

***17–24.**

The door has a weight of 200 lb and a center of gravity at G . Determine how far the door moves in 2 s, starting from rest, if a man pushes on it at C with a horizontal force $F = 30$ lb. Also, find the vertical reactions at the rollers A and B .

SOLUTION

$$\pm \Sigma F_x = m(a_G)_x; \quad 30 = \left(\frac{200}{32.2}\right)a_G$$

$$a_G = 4.83 \text{ ft/s}^2$$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_k)_A; \quad N_B(12) - 200(6) + 30(9) = \left(\frac{200}{32.2}\right)(4.83)(7)$$

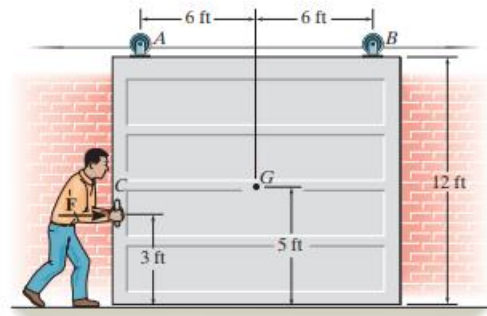
$$N_B = 95.0 \text{ lb}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 95.0 - 200 = 0$$

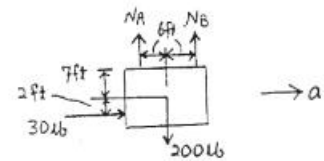
$$N_A = 105 \text{ lb}$$

$$(\pm) \quad s = s_0 + v_0 t + \frac{1}{2}a_G t^2$$

$$s = 0 + 0 + \frac{1}{2}(4.83)(2)^2 = 9.66 \text{ ft}$$



Ans.



Ans.

Ans.

17-43.

Determine the acceleration of the 150-lb cabinet and the normal reaction under the legs A and B if $P = 35$ lb. The coefficients of static and kinetic friction between the cabinet and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively. The cabinet's center of gravity is located at G .

SOLUTION

Equations of Equilibrium: The free-body diagram of the cabinet under the static condition is shown in Fig. a , where P is the unknown minimum force needed to move the cabinet. We will assume that the cabinet slides before it tips. Then, $F_A = \mu_s N_A = 0.2N_A$ and $F_B = \mu_s N_B = 0.2N_B$.

$$\rightarrow \Sigma F_x = 0; \quad P - 0.2N_A - 0.2N_B = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad N_A + N_B - 150 = 0 \quad (2)$$

$$+\Sigma M_A = 0; \quad N_B(2) - 150(1) - P(4) = 0 \quad (3)$$

Solving Eqs. (1), (2), and (3) yields

$$P = 30 \text{ lb} \quad N_A = 15 \text{ lb} \quad N_B = 135 \text{ lb}$$

Since $P < 35$ lb and N_A is positive, the cabinet will slide.

Equations of Motion: Since the cabinet is in motion, $F_A = \mu_k N_A = 0.15N_A$ and $F_B = \mu_k N_B = 0.15N_B$. Referring to the free-body diagram of the cabinet shown in Fig. b ,

$$\rightarrow \Sigma F_x = m(a_G)_x; \quad 35 - 0.15N_A - 0.15N_B = \left(\frac{150}{32.2}\right)a \quad (4)$$

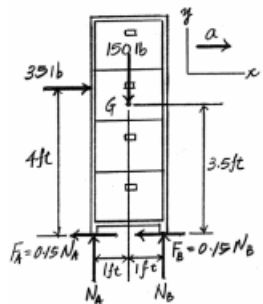
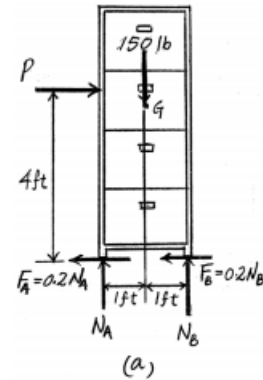
$$\rightarrow \Sigma F_x = m(a_G)_x; \quad N_A + N_B - 150 = 0 \quad (5)$$

$$+\Sigma M_G = 0; \quad N_B(1) - 0.15N_B(3.5) - 0.15N_A(3.5) - N_A(1) - 35(0.5) = 0 \quad (6)$$

Solving Eqs. (4), (5), and (6) yields

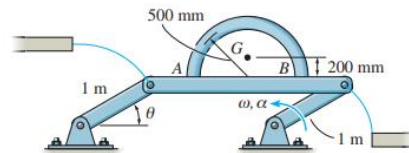
$$a = 2.68 \text{ ft/s}^2 \quad \text{Ans.}$$

$$N_A = 26.9 \text{ lb} \quad N_B = 123 \text{ lb} \quad \text{Ans.}$$



17-53.

