane as the positive  $x$  axis;  $\cos u_x$  and  $F_x$  will then be positive. An angle  $u_x$  larger than  $90^\circ$  (obtuse) indicates

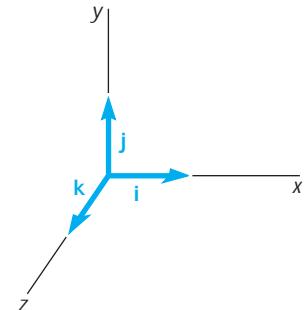


Fig. 2.32

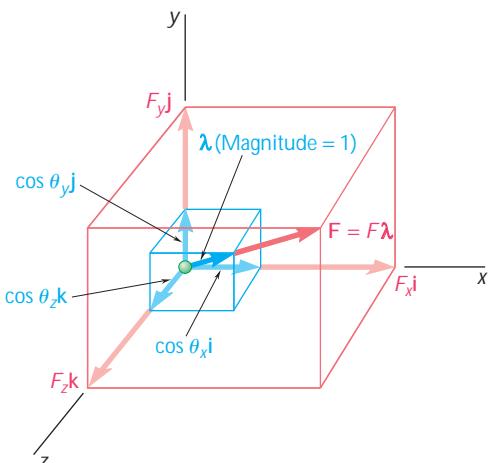


Fig. 2.33

that  $\mathbf{F}$  is on the other side of the  $yz$  plane;  $\cos u_x$  and  $F_x$  will then be negative. In Example 1 the angles  $u_x$  and  $u_y$  are acute, while  $u_z$  is obtuse; consequently,  $F_x$  and  $F_y$  are positive, while  $F_z$  is negative.

Substituting into (2.20) the expressions obtained for  $F_x$ ,  $F_y$ ,  $F_z$  in (2.19), we write

$$\mathbf{F} = F(\cos u_x \mathbf{i} + \cos u_y \mathbf{j} + \cos u_z \mathbf{k}) \quad (2.21)$$

which shows that the force  $\mathbf{F}$  can be expressed as the product of the scalar  $F$  and the vector

$$\lambda = \cos u_x \mathbf{i} + \cos u_y \mathbf{j} + \cos u_z \mathbf{k} \quad (2.22)$$

Clearly, the vector  $\lambda$  is a vector whose magnitude is equal to 1 and whose direction is the same as that of  $\mathbf{F}$  (Fig. 2.33). The vector  $\lambda$  is referred to as the *unit vector* along the line of action of  $\mathbf{F}$ . It follows from (2.22) that the components of the unit vector  $\lambda$  are respectively equal to the direction cosines of the line of action of  $\mathbf{F}$ :

$$l_x = \cos u_x \quad l_y = \cos u_y \quad l_z = \cos u_z \quad (2.23)$$

We should observe that the values of the three angles  $u_x$ ,  $u_y$ ,  $u_z$  are not independent. Recalling that the sum of the squares of the components of a vector is equal to the square of its magnitude, we write

$$l_x^2 + l_y^2 + l_z^2 = 1$$

or, substituting for  $l_x$ ,  $l_y$ ,  $l_z$  from (2.23),

$$\cos^2 u_x + \cos^2 u_y + \cos^2 u_z = 1 \quad (2.24)$$

In Example 1, for instance, once the values  $u_x = 60^\circ$  and  $u_y = 45^\circ$  have been selected, the value of  $u_z$  *must* be equal to  $60^\circ$  or  $120^\circ$  in order to satisfy identity (2.24).

When the components  $F_x$ ,  $F_y$ ,  $F_z$  of a force  $\mathbf{F}$  are given, the magnitude  $F$  of the force is obtained from (2.18).† The relations (2.19) can then be solved for the direction cosines,

$$\cos u_x = \frac{F_x}{F} \quad \cos u_y = \frac{F_y}{F} \quad \cos u_z = \frac{F_z}{F} \quad (2.25)$$

and the angles  $u_x$ ,  $u_y$ ,  $u_z$  characterizing the direction of  $\mathbf{F}$  can be found.

**EXAMPLE 2.** A force  $\mathbf{F}$  has the components  $F_x = 20 \text{ lb}$ ,  $F_y = -30 \text{ lb}$ ,  $F_z = 60 \text{ lb}$ . Determine its magnitude  $F$  and the angles  $u_x$ ,  $u_y$ ,  $u_z$  it forms with the coordinate axes.

From formula (2.18) we obtain†

$$\begin{aligned} F &= \sqrt{F_x^2 + F_y^2 + F_z^2} \\ &= \sqrt{(20 \text{ lb})^2 + (-30 \text{ lb})^2 + (60 \text{ lb})^2} \\ &= 70 \text{ lb} \end{aligned}$$

†With a calculator programmed to convert rectangular coordinates into polar coordinates, the following procedure will be found more expeditious for computing  $F$ : First determine  $F_h$  from its two rectangular components  $F_x$  and  $F_z$  (Fig. 2.30c), then determine  $F$  from its two rectangular components  $F_h$  and  $F_y$  (Fig. 2.30b). The actual order in which the three components  $F_x$ ,  $F_y$ ,  $F_z$  are entered is immaterial.

Substituting the values of the components and magnitude of  $\mathbf{F}$  into Eqs. (2.25), we write

$$\cos u_x = \frac{F_x}{F} = \frac{20 \text{ lb}}{70 \text{ lb}} \quad \cos u_y = \frac{F_y}{F} = \frac{-30 \text{ lb}}{70 \text{ lb}} \quad \cos u_z = \frac{F_z}{F} = \frac{60 \text{ lb}}{70 \text{ lb}}$$

Calculating successively each quotient and its arc cosine, we obtain

$$u_x = 73.4^\circ \quad u_y = 115.4^\circ \quad u_z = 31.0^\circ$$

These computations can be carried out easily with a calculator. ■

## 2.13 FORCE DEFINED BY ITS MAGNITUDE AND TWO POINTS ON ITS LINE OF ACTION

In many applications, the direction of a force  $\mathbf{F}$  is defined by the coordinates of two points,  $M(x_1, y_1, z_1)$  and  $N(x_2, y_2, z_2)$ , located on its line of action (Fig. 2.34). Consider the vector  $\overrightarrow{MN}$  joining  $M$  and  $N$

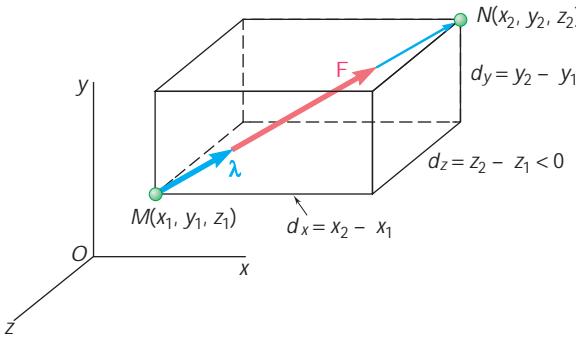


Fig. 2.34

and of the same sense as  $\mathbf{F}$ . Denoting its scalar components by  $d_x$ ,  $d_y$ ,  $d_z$ , respectively, we write

$$\overrightarrow{MN} = d_x \mathbf{i} + d_y \mathbf{j} + d_z \mathbf{k} \quad (2.26)$$

The unit vector  $\lambda$  along the line of action of  $\mathbf{F}$  (i.e., along the line  $MN$ ) may be obtained by dividing the vector  $\overrightarrow{MN}$  by its magnitude  $MN$ . Substituting for  $\overrightarrow{MN}$  from (2.26) and observing that  $MN$  is equal to the distance  $d$  from  $M$  to  $N$ , we write

$$\lambda = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d} (d_x \mathbf{i} + d_y \mathbf{j} + d_z \mathbf{k}) \quad (2.27)$$

Recalling that  $\mathbf{F}$  is equal to the product of  $F$  and  $\lambda$ , we have

$$\mathbf{F} = F \lambda = \frac{F}{d} (d_x \mathbf{i} + d_y \mathbf{j} + d_z \mathbf{k}) \quad (2.28)$$

from which it follows that the scalar components of  $\mathbf{F}$  are, respectively,

$$F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d} \quad (2.29)$$

The relations (2.29) considerably simplify the determination of the components of a force  $\mathbf{F}$  of given magnitude  $F$  when the line of action of  $\mathbf{F}$  is defined by two points  $M$  and  $N$ . Subtracting the coordinates of  $M$  from those of  $N$ , we first determine the components of the vector  $\overrightarrow{MN}$  and the distance  $d$  from  $M$  to  $N$ :

$$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1$$

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

Substituting for  $F$  and for  $d_x$ ,  $d_y$ ,  $d_z$ , and  $d$  into the relations (2.29), we obtain the components  $F_x$ ,  $F_y$ ,  $F_z$  of the force.

The angles  $u_x$ ,  $u_y$ ,  $u_z$  that  $\mathbf{F}$  forms with the coordinate axes can then be obtained from Eqs. (2.25). Comparing Eqs. (2.22) and (2.27), we can also write

$$\cos u_x = \frac{d_x}{d} \quad \cos u_y = \frac{d_y}{d} \quad \cos u_z = \frac{d_z}{d} \quad (2.30)$$

and determine the angles  $u_x$ ,  $u_y$ ,  $u_z$  directly from the components and magnitude of the vector  $MN$ .

## 2.14 ADDITION OF CONCURRENT FORCES IN SPACE

The resultant  $\mathbf{R}$  of two or more forces in space will be determined by summing their rectangular components. Graphical or trigonometric methods are generally not practical in the case of forces in space.

The method followed here is similar to that used in Sec. 2.8 with coplanar forces. Setting

$$\mathbf{R} = \Sigma \mathbf{F}$$

we resolve each force into its rectangular components and write

$$R_x \mathbf{i} + R_y \mathbf{j} + R_z \mathbf{k} = \Sigma(F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k})$$

$$= (\Sigma F_x) \mathbf{i} + (\Sigma F_y) \mathbf{j} + (\Sigma F_z) \mathbf{k}$$

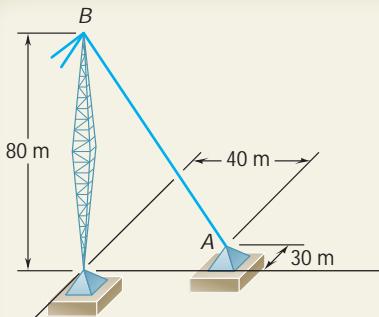
from which it follows that

$$R_x = \Sigma F_x \quad R_y = \Sigma F_y \quad R_z = \Sigma F_z \quad (2.31)$$

The magnitude of the resultant and the angles  $u_x$ ,  $u_y$ ,  $u_z$  that the resultant forms with the coordinate axes are obtained using the method discussed in Sec. 2.12. We write

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \quad (2.32)$$

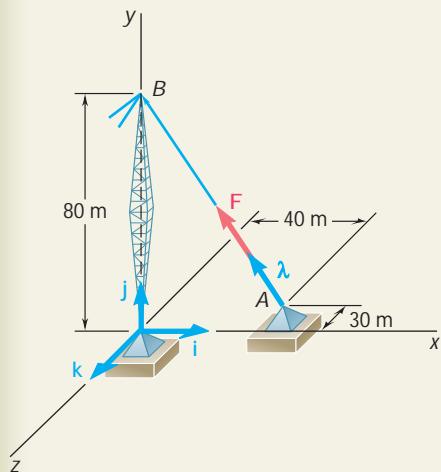
$$\cos u_x = \frac{R_x}{R} \quad \cos u_y = \frac{R_y}{R} \quad \cos u_z = \frac{R_z}{R} \quad (2.33)$$



## SAMPLE PROBLEM 2.7

A tower guy wire is anchored by means of a bolt at A. The tension in the wire is 2500 N. Determine (a) the components  $F_x$ ,  $F_y$ ,  $F_z$  of the force acting on the bolt, (b) the angles  $u_x$ ,  $u_y$ ,  $u_z$  defining the direction of the force.

