Problem ①

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Given : lead screw Mechanism

$D_{LS} = 14\,mm = 0.014\,m$

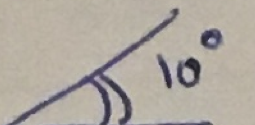
$length = 500\,mm = 0.5\,m$

$pitch = 0.5\,rev/mm = 500\,rev/m$

$\eta = 0.45$

$\mu = 0.1$

$m_L = 6\,kg$

$\theta = 10^\circ$ 

$\frac{J_{machine}}{J_{motor}} = \frac{4}{1}$

Trapezoidal $\Rightarrow$ $(1/6 - 2/3 - 1/6)$

distance = 8 mm = 0.008 m

$t_{move} = 0.2$, $t_{dwell} = 0.1\,sec$

Solution :

$$\boxed{T_m = T_R + J_t \ddot{\theta}_m}$$

① to get The inertia :

$$J_t = J_m + J_{\substack{Lead\\screw}} + J_{Load}$$

(a) $J_{\substack{Lead\\screw}} \Rightarrow$ It is a cylinder shape

$= \frac{1}{2} m r^2 = \frac{1}{2} \rho \pi r^4 L = \frac{1}{2}(7800)(\pi)(0.007)^4(0.5)$

$= 1.47 \times 10^{-5}\ Kg\,m^2$

(b) $J_{Load} = \frac{m_L}{2(2\pi P)^2} = \frac{6}{2(2\pi \times 500)^2} = 1.35 \times 10^{-6}$

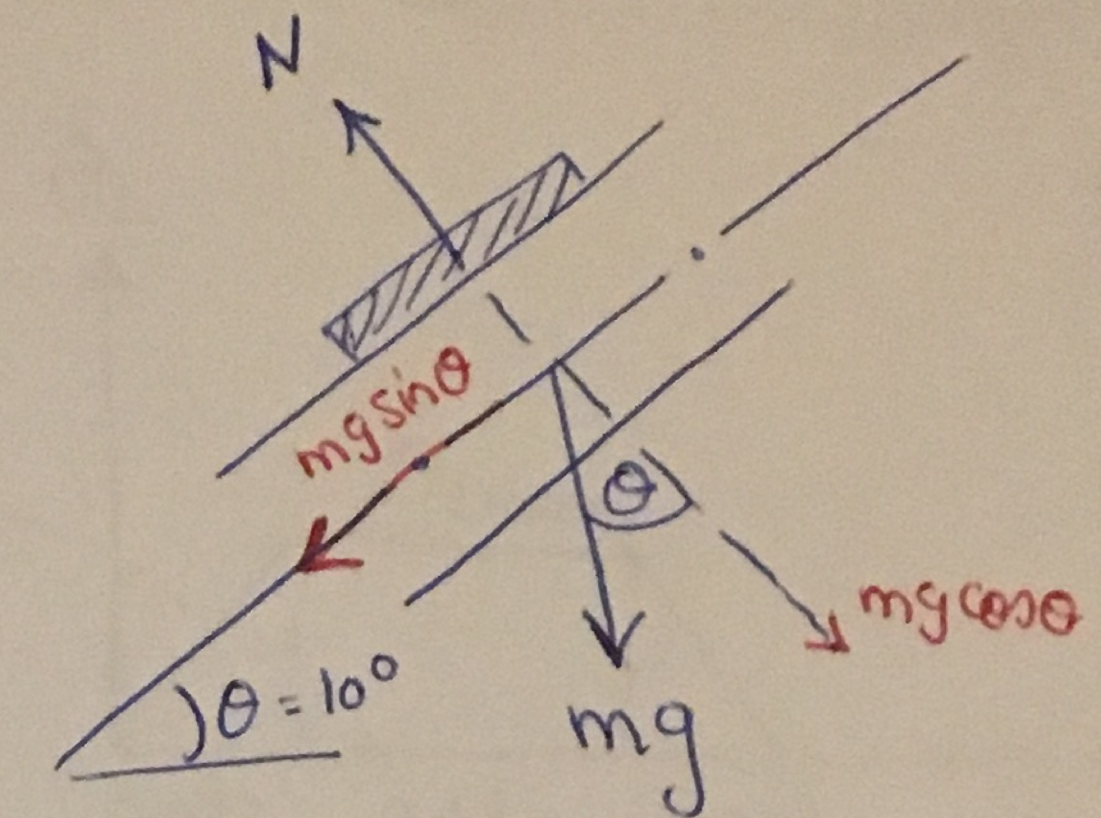
(c) $J_{machine} = J_{Load} + J_{\substack{lead\\screw}} = 1.605 \times 10^{-5}\ Kg\,m^2$

$\therefore\ J_m = \frac{J_{machine}}{4} = 4 \times 10^{-6}\ Kg\,m^2$

$\therefore\ \boxed{J_{total} = 2 \times 10^{-5}\ Kg\,m^2}$

② To get the T_R (resisting Torque)

T_f = Torque due to friction



$$= \frac{\mu\, mg\cos\theta}{2\,(2\pi P)}$$

$$= \frac{0.1 \times 6 \times 9.8 \times \cos 10}{0.45 \times (2\pi \times 500)}$$

