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# Stock Selection Case Study

**Efficient Frontier - Harry Markowitz**

## Introduction:

- The financial-analytical team will be using Markowitz's Efficient Frontier Portfolio optimization model. The purpose of this portfolio is to have an investor, buy a combination of the three stocks Alpha, Bravo, and Charlie. The first step in deciding where to invest your money, is how strongly your beliefs stand in the future performances of the securities.This means to invest your money to where you believe your money will grow. So, we must consider what the investor desires using the Markowitz's Portfolio model.

## Problem Statement:

- The financial-analytical team needed to present to an investor the optimal combination of the three stocks, Alpha, Bravo and Charlie. To create this portfolio they were told to apply Markowitz's Efficient Frontier Portfolio optimization model. With these three stocks (Alpha, Bravo and Charlie) in mind, the investor's goal is to invest their money with the lowest amount of risk. Using these three stocks, they needed to develop an optimal or feasible solution. Since there is a minimum return of 10%, the investor wants this portfolio to include the proportion of the investments to be made across the three stocks. These investments must sum together to a total of 1.0 (or 100%). The overall goal for the team to figure out is: to present the investor some combination of the three stocks that gives at least a 10% return at the lowest possible risk. Questions in addition include: how much does the investor want to invest, how to maximize your profit and how to minimize your risk with securities.

❖ **Data Sources:**

➢ The source of the data is the "Building a Portfolio Optimization Model". This is a given document from the Professor.

➢ Another given document is the "Portfolio Selection" by Harry Markowitz in The Journal of Finance (Mar.1952) pp (77-91).

❖ **Data Description:**

➢ Since the financial-analytical team is utilizing the Markowitz efficient frontier, we needed to pick the stocks (Alpha, Bravo and Charlie) that give the portfolio with the minimum variance for a given return and maximum return for a given variance. A portfolio's variance depends on how each individual stock's price fluctuations are correlated with every other stock's, known as the covariance of returns.

➢ First the Financial- analytical team created a spreadsheet using data from the stocks. To display our portfolio in a simple and cohesive way is to use colors from certain cells. Expected Return yellow, Risk Variance of return orange, Variance-covariance matrix green, Portfolio Weight blue, Minimum Expected Return yellow, Portfolio Return orange, and risk variance gray. When developing an equation to define our objective which includes the data cells and the changing cells. By identifying the objective cell and the changing cells using Solver from the Data tab. We selected the cell to be optimized in the set objective window. We then choose to minimize the objective cell. Then to select changing cells in the by changing variable cells windows. Then in the solving method tab we did the GRG Nonlinear. The nonlinear Solver is used to solve nonlinear problems.

❖ ***I. verbal***

Decision variables need to be made on the levels subject to these number of activities. The activities in this problem are the three stocks Alpha, Bravo and Charlie. Then, the decision variables for this portfolio are the optimal weights, the variance-covariance matrix of returns and the expected return. With these decision variables, this is how you obtain the objective function to reach the goal to minimize risk in your portfolio.

For problems that are non linear programming, the objective function/goal can include cross product terms involving the product of two or more decision variables. The nature of this objective function is nonlinear because it has more than 2 variables. In this case we have the ultimate decision variable which is to obtain the optimal combined weights of the three stocks (Alpha, Bravo and Charlie) for this portfolio. In order to obtain the optimal weights of each stock, we must first have the covariance matrix.

As what the data has described before the portfolio's variance, depends on how each stock's price fluctuates are correlated with every other stock's (the covariance of the returns). You also need the expected return of each stock to calculate the optimal weights of each stock. The expected return for Alpha is 6.2% with a risk variance of return of 1.46%. The expected return of Bravo is 14.6% with a risk variance of return of 8.54%. The expected return of Charlie is 12.8% with a risk variance of return of 2.89%.

Keeping in mind the variances and the expected return we need these activity levels to be greater or equal to zero in order to satisfy the constraints. Each constraint described a restriction on the feasible values for the levels of the activities. Constraints can have a mathematical sign to show the constraint that you would like to achieve. One constraint in this problem is to have a minimum expected return greater than 10%. Second constraint in this portfolio is that the total weights of each stock must sum to a total of 1.0 or (100%). The decisions on activity levels are to be based on the overall measure of performance which is the objective cell/function of the portfolio. The nature of the constraints, is that they are integers because they are continuous. In general the goal is to either maximize or minimize the objective. In this portfolio our goal/objective is to minimize risk subject to our constraints.