## SOLUTION



**a. Components of the Force.** The line of action of the force acting on the bolt passes through A and  $\overrightarrow{B}$ , and the force is directed from A to B. The components of the vector  $\overrightarrow{AB}$ , which has the same direction as the force, are

$$d_x = -40 \text{ m} \quad d_y = +80 \text{ m} \quad d_z = +30 \text{ m}$$

The total distance from A to B is

$$AB = d = \sqrt{d_x^2 + d_y^2 + d_z^2} = 94.3 \text{ m}$$

Denoting by  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$  the unit vectors along the coordinate axes, we have

$$\overrightarrow{AB} = -(40 \text{ m})\mathbf{i} + (80 \text{ m})\mathbf{j} + (30 \text{ m})\mathbf{k}$$

Introducing the unit vector  $\mathbf{L} = \overrightarrow{AB}/AB$ , we write

$$\mathbf{F} = F\mathbf{L} = F \frac{\overrightarrow{AB}}{AB} = \frac{2500 \text{ N}}{94.3 \text{ m}} \overrightarrow{AB}$$

Substituting the expression found for  $\overrightarrow{AB}$ , we obtain

$$\mathbf{F} = \frac{2500 \text{ N}}{94.3 \text{ m}} [-(40 \text{ m})\mathbf{i} + (80 \text{ m})\mathbf{j} + (30 \text{ m})\mathbf{k}]$$

$$\mathbf{F} = -(1060 \text{ N})\mathbf{i} + (2120 \text{ N})\mathbf{j} + (795 \text{ N})\mathbf{k}$$

The components of  $\mathbf{F}$ , therefore, are

$$F_x = -1060 \text{ N} \quad F_y = +2120 \text{ N} \quad F_z = +795 \text{ N} \quad \blacktriangleleft$$

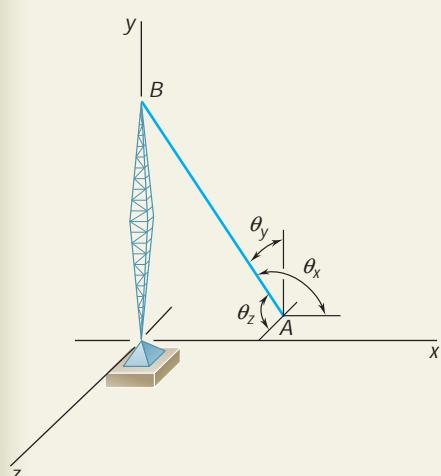
**b. Direction of the Force.** Using Eqs. (2.25), we write

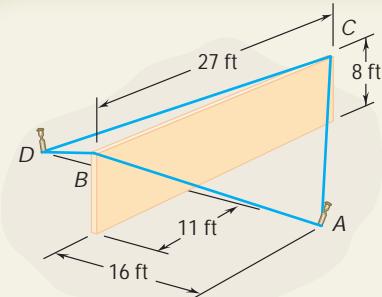
$$\begin{aligned} \cos u_x &= \frac{F_x}{F} = \frac{-1060 \text{ N}}{2500 \text{ N}} & \cos u_y &= \frac{F_y}{F} = \frac{+2120 \text{ N}}{2500 \text{ N}} \\ \cos u_z &= \frac{F_z}{F} = \frac{+795 \text{ N}}{2500 \text{ N}} \end{aligned}$$

Calculating successively each quotient and its arc cosine, we obtain

$$u_x = 115.1^\circ \quad u_y = 32.0^\circ \quad u_z = 71.5^\circ \quad \blacktriangleleft$$

(Note. This result could have been obtained by using the components and magnitude of the vector  $\overrightarrow{AB}$  rather than those of the force  $\mathbf{F}$ .)





## SAMPLE PROBLEM 2.8

A wall section of precast concrete is temporarily held by the cables shown. Knowing that the tension is 840 lb in cable  $AB$  and 1200 lb in cable  $AC$ , determine the magnitude and direction of the resultant of the forces exerted by cables  $AB$  and  $AC$  on stake A.

### SOLUTION

**Components of the Forces.** The force exerted by each cable on stake A will be resolved into  $x$ ,  $y$ , and  $z$  components. We first determine the components and magnitude of the vectors  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$ , measuring them from A toward the wall section. Denoting by  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$  the unit vectors along the coordinate axes, we write

$$\begin{aligned}\overrightarrow{AB} &= -(16 \text{ ft})\mathbf{i} + (8 \text{ ft})\mathbf{j} + (11 \text{ ft})\mathbf{k} & AB = 21 \text{ ft} \\ \overrightarrow{AC} &= -(16 \text{ ft})\mathbf{i} + (8 \text{ ft})\mathbf{j} - (16 \text{ ft})\mathbf{k} & AC = 24 \text{ ft}\end{aligned}$$

Denoting by  $\lambda_{AB}$  the unit vector along  $AB$ , we have

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{840 \text{ lb}}{21 \text{ ft}}\overrightarrow{AB}$$

Substituting the expression found for  $\overrightarrow{AB}$ , we obtain

$$\begin{aligned}\mathbf{T}_{AB} &= \frac{840 \text{ lb}}{21 \text{ ft}}[-(16 \text{ ft})\mathbf{i} + (8 \text{ ft})\mathbf{j} + (11 \text{ ft})\mathbf{k}] \\ \mathbf{T}_{AB} &= -(640 \text{ lb})\mathbf{i} + (320 \text{ lb})\mathbf{j} + (440 \text{ lb})\mathbf{k}\end{aligned}$$

Denoting by  $\lambda_{AC}$  the unit vector along  $AC$ , we obtain in a similar way

$$\begin{aligned}\mathbf{T}_{AC} &= T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{1200 \text{ lb}}{24 \text{ ft}}\overrightarrow{AC} \\ \mathbf{T}_{AC} &= -(800 \text{ lb})\mathbf{i} + (400 \text{ lb})\mathbf{j} - (800 \text{ lb})\mathbf{k}\end{aligned}$$

**Resultant of the Forces.** The resultant  $\mathbf{R}$  of the forces exerted by the two cables is

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = -(1440 \text{ lb})\mathbf{i} + (720 \text{ lb})\mathbf{j} - (360 \text{ lb})\mathbf{k}$$

The magnitude and direction of the resultant are now determined:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(-1440)^2 + (720)^2 + (-360)^2}$$

$$R = 1650 \text{ lb} \quad \blacktriangleleft$$

From Eqs. (2.33) we obtain

$$\cos u_x = \frac{R_x}{R} = \frac{-1440 \text{ lb}}{1650 \text{ lb}} \quad \cos u_y = \frac{R_y}{R} = \frac{+720 \text{ lb}}{1650 \text{ lb}}$$

$$\cos u_z = \frac{R_z}{R} = \frac{-360 \text{ lb}}{1650 \text{ lb}}$$

Calculating successively each quotient and its arc cosine, we have

$$u_x = 150.8^\circ \quad u_y = 64.1^\circ \quad u_z = 102.6^\circ \quad \blacktriangleleft$$

# SOLVING PROBLEMS ON YOUR OWN

In this lesson we saw that *forces in space* may be defined by their magnitude and direction or by the three rectangular components  $F_x$ ,  $F_y$ , and  $F_z$ .

**A. When a force is defined by its magnitude and direction,** its rectangular components  $F_x$ ,  $F_y$ , and  $F_z$  may be found as follows:

**Case 1.** If the direction of the force  $\mathbf{F}$  is defined by the angles  $u_y$  and  $f$  shown in Fig. 2.30, projections of  $\mathbf{F}$  through these angles or their complements will yield the components of  $\mathbf{F}$  [Eqs. (2.17)]. Note that the  $x$  and  $z$  components of  $\mathbf{F}$  are found by first projecting  $\mathbf{F}$  onto the horizontal plane; the projection  $\mathbf{F}_h$  obtained in this way is then resolved into the components  $\mathbf{F}_x$  and  $\mathbf{F}_z$  (Fig. 2.30c).

**Case 2.** If the direction of the force  $\mathbf{F}$  is defined by the angles  $u_x$ ,  $u_y$ ,  $u_z$  that  $\mathbf{F}$  forms with the coordinate axes, each component can be obtained by multiplying the magnitude  $F$  of the force by the cosine of the corresponding angle [Example 1]:

$$F_x = F \cos u_x \quad F_y = F \cos u_y \quad F_z = F \cos u_z$$

**Case 3.** If the direction of the force  $\mathbf{F}$  is defined by two points  $M$  and  $N$  located on its line of action (Fig. 2.34), you will first express the vector  $\overrightarrow{MN}$  drawn from  $M$  to  $N$  in terms of its components  $d_x$ ,  $d_y$ ,  $d_z$  and the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ :

$$\overrightarrow{MN} = d_x \mathbf{i} + d_y \mathbf{j} + d_z \mathbf{k}$$

Next, you will determine the unit vector  $\lambda$  along the line of action of  $\mathbf{F}$  by dividing the vector  $\overrightarrow{MN}$  by its magnitude  $MN$ . Multiplying  $\lambda$  by the magnitude of  $\mathbf{F}$ , you will obtain the desired expression for  $\mathbf{F}$  in terms of its rectangular components [Sample Prob. 2.7]:

$$\mathbf{F} = F \lambda = \frac{F}{d} (d_x \mathbf{i} + d_y \mathbf{j} + d_z \mathbf{k})$$

It is advantageous to use a consistent and meaningful system of notation when determining the rectangular components of a force. The method used in this text is illustrated in Sample Prob. 2.8 where, for example, the force  $\mathbf{T}_{AB}$  acts from stake  $A$  toward point  $B$ . Note that the subscripts have been ordered to agree with the direction of the force. It is recommended that you adopt the same notation, as it will help you identify point 1 (the first subscript) and point 2 (the second subscript).

When forming the vector defining the line of action of a force, you may think of its scalar components as the number of steps you must take in each coordinate direction to go from point 1 to point 2. It is essential that you always remember to assign the correct sign to each of the components.