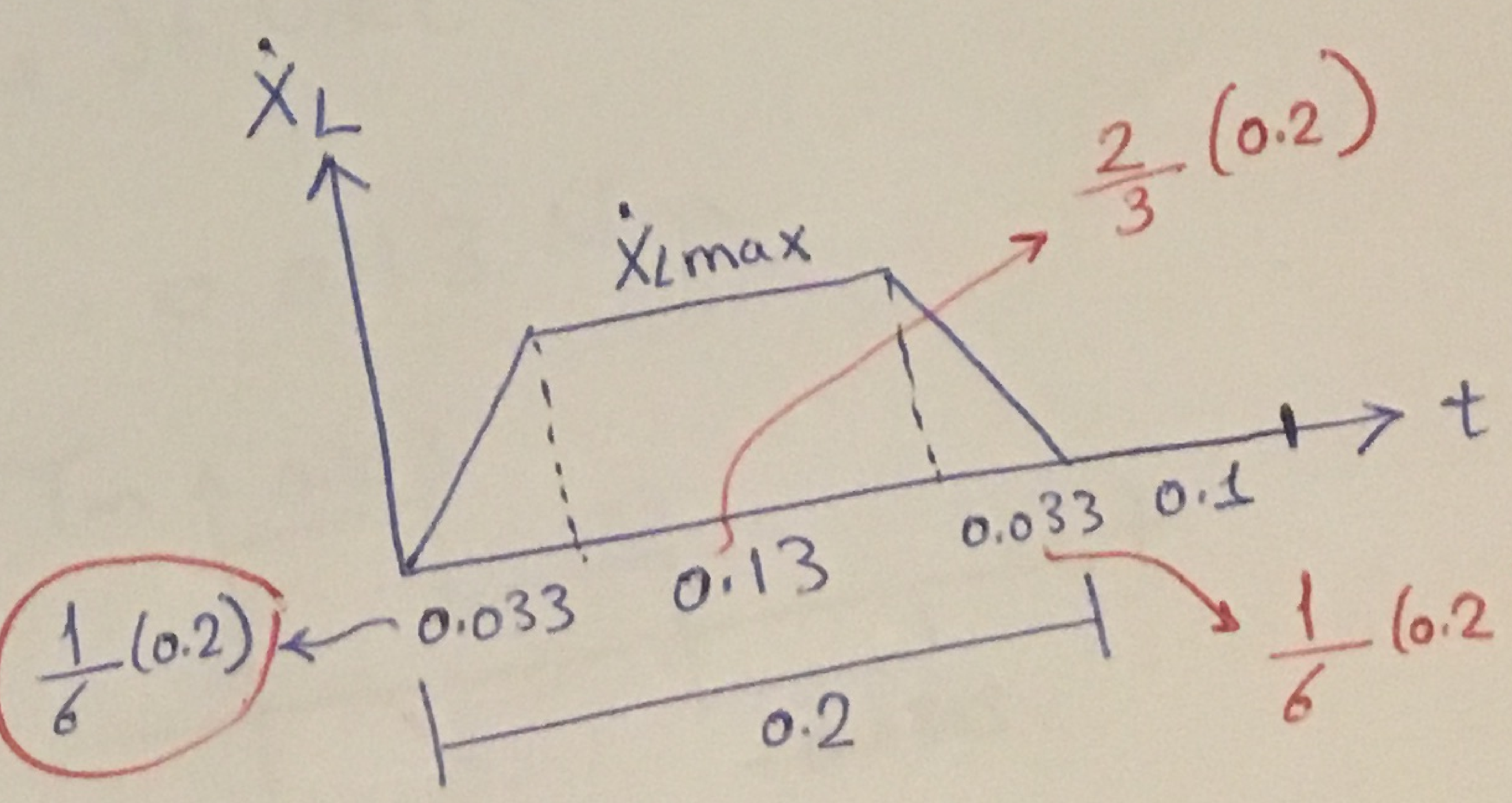
$$T_f = \boxed{4.096 \times 10^{-3}}$$

To get external Torque $\Longrightarrow$ due to load

$$T_L = \frac{mg\sin\theta}{(2\pi P)\,2} = \frac{6 \times 9.8 \times \sin 10}{2\pi \times 500 \times 0.45}$$

$$T_L = \boxed{7.22 \times 10^{-3}}$$

$$\boxed{\therefore\ T_R = T_f + T_L = 0.0113\ Nm}$$



③ To get $\ddot{\theta}_m$

$$X_{L\,max} = \int_0^t \dot{x}_L(t)\,dt$$

$$X_{L\,max} = \underbrace{\frac{0.2 + 0.13}{2}}_{\text{area}}\ \dot{X}_{L\,max}$$

distance = 0.008m

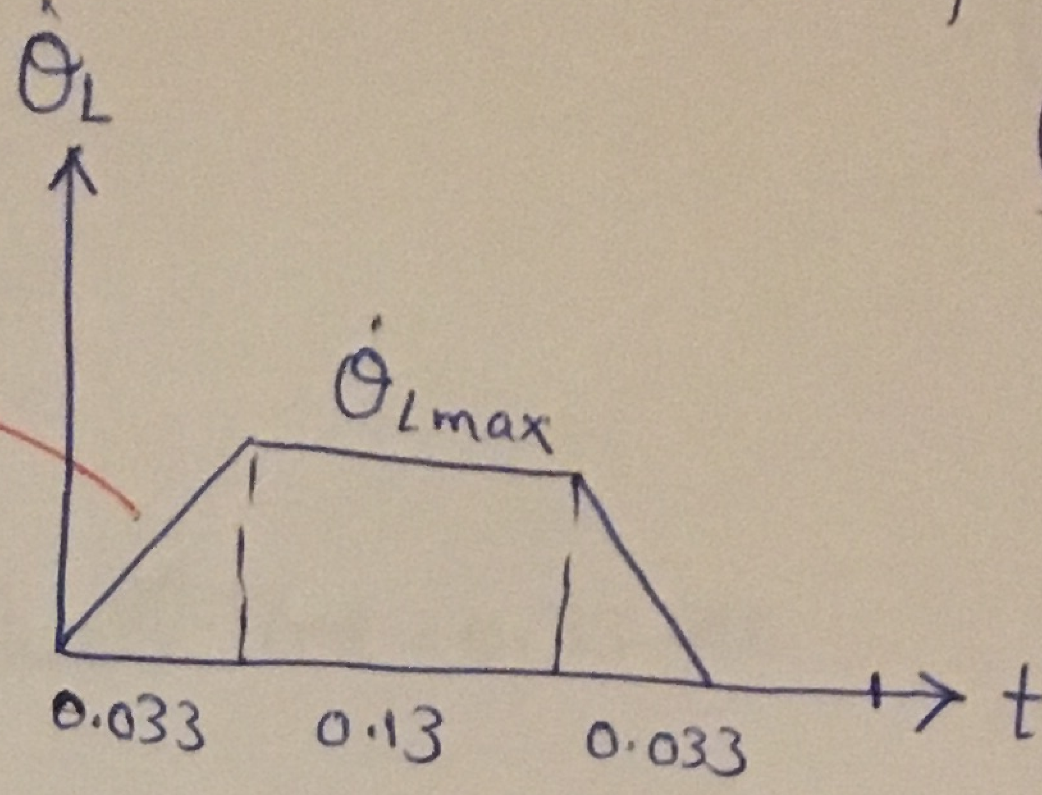
$$\dot{X}_{L\,max} = \frac{0.008 \times 2}{0.2 + 0.13} = 0.048\ m/s \longrightarrow \text{linear speed}$$

