

Summer 2021

Course Code: PHM 212s

Time: 2.00 Hrs.

Complex Variables, Special functions and Numerical Analysis

The Exam Consists of FOUR Questions in TWO Pages.

Total Marks: 60 Marks

1/2

Important Rules:

- Having a mobile, Smart Watch or earphones inside the examination hall is forbidden and is considered as a cheating behavior.
- It is forbidden to have any references, notes, books, or any other materials even if it is not related to the exam content with you in the examination hall
- It is not allowable to use programable or graphical calculators.

تعليمات هامة

- حيازة (المحمول- الساعات الذكية - سماعة الأذن) داخل لجنة الامتحان يعتبر حالة غش تستوجب العقاب .
- لايسمح بدخول أي كتب أو ملازم أو أوراق داخل اللجنة والمخالفة تعتبر حالة غش.
- ممنوع استخدام الآلات الحاسبة المبرمجة و التي تستطيع الرسم.

Question (1): (14 Marks)

(A) Solve in terms of the Gamma function $\int_0^{\infty} x^m k^{-x} dx$ where m and k are any real numbers.

(State the conditions on m and k such that the integral converges).

[7 Marks]

(B) Solve in terms of Bessel functions the following differential equation

$$x y'' + 5 y' + x y = 0$$

[7 Marks]

Question (2): (12 Marks)

Find two linearly independent solutions in powers of "x" for the following differential equation:

$$(1 - x^2) y'' - 2x y' + 20 y = 0$$

Question (3): (14 Marks)

(A) Show that $\frac{d}{dx} (x^n J_n(x)) = x^n J_{n-1}(x)$. Hence, evaluate $\int x^5 J_2(x) dx$.

[7 Marks]

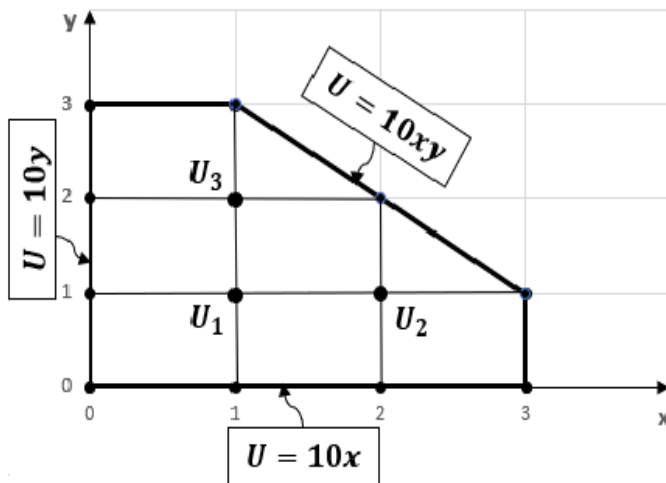
(B) evaluate $\int J_7(x) dx$. (Hint: you may use $J'_n = \frac{1}{2}(J_{n-1} - J_{n+1})$).

[7 Marks]

Question (4): (20 Marks)

Use 4 decimal places in your calculations.

- (A) Find the solution (accurate to 3D) for the Dirichlet boundary value problem for the Poisson equation $\nabla^2 U = 10(x^2 - y^2)$ in the region and for the boundary conditions shown in the figure below. Use Gauss-Seidel method with zero initial approximations for the interior points.



[10 Marks]

- (B) Using two steps of Runge Kutta Method, solve the following system of ODEs:

$$x' = x - y - t, \quad y' = 4x - 2y, \quad x(0) = 1, \quad y(0) = 0$$

Find $x(0.2)$ & $y(0.2)$

[10 Marks]

End of Exam, Best Wishes.