(continued)

**B. When a force is defined by its rectangular components  $F_x$ ,  $F_y$ ,  $F_z$  you can obtain its magnitude  $F$  by writing**

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

You can determine the direction cosines of the line of action of  $\mathbf{F}$  by dividing the components of the force by  $F$ :

$$\cos u_x = \frac{F_x}{F} \quad \cos u_y = \frac{F_y}{F} \quad \cos u_z = \frac{F_z}{F}$$

From the direction cosines you can obtain the angles  $u_x$ ,  $u_y$ ,  $u_z$  that  $\mathbf{F}$  forms with the coordinate axes [Example 2].

**C. To determine the resultant  $R$  of two or more forces** in three-dimensional space, first determine the rectangular components of each force by one of the procedures described above. Adding these components will yield the components  $R_x$ ,  $R_y$ ,  $R_z$  of the resultant. The magnitude and direction of the resultant may then be obtained as indicated above for a force  $\mathbf{F}$  [Sample Prob. 2.8].

# PROBLEMS

- 2.71** Determine (a) the  $x$ ,  $y$ , and  $z$  components of the 900-N force, (b) the angles  $u_x$ ,  $u_y$ , and  $u_z$  that the force forms with the coordinate axes.

- 2.72** Determine (a) the  $x$ ,  $y$ , and  $z$  components of the 750-N force, (b) the angles  $u_x$ ,  $u_y$ , and  $u_z$  that the force forms with the coordinate axes.

- 2.73** A gun is aimed at a point  $A$  located  $35^\circ$  east of north. Knowing that the barrel of the gun forms an angle of  $40^\circ$  with the horizontal and that the maximum recoil force is 400 N, determine (a) the  $x$ ,  $y$ , and  $z$  components of that force, (b) the values of the angles  $u_x$ ,  $u_y$ , and  $u_z$  defining the direction of the recoil force. (Assume that the  $x$ ,  $y$ , and  $z$  axes are directed, respectively, east, up, and south.)

- 2.74** Solve Prob. 2.73, assuming that point  $A$  is located  $15^\circ$  north of west and that the barrel of the gun forms an angle of  $25^\circ$  with the horizontal.

- 2.75** Cable  $AB$  is 65 ft long, and the tension in that cable is 3900 lb. Determine (a) the  $x$ ,  $y$ , and  $z$  components of the force exerted by the cable on the anchor  $B$ , (b) the angles  $u_x$ ,  $u_y$ , and  $u_z$  defining the direction of that force.

- 2.76** Cable  $AC$  is 70 ft long, and the tension in that cable is 5250 lb. Determine (a) the  $x$ ,  $y$ , and  $z$  components of the force exerted by the cable on the anchor  $C$ , (b) the angles  $u_x$ ,  $u_y$ , and  $u_z$  defining the direction of that force.

- 2.77** The end of the coaxial cable  $AE$  is attached to the pole  $AB$ , which is strengthened by the guy wires  $AC$  and  $AD$ . Knowing that the tension in wire  $AC$  is 120 lb, determine (a) the components of the force exerted by this wire on the pole, (b) the angles  $u_x$ ,  $u_y$ , and  $u_z$  that the force forms with the coordinate axes.

- 2.78** The end of the coaxial cable  $AE$  is attached to the pole  $AB$ , which is strengthened by the guy wires  $AC$  and  $AD$ . Knowing that the tension in wire  $AD$  is 85 lb, determine (a) the components of the force exerted by this wire on the pole, (b) the angles  $u_x$ ,  $u_y$ , and  $u_z$  that the force forms with the coordinate axes.

- 2.79** Determine the magnitude and direction of the force  $\mathbf{F} = (690 \text{ lb})\mathbf{i} + (300 \text{ lb})\mathbf{j} - (580 \text{ lb})\mathbf{k}$ .

- 2.80** Determine the magnitude and direction of the force  $\mathbf{F} = (650 \text{ N})\mathbf{i} - (320 \text{ N})\mathbf{j} + (760 \text{ N})\mathbf{k}$ .

- 2.81** A force acts at the origin of a coordinate system in a direction defined by the angles  $u_x = 75^\circ$  and  $u_z = 130^\circ$ . Knowing that the  $y$  component of the force is  $+300 \text{ lb}$ , determine (a) the angle  $u_y$ , (b) the other components and the magnitude of the force.

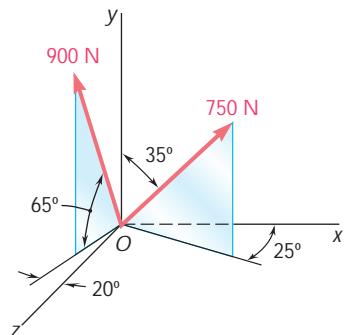


Fig. P2.71 and P2.72

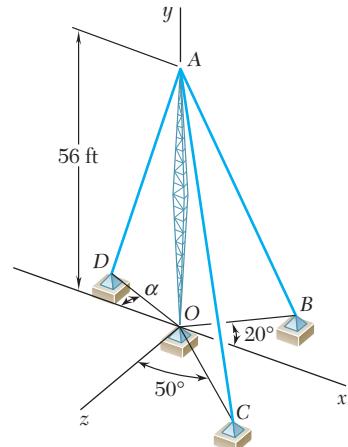


Fig. P2.75 and P2.76

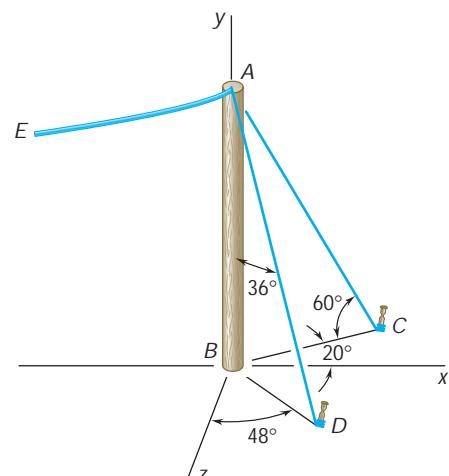


Fig. P2.77 and P2.78

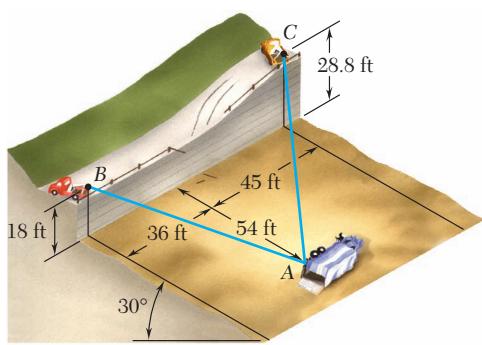


Fig. P2.85 and P2.86

- 2.82** A force acts at the origin of a coordinate system in a direction defined by the angles  $u_y = 55^\circ$  and  $u_z = 45^\circ$ . Knowing that the  $x$  component of the force is  $-500$  N, determine (a) the angle  $u_x$ , (b) the other components and the magnitude of the force.

- 2.83** A force  $\mathbf{F}$  of magnitude  $230$  N acts at the origin of a coordinate system. Knowing that  $u_x = 32.5^\circ$ ,  $F_y = -60$  N, and  $F_z > 0$ , determine (a) the components  $F_x$  and  $F_z$ , (b) the angles  $u_y$  and  $u_z$ .

- 2.84** A force  $\mathbf{F}$  of magnitude  $210$  N acts at the origin of a coordinate system. Knowing that  $F_x = 80$  N,  $u_z = 151.2^\circ$ , and  $F_y < 0$ , determine (a) the components  $F_y$  and  $F_z$ , (b) the angles  $u_x$  and  $u_y$ .

- 2.85** In order to move a wrecked truck, two cables are attached at  $A$  and pulled by winches  $B$  and  $C$  as shown. Knowing that the tension in cable  $AB$  is  $2$  kips, determine the components of the force exerted at  $A$  by the cable.

- 2.86** In order to move a wrecked truck, two cables are attached at  $A$  and pulled by winches  $B$  and  $C$  as shown. Knowing that the tension in cable  $AC$  is  $1.5$  kips, determine the components of the force exerted at  $A$  by the cable.

- 2.87** Knowing that the tension in cable  $AB$  is  $1425$  N, determine the components of the force exerted on the plate at  $B$ .

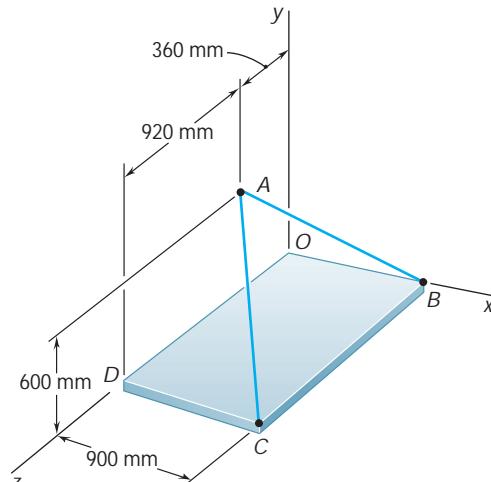


Fig. P2.87 and P2.88

- 2.88** Knowing that the tension in cable  $AC$  is  $2130$  N, determine the components of the force exerted on the plate at  $C$ .

- 2.89** A frame  $ABC$  is supported in part by cable  $DBE$  that passes through a frictionless ring at  $B$ . Knowing that the tension in the cable is  $385$  N, determine the components of the force exerted by the cable on the support at  $D$ .

- 2.90** For the frame and cable of Prob. 2.89, determine the components of the force exerted by the cable on the support at  $E$ .

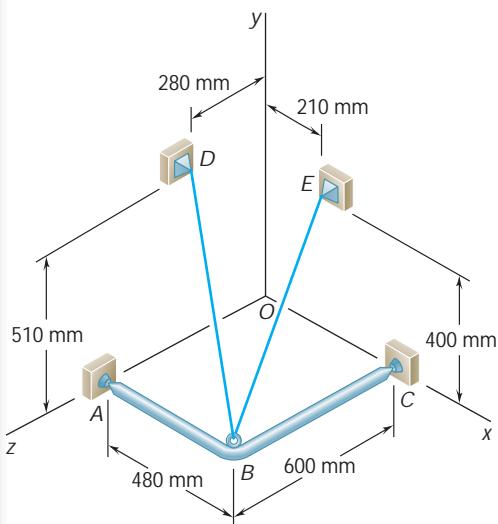


Fig. P2.89

- 2.91** Find the magnitude and direction of the resultant of the two forces shown knowing that  $P = 600$  N and  $Q = 450$  N.

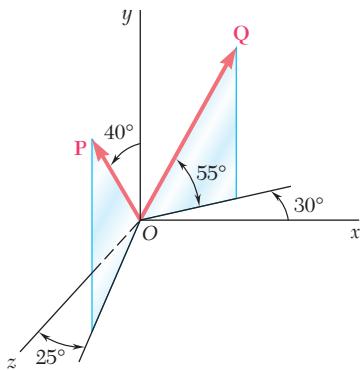


Fig. P2.91 and P2.92

- 2.92** Find the magnitude and direction of the resultant of the two forces shown knowing that  $P = 450$  N and  $Q = 600$  N.

- 2.93** Knowing that the tension is 425 lb in cable  $AB$  and 510 lb in cable  $AC$ , determine the magnitude and direction of the resultant of the forces exerted at  $A$  by the two cables.