$$\dot{X}_{L\,max} = \frac{\dot{\theta}_{L\,max}}{2\pi P} \Longrightarrow \boxed{\dot{\theta}_{max} = 48\pi\ rad/sec}$$

Now we want to see the $\dot{\theta}_{L max}$ ← Important

(27)

Equation of line



$$\boxed{\dot{\theta}_L = \frac{\dot{\theta}_{Lmax}}{0.033} t}$$

$$\ddot{\theta}_{acc} = \frac{d\dot{\theta}_L}{dt} = \frac{\dot{\theta}_{Lmax}}{0.033} = 1460.4\pi \ rad/sec^2$$

$$\ddot{\theta}_{dec} = -1460.4\pi \ r/sec^2$$

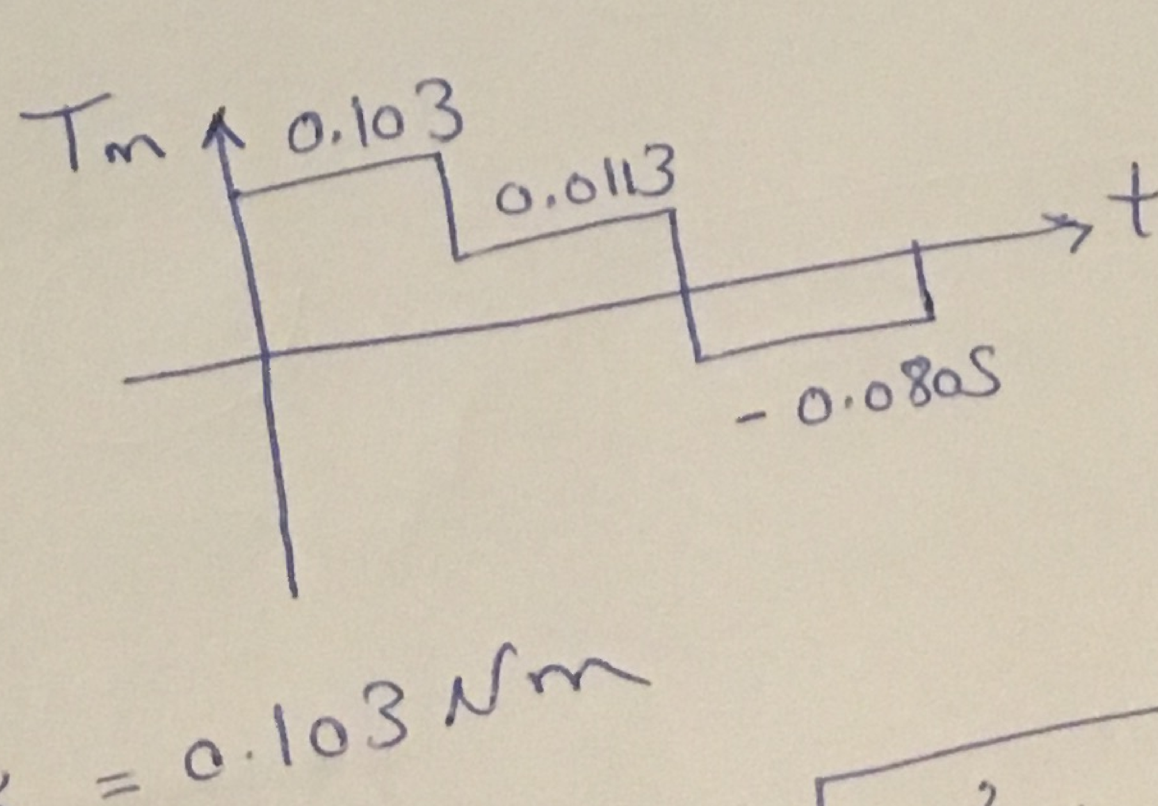
$$\ddot{\theta}_{ss} = 0$$

(3) To get The Motor Torque

$$T_{acc} = T_R + J_t \ddot{\theta}_{acc} = 0.103 \ Nm$$

$$T_{dec} = T_R + J_t \ddot{\theta}_{dec} = -0.0805 \ Nm$$

$$T_{ss} = T_R = 0.0113 \ Nm$$



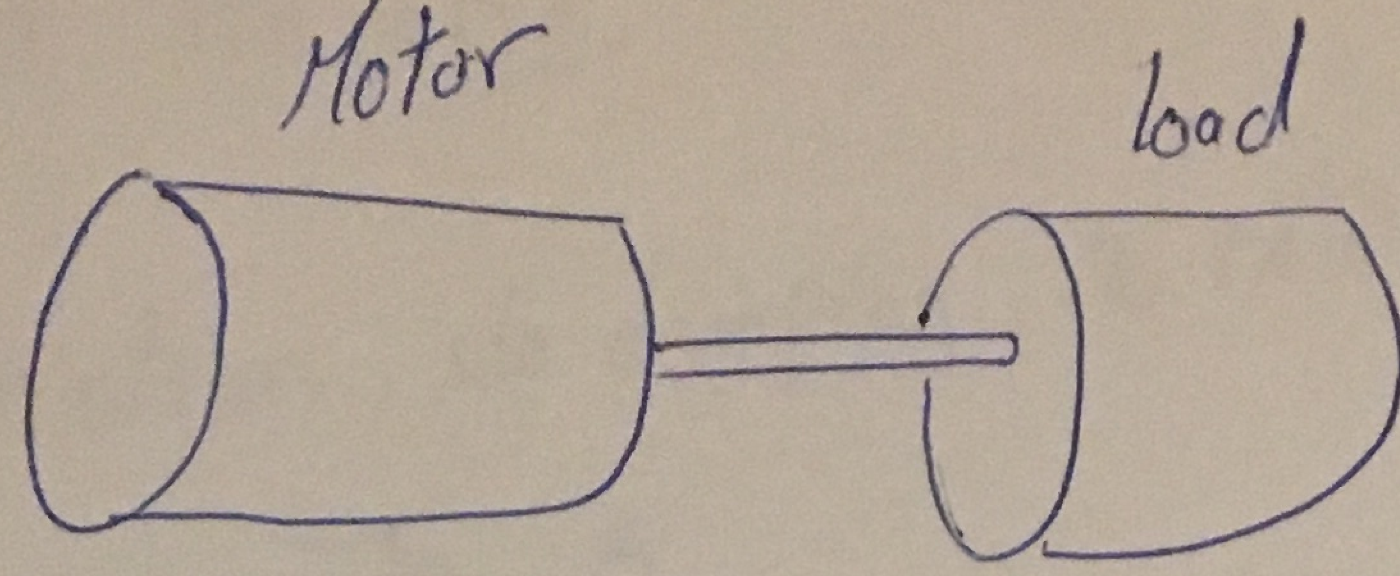
Thus $T_{peak} = 0.103 \ Nm$

$$T_{rms} = \sqrt{\frac{\int_0^{T_m} T_m^2(t)\,dt}{t_{cycle}}} = \sqrt{\frac{T_{acc}^2 t_a + T_{dec}^2 t_d + T_{ss}^2 t_{ss}}{t_{cycle}}}$$

