Actuator Sizing

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- **Sizing Algorithm**

Actuator Sizing Algorithm:

1. Define the geometric relationship between the actuator and load. In other words, select the type of motion transmission mechanism between the motor and load (*N*=reduction ratio).
2. Define the inertia and torque\force characteristics of the load and transmission mechanisms, i.e. define the inertia of the tool as well as the inertia of the gear reducer mechanisms (J_l, T_l).
3. Define the desired cyclic motion profile in the load speed versus time ($\theta'_l(t)$).
4. Using the reflection equations developed above, calculate the reflected load inertia and torque/force (J_{eff}, T_{eff}) that will effectively act on the actuator shaft as well as the desired motion at the actuator shaft ($\theta'_m(t)$).
5. Guess a actuator/motor inertia from an available list (catalog) (or make the first calculation with zero motor inertia assumption), and calculate the torque history, $T_m(t)$, for the desired motion cycle. Then calculate the peak torque and RMS torque from $T_m(t)$.

- **Sizing Algorithm**

Actuator Sizing Algorithm Cont.:

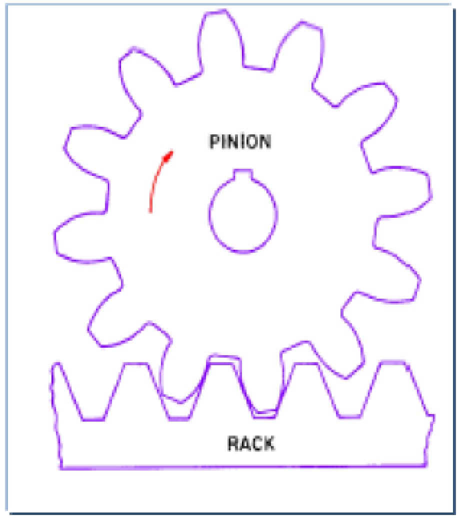
6. Check if the actuator size meets the required performance in terms of peak and RMS torque, and maximum speed capacity (T_p, T_{rms}, θ'_{max}). If the above selected actuator/motor from the available list does not meet the requirements (i.e. too small or too large), repeat the previous step by selecting a different motor. It should be noted that if a stepper motor is used, the torque capacity of the stepper motor is rated only in terms of the continuous rating, not peak. Therefore, the required peak and RMS torque must be smaller than the continuous torque capacity of the step motor.
7. Most servo motor continuous torque capacity rating is given for $25^o C$. If ambient temperature is different than $25^o C$, the continuous (RMS) torque capacity of the motor should be derated using the following equation for a temperature,

$$T_{rms} = T_{rms}(25^o C)\sqrt{\frac{(155 - Temp^o C)}{130}}$$

- **Sizing Algorithm**
- **Rotary to Linear**

Conversion of Rotary to linear Motion

1. Rack and pinion drives,
2. Power (lead) screws,
3. Linkages.



$$V = R\omega$$

If the load attached to the rack has mass *m, then,* total equivalent moment of inertia equals

$$J_{\text{eq}} = J_1 + mR^2 = J_1 + m\left(\frac{V}{\omega}\right)^2$$

Conversely, if the rack is the driver, then the moment of inertia *J1 attached* to the pinion shaft must be reflected back to the rack, and the equivalent linear inertia as felt by the pinion driving the rack is

$$m_{\text{eq}} = m + \frac{J_1}{R^2}.$$

- **Sizing Algorithm**
- **Rotary to Linear**

Mechanisms and Drives

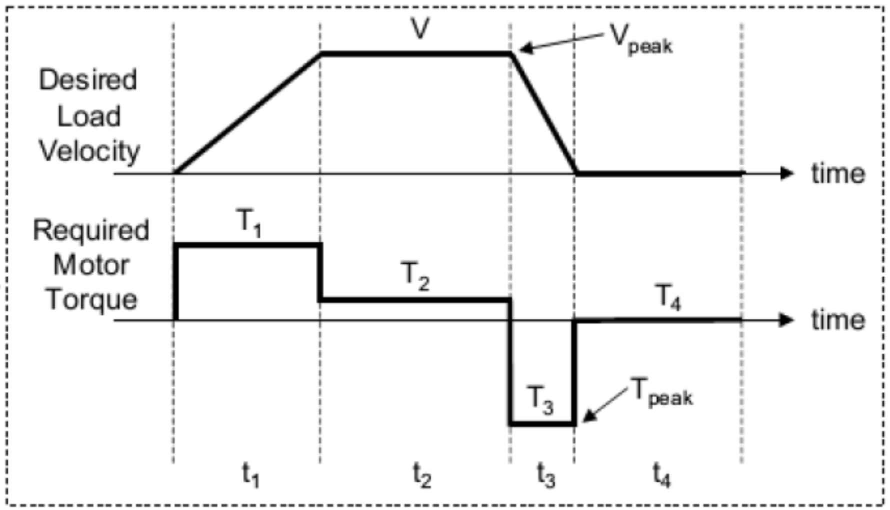
$$J_{total} = J_{motor} + J_{coupling} + J_{load}$$