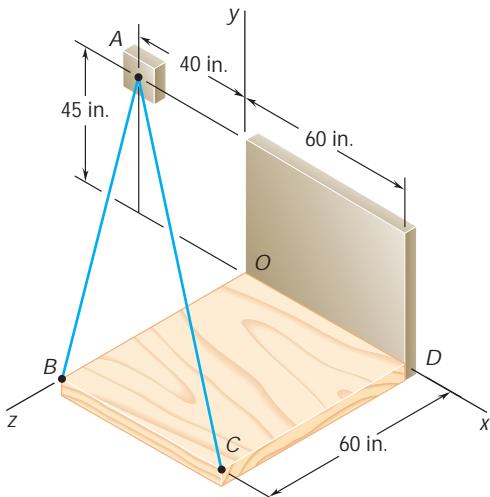


Fig. P2.93 and P2.94

- 2.94** Knowing that the tension is 510 lb in cable  $AB$  and 425 lb in cable  $AC$ , determine the magnitude and direction of the resultant of the forces exerted at  $A$  by the two cables.

- 2.95** For the frame of Prob. 2.89, determine the magnitude and direction of the resultant of the forces exerted by the cable at  $B$  knowing that the tension in the cable is 385 N.

**2.96** For the cables of Prob. 2.87, knowing that the tension is 1425 N in cable  $AB$  and 2130 N in cable  $AC$ , determine the magnitude and direction of the resultant of the forces exerted at  $A$  by the two cables.

**2.97** The boom  $OA$  carries a load  $\mathbf{P}$  and is supported by two cables as shown. Knowing that the tension in cable  $AB$  is 183 lb and that the resultant of the load  $\mathbf{P}$  and of the forces exerted at  $A$  by the two cables must be directed along  $OA$ , determine the tension in cable  $AC$ .

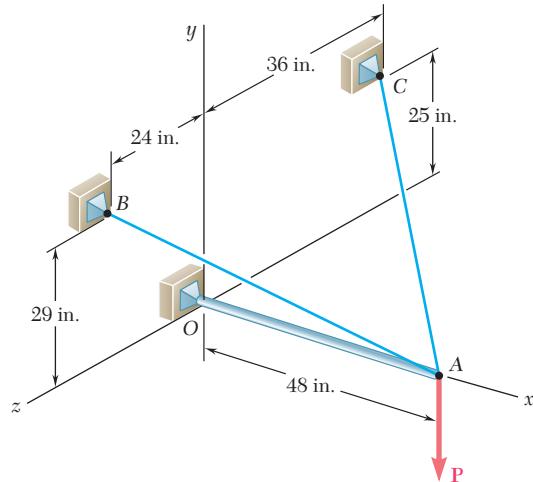


Fig. P2.97

**2.98** For the boom and loading of Prob. 2.97, determine the magnitude of the load  $\mathbf{P}$ .

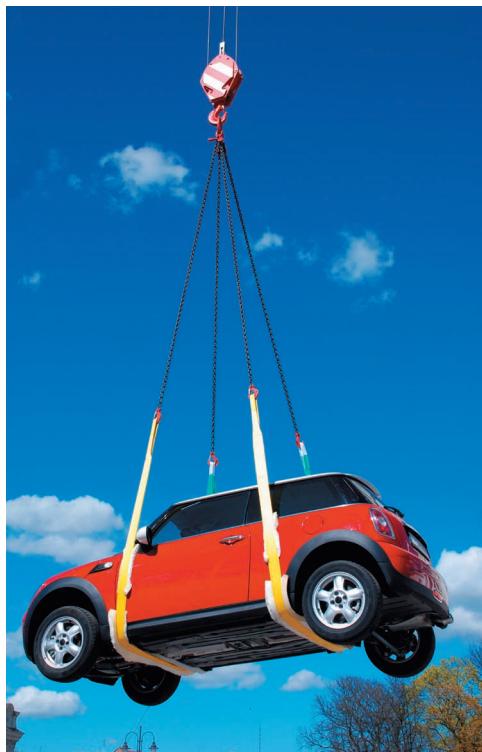
## 2.15 EQUILIBRIUM OF A PARTICLE IN SPACE

According to the definition given in Sec. 2.9, a particle  $A$  is in equilibrium if the resultant of all the forces acting on  $A$  is zero. The components  $R_x$ ,  $R_y$ ,  $R_z$  of the resultant are given by the relations (2.31); expressing that the components of the resultant are zero, we write

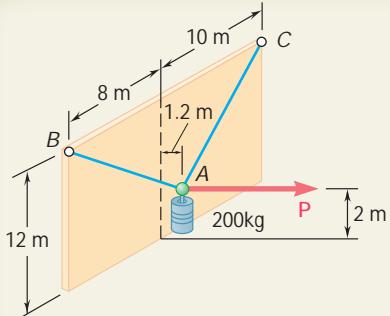
$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0 \quad (2.34)$$

Equations (2.34) represent the necessary and sufficient conditions for the equilibrium of a particle in space. They can be used to solve problems dealing with the equilibrium of a particle involving no more than three unknowns.

To solve such problems, you first should draw a free-body diagram showing the particle in equilibrium and *all* the forces acting on it. You can then write the equations of equilibrium (2.34) and solve them for three unknowns. In the more common types of problems, these unknowns will represent (1) the three components of a single force or (2) the magnitude of three forces, each of known direction.



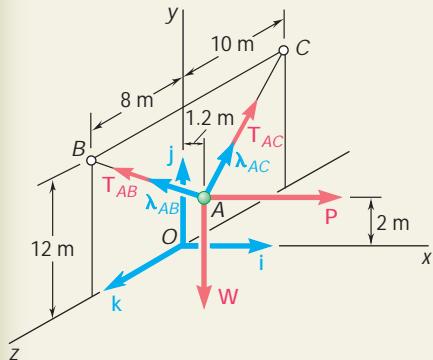
**Photo 2.2** While the tension in the four cables supporting the car cannot be found using the three equations of (2.34), a relation between the tensions can be obtained by considering the equilibrium of the hook.



## SAMPLE PROBLEM 2.9

A 200-kg cylinder is hung by means of two cables  $AB$  and  $AC$ , which are attached to the top of a vertical wall. A horizontal force  $\mathbf{P}$  perpendicular to the wall holds the cylinder in the position shown. Determine the magnitude of  $\mathbf{P}$  and the tension in each cable.

## SOLUTION



**Free-Body Diagram.** Point A is chosen as a free body; this point is subjected to four forces, three of which are of unknown magnitude.

Introducing the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ , we resolve each force into rectangular components.

$$\begin{aligned}\mathbf{P} &= P\mathbf{i} \\ \mathbf{W} &= -mg\mathbf{j} = -(200 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(1962 \text{ N})\mathbf{j}\end{aligned}\quad (1)$$

In the case of  $\mathbf{T}_{AB}$  and  $\mathbf{T}_{AC}$ , it is necessary first to determine the components and magnitudes of the vectors  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$ . Denoting by  $\mathbf{l}_{AB}$  the unit vector along  $AB$ , we write

$$\begin{aligned}\overrightarrow{AB} &= -(1.2 \text{ m})\mathbf{i} + (10 \text{ m})\mathbf{j} + (8 \text{ m})\mathbf{k} \quad AB = 12.862 \text{ m} \\ \mathbf{l}_{AB} &= \frac{\overrightarrow{AB}}{12.862 \text{ m}} = -0.09330\mathbf{i} + 0.7775\mathbf{j} + 0.6220\mathbf{k}\end{aligned}$$

$$\mathbf{T}_{AB} = T_{AB}\mathbf{l}_{AB} = -0.09330T_{AB}\mathbf{i} + 0.7775T_{AB}\mathbf{j} + 0.6220T_{AB}\mathbf{k} \quad (2)$$

Denoting by  $\mathbf{l}_{AC}$  the unit vector along  $AC$ , we write in a similar way

$$\overrightarrow{AC} = -(1.2 \text{ m})\mathbf{i} + (10 \text{ m})\mathbf{j} - (10 \text{ m})\mathbf{k} \quad AC = 14.193 \text{ m}$$

$$\mathbf{l}_{AC} = \frac{\overrightarrow{AC}}{14.193 \text{ m}} = -0.08455\mathbf{i} + 0.7046\mathbf{j} - 0.7046\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\mathbf{l}_{AC} = -0.08455T_{AC}\mathbf{i} + 0.7046T_{AC}\mathbf{j} - 0.7046T_{AC}\mathbf{k} \quad (3)$$

**Equilibrium Condition.** Since A is in equilibrium, we must have

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{P} + \mathbf{W} = 0$$

or, substituting from (1), (2), (3) for the forces and factoring  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$ ,

$$\begin{aligned}(-0.09330T_{AB} - 0.08455T_{AC} + P)\mathbf{i} \\ + (0.7775T_{AB} + 0.7046T_{AC} - 1962 \text{ N})\mathbf{j} \\ + (0.6220T_{AB} - 0.7046T_{AC})\mathbf{k} = 0\end{aligned}$$

Setting the coefficients of  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$  equal to zero, we write three scalar equations, which express that the sums of the  $x$ ,  $y$ , and  $z$  components of the forces are respectively equal to zero.

$$(\Sigma F_x = 0): \quad -0.09330T_{AB} - 0.08455T_{AC} + P = 0$$

$$(\Sigma F_y = 0): \quad +0.7775T_{AB} + 0.7046T_{AC} - 1962 \text{ N} = 0$$

$$(\Sigma F_z = 0): \quad +0.6220T_{AB} - 0.7046T_{AC} = 0$$

Solving these equations, we obtain

$$P = 235 \text{ N} \quad T_{AB} = 1402 \text{ N} \quad T_{AC} = 1238 \text{ N}$$

# SOLVING PROBLEMS ON YOUR OWN

We saw earlier that when a particle is in *equilibrium*, the resultant of the forces acting on the particle must be zero. Expressing this fact in the case of the equilibrium of a *particle in three-dimensional space* will provide you with three relations among the forces acting on the particle. These relations may be used to determine three unknowns—usually the magnitudes of three forces.

Your solution will consist of the following steps:

- 1. Draw a free-body diagram of the particle.** This diagram shows the particle and all the forces acting on it. Indicate on the diagram the magnitudes of known forces, as well as any angles or dimensions that define the direction of a force. Any unknown magnitude or angle should be denoted by an appropriate symbol. Nothing else should be included in your free-body diagram.
- 2. Resolve each of the forces into rectangular components.** Following the method used in the preceding lesson, you will determine for each force  $\mathbf{F}$  the unit vector  $\mathbf{\lambda}$  defining the direction of that force and express  $\mathbf{F}$  as the product of its magnitude  $F$  and the unit vector  $\mathbf{\lambda}$ . You will obtain an expression of the form

$$\mathbf{F} = F\mathbf{L} = \frac{F}{d} (d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k})$$

where  $d$ ,  $d_x$ ,  $d_y$ , and  $d_z$  are dimensions obtained from the free-body diagram of the particle. If a force is known in magnitude as well as in direction, then  $F$  is known and the expression obtained for  $\mathbf{F}$  is well defined; otherwise  $F$  is one of the three unknowns that should be determined.