The arched pipe has a mass of 80 kg and rests on the surface of the platform. As it is hoisted from one level to the next, $\alpha = 0.25 \text{ rad/s}^2$ and $\omega = 0.5 \text{ rad/s}$ at the instant $\theta = 30^\circ$. If it does not slip, determine the normal reactions of the arch on the platform at this instant.



SOLUTION

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 80(9.81) = 20 \sin 60^\circ - 20 \cos 60^\circ$$

$$N_A + N_B = 792.12$$

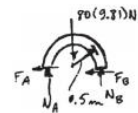
$$\zeta + \Sigma M_A = \Sigma (M_k)_A; \quad N_B(1) - 80(9.81)(0.5) = 20 \cos 60^\circ(0.2) + 20 \sin 60^\circ(0.5) - 20 \cos 60^\circ(0.5) + 20 \sin 60^\circ(0.2)$$

$$N_B = 402 \text{ N}$$

Ans.

$$N_A = 391 \text{ N}$$

Ans.



$$80(9.81)(1) = 20$$

$$30(0.5)(1) = 20$$

17-59.

The uniform slender rod has a mass m . If it is released from rest when $\theta = 0^\circ$, determine the magnitude of the reactive force exerted on it by pin B when $\theta = 90^\circ$.

SOLUTION

Equations of Motion: Since the rod rotates about a fixed axis passing through point B , $(a_G)_t = \alpha r_G = \alpha \left(\frac{L}{6}\right)$ and $(a_G)_n = \omega^2 r_G = \omega^2 \left(\frac{L}{6}\right)$. The mass moment of inertia of the rod about its G is $I_G = \frac{1}{12}mL^2$. Writing the moment equation of motion about point B ,

$$+\Sigma M_B = \Sigma (M_k)_B; \quad -mg \cos \theta \left(\frac{L}{6}\right) = -m \left[\alpha \left(\frac{L}{6}\right) \right] \left(\frac{L}{6}\right) - \left(\frac{1}{12}mL^2\right) \alpha$$

$$\alpha = \frac{3g}{2L} \cos \theta$$

This equation can also be obtained by applying $\Sigma M_B = I_B \alpha$, where $I_B = \frac{1}{12}mL^2 + m \left(\frac{L}{6}\right)^2 = \frac{1}{9}mL^2$. Thus,

$$+\Sigma M_B = I_B \alpha; \quad -mg \cos \theta \left(\frac{L}{6}\right) = -\left(\frac{1}{9}mL^2\right) \alpha$$

$$\alpha = \frac{3g}{2L} \cos \theta$$

Using this result and writing the force equation of motion along the n and t axes,

$$\Sigma F_t = m(a_G)_t; \quad mg \cos \theta - B_t = m \left[\left(\frac{3g}{2L} \cos \theta\right) \left(\frac{L}{6}\right) \right]$$

$$B_t = \frac{3}{4}mg \cos \theta \quad (1)$$

$$\Sigma F_n = m(a_G)_n; \quad B_n - mg \sin \theta = m \left[\omega^2 \left(\frac{L}{6}\right) \right]$$

$$B_n = \frac{1}{6}m\omega^2 L + mg \sin \theta \quad (2)$$

Kinematics: The angular velocity of the rod can be determined by integrating

$$\int \omega d\omega = \int \alpha d\theta$$

$$\int_0^\omega \omega d\omega = \int_0^\theta \frac{3g}{2L} \cos \theta d\theta$$

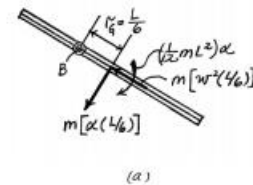
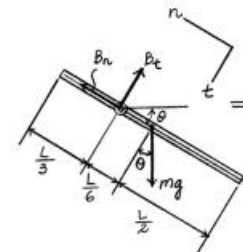
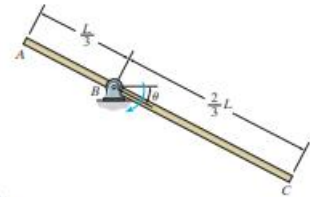
$$\omega = \sqrt{\frac{3g}{L} \sin \theta}$$