- ***ii. Mathematically***
  - **Algebraic Model For Building a Portfolio Optimization Model:**
    - The financial-analytical team approach is to formulate a mathematical representation that expresses in symbols and expressions that describes the essence of Markowitz's Portfolio optimization method. In terms of making the following decision;

    - Decision to be made: The purchase of the combination of 3 stocks (*Let X1 = Alpha Stock Let, X2 = Bravo Stock, X3 = Charlie Stock* ) to build a portfolio that minimizes the risk of obtaining a high return $\geq$ 10%.

| |
|---|
| **Total Portfolio Weight** = Alpha Weight + Bravo Weight + Charlie Weight |

***The total portfolio weight must sum to (100%)***

Following the objective function and since the number is not yet known,an algebraic variable **R(*V*)** is a representation of this quantity. Thus,

**R(V)=** Minimum Possible Risk.

Where R(V) is referred to as a decision variable. Then, using the data given in "Building a Portfolio Optimization Model" the problem can be presented in the following form; where the values of the covariance-matrix and portfolio weights, will help the team to minimize the risk at the highest return.

**Risk (Variance) =** (0.0146*Alpha Weight*Alpha Weight)+ (0.0187*Alpha Weigh*Bravo Weight) + (0.0145*Alpha Weight*Charlie Weight)+( 0.0187*Bravo Weight*Alpha Weight) + (0.0854*Bravo Weight*Bravo Weight) + (0.0104*Bravo Weight*Charlie Weight)+( 0.0145*Charlie Weight*Alpha Weight) + (0.0104*Charlie Weight*Bravo Weight) + (0.0289*Charlie Weight*Charlie Weight).

Subject to satisfying all the following constraints where

- Total Weights = 1
- Minimum Expected Return $\geq$ 10%

And each stock weight (Alpha, Bravo, and Charlie) has to be $\geq$ 0.

### ❖ ***iii. In a Spreadsheet***

Figure 1.1 interprets the data from the mathematical model before running solver function. In other words, the mathematical model helped us to allocate all the information necessary in the worksheet to obtain the results.

**Markowitz's Portfolio Optimization Method (Nonlinear Programming)**

| | Alpha | Bravo | Charlie |
|---|---|---|---|
| Expected Return | 0.062 | 0.146 | 0.128 |

| | | | |
|---|---|---|---|
| Risk Variance of Returns | 1.46% | 8.54% | 2.89% |

| Variance-Covariance Matrix | Alpha | Bravo | Charlie |
|---|---|---|---|
| Alpha | 0.0146 | 0.0187 | 0.0145 |
| Bravo | 0.0187 | 0.0854 | 0.0104 |
| Charlie | 0.0145 | 0.0104 | 0.0289 |

| | Alpha | Bravo | Charlie | | Total |
|---|---|---|---|---|---|
| Portfolio Weights | 0.333333 | 0.333333 | 0.3333333 | | 1 |

| | Minimum Expected Return | | Portfolio Return |
|---|---|---|---|
| Expected Return | 10% | ≤ | 11% |

| | |
|---|---|
| Risk (Variance) | 2.401% |

| Range Name | Cells |
|---|---|
| Expected Return | E5:G5 |
| Risk Variance of Returns | E7:G7 |
| Variance-Covariance Matrix | E11:G13 |
| Portfolio Weights | D16:F16 |
| Total Portfolio Weights | H15 |
| Minimum Expected Rerturn | D21 |
| Portfolio Return | F21 |
| Risk (Variance) | D23 |

**Solver Parameters**

**Set Objective Cell**: Minumum Risk

**To**: Min

**By Changing Variables Cell**

Portfolio Wieghts

**Subject to the Constrainst:**

PortfolioReturn >= MinimumExpectedReturn

TotalWeights = 1

**Solver Function:**

Make Variables Nonnegative

Solving Method: GRG Nonlinear

Figure 1.1

❖ **Results: what is the outcome of your analysis across the different models?**

Figure 1.2 shows the corresponding spreadsheet model after having applied Solver. For ease of interpretation, the changing cells Portfolio Weights (D16:F16) gives the values of Alpha, Bravo, and Charlie as percentages. These cells indicate that the optimal solution for the investor, is to purchase some combinations of the 3 stocks built in a portfolio that gives at least 10% (F21) at the lowest possible risk 1.814% (D23) and which the weight of three stocks was the following,