- 3. Set the resultant, or sum, of the forces exerted on the particle equal to zero.** You will obtain a vectorial equation consisting of terms containing the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$ , or  $\mathbf{k}$ . You will group the terms containing the same unit vector and factor that vector. For the vectorial equation to be satisfied, the coefficient of each of the unit vectors must be set equal to zero. This will yield three scalar equations that you can solve for no more than three unknowns [Sample Prob. 2.9].

# PROBLEMS

## FREE BODY PRACTICE PROBLEMS

- 2.F5** A 36-lb triangular plate is supported by three cables as shown. Draw the free-body diagram needed to determine the tension in each wire.

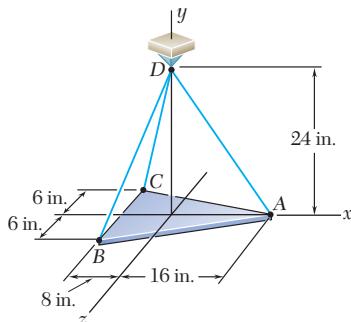


Fig. P2.F5

- 2.F6** A 70-kg cylinder is supported by two cables  $AC$  and  $BC$ , which are attached to the top of vertical posts. A horizontal force  $P$ , perpendicular to the plane containing the posts, holds the cylinder in the position shown. Draw the free-body diagram needed to determine the magnitude of  $P$  and the force in each cable.

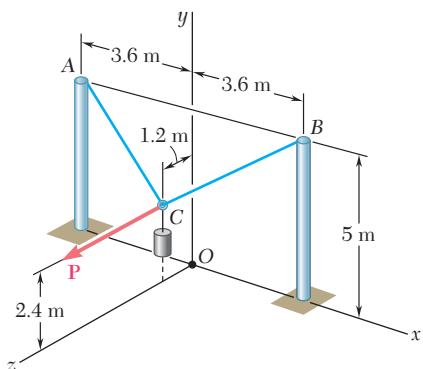


Fig. P2.F6

- 2.F7** Three cables are connected at point  $D$ , which is located 18 in. below the T-shaped pipe support  $ABC$ . The cables support a 180-lb cylinder as shown. Draw the free-body diagram needed to determine the tension in each cable.

- 2.F8** A 100-kg container is suspended from ring  $A$ , to which cables  $AC$  and  $AE$  are attached. A force  $P$  is applied to end  $F$  of a third cable that passes over a pulley at  $B$  and through ring  $A$  and then is attached to a support at  $D$ . Draw the free-body diagram needed to determine the magnitude of  $P$ . (Hint: The tension is the same in all portions of cable  $FBAD$ .)

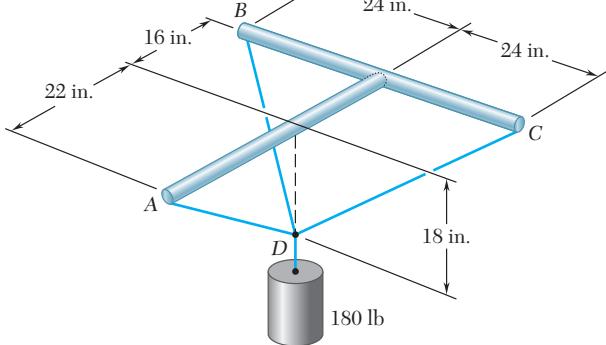


Fig. P2.F7

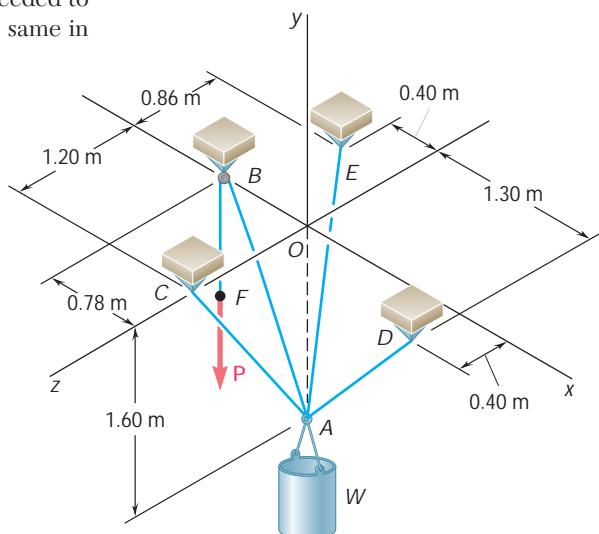


Fig. P2.F8

## END-OF-SECTION PROBLEMS

- 2.99** A container is supported by three cables that are attached to a ceiling as shown. Determine the weight  $W$  of the container, knowing that the tension in cable  $AB$  is 6 kN.

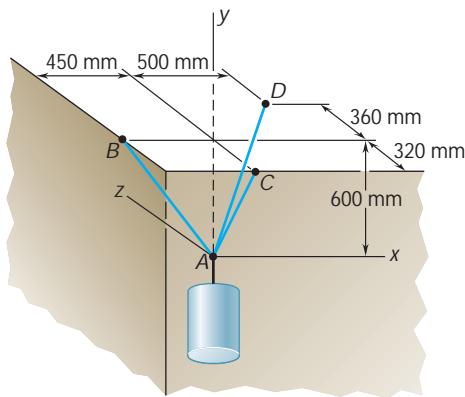


Fig. P2.99 and P2.100

- 2.100** A container is supported by three cables that are attached to a ceiling as shown. Determine the weight  $W$  of the container, knowing that the tension in cable  $AD$  is 4.3 kN.

- 2.101** Three cables are used to tether a balloon as shown. Determine the vertical force  $\mathbf{P}$  exerted by the balloon at  $A$  knowing that the tension in cable  $AD$  is 481 N.

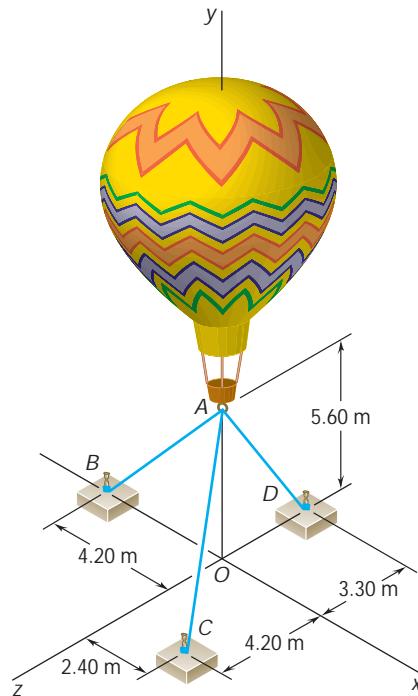


Fig. P2.101 and P2.102

- 2.102** Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800-N vertical force at  $A$ , determine the tension in each cable.

- 2.103** A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable  $AB$  is 750 lb.

- 2.104** A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable  $AD$  is 616 lb.

- 2.105** A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable  $AC$  is 544 lb.

- 2.106** A 1600-lb crate is supported by three cables as shown. Determine the tension in each cable.

- 2.107** Three cables are connected at  $A$ , where the forces  $\mathbf{P}$  and  $\mathbf{Q}$  are applied as shown. Knowing that  $Q = 0$ , find the value of  $P$  for which the tension in cable  $AD$  is 305 N.

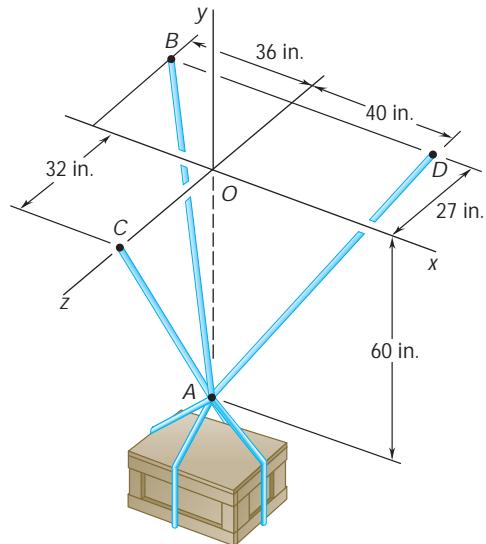


Fig. P2.103, P2.104, P2.105, and P2.106

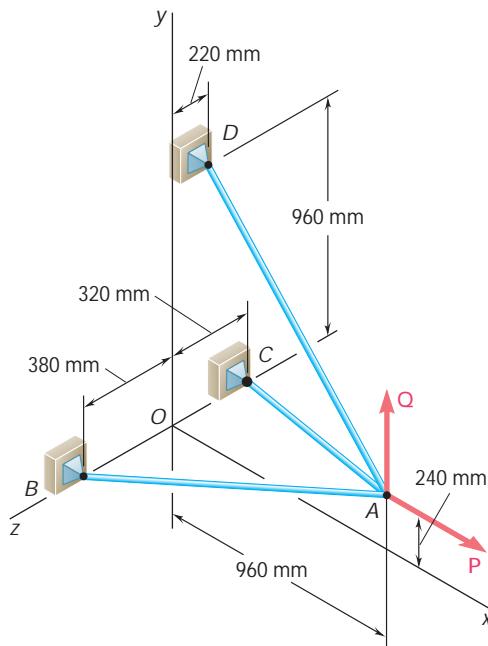


Fig. P2.107 and P2.108

- 2.108** Three cables are connected at  $A$ , where the forces  $\mathbf{P}$  and  $\mathbf{Q}$  are applied as shown. Knowing that  $P = 1200$  N, determine the values of  $Q$  for which cable  $AD$  is taut.

- 2.109** A rectangular plate is supported by three cables as shown. Knowing that the tension in cable  $AC$  is 60 N, determine the weight of the plate.

- 2.110** A rectangular plate is supported by three cables as shown. Knowing that the tension in cable  $AD$  is 520 N, determine the weight of the plate.

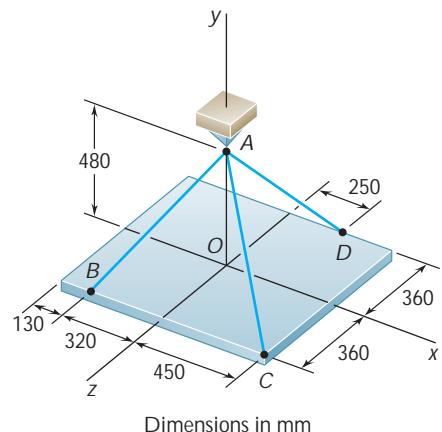
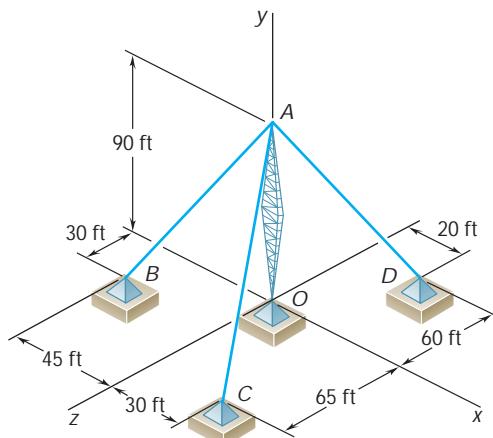


Fig. P2.109 and P2.110

- 2.111** A transmission tower is held by three guy wires attached to a pin at *A* and anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AB* is 630 lb, determine the vertical force **P** exerted by the tower on the pin at *A*.