When $\theta = 90^\circ$, $\omega = \sqrt{\frac{3g}{L}}$. Substituting this result and $\theta = 90^\circ$ into Eqs. (1) and (2),

$$B_t = \frac{3}{4}mg \cos 90^\circ = 0$$

$$B_n = \frac{1}{6}m \left(\frac{3g}{L}\right) L + mg \sin 90^\circ = \frac{3}{2}mg$$

$$F_B = \sqrt{B_t^2 + B_n^2} = \sqrt{0^2 + \left(\frac{3}{2}mg\right)^2} = \frac{3}{2}mg \quad \text{Ans.}$$



17-79.

The two blocks A and B have a mass of 5 kg and 10 kg, respectively. If the pulley can be treated as a disk of mass 3 kg and radius 0.15 m, determine the acceleration of block A . Neglect the mass of the cord and any slipping on the pulley.



SOLUTION

Kinematics: Since the pulley rotates about a fixed axis passes through point O , its angular acceleration is

$$\alpha = \frac{a}{r} = \frac{a}{0.15} = 6.6667a$$

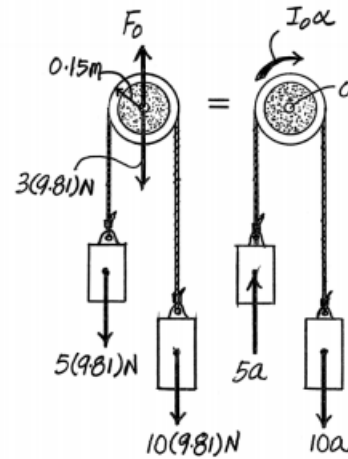
The mass moment of inertia of the pulley about point O is

$$I_o = \frac{1}{2}Mr^2 = \frac{1}{2}(3)(0.15^2) = 0.03375 \text{ kg} \cdot \text{m}^2$$

Equation of Motion: Write the moment equation of motion about point O by referring to the free-body and kinetic diagram of the system shown in Fig. a ,

$$\begin{aligned} \zeta + \Sigma M_o &= \Sigma (M_k)_o; & 5(9.81)(0.15) - 10(9.81)(0.15) \\ & & = -0.03375(6.6667a) - 5a(0.15) - 10a(0.15) \\ a &= 2.973 \text{ m/s}^2 = 2.97 \text{ m/s}^2 \end{aligned}$$

Ans.



*17-96.

The spool has a mass of 100 kg and a radius of gyration of $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 50$ N.

SOLUTION

$$\rightarrow \Sigma F_x = m(a_G)_x; \quad 50 + F_A = 100a_G$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 100(9.81) = 0$$

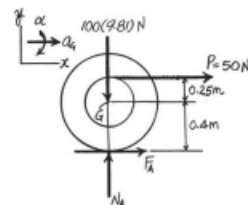
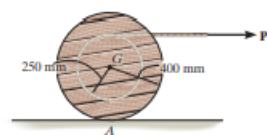
$$\zeta + \Sigma M_G = I_G \alpha; \quad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$$

Assume no slipping: $a_G = 0.4\alpha$

$$\alpha = 1.30 \text{ rad/s}^2$$

$$a_G = 0.520 \text{ m/s}^2 \quad N_A = 981 \text{ N} \quad F_A = 2.00 \text{ N}$$

Since $(F_A)_{\max} = 0.2(981) = 196.2 \text{ N} > 2.00 \text{ N}$



Ans.

OK

*17-120.

The 30-kg slender rod AB rests in the position shown when the horizontal force $P = 50 \text{ N}$ is applied. Determine the initial angular acceleration of the rod. Neglect the mass of the rollers.