$$= \sqrt{\frac{(0.103)^2(0.03) + (0.0805)^2(0.03) + (0.0113)^2(0.133)}{0.3}}$$

$$\boxed{T_{rms} = 0.044 \ Nm}$$

Problem 4



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Given: cylindrical shape: $d = 50\,mm = 0.05\,m$

$l = 60\,mm = 0.06\,m$

$\rho = 7800\,kg/m^3$

$\theta_{max} = \frac{1}{4}\,rev = \frac{\pi}{2}\,rad$

$t_{cycle} = 0.25\,sec$

$t_{dwell} = 0.1\,sec$

$t_{motion} = 0.15\,sec$

Periodic $= \left(\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}\right) \Longrightarrow$ acceleration $K(e^{10t} - 1)$

Solution

① to get J_t :-

$$J_t \ddot{\theta}_m = T_m - T_R$$

$$\boxed{T_m = T_R + J_t \ddot{\theta}_m}$$

$$J_t = J_m + J_{load}$$

$$J_L = \frac{1}{2} m r^2 = \frac{1}{2} \rho \pi r^4 L = \frac{1}{2} (7800)(\pi)(0.025)^4 (0.06)$$

$$= 2.87 \times 10^{-4}\ kg\,m^2$$

& assume $J_m = J_L \longrightarrow$ i.e ration is 1:1

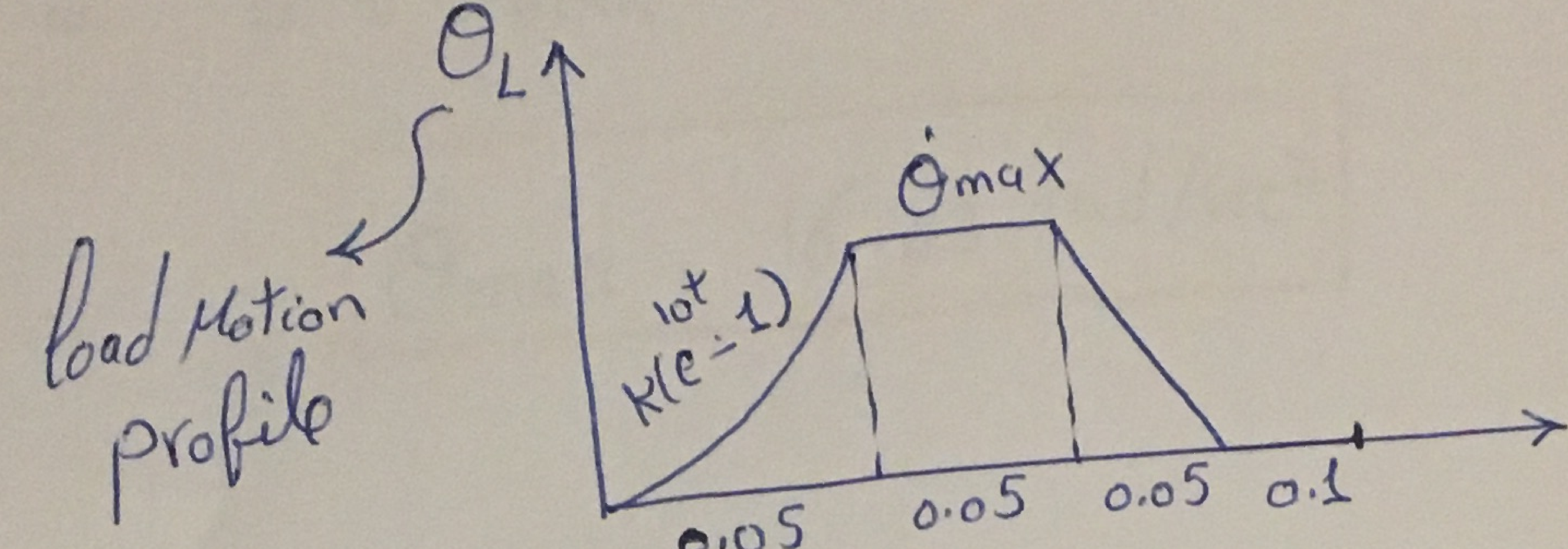
$$\boxed{J_t = 5.74 \times 10^{-4}\ kg\,m^2}$$

