CSE 681/617 Assignment 1 (Due Nov 5, 2024)

1. Equalize the histogram of the 8×8 image below. The image has grey levels $0, 1, \ldots, 7$.

$$\begin{array}{cccccccc} 4 & 4 & 4 & 4 & 4 & 4 & 4 & 0 \\ 4 & 5 & 5 & 5 & 5 & 5 & 4 & 0 \\ 4 & 5 & 6 & 6 & 6 & 5 & 4 & 0 \\ 4 & 5 & 6 & 7 & 6 & 5 & 4 & 0 \\ 4 & 5 & 6 & 6 & 6 & 5 & 4 & 0 \\ 4 & 5 & 5 & 5 & 5 & 5 & 4 & 0 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 & 0 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 & 0 \end{array}$$

2. Assume that we have many noisy versions $g_i(x, y)$ of the same image $f(x, y)$, i.e.

$$g_i(x, y) = f(x, y) + \eta_i(x, y)$$

where the noise η_i is zero-mean and all point-pairs (x, y) are uncorrelated. Then we can reduce noise by taking the mean of all the noisy images

$$\bar{g}(x, y) = \frac{1}{M} \sum_{i=1}^{M} g_i(x, y).$$

Prove that

$$E\{\bar{g}(x, y)\} = f(x, y)$$

and

$$\sigma^2_{\bar{g}(x,y)} = \frac{1}{M} \sigma^2_{\eta(x,y)}$$

where $\sigma^2_{\eta(x,y)}$ is the variance of η and $\sigma^2_{\bar{g}(x,y)}$ the variance of $\bar{g}(x, y)$.

3. A two-variable continuous function's Laplace-operator is

$$\bigtriangledown^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}.$$

From this, it follows that

$$\mathcal{F}\{\bigtriangledown^2 f(x, y)\} = -(2\pi)^2 (u^2 + v^2) F(u, v),$$

where $F(u, v) = \mathcal{F}\{f(x, y)\}$. Determine the corresponding operator and the Fourier transform in the discrete case. Compare the result obtained to the continuous case.

4. Show that subtracting the Laplacian from an image is proportional to unsharp masking. Use the definition for the Laplacian in the discrete case.

5. Implement and test your algorithm.
(a) Equalize the image EightAM.png so that histogram g is approximately uniform. You can compare the results you get to those produced by the cv2 .
(b) Match the brightness histogram for EightAM.png to the histogram for LENA.png. Your program should also create plots of the histograms of EightAM before the transformation, EightAM after the transformation, and the histogram of Lena.