**Fig. P2.111 and P2.112**

- 2.112** A transmission tower is held by three guy wires attached to a pin at *A* and anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AC* is 920 lb, determine the vertical force **P** exerted by the tower on the pin at *A*.

- 2.113** In trying to move across a slippery icy surface, a 180-lb man uses two ropes *AB* and *AC*. Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

- 2.114** Solve Prob. 2.113, assuming that a friend is helping the man at *A* by pulling on him with a force **P** =  $-(60 \text{ lb})\mathbf{k}$ .

- 2.115** For the rectangular plate of Probs. 2.109 and 2.110, determine the tension in each of the three cables knowing that the weight of the plate is 792 N.

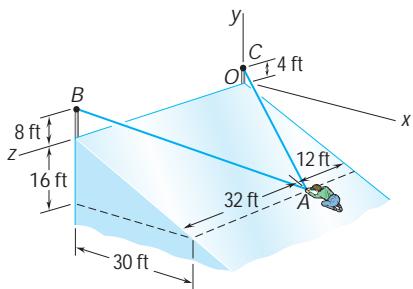
- 2.116** For the cable system of Probs. 2.107 and 2.108, determine the tension in each cable knowing that  $P = 2880 \text{ N}$  and  $Q = 0$ .

- 2.117** For the cable system of Probs. 2.107 and 2.108, determine the tension in each cable knowing that  $P = 2880 \text{ N}$  and  $Q = 576 \text{ N}$ .

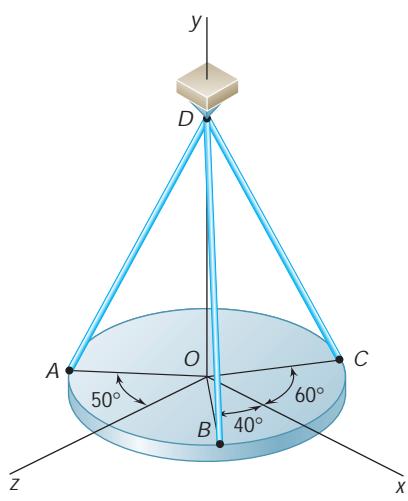
- 2.118** For the cable system of Probs. 2.107 and 2.108, determine the tension in each cable knowing that  $P = 2880 \text{ N}$  and  $Q = -576 \text{ N}$  (**Q** is directed downward).

- 2.119** For the transmission tower of Probs. 2.111 and 2.112, determine the tension in each guy wire knowing that the tower exerts on the pin at *A* an upward vertical force of 2100 lb.

- 2.120** A horizontal circular plate weighing 60 lb is suspended as shown from three wires that are attached to a support at *D* and form  $30^\circ$  angles with the vertical. Determine the tension in each wire.



**Fig. P2.113**



**Fig. P2.120**

- 2.121** Cable  $BAC$  passes through a frictionless ring  $A$  and is attached to fixed supports at  $B$  and  $C$ , while cables  $AD$  and  $AE$  are both tied to the ring and are attached, respectively, to supports at  $D$  and  $E$ . Knowing that a 200-lb vertical load  $\mathbf{P}$  is applied to ring  $A$ , determine the tension in each of the three cables.

- 2.122** Knowing that the tension in cable  $AE$  of Prob. 2.121 is 75 lb, determine (a) the magnitude of the load  $\mathbf{P}$ , (b) the tension in cables  $BAC$  and  $AD$ .

- 2.123** A container of weight  $W$  is suspended from ring  $A$ . Cable  $BAC$  passes through the ring and is attached to fixed supports at  $B$  and  $C$ . Two forces  $\mathbf{P} = P\mathbf{i}$  and  $\mathbf{Q} = Q\mathbf{k}$  are applied to the ring to maintain the container in the position shown. Knowing that  $W = 376$  N, determine  $P$  and  $Q$ . (Hint: The tension is the same in both portions of cable  $BAC$ .)

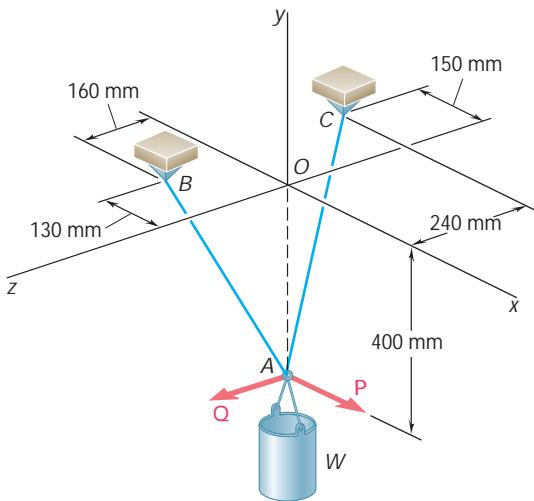


Fig. P2.123

- 2.124** For the system of Prob. 2.123, determine  $W$  and  $Q$  knowing that  $P = 164$  N.

- 2.125** Collars  $A$  and  $B$  are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force  $\mathbf{P} = (341 \text{ N})\mathbf{j}$  is applied to collar  $A$ , determine (a) the tension in the wire when  $y = 155$  mm, (b) the magnitude of the force  $\mathbf{Q}$  required to maintain the equilibrium of the system.

- 2.126** Solve Prob. 2.125 assuming that  $y = 275$  mm.

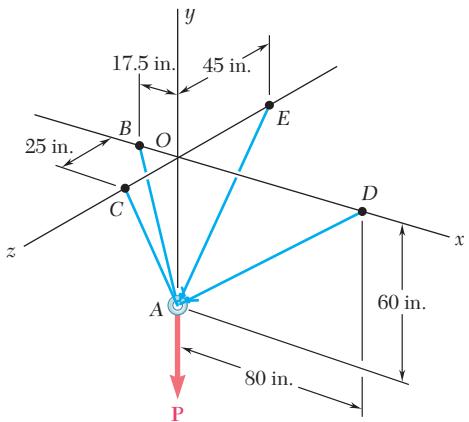


Fig. P2.121

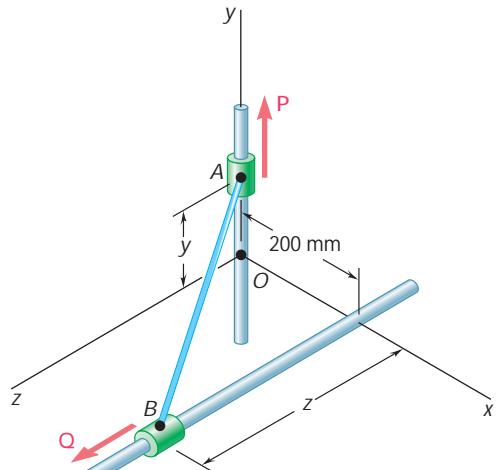


Fig. P2.125

# REVIEW AND SUMMARY

In this chapter we have studied the effect of forces on particles, i.e., on bodies of such shape and size that all forces acting on them may be assumed applied at the same point.

## Resultant of two forces

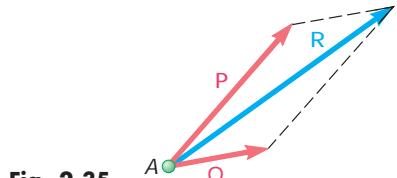


Fig. 2.35

## Components of a force

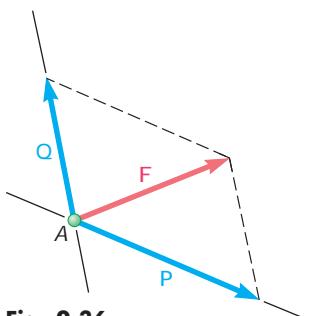


Fig. 2.36

## Rectangular components Unit vectors

Forces are *vector quantities*; they are characterized by a *point of application*, a *magnitude*, and a *direction*, and they add according to the *parallelogram law* (Fig. 2.35). The magnitude and direction of the resultant **R** of two forces **P** and **Q** can be determined either graphically or by trigonometry, using successively the law of cosines and the law of sines [Sample Prob. 2.1].

Any given force acting on a particle can be resolved into two or more *components*, i.e., it can be replaced by two or more forces which have the same effect on the particle. A force **F** can be resolved into two components **P** and **Q** by drawing a parallelogram which has **F** for its diagonal; the components **P** and **Q** are then represented by the two adjacent sides of the parallelogram (Fig. 2.36) and can be determined either graphically or by trigonometry [Sec. 2.6].

A force **F** is said to have been resolved into two *rectangular components* if its components  $F_x$  and  $F_y$  are perpendicular to each other and are directed along the coordinate axes (Fig. 2.37). Introducing the *unit vectors* **i** and **j** along the *x* and *y* axes, respectively, we write [Sec. 2.7]

$$\mathbf{F}_x = F_x \mathbf{i} \quad \mathbf{F}_y = F_y \mathbf{j} \quad (2.6)$$

and

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} \quad (2.7)$$

where  $F_x$  and  $F_y$  are the *scalar components* of **F**. These components, which can be positive or negative, are defined by the relations

$$F_x = F \cos \alpha \quad F_y = F \sin \alpha \quad (2.8)$$

When the rectangular components  $F_x$  and  $F_y$  of a force **F** are given, the angle  $\alpha$  defining the direction of the force can be obtained by writing

$$\tan \alpha = \frac{F_y}{F_x} \quad (2.9)$$

The magnitude  $F$  of the force can then be obtained by solving one of the equations (2.8) for  $F$  or by applying the Pythagorean theorem and writing

$$F = \sqrt{F_x^2 + F_y^2} \quad (2.10)$$

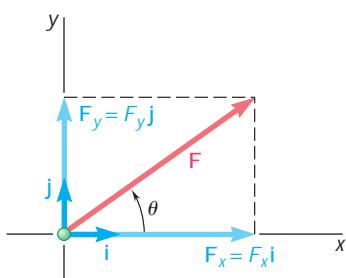


Fig. 2.37

When *three or more coplanar forces* act on a particle, the rectangular components of their resultant  $\mathbf{R}$  can be obtained by adding algebraically the corresponding components of the given forces [Sec. 2.8]. We have

$$R_x = \sum F_x \quad R_y = \sum F_y \quad (2.13)$$

The magnitude and direction of  $\mathbf{R}$  can then be determined from relations similar to Eqs. (2.9) and (2.10) [Sample Prob. 2.3].