2 to get the T_R:-

No Friction, No damping, no external torque

$T_R = 0$

3 to get $\ddot{\theta}_m$ (هاام جدا =١)



→ First Try to remove K:-

$$\dot{\theta}_L = K(e^{10t} - 1) \longrightarrow (1)$$

put $t = 0.05$ sec get $\dot{\theta}_L = \dot{\theta}_{max}$

$$\dot{\theta}_{max} = K(e^{10(0.05)} - 1) \rightarrow \boxed{K = \frac{\dot{\theta}_{max}}{0.65}}$$

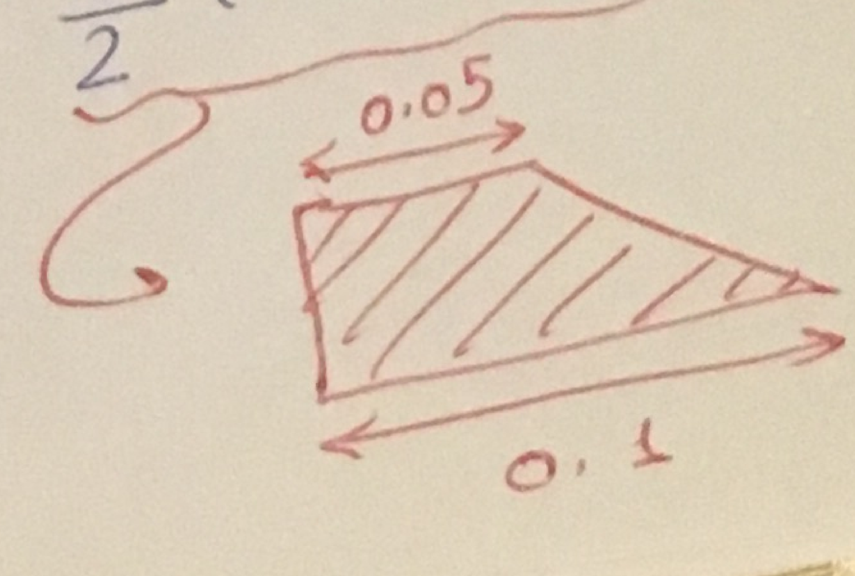
SubsiTute in (1)

$$\boxed{\dot{\theta}_L = \frac{\dot{\theta}_{max}}{0.65}(e^{10t} - 1)} \longrightarrow (2)$$

to get θ_{max} is The area under The curve

$$\theta_{max} = \int_0^t \dot{\theta}_L \, dt$$

$$\theta_{max} = \int_0^{0.05} \frac{\dot{\theta}_{max}}{0.65}(e^{10t} - 1)\,dt + \frac{1}{2}(0.1 + 0.05)\,\dot{\theta}_{max}$$



$$\theta_{max} = \left[\frac{\dot\theta_{max}}{0.65 \times 10} e^{10t} - \frac{\dot\theta_{max}}{0.65} t \right]_0^{0.05} + 0.075\, \dot\theta_{max} \quad (32)$$

$$\frac{\pi}{2} = \left[\frac{\dot\theta_{max}}{6.5} e^{10(0.05)} - \frac{\dot\theta_{max}(0.05)}{0.65} \right] - \frac{\dot\theta_{max}}{0.65} + 0.075\, \dot\theta_{max}$$

$$\frac{\pi}{2} = 0.1\, \dot\theta_{max} - 0.077\, \dot\theta_{max} + 0.075\, \dot\theta_{max}$$

$$\boxed{\dot\theta_{max} = 16.03 \text{ rad/sec}^2}$$

in equation (2)

$$\ddot\theta_a = \frac{d}{dt} \dot\theta_a = \frac{d}{dt} \left[24.65 \left(e^{10t} - 1 \right) \right]$$

$$\ddot\theta_a = 246.6\, e^{10t}$$

$$\ddot\theta_d = -\frac{\dot\theta_{max}}{0.05} = -320.6 \text{ rad/sec}^2$$

$$\ddot\theta_{s.s} = 0$$

Torque

$$T_a = J_t \ddot\theta_m = 0.1415\, e^{10t} \text{ Nm}$$

$$T_d = -0.184 \text{ Nm}$$

$$T_{ss} = 0$$

The graph becomes

To get the peak, put $t = 0.05$

عشان انا بحسب عند ال Max أذن لازم أعوض بـ $T = 0.05\,S$

$$T_a = 0.1415\,e^{10(0.05)} = 0.233\ Nm$$

د. حاتم

To get the Trms

$$T_{rms} = \sqrt{\frac{\int (0.1415\,e^{10t})^2\,dt + (0.184)^2 \times 0.05}{0.25}}$$

$$= \sqrt{\frac{\frac{0.02}{20}\,e^{20t}\Big|_0^{0.05} + 1.7 \times 10^{-3}}{0.25}}$$

$$\therefore \boxed{T_{rms} = 0.117\ Nm}$$

Problem ⑥

Given: Conveyor + Gear Box + rotary Motor

Bags = 15 Kg

$D_r = 300\,mm = 0.3\,m$

$L_r = 1000\,mm = 1\,m$

$N = 50:1$ (reduction ratio)

