AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING
SPECIALIZED ENGINEERING PROGRAMS
Communication Junior

Spring 2021	Quiz- 1	Time Allowed: 20 min.

PHM 212s: Special Functions [5 Marks]

Name: **ID:** **G:** **S:**

Question 1:

Evaluate the following integral in terms of Gamma function.

$$\int_0^{\pi/2} \left(\csc^4 x - \csc^3 x\right)^{1/6} \cos x \, dx$$

[2 marks]

Question 2: (*Please start the solution in the Back of the paper)*

Find a series solution for the following D.E. in powers of x.

$$\left(3 - x^2\right) y'' - x y' + 4y = 0$$

[3 marks]

Model Answer

Q1

$$\to t=\sin\theta \Rightarrow dt=\cos\theta d\theta \Rightarrow \csc\theta = {}^{1}\!/_{t} = t^{-1} \cdots\cdots \boxed{0.5}$$

$$\Rightarrow I=\int_0^1 \left(t^{-4}-t^{-3}\right)^{1/6} dt \;= \int_0^1 t^{-4/6}\left(1-t\right)^{1/6} dt \quad \cdots\cdots \boxed{0.5}$$

$$\to x-1 = -4/6,\, y-1 = 1/6 \quad \cdots\cdots \boxed{0.5}$$

$$\Rightarrow I=\beta\left(\frac{2}{6},\frac{7}{6}\right) \;= \frac{\Gamma\left(\frac{2}{6}\right)\Gamma\left(\frac{7}{6}\right)}{\Gamma\left(\frac{9}{6}\right)} \quad \cdots\cdots \boxed{0.5}$$

Q2

$$\to p(x)=\frac{-x}{3-x^2},\, q(x)=\frac{16}{3-x^2} \Rightarrow p(x)\,\&\, q(x) \text{ are both analytic at } x_0=0$$

$$\Rightarrow x_0=0 \text{ is an ordinary point } \Rightarrow \text{ Power series Method } \cdots\cdots \boxed{0.5}$$

$$let\ \mathrm{y}=\sum_{n=0}^{\infty} a_n x^n \Rightarrow y'=\sum_{n=1}^{\infty} n a_n x^{n-1} \Rightarrow y''=\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$$

$$\Rightarrow (3-x^2)\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - x\sum_{n=1}^{\infty} n a_n x^{n-1} + 4\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\Rightarrow \sum_{n=2}^{\infty} 3n(n-1)a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1)a_n x^n - \sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} 4a_n x^n = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} 3(n+2)(n+1)a_{n+2} x^n - \sum_{n=0}^{\infty} n(n-1)a_n x^n - \sum_{n=0}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} 4a_n x^n = 0 \cdots\cdots \boxed{0.5}$$

$$coef\,(x^n)=0$$

$$\Rightarrow 3(n+2)(n+1)a_{n+2} - n(n-1)a_n - n a_n + 4a_n = 0$$

$$\Rightarrow 3(n+2)(n+1)a_{n+2} = \left[n(n-1)+n-4\right]a_n$$

$$\Rightarrow a_{n+2} = \frac{(n-2)(n+2)}{3(n+2)(n+1)}a_n,\, n\ge 0 \cdots\cdots \boxed{0.5}$$

$$n=0 \Rightarrow a_2 = \frac{-2.2}{3.2.1}a_0 = \frac{-2}{3}a_0,\, n=2 \Rightarrow a_4 = 0 = a_6 = a_8 = \ldots = a_{2m}, \cdots\cdots \boxed{0.5}$$

$$n=1 \Rightarrow a_3 = \frac{-1.3}{3.3.2}a_1,\, n=3 \Rightarrow a_5 = \frac{1.5}{3.5.4}a_3 = \frac{(-1.1)(3.5)}{3^2(5.4.3.2)}a_1,$$

$$\Rightarrow a_{2m+1} = \frac{\left[-1.1\ldots.(2m-3)\right]\left[3.5.7.9\ldots(2m+1)\right]}{3^m(2m+1)!}a_1 \quad \cdots\cdots \boxed{0.5}$$

$$\Rightarrow y = a_0\left(1-\frac{2}{3}x^2\right) + a_1\left(x + \sum_{m=1}^{\infty} a_{2m+1}x^{2m+1}\right) \cdots\cdots \boxed{0.5}$$