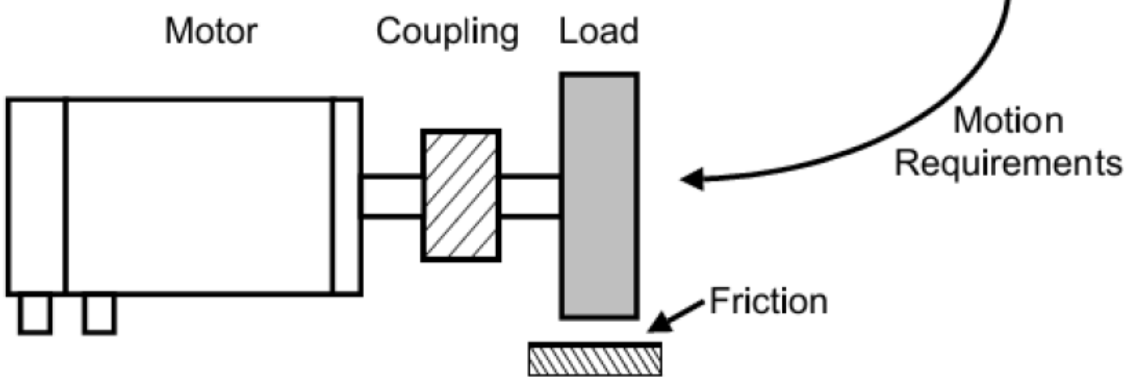
$$T_{acc} = J_{total} \frac{dn}{dt}$$

$$T_{motor} = T_{acc} + T_{friction}$$

$$T_{r.m.s.} = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + T_3^2 t_3 + T_4^2 t_4}{t_1 + t_2 + t_3 + t_4}}$$

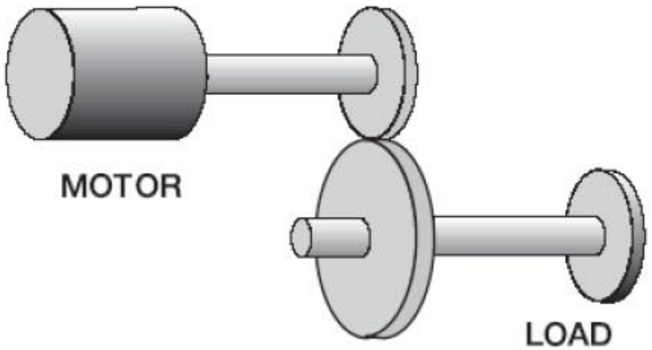
$$n_{average} = \frac{\frac{1}{2}V t_1 + V t_2 + \frac{1}{2}V t_3}{t_1 + t_2 + t_3 + t_4}$$





- **Sizing Algorithm**
- **Rotary to Linear**
- **Mechanisms**

Gear Drive:



where
S_m = motor speed (rpm)
S_1 = load speed (rpm)
N = gear ratio
N_1 = number of load gear teeth
N_m = number of motor gear teeth
T_m = motor torque (lb-in)
T_1 = load torque (lb-in)
e = efficiency
J_t = total inertia (lb-in-s²)
J_1 = load inertia (lb-in-s²)
J_m = motor inertia (lb-in-s²)