$\theta = 20$ (orientation)

Conveyor weight = 10 Kg

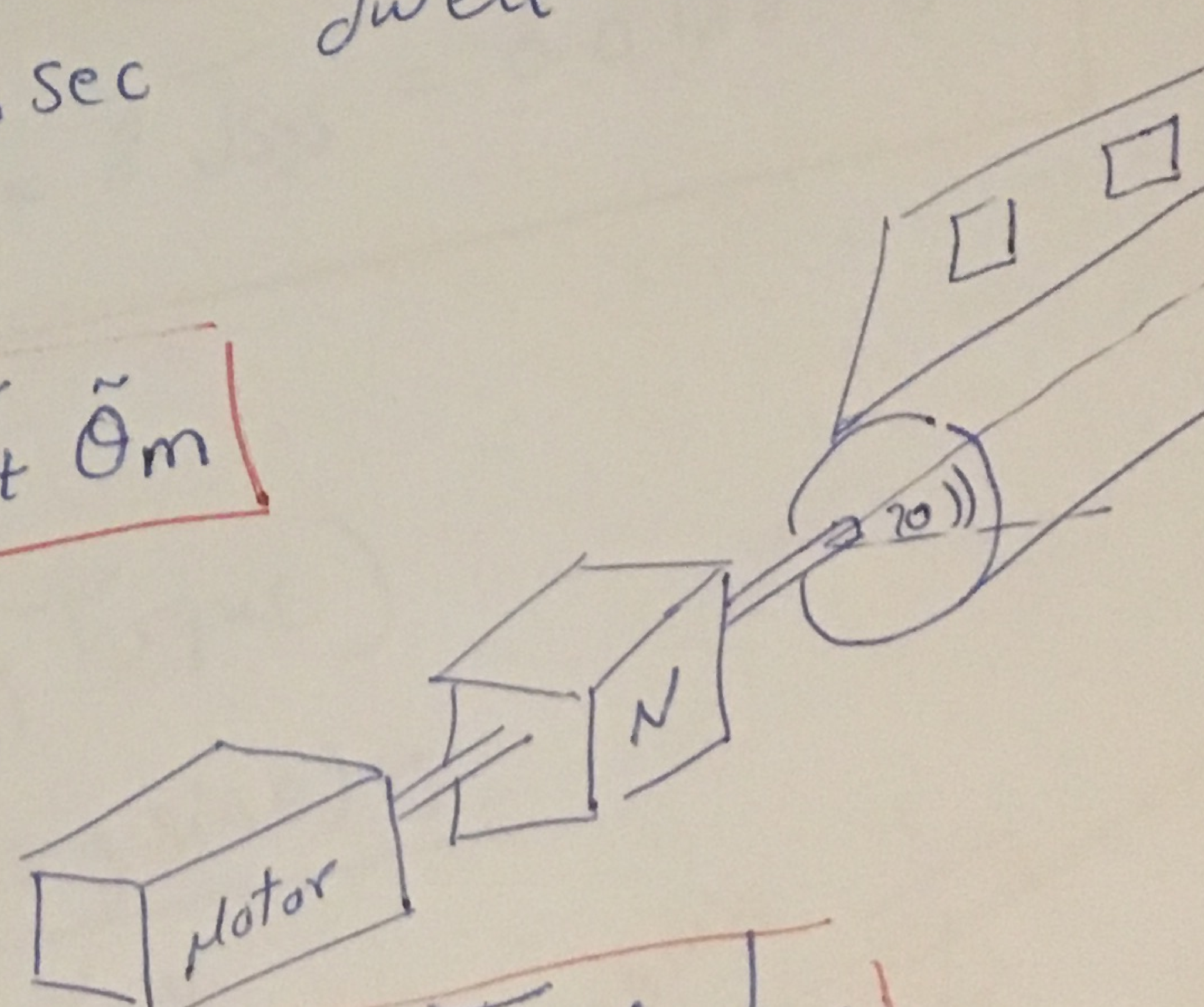
$T_{working} = 35\,°C$

$n = 10$ Bags

Motion profile $(\frac{1}{10} - \frac{4}{5} - \frac{1}{10})$ Trapezoid

distance = 500 mm = 0.5 m

Move Time = 30 Sec   dwell time = 3 sec



$$\boxed{T_m = T_R + J_t \ddot{\theta}_m}$$

Solution:

