Probability Basics

Probability provides a mathematical framework to quantify uncertainty. It helps us measure how likely an event is to occur.

Key Terminology

- **Experiment**: A process that produces an outcome (e.g., tossing a coin).
- **Outcome**: A single possible result (e.g., getting heads).
- **Event**: A set of outcomes (e.g., getting an even number on a die roll).
- **Sample Space (S)**: The set of all possible outcomes.

Formula

$$P(A) = \frac{Number\ of\ Favourable\ Outcome}{Total\ Number\ of\ Favourable\ Outcomes}$$

1. Basic Probability Rules

A. Addition Rule

- For **mutually exclusive events** (cannot happen together):
  - $P(A \cup B) = P(A) + P(B)$

- For **non-mutually exclusive events:**
  - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- **Example:** Rolling a die → Probability of 2 or 3 = 1/6 + 1/6 = 2/6

B. Multiplication Rule

- For **independent events**:
  - $P(A \cap B) = P(A) \times P(B)$

- For **dependent events**:
  - $P(A \cap B) = P(A) \times P(B|A)$

- **Example:** Drawing 2 aces from a deck without replacement.

C. Complement Rule

- The probability of an event **not happening**:
  - $P(A^c) = 1 - P(A)$

- **Example:** Probability of not rolling a 6 on a die = 1 – 1/6 = 5/6

2. Conditional Probability

Conditional probability is the probability of event A given that event B has occurred.

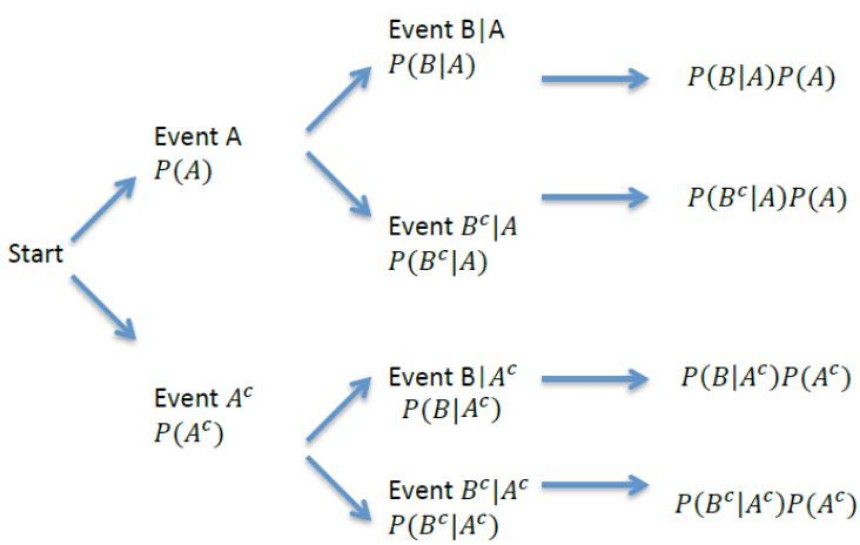
Formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example:

- Probability a person is left-handed given they are an artist.

Tree diagram:



3. Bayes’ Theorem

A powerful rule that allows us to update probabilities based on new information.

Formula:

$$P(A|B) = \frac{P(A \bigcap B)}{P(B)} = \frac{P(A) \cdot P(B|A)}{P(B)}$$

where:

$P(A) =$ The probability of A occurring

$P(B) =$ The probability of B occurring

$P(A|B) =$ The probability of A given B

$P(B|A) =$ The probability of B given A

$P\left(A \bigcap B\right)) =$ The probability of both A and B occurri

Example:

Medical Testing

- Disease prevalence = 1%
- Test sensitivity = 99%
- Test specificity = 95%

Question: If a person tests positive, what is the probability they actually have the disease?

(Shows how prior knowledge updates posterior probability.)