A force  $\mathbf{F}$  in *three-dimensional space* can be resolved into rectangular components  $\mathbf{F}_x$ ,  $\mathbf{F}_y$ , and  $\mathbf{F}_z$  [Sec. 2.12]. Denoting by  $u_x$ ,  $u_y$ , and  $u_z$ , respectively, the angles that  $\mathbf{F}$  forms with the  $x$ ,  $y$ , and  $z$  axes (Fig. 2.38), we have

$$F_x = F \cos u_x \quad F_y = F \cos u_y \quad F_z = F \cos u_z \quad (2.19)$$

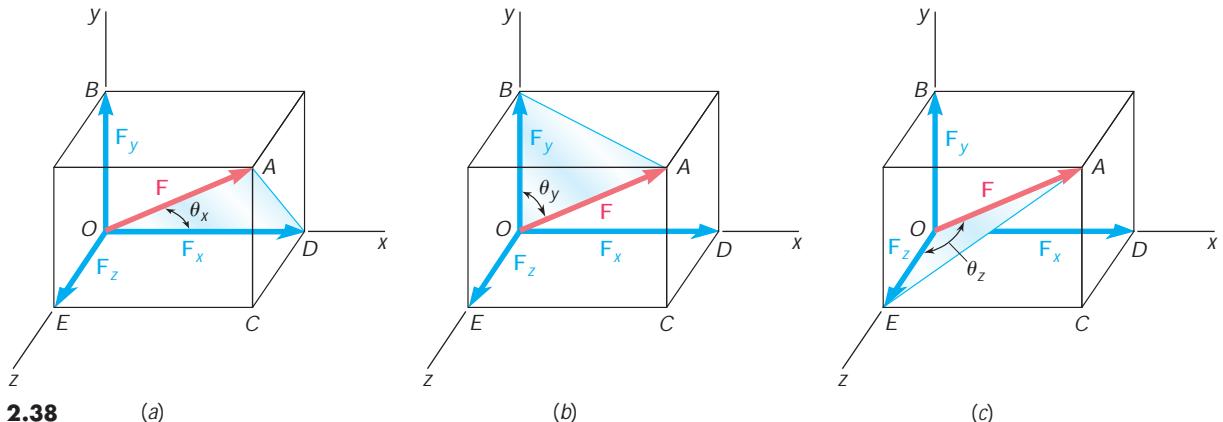


Fig. 2.38 (a) (b) (c)

The cosines of  $u_x$ ,  $u_y$ ,  $u_z$  are known as the *direction cosines* of the force  $\mathbf{F}$ . Introducing the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$  along the coordinate axes, we write

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k} \quad (2.20)$$

or

$$\mathbf{F} = F(\cos u_x \mathbf{i} + \cos u_y \mathbf{j} + \cos u_z \mathbf{k}) \quad (2.21)$$

which shows (Fig. 2.39) that  $\mathbf{F}$  is the product of its magnitude  $F$  and the unit vector

$$\boldsymbol{\lambda} = \cos u_x \mathbf{i} + \cos u_y \mathbf{j} + \cos u_z \mathbf{k}$$

Since the magnitude of  $\boldsymbol{\lambda}$  is equal to unity, we must have

$$\cos^2 u_x + \cos^2 u_y + \cos^2 u_z = 1 \quad (2.24)$$

When the rectangular components  $F_x$ ,  $F_y$ ,  $F_z$  of a force  $\mathbf{F}$  are given, the magnitude  $F$  of the force is found by writing

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \quad (2.18)$$

and the direction cosines of  $\mathbf{F}$  are obtained from Eqs. (2.19). We have

$$\cos u_x = \frac{F_x}{F} \quad \cos u_y = \frac{F_y}{F} \quad \cos u_z = \frac{F_z}{F} \quad (2.25)$$

## Resultant of several coplanar forces

### Forces in space

### Direction cosines

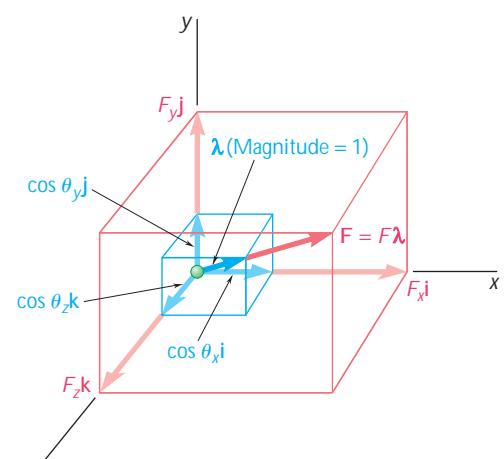


Fig. 2.39

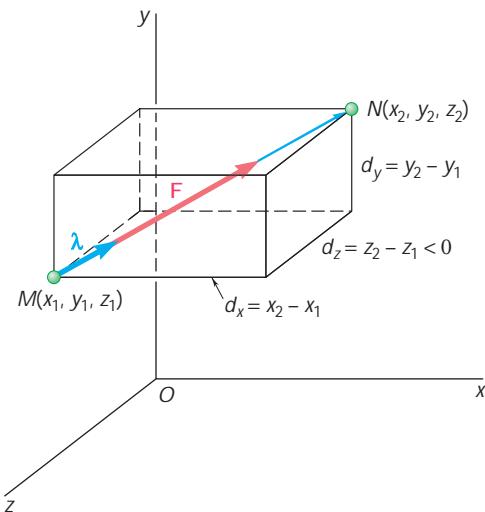


Fig. 2.40

### Resultant of forces in space

When a force  $\mathbf{F}$  is defined in three-dimensional space by its magnitude  $F$  and two points  $M$  and  $N$  on its line of action [Sec. 2.13], its rectangular components can be obtained as follows. We first express the vector  $\overrightarrow{MN}$  joining points  $M$  and  $N$  in terms of its components  $d_x$ ,  $d_y$ , and  $d_z$  (Fig. 2.40); we write

$$\overrightarrow{MN} = d_x \mathbf{i} + d_y \mathbf{j} + d_z \mathbf{k} \quad (2.26)$$

We next determine the unit vector  $\lambda$  along the line of action of  $\mathbf{F}$  by dividing  $\overrightarrow{MN}$  by its magnitude  $MN = d$ :

$$\lambda = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d}(d_x \mathbf{i} + d_y \mathbf{j} + d_z \mathbf{k}) \quad (2.27)$$

Recalling that  $\mathbf{F}$  is equal to the product of  $F$  and  $\lambda$ , we have

$$\mathbf{F} = F\lambda = \frac{F}{d}(d_x \mathbf{i} + d_y \mathbf{j} + d_z \mathbf{k}) \quad (2.28)$$

from which it follows [Sample Probs. 2.7 and 2.8] that the scalar components of  $\mathbf{F}$  are, respectively,

$$F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d} \quad (2.29)$$

When *two or more forces* act on a particle in *three-dimensional space*, the rectangular components of their resultant  $\mathbf{R}$  can be obtained by adding algebraically the corresponding components of the given forces [Sec. 2.14]. We have

$$R_x = \Sigma F_x \quad R_y = \Sigma F_y \quad R_z = \Sigma F_z \quad (2.31)$$

The magnitude and direction of  $\mathbf{R}$  can then be determined from relations similar to Eqs. (2.18) and (2.25) [Sample Prob. 2.8].

A particle is said to be in *equilibrium* when the resultant of all the forces acting on it is zero [Sec. 2.9]. The particle will then remain at rest (if originally at rest) or move with constant speed in a straight line (if originally in motion) [Sec. 2.10].

To solve a problem involving a particle in equilibrium, one first should draw a *free-body diagram* of the particle showing all the forces acting on it [Sec. 2.11]. If *only three coplanar forces* act on the particle, a *force triangle* may be drawn to express that the particle is in equilibrium. Using graphical methods of trigonometry, this triangle can be solved for no more than two unknowns [Sample Prob. 2.4]. If *more than three coplanar forces* are involved, the equations of equilibrium

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad (2.15)$$

should be used. These equations can be solved for no more than two unknowns [Sample Prob. 2.6].

When a particle is in *equilibrium in three-dimensional space* [Sec. 2.15], the three equations of equilibrium

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0 \quad (2.34)$$

should be used. These equations can be solved for no more than three unknowns [Sample Prob. 2.9].

### Equilibrium of a particle

### Free-body diagram

### Equilibrium in space

# REVIEW PROBLEMS

- 2.127** Two structural members *A* and *B* are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 15 kN in member *A* and 10 kN in member *B*, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members *A* and *B*.

- 2.128** Member *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 300-lb horizontal component, determine (a) the magnitude of the force **P**, (b) its vertical component.

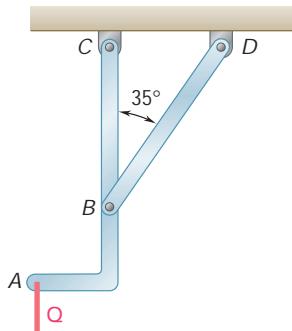


Fig. P2.128

- 2.129** Determine (a) the required tension in cable *AC*, knowing that the resultant of the three forces exerted at point *C* of boom *BC* must be directed along *BC*, (b) the corresponding magnitude of the resultant.

- 2.130** Two cables are tied together at *C* and are loaded as shown. Determine the tension (a) in cable *AC*, (b) in cable *BC*.

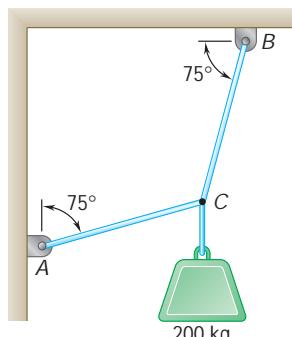


Fig. P2.130

- 2.131** A welded connection is in equilibrium under the action of the four forces shown. Knowing that  $F_A = 8 \text{ kN}$  and  $F_B = 16 \text{ kN}$ , determine the magnitudes of the other two forces.

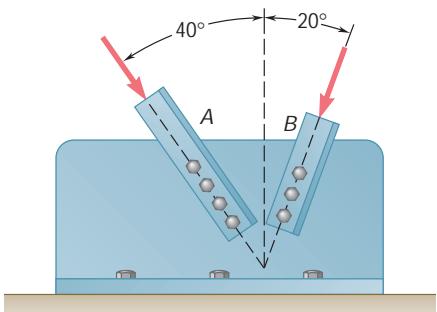


Fig. P2.127

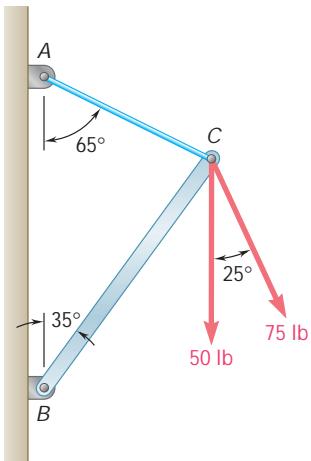


Fig. P2.129

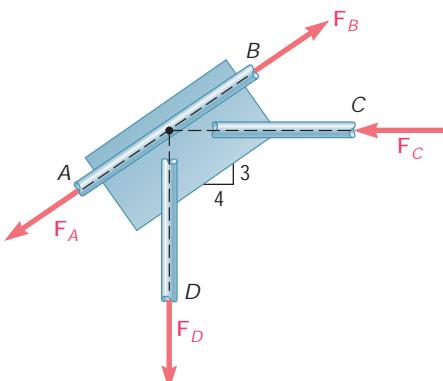


Fig. P2.131

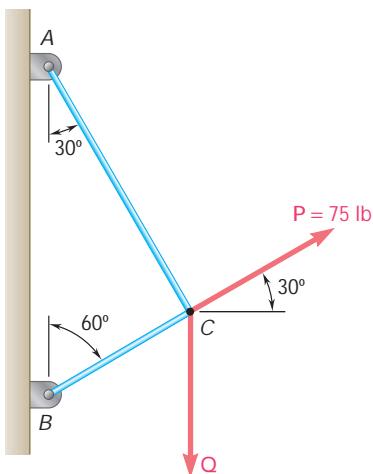


Fig. P2.132

**2.132** Two cables tied together at  $C$  are loaded as shown. Determine the range of values of  $Q$  for which the tension will not exceed 60 lb in either cable.