SOLUTION

Equations of Motion: Here, the mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(30)(1.5^2) = 5.625 \text{ kg} \cdot \text{m}^2$. Writing the moment equations of motion about the intersection point A of the lines of action of \mathbf{N}_A and \mathbf{N}_B and using Fig. a ,

$$+\Sigma M_A = \Sigma (M_k)_A; \quad -50(0.15) = 30(a_G)_x(0.75) - 5.625\alpha$$

$$5.625\alpha - 22.5(a_G)_x = 75 \quad (1)$$

Kinematics: Applying the relative acceleration equation to points A and G , Fig. b ,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/A}$$

$$(a_G)_x \mathbf{i} + (a_G)_y \mathbf{j} = -a_A \mathbf{j} + (-\alpha \mathbf{k}) \times (-0.75 \mathbf{j}) - 0$$

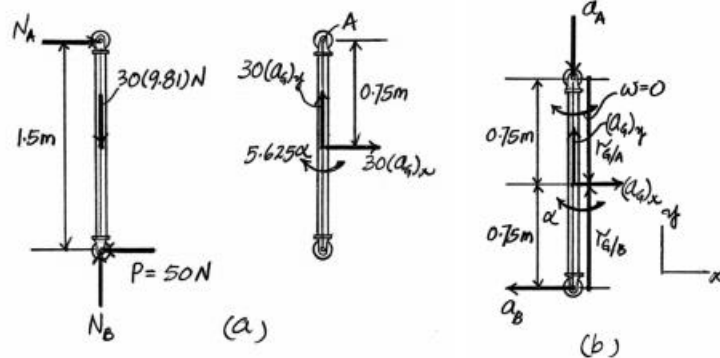
$$(a_G)_x \mathbf{i} + (a_G)_y \mathbf{j} = -0.75\alpha \mathbf{i} - a_A \mathbf{j}$$

Equating the i components,

$$(a_G)_x = -0.75\alpha \quad (2)$$

Substituting Eq. (2) into Eq. (1),

$$\alpha = 3.333 \text{ rad/s}^2 = 3.33 \text{ rad/s}^2 \quad \text{Ans.}$$



***18-8.**

The double pulley consists of two parts that are attached to one another. It has a weight of 50 lb and a centroidal radius of gyration of $k_O = 0.6$ ft and is turning with an angular velocity of 20 rad/s clockwise. Determine the angular velocity of the pulley at the instant the 20-lb weight moves 2 ft downward.

SOLUTION

Kinetic Energy and Work: Since the pulley rotates about a fixed axis, $v_A = \omega r_A = \omega(1)$ and $v_B = \omega r_B = \omega(0.5)$. The mass moment of inertia of the pulley about point O is $I_O = mk_O^2 = \left(\frac{50}{32.2}\right)(0.6^2) = 0.5590 \text{ slug} \cdot \text{ft}^2$. Thus, the kinetic energy of the system is

$$\begin{aligned} T &= \frac{1}{2} I_O \omega^2 + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \\ &= \frac{1}{2} (0.5590) \omega^2 + \frac{1}{2} \left(\frac{20}{32.2} \right) [\omega(1)]^2 + \frac{1}{2} \left(\frac{30}{32.2} \right) [\omega(0.5)]^2 \\ &= 0.7065 \omega^2 \end{aligned}$$

Thus, $T_1 = 0.7065(20^2) = 282.61 \text{ ft} \cdot \text{lb}$. Referring to the FBD of the system shown in Fig. *a*, we notice that \mathbf{O}_x , \mathbf{O}_y , and \mathbf{W}_P do no work while \mathbf{W}_A does positive work and \mathbf{W}_B does negative work. When A moves 2 ft downward, the pulley rotates

$$\begin{aligned} \theta &= \frac{S_A}{r_A} = \frac{S_B}{r_B} \\ \frac{2}{1} &= \frac{S_B}{0.5} \\ S_B &= 2(0.5) = 1 \text{ ft} \uparrow \end{aligned}$$

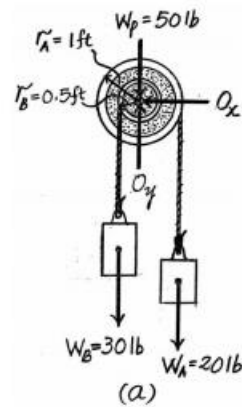
Thus, the work of \mathbf{W}_A and \mathbf{W}_B are

$$\begin{aligned} U_{W_A} &= W_A S_A = 20(2) = 40 \text{ ft} \cdot \text{lb} \\ U_{W_B} &= -W_B S_B = -30(1) = -30 \text{ ft} \cdot \text{lb} \end{aligned}$$

Principle of Work and Energy:

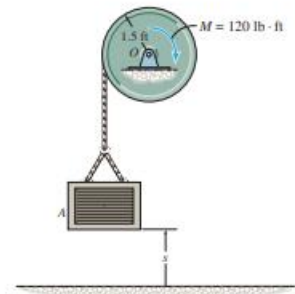
$$\begin{aligned} T_1 + U_{1 \rightarrow 2} &= T_2 \\ 282.61 + [40 + (-30)] &= 0.7065 \omega^2 \\ \omega &= 20.4 \text{ rad/s} \end{aligned}$$

Ans.