① to get J_t

$$\boxed{J_t = J_m + J_{conv}\Big|_{eff} + J_{load}\Big|_{eff} + J_{Roller}\Big|_{eff}}$$

$$\therefore J_{Roller}\Big|_{eff} = \frac{2\left[\frac{1}{2} m_r r_r^2\right]}{\eta N^2} = \frac{\rho \pi r_r^4 L_r}{N^2}$$

$$= \frac{7800 \times \pi \times (0.15)^4 \times 1}{(50)^2} = 4.96 \times 10^{-3}\ Kg\,m^2$$

$$J_{conv}\Big|_{eff} = \frac{m_c r_r^2}{2N^2} = \frac{10 \times (0.15)^2}{(50)^2} = 9 \times 10^{-5}\ Kg\,m^2$$

no of bags

$$J_L\Big|_{eff} = \frac{(10 \times 15)(0.15)^2}{(50)^2} = 1.35 \times 10^{-3}\ Kg\,m^3$$

$$\frac{m_L r^2}{2N^2}$$

So $J_{sys} = J_{conv}\Big|_{eff} + J_{Load}\Big|_{eff} + J_{Roller}\Big|_{eff}$

$= 6.4 \times 10^{-3}$

Now we want to assume the Motor inertia So $J_m = J_{System}$

So $\boxed{J_{total} = 2J_{sys} = 0.0128\ Kg\,m^2}$

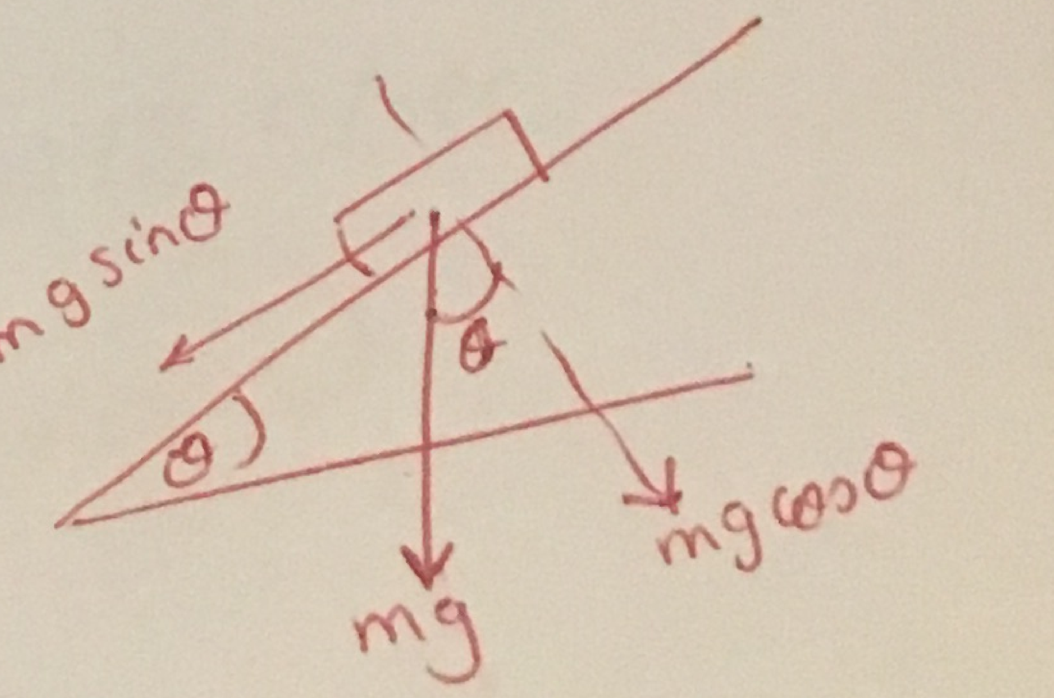
② to get the T_R (resisting Torque)

$$T_R = T_L\Big|_{eff} = \frac{(m_L g \sin\theta)\, r_r}{\eta N}$$

mass of load only

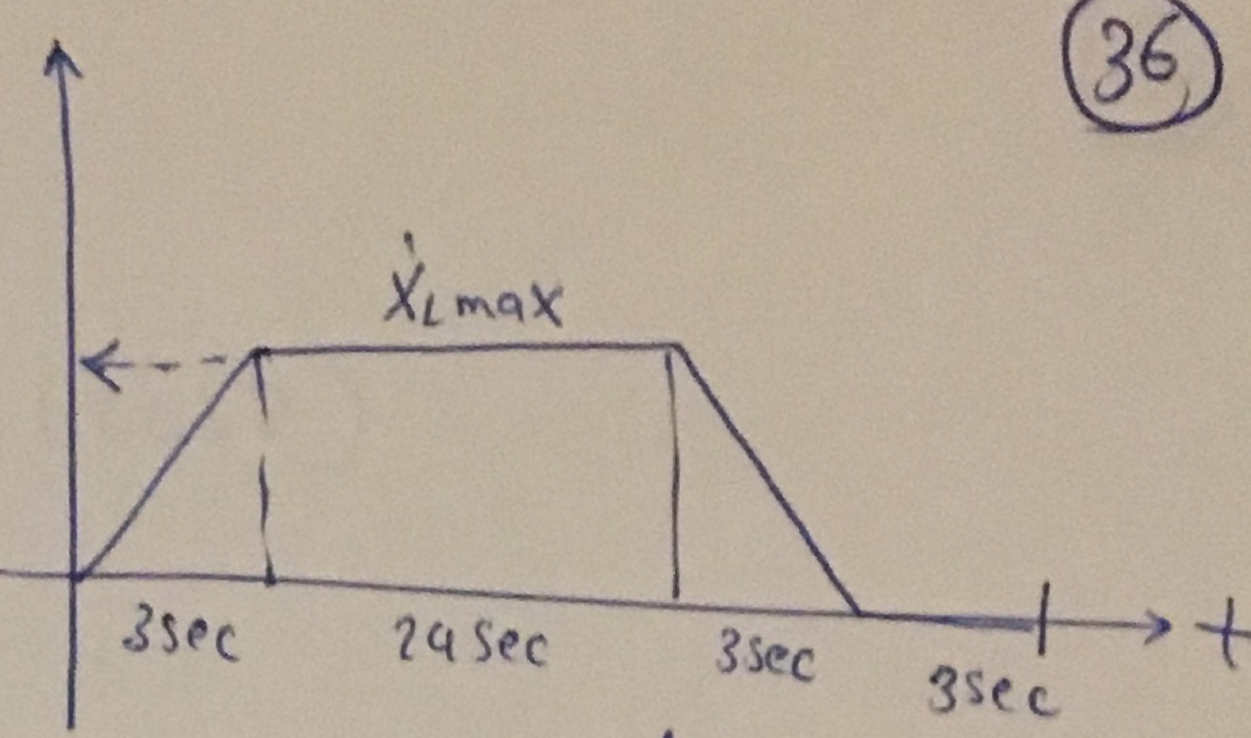
$$= \frac{(10 \times 15 \times 9.8) \sin 20 \times 0.15}{50}$$