- X1 = 45.26%, Allocate 45.6% of the portfolio to Alpha Stock.
- X2 = 10.40%, Allocate 10.40% of the portfolio to Bravo Stock.
- X3 = 44.34%, Allocate 44.34% of the portfolio to Charlie Stock.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | |
| 2 | | **Markowitz's Portfolio Optimization Method (Nonlinear Programming)** | | | | | | |
| 3 | | | | | | | | |
| 4 | | | | | Alpha | Bravo | Charlie | |
| 5 | | | Expected Return | | 0.062 | 0.146 | 0.128 | |
| 6 | | | | | | | | |
| 7 | | | Risk Variance of Returns | | 1.46% | 8.54% | 2.89% | |
| 8 | | | | | | | | |
| 9 | | | Variance-Covariance Matrix | | | | | |
| 10 | | | | | Alpha | Bravo | Charlie | |
| 11 | | | | Alpha | 0.0146 | 0.0187 | 0.0145 | |
| 12 | | | | Bravo | 0.0187 | 0.0854 | 0.0104 | |
| 13 | | | | Charlie | 0.0145 | 0.0104 | 0.0289 | |
| 14 | | | | | | | | |
| 15 | | | | Alpha | Bravo | Charlie | | Total |
| 16 | | Portfolio Weights | | 45.26% | 10.40% | 44.34% | | 100% |
| 17 | | | | | | | | |
| 18 | | | | Minimum | | | | |
| 19 | | | | Expected | | Portfolio | | |
| 20 | | | | Return | | Return | | |
| 21 | | Expected Return | | 10% | ≤ | 10.00% | | |
| 22 | | | | | | | | |
| 23 | | Risk (Variance) | | 1.814% | | | Figure 1.2 | |

| Range Name | Cells |
|---|---|
| Expected Return | E5:G5 |
| Risk Variance of Returns | E7:G7 |
| Variance-Covariance Matrix | E11:G13 |
| Portfolio Weights | D16:F16 |
| Total Portfolio Weights | H15 |
| Minimum Expected Rerturn | D21 |
| Portfolio Return | F21 |
| Risk (Variance) | D23 |

| Solver Parameters |
|---|
| **Set Objective Cell**: Minumum Risk |
| **To**: Min |
| **By Changing Variables Cell** |
| Portfolio Wieghts |
| **Subject to the Constraint:** |
| PortfolioReturn >= MinimumExpectedReturn |
| TotalWeights = 1 |
| |
| **Solver Function:** |
| Make Variables Nonnegative |
| Solving Method: GRG Nonlinear |

Then, the answer report (Figure 1.3) demonstrates the initial and final values of the objective (Risk Variance) and all variables (Portfolio Weights). This report also reveals the optimal values for all constraints, including a note suggesting which constraints were binding. The slack on a constraint tells you how far away a constraint is from becoming a binding constraint.

**Microsoft Excel 16.0 Answer Report**
**Worksheet: [Project 2-QNT 2020.xlsx]Investement Portfolio 2**
**Report Created: 12/11/2020 10:11:13 PM**
**Result: Solver found a solution. All Constraints and optimality conditions are satisfied.**
**Solver Engine**
Engine: GRG Nonlinear
Solution Time: 0.25 Seconds.
Iterations: 4 Subproblems: 0
**Solver Options**
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Convergence 0.0001, Population Size 100, Random Seed 0, Derivatives Forward, Require Bounds
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

| Cell | Name | Original Value | Final Value |
|---|---|---|---|
| $D$23 | MinimumRisk | 2.401% | 1.814% |

Variable Cells

| Cell | Name | Original Value | Final Value | Integer |
|---|---|---|---|---|
| $D$16 | Portfolio Weights Alpha | 33.33% | 45.26% | Contin |
| $E$16 | Portfolio Weights Bravo | 33.33% | 10.40% | Contin |
| $F$16 | Portfolio Weights Charlie | 33.33% | 44.34% | Contin |

Constraints

| Cell | Name | Cell Value | Formula | Status | Slack |
|---|---|---|---|---|---|
| $F$21 | PortfolioReturn | 10.00% | $F$21>=$D$21 | Binding | 0.00% |
| $H$16 | TotalWeights | 100% | $H$16=1 | Binding | 0 |

Figure 1.3

- Conclusion:

  - The Financial-analytical team determined that the investor would invest a certain amount of money in each stock. If the investor invests a million dollars toward stocks, we will recommend the investor to invest;$452,600.00 (45.26%) to Alpha, $104,00 (10.40%) to Bravo, $443,400.00 (44.34%) to Charlie. All the currency should be a sum total of $1M just like the percentages of each stock should sum to 100%. Overall this is the recommendation that we recommend given the financial information available. If the financial circumstances change, our recommendation will change as well.