18-29.

A motor supplies a constant torque or twist of $M = 120 \text{ lb} \cdot \text{ft}$ to the drum. If the drum has a weight of 30 lb and a radius of gyration of $k_O = 0.8 \text{ ft}$, determine the speed of the 15-lb crate A after it rises $s = 4 \text{ ft}$ starting from rest. Neglect the mass of the cord.



SOLUTION

Free Body Diagram: The weight of the crate does *negative* work since it acts in the opposite direction to that of its displacement s_w . Also, the couple moment \mathbf{M} does positive work as it acts in the same direction of its angular displacement θ . The reactions O_x , O_y and the weight of the drum do no work since point O does not displace.

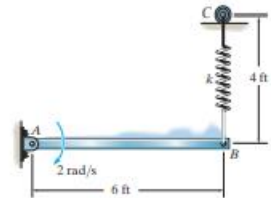
Kinematic: Since the drum rotates about point O , the angular velocity of the drum and the speed of the crate can be related by $\omega_D = \frac{v_A}{r_D} = \frac{v_A}{1.5} = 0.6667 v_A$. When the crate rises $s = 4 \text{ ft}$, the angular displacement of the drum is given by $\theta = \frac{s}{r_D} = \frac{4}{1.5} = 2.667 \text{ rad}$.

Principle of Work and Energy: The mass moment of inertia of the drum about point O is $I_O = mk_O^2 = \left(\frac{30}{32.2}\right)(0.8^2) = 0.5963 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 18-13, we have

$$\begin{aligned} T_1 + \sum U_{1-2} &= T_2 \\ 0 + M\theta - W_C s_C &= \frac{1}{2} I_O \omega^2 + \frac{1}{2} m_C v_C^2 \\ 0 + 120(2.667) - 15(4) &= \frac{1}{2} (0.5963)(0.6667 v_A)^2 + \frac{1}{2} \left(\frac{15}{32.2}\right) v_A^2 \\ v_A &= 26.7 \text{ ft/s} \end{aligned}$$

Ans.

At the instant shown, the 50-lb bar rotates clockwise at 2 rad/s . The spring attached to its end always remains vertical due to the roller guide at C . If the spring has an unstretched length of 2 ft and a stiffness of $k = 6 \text{ lb/ft}$, determine the angular velocity of the bar the instant it has rotated 30° clockwise.

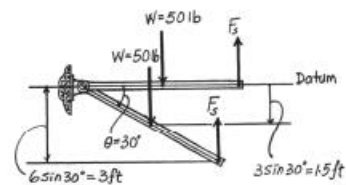


SOLUTION

Datum through A .

$$\begin{aligned} T_1 + V_1 &= T_2 + V_2 \\ \frac{1}{2} \left[\frac{1}{3} \left(\frac{50}{32.2} \right) (6)^2 \right] (2)^2 + \frac{1}{2} (6)(4 - 2)^2 &= \frac{1}{2} \left[\frac{1}{3} \left(\frac{50}{32.2} \right) (6)^2 \right] \omega^2 \\ &+ \frac{1}{2} (6)(7 - 2)^2 - 50(1.5) \\ \omega &= 2.30 \text{ rad/s} \end{aligned}$$

Ans.



18–47.

At the instant the spring becomes undeformed, the center of the 40-kg disk has a speed of 4 m/s. From this point determine the distance d the disk moves down the plane before momentarily stopping. The disk rolls without slipping.

SOLUTION

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} \left[\frac{1}{2} (40) (0.3)^2 \right] \left(\frac{4}{0.3} \right)^2 + \frac{1}{2} (40) (4)^2 + 40(9.81)d \sin 30^\circ = 0 + \frac{1}{2} (200) d^2$$

$$100d^2 - 196.2d - 480 = 0$$

Solving for the positive root

$$d = 3.38 \text{ m}$$

Ans.

