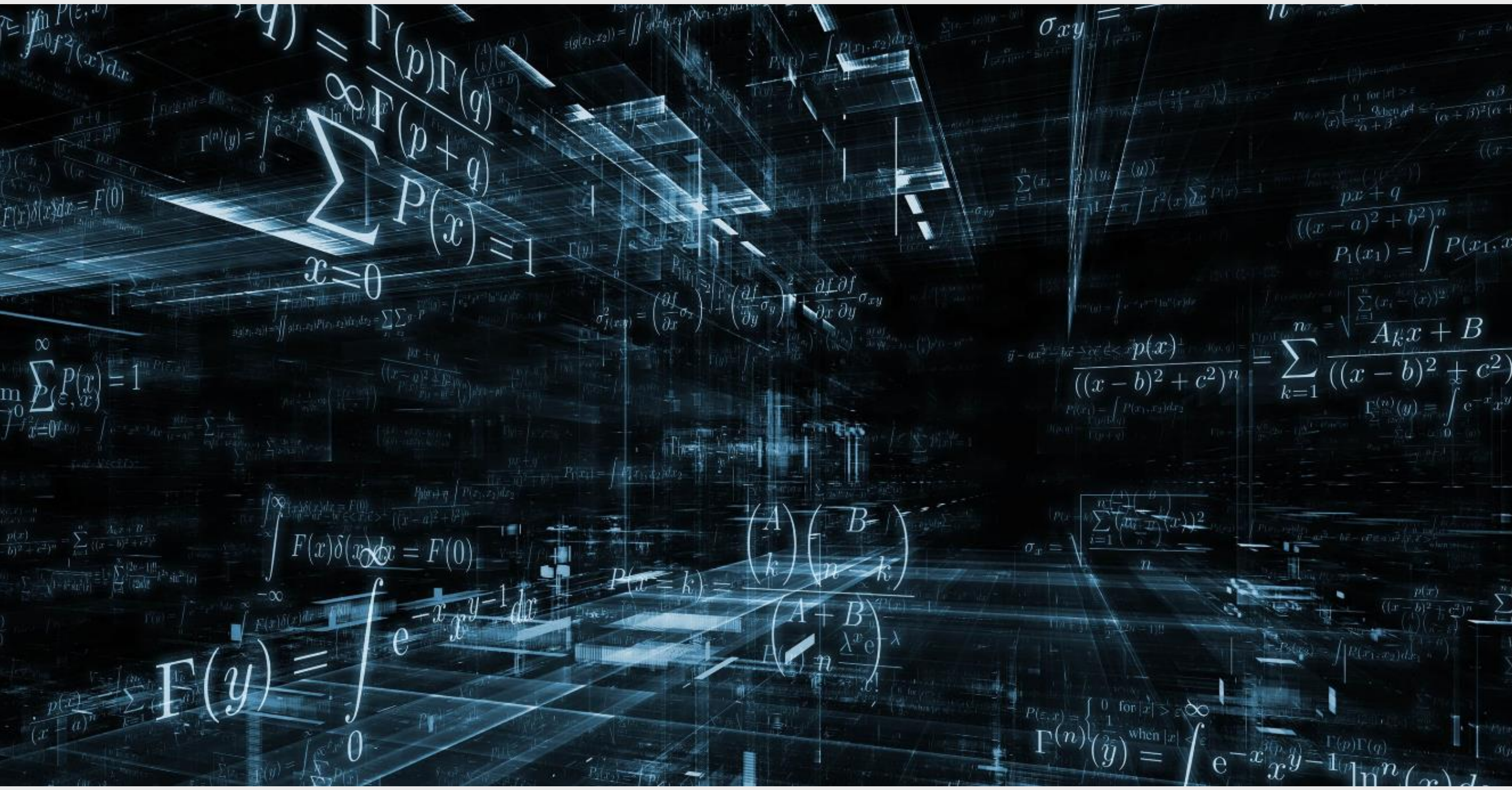
PHM212s: Complex, Special Functions and Numerical Analysis

Exercise Sheet

Numerical Analysis



"All the world's a differential equation, and

the men and women are merely variables."

-Ben Orlin

[1] Use Runge – Kutta method to:

i. Solve the differential equation: $\frac{dy}{dx} = x^2 - y$, over the intervals $x = 0$ to $x = 0.5$ with step size 0.5, where $y(0) = 1$ and from $x = 0.5$ to $x = 1.5$ with step size 1.0. [Final 2015]

ii. Solve the differential equation: $y' = x + y^2$ to **find** $y(1)$. ; $y(0.5) = 1$ & $h = 0.25$

[Final 2017] [Spring 2019]

iii. Given $y' = x^2 - y, y(1) = 0.5$. **Find** $y(1.2)$ with $h = 0.1$. [Spring 2021]

iv. Using <u>two steps</u>, **Solve** the following system of ODEs to **Find** $x(0.2)$ & $y(0.2)$

$$x' = x - y - t,\quad y' = 4x - 2y, \qquad x(0) = 1, y(0) = 0$$

[Summer 2021]

[2] Consider the Dirichlet problem $\nabla^2 u(x, y) = 18(x^2 + y^2)$ over the square $0 \le x \le 1$ and $0 \le y \le 1$ Such that: $u(x, y) = 0$ on the top of the square and $u(x, y) = 100$ on the sides and bottom of the square. [Spring 2019]

(i) **Generate** a grid by taking $h = \frac{1}{3}$ and applying the finite difference method

(ii) **Solve** the resulting system of equations using Gauss – Seidel method assuming zero initial values. **Obtain** the first three successive approximations of the solutions.

[3] Solve (using $h = \frac{1}{4}$) the Dirichlet boundary value problem for the Laplace equation: $\nabla^2 u = 0$ in the region $x > 0, y > 0, x + y < 1$.

With the boundary conditions defined by:

$$u(0, y) = 0, \qquad u(x, 1 - x) = 0, \qquad u(x, 0) = x(1 - x)$$

Use Gauss – Seidel method to **solve** the resulting system of linear equations starting with zero values at interior points. **Perform** <u>three</u> iterations. *[Spring 2021]*

[4] Find the solution *(accurate to 3D)* for the Dirichlet boundary value problem for the Poisson equation $\nabla^2 u(x, y) = 10(x^2 + y^2)$ in the region and for the boundary conditions shown in the figure below. **Use** Gauss – Seidel method with zero initial approximations for the interior points. *[Summer 2021]*