P(Disease | Positive) = (0.99×0.01) / ((0.99×0.01) + (0.05×0.99)) ≈ 16.7%

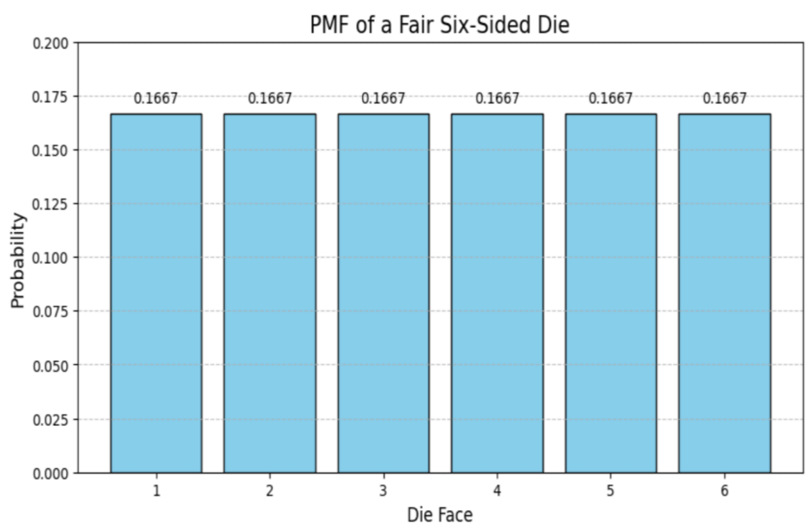
4. Random Variables

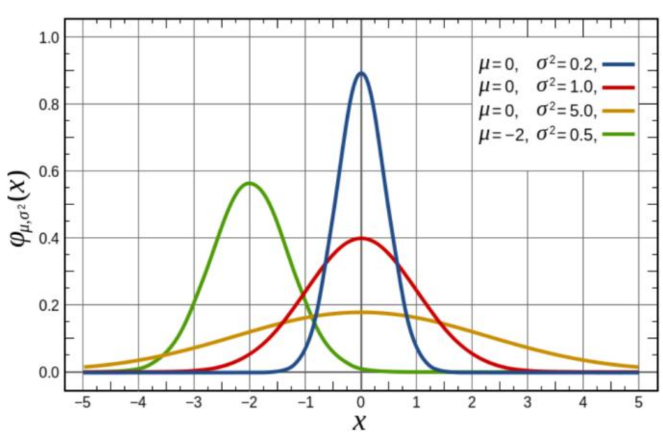
Discrete Random Variables

- Take countable values.
- Represented by **Probability Mass Function (PMF)**.
- Example: Number of heads in 5 coin tosses.

Continuous Random Variables

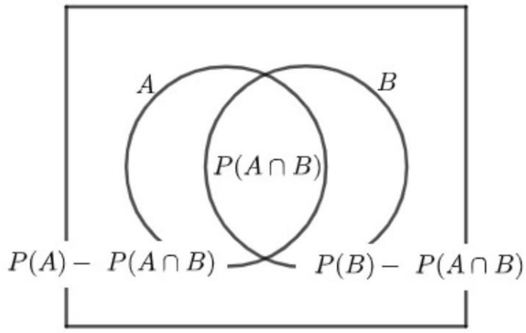
- Take infinitely many values in a range.
- Represented by **Probability Density Function (PDF)**.
- Example: Height of students.



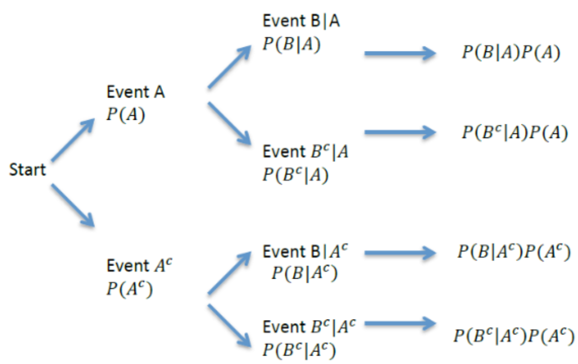


5. Visual Examples

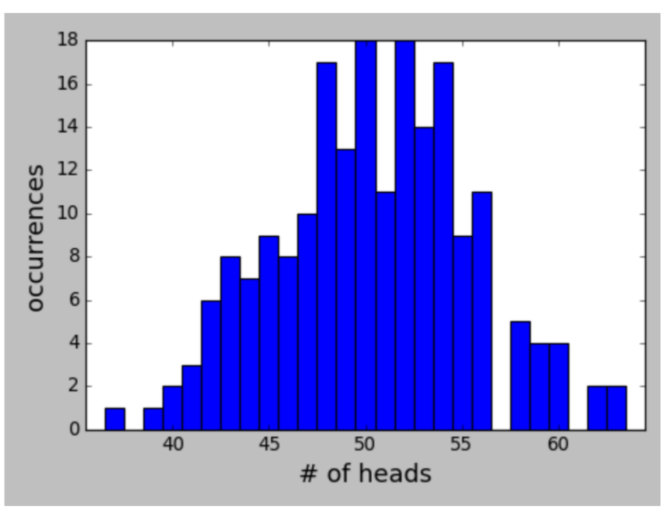
- **Venn Diagram**: For addition & intersection of events.



- **Tree Diagram:** For conditional probability.



- **PMF Graph: Coin toss distribution.**



6. Common Mistakes

- Assuming events are independent when they are not.
- Forgetting to subtract overlap in the addition rule.
- Misinterpreting conditional probability.
- Believing all real-world data follows a normal distribution.