speed (motor) = speed (load) x gear ratio

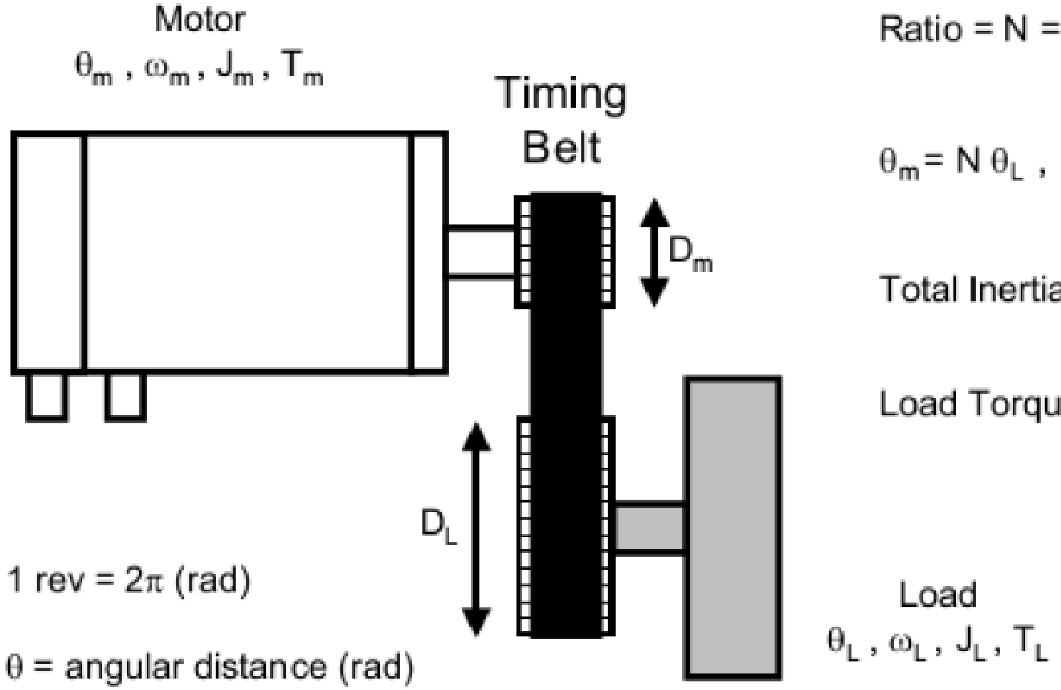
$S_m = S_1 \times N$

or $S_m = S_1 \times N_1 \div N_m$

torque at motor = torque at load ÷ gear ratio

$$T_m = \frac{T_1}{Ne}$$

- **Sizing Algorithm**
- **Rotary to Linear**
- **Mechanisms**



1 rev = 2π (rad)

θ = angular distance (rad)

ω = angular velocity (rad/s)

$$\text{Ratio} = N = \frac{\text{motor velocity}}{\text{load velocity}} = \frac{D_L}{D_m}$$

$$\theta_m = N\,\theta_L \;,\quad \omega_m = N\,\omega_L$$

$$\text{Total Inertia} = J_{total} = J_m + \frac{1}{N^2} J_L$$

$$\text{Load Torque Reflected to Motor} = \frac{1}{N} T_L$$

- **Sizing Algorithm**
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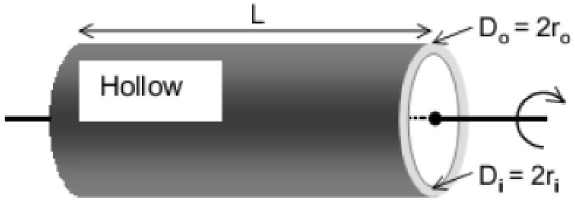
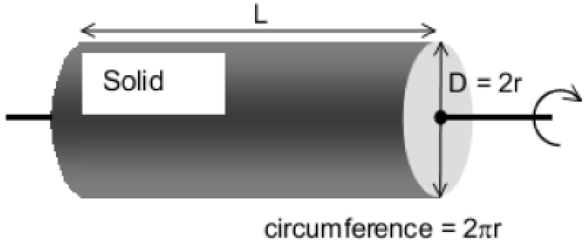
- **Mechanisms**
- **Useful Info**

In addition to knowing the relationships for common mechanical transmissions, it is also very useful to know the relationships for some common shapes as follows:



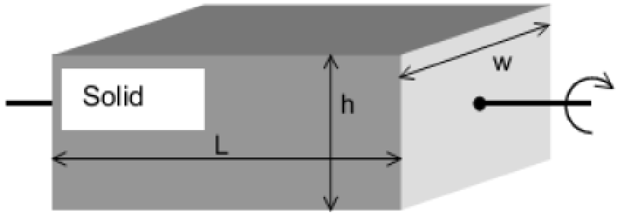
$$J = \frac{m\,(D_o^2 + D_i^2)}{8} = \frac{W\,(r_o^2 + r_i^2)}{2g} = \frac{\pi\, L\, \rho (r_o^4 - r_i^4)}{2g}$$

$$\text{Volume} = \frac{\pi}{4}(D_o^2 - D_i^2)\, L$$



$$J = \frac{m\,D^2}{8} = \frac{W\,r^2}{2\,g} = \frac{\pi\, L\, \rho\, r^4}{2\,g}$$

$$\text{Volume} = \pi\, r^2\, L$$



$$J = \frac{W}{12\,g}(h^2 + w^2)$$

$$\text{Volume} = L\, h\, w$$

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Material Densities

	oz/in³	lb/in³	gm/cm³
Aluminum	1.57	0.098	2.72
Brass	4.96	0.31	8.6
Bronze	4.72	0.295	8.17
Copper	5.15	0.322	8.91
Plastic	0.64	0.04	1.11
Steel	4.48	0.28	7.75

Friction Coefficients

(Sliding)	μ
Steel on Steel	0.58
Steel on Steel (Greased)	0.15
Aluminum on Steel	0.45
Copper on Steel	0.36
Brass on Steel	0.40
Plastic on Steel	0.20
Linear Bearings	0.001

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