Ain Shams University
Faculty of Engineering
Eng.Physics & Math.Departement

Engineering Math(PHM 663)
Spring 2020-2021
Allowed Time: 2 Hrs.

Total mark (70)

The exam is composed of 6 questions in one page. The mark for each question is (20) marks

Part (1): Answer TWO questions only

1) a) Show that the function: $u(x,y) = 8x^3 - 24xy^2 + 10xy + 6y + 8e^x \cos y$, is *harmonic and find* it's corresponding analytic function $f(z) = u + iv$. *find* $f'(z)$.

b) Find all values of Z for the following:

i. $e^{2z+2i} = 4 - 4i$ ii. $z = \left(2 + 2\sqrt{3i}\right)^{2i}$

2) a) Find all Laurent series that represent the function $f(z) = \frac{16z}{z^2+2z-15}$ in different domains.

b) Evaluate the following integrals :

i. $\oint_c \frac{e^{2z+1}}{z^3-2iz^2}\, dz$, *where* $c: |Z| = 5$ ii. $\int_{-\infty}^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$

iii. If c is the circle $|z|= 7$ and $g(z_0)= \oint_c \frac{z^4+4z^2+5}{(z-z_0)^3}\, dZ$. **Find:** (1) $g(3i)$ (2) $g(10+3i)$

3) a) Find all value of Z such that:

I. $\sin 2z = 30i$

II. $(2 + 2i)^{3i}$

b) Evaluate the following integrals.

I. $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)}$

II. $\oint (z^3 + 3z^2 + 5)\, e^{\frac{1}{z}}\, dz$

III. $\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta}$

Part (2): Answer TWO questions only

4) a) Evaluate the following integral $\int_3^{\infty} e^{(6x-x^2)}\, dx$.

b) Find the series solution of : $y'' + xy = 0$, near the ordinary $x = 0$.

5) a) Find the area enclosed by the equation of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

b) Evaluate : $L^{-1}\left(\frac{s}{(s^2+1)^2}\right)$ and hence evaluate : $L^{-1}\left(\frac{1}{(s^2+1)^2}\right)$

c)Solve the initial value problem : $3y' - 4y = \sin(2t)$, given that : $y(0) = \frac{1}{3}$,

using **LaPlace Transform**

6)a) Solve the equation: $x(t) = \sin(t) + \int_0^t x(u)\sinh(t-u)\, du$

b) Find $f(t) = L^{-1}\left[\frac{3}{s} + \frac{2e^{-2s}}{s^2} - \frac{2e^{-5s}}{s^2}\right]$ and sketch the graph of this function

c) Find **LaPlace Transform** of the half – wave rectifier wave form defined by :

$$f(t) = \begin{cases} a\sin(t) & ; \quad 0 < t < \pi \\ 0 & ; \quad \pi < t < 2\pi \end{cases}, \quad f(t + 2\pi) = f(t).$$

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