2024-2025 - IngéSUP - Technical Systems

Final Exam - Mechanical Systems

Time: 2 hours

ESME Bordeaux-Lille-Lyon-Paris

First Name/ Last Name

____

Responses will be reported exclusively on the response document. No other document will be corrected. Be as concise as possible and use the available spacing wisely.

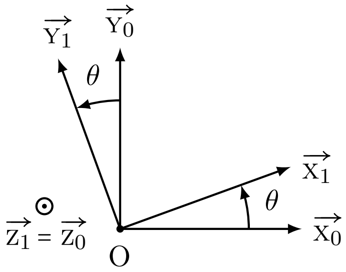
Use of documents is not authorized.

Means of calculation not authorized.

Any candidate who detects what he/she thinks was a statement error must indicate the means and initiatives he/she takes to continue his/her work.

Exercise 1 : Dot and cross products (2pts)

Let a direct orthonormal frame $R_1(O, \overrightarrow{X_1}, \overrightarrow{Y_1}, \overrightarrow{Z_1})$ be oriented at an angle θ relatively to a second direct orthonormal frame $R_0(O, \overrightarrow{X_0}, \overrightarrow{Y_0}, \overrightarrow{Z_0})$ such that θ is the angle between the basis vectors $(\overrightarrow{X_0}, \overrightarrow{X_1})$ and $(\overrightarrow{Y_0}, \overrightarrow{Y_1})$ and such that $\overrightarrow{Z_0} = \overrightarrow{Z_1}$.



Q1. Give the result of the following three dot products: $\overrightarrow{X_0} \cdot \overrightarrow{X_1}$, $\overrightarrow{X_1} \cdot \overrightarrow{Y_1}$ and $\overrightarrow{Z_0} \cdot \overrightarrow{Z_1}$ [0.5 pt]

Q2. Give the result of the following three cross products: $\overrightarrow{X_0} \times \overrightarrow{Y_1}$, $\overrightarrow{X_1} \times \overrightarrow{Y_1}$ and $\overrightarrow{Z_0} \times \overrightarrow{Z_1}$ [0.5 pt]

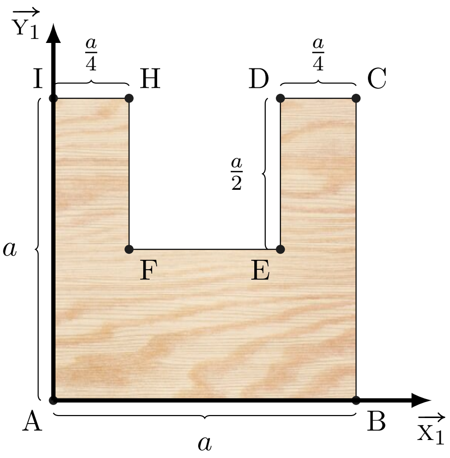
We define a position vector $\overrightarrow{OA}$ such that $\boldsymbol{\overrightarrow{OA} = \overrightarrow{X_1} + 2\overrightarrow{Y_1}}$

Q3. Give the result of the folowing three dot products: $\overrightarrow{OA} \cdot \overrightarrow{X_0}$, $\overrightarrow{OA} \cdot \overrightarrow{Y_0}$ and $\overrightarrow{OA} \cdot \overrightarrow{X_1}$ [0.5 pt]

Q4. Give the result of the folowing three cross products: $\overrightarrow{OA} \times \overrightarrow{X_0}$, $\overrightarrow{OA} \times \overrightarrow{Y_0}$ and $\overrightarrow{OA} \times \overrightarrow{X_1}$ [0.5 pt]

Exercise 2 : Vector calculus and center of mass (5pts)

We study a wooden plate (of negligible thickness) of homogeneous surface mass. The mass geometry will be given in the associated frame $R_1(\mathrm{A}, \overrightarrow{\mathrm{X_1}}, \overrightarrow{\mathrm{Y_1}}, \overrightarrow{\mathrm{Z_1}})$ as shown below. Reminder: the mass of a plate of surface S of this material will be given by $m = \sigma S$, where σ is the surface mass.



Q1. Give the position vectors of points C, D and F in the frame $R_1(\mathrm{A}, \overrightarrow{\mathrm{X_1}}, \overrightarrow{\mathrm{Y_1}}, \overrightarrow{\mathrm{Z_1}})$ [1 pt]

Q2. Give the total mass of the plate as a function of the plate parameters, σ and a [1 pt]

We recall that the center of mass $\overrightarrow{\mathrm{AG}}$ of a set of solids $\mathcal{S}_i$ of mass m_i and center of mass G_i is given by:

$$\overrightarrow{\mathrm{AG}} = \frac{1}{M} \sum_i m_i \overrightarrow{\mathrm{AG}_i}.$$

For example, a solid composed of two rectangular plates has a center of mass given by the position vector $\overrightarrow{\mathrm{AG}} = \dfrac{1}{M}\left(m_1\overrightarrow{\mathrm{AG}_1} + m_2\overrightarrow{\mathrm{AG}_2}\right)$, where $M = m_1 + m_2$.

Q3. Give the center of mass of the plate, represented by the position vector $\overrightarrow{\mathrm{AG}}$, in the frame $R_1(\mathrm{A}, \overrightarrow{\mathrm{X}_1}, \overrightarrow{\mathrm{Y}_1}, \overrightarrow{\mathrm{Z}_1})$ [1 pt]

We now consider the rotation of this plate around the axis $(\mathrm{A}, \overrightarrow{\mathrm{Z}_0})$ of a fixed frame $R_0(\mathrm{A}, \overrightarrow{\mathrm{X}_0}, \overrightarrow{\mathrm{Y}_0}, \overrightarrow{\mathrm{Z}_0})$. We will denote θ, the angle between the basis vectors $(\overrightarrow{\mathrm{X}_0}, \overrightarrow{\mathrm{X}_1})$ and $(\overrightarrow{\mathrm{Y}_0}, \overrightarrow{\mathrm{Y}_1})$.

Q4. Draw the plane figure to represent this rotation. [1 pt]

Q5. From this representation, give the position vector $\overrightarrow{\mathrm{AG}}$ in the fixed frame $R_0(\mathrm{A}, \overrightarrow{\mathrm{X}_0}, \overrightarrow{\mathrm{Y}_0}, \overrightarrow{\mathrm{Z}_0})$. [1 pt]