$= 1.508$ Nm



3 to Find $\ddot{\theta}_m$

$$X_{L\,max} = \int_0^t \dot{X}_L(t)\,dt$$



$$0.5 = \frac{24+30}{2}\,\dot{X}_{L\,max} \longrightarrow \text{linear speed of The load}$$

$$\boxed{\therefore \dot{X}_{L\,max} = 0.0185 \text{ m/sec}}$$

$$\dot{X}_{L\,max} = \dot{\theta}_{L\,max}\, r$$

$$V = \omega\, r$$

$$\therefore \dot{\theta}_{L\,max} = \frac{0.0189}{\underbrace{r_{roller}}_{0.15}} = 0.123 \text{ r/sec}$$

angular speed of The load

كدة أنا عديت الـ Conveyor
ووقفت عند الـ gear Box

But This speed (angular speed) is The load speed (Roller) so To Transfer J to The Motor Multiply By N

$$\dot{\theta}_{max} = N\,\dot{\theta}_{L\,max} = 50 \times 0.123 = 6.173 \text{ r/sec}$$

كدة أنا وقفت عند الـ gear Box وعديته ووقفت عند الـ Motor

we can get $\ddot{\theta}_m$

$$\ddot{\theta}_{acc} = \frac{\dot{\theta}_{max}}{3} = 2.058 \text{ r/sec}^2$$

$$\ddot{\theta}_{dec} = -2.058 \text{ r/sec}^2$$

$$\ddot{\theta}_{ss} = 0$$

then substitute in The Main equation:-

$$T_a = T_R + J_t \ddot{\theta}_{acc}$$

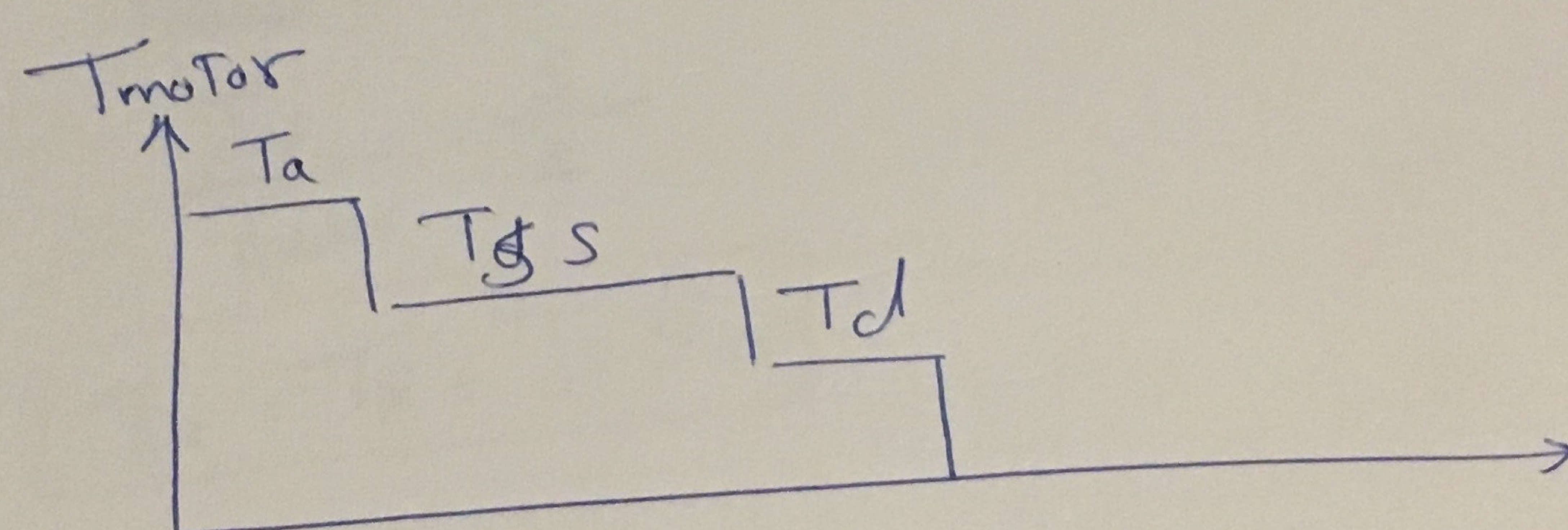
$$= 1.508 + (0.0128)(2.058)$$

$$\boxed{T_a = 1.5343}$$

$$T_d = T_R + J_t \ddot{\theta}_d$$

$$= 1.508 - (0.0128)(2.058) = \boxed{1.4816\ Nm}$$

$$T_{s.s} = T_R = \boxed{1.508\ Nm}$$



$$\boxed{T_{peak} = T_{acc} = 1.5343\ Nm}$$

$$T_{rms} = \sqrt{\frac{\int_0^{t_{as}} t_m^2\, dt}{t_{cycle}}} = \sqrt{\frac{(1.5343)^2(3) + (1.4816)^2(3) + (1.508)^2(24)}{33}}$$

$$T_{rms} = 0.147 \quad @ \ 25^\circ C$$

Nm