**2.133** A horizontal circular plate is suspended as shown from three wires that are attached to a support at  $D$  and form 30° angles with the vertical. Knowing that the  $x$  component of the force exerted by wire  $AD$  on the plate is 110.3 N, determine (a) the tension in wire  $AD$ , (b) the angles  $u_x$ ,  $u_y$ , and  $u_z$  that the force exerted at  $A$  forms with the coordinate axes.

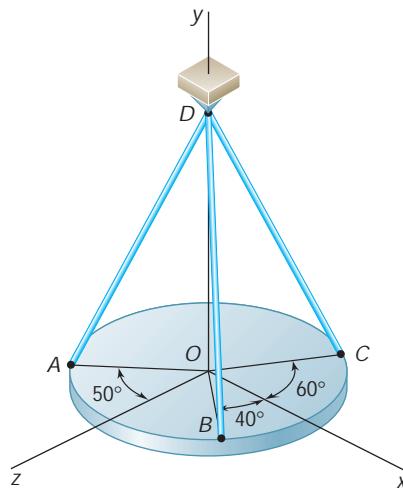


Fig. P2.133

**2.134** A force acts at the origin of a coordinate system in a direction defined by the angles  $u_y = 55^\circ$  and  $u_z = 45^\circ$ . Knowing that the  $x$  component of the force is  $-500$  lb, determine (a) the angle  $u_x$ , (b) the other components and the magnitude of the force.

**2.135** Find the magnitude and direction of the resultant of the two forces shown knowing that  $P = 300$  N and  $Q = 400$  N.

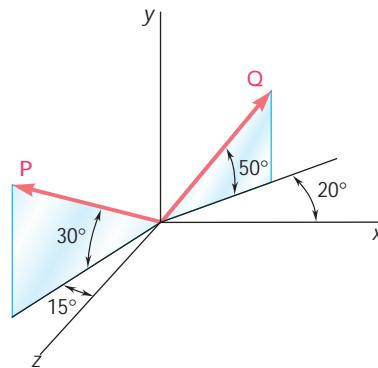


Fig. P2.135

- 2.136** Three cables are used to tether a balloon as shown. Determine the vertical force  $\mathbf{P}$  exerted by the balloon at  $A$  knowing that the tension in cable  $AC$  is 444 N.

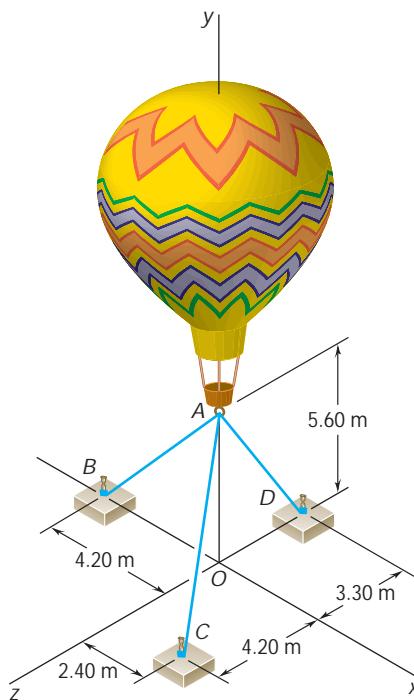


Fig. P2.136

- 2.137** Collars  $A$  and  $B$  are connected by a 25-in.-long wire and can slide freely on frictionless rods. If a 60-lb force  $\mathbf{Q}$  is applied to collar  $B$  as shown, determine (a) the tension in the wire when  $x = 9$  in., (b) the corresponding magnitude of the force  $\mathbf{P}$  required to maintain the equilibrium of the system.

- 2.138** Collars  $A$  and  $B$  are connected by a 25-in.-long wire and can slide freely on frictionless rods. Determine the distances  $x$  and  $z$  for which the equilibrium of the system is maintained when  $P = 120$  lb and  $Q = 60$  lb.

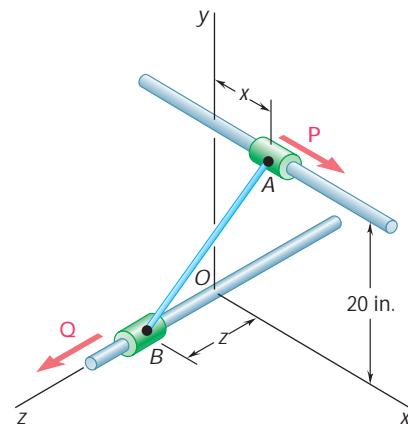


Fig. P2.137 and P2.138

# COMPUTER PROBLEMS

**2.C1** Write a computer program that can be used to determine the magnitude and direction of the resultant of  $n$  coplanar forces applied at a point A. Use this program to solve Probs. 2.32, 2.33, 2.35, and 2.38.

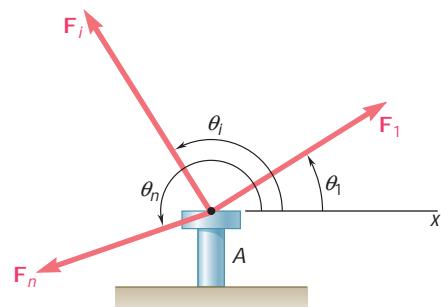


Fig. P2.C1

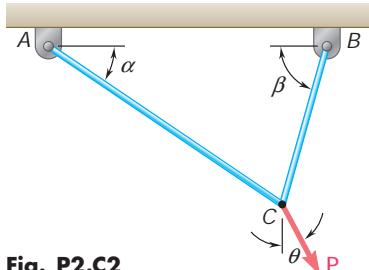


Fig. P2.C2

**2.C2** A load  $\mathbf{P}$  is supported by two cables as shown. Write a computer program that can be used to determine the tension in each cable for any given value of  $P$  and for values of  $\theta$  ranging from  $\theta_1 = b - 90^\circ$  to  $\theta_2 = 90^\circ - a$ , using given increments  $\Delta\theta$ . Use this program to determine for the following three sets of numerical values (a) the tension in each cable for values of  $\theta$  ranging from  $\theta_1$  to  $\theta_2$ , (b) the value of  $\theta$  for which the tension in the two cables is as small as possible, (c) the corresponding value of the tension:

- (1)  $a = 35^\circ$ ,  $b = 75^\circ$ ,  $P = 400$  lb,  $\Delta\theta = 5^\circ$
- (2)  $a = 50^\circ$ ,  $b = 30^\circ$ ,  $P = 600$  lb,  $\Delta\theta = 10^\circ$
- (3)  $a = 40^\circ$ ,  $b = 60^\circ$ ,  $P = 250$  lb,  $\Delta\theta = 5^\circ$

**2.C3** An acrobat is walking on a tightrope of length  $L = 20.1$  m attached to supports A and B at a distance of 20.0 m from each other. The combined weight of the acrobat and his balancing pole is 800 N, and the friction between his shoes and the rope is large enough to prevent him from slipping. Neglecting the weight of the rope and any elastic deformation, write a computer program to calculate the deflection  $y$  and the tension in portions AC and BC of the rope for values of  $x$  from 0.5 m to 10.0 m using 0.5-m increments. From the data obtained, determine (a) the maximum deflection of the rope, (b) the maximum tension in the rope, (c) the smallest values of the tension in portions AC and BC of the rope.

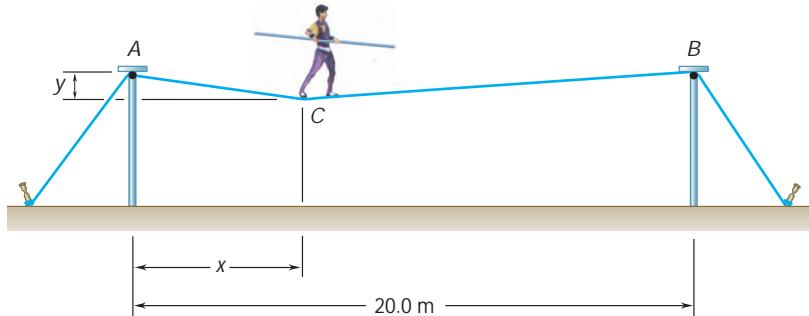


Fig. P2.C3

**2.C4** Write a computer program that can be used to determine the magnitude and direction of the resultant of  $n$  forces  $\mathbf{F}_i$ , where  $i = 1, 2, \dots, n$ , that are applied at point  $A_0$  of coordinates  $x_0, y_0$ , and  $z_0$ , knowing that the line of action of  $\mathbf{F}_i$  passes through point  $A_i$  of coordinates  $x_i, y_i$ , and  $z_i$ . Use this program to solve Probs. 2.93, 2.94, and 2.95.

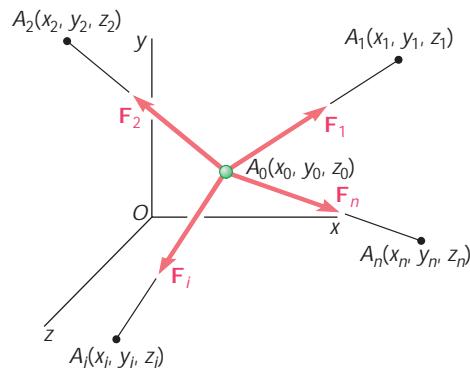


Fig. P2.C4

**2.C5** Three cables are attached at points  $A_1, A_2$ , and  $A_3$ , respectively, and are connected at point  $A_0$ , to which a given load  $\mathbf{P}$  is applied as shown. Write a computer program that can be used to determine the tension in each of the cables. Use this program to solve Probs. 2.102, 2.106, 2.107, and 2.115.

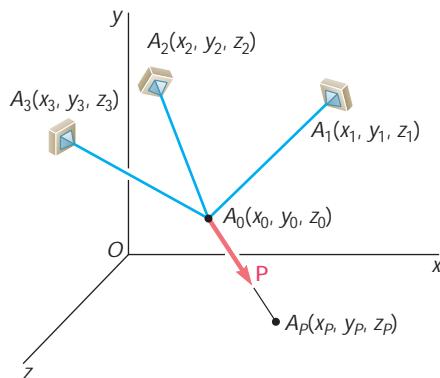


Fig. P2.C5