Section 7

3 properties of integration: → for any proof of integration, prove the differentiation and then integrate

① $2J_n' = J_{n-1} - J_{n+1} \Rightarrow \int J_{n+1}\,dx = \int J_{n-1}\,dx - 2J_n$

$J_0' = -J_1 \Rightarrow \int J_1\,dx = -J_0 + c$

exp: $\int J_3(x)\,dx = \int J_1\,dx - 2J_2 = -J_0 - 2J_2 + c$

$n=2$

$\int J_5(x)\,dx = \int J_3\,dx - 2J_4$

$n=4$

$= -J_0 - 2J_2 - 2J_4 + c$

$\int J_4(x)\,dx = \int J_2\,dx - 2J_3$

$(\underbrace{\int J_0\,dx}_{\text{numerically Solved}} - 2J_1) - 2J_3 + c$

$\int x^n J_n$ <
- Power - Index = 1: $m - n$
- $m + n = 1$

② $\frac{d}{dx}(x^n J_n) = x^n J_{n-1} \Rightarrow \int x^n J_{n-1}\,dx = x^n J_n + c$

$\frac{d}{dx}(x^{-n} J_n) = -x^{-n} J_{n+1} \Rightarrow \int x^{-n} J_{n+1}\,dx = -x^{-n} J_n + c$

exp: $\int x^3 J_2\,dx = x^3 J_3 + c$

$\int x^{-3} J_4\,dx = -x^{-3} J_3 + c$

$$\int x^{3/2} \sqrt{\frac{2}{\pi x}} \sin x \, dx = \sqrt{\frac{2}{\pi}} \int x \sin x \, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[x(-\cos x) - 1 \sin x \right]$$

ممكن اضرب في $\frac{\sqrt{x}}{\sqrt{x}}$ واجيبها لل J

$$\int x^{3/2} J_{1/2} \, dx = x^{3/2} J_{3/2} + c$$

$$\int x^{1/2} J_{\frac{1}{2}} \, dx = -x^{1/2} J_{-\frac{1}{2}}$$

Power > index

exp: $\int x^5 J_0 \, dx$

$u = x^4 \qquad dv = x J_0 \, dx$

$du = 4x^3 dx \qquad v = x J_1$

$2 > 1$

Power > index

the difference became less

$I = x^5 J_1 - 4 \int x^4 J_1 \, dx$ لازم تجرب وتشوف انهي هيطلع تكامله احسن

$u = x \qquad dv = x J_1$

$du = 2x\,dx \qquad v = x^2 J_2$

$$I = x^5 J_1 - 4\left[x^4 J_2 - 2\int x^3 J_2 \, dx\right]$$

$$= x^5 J_1 - 4x^4 J_2 + 8x^3 J_3 + C$$

$\to m+n=1$
$m-n=1$

NB: $\int x^1 J_0 \, dx = x J_1 = -x J_{-1} = -x(-J_1) = x J_1$

$4, -2$ #

Power < index

$- \int x^0 J_3 \, dx$

$u = x^2 \qquad dv = x J_3 \, dx$

$du = 2x\,dx \qquad v = -x^2 J_2$

$$I = -J_2 + 2\int x^{-1} J_2 \, dx$$

$$= -J_2 + 2(-x^{-1} J_1) + C$$

Complex.

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$f(z) = z^2 < \begin{matrix} z = x+iy \\ z = re^{i\theta} \end{matrix} \quad \text{or} \quad f(z) = (re^{i\theta})^2$$

$$= r^2 e^{i2\theta}$$

$$= (x+iy)^2 \qquad = r^2[\cos(2\theta) + i\sin(2\theta)]$$

$$= \underbrace{x^2 - y^2}_{Re\{z\}} + \underbrace{(2xy)}_{Im\{z\}} i \qquad = \underbrace{r^2\cos 2\theta}_{Re\{z\}} + i\underbrace{(r^2 \sin 2\theta)}_{Im\{z\}}$$

NB. Both are rectangular except on is in Cartesian & one is in polar coordinates.

$$f(z) = \frac{z+i}{z^2+i} \qquad z = x+iy$$

$$= r\cos\theta + i\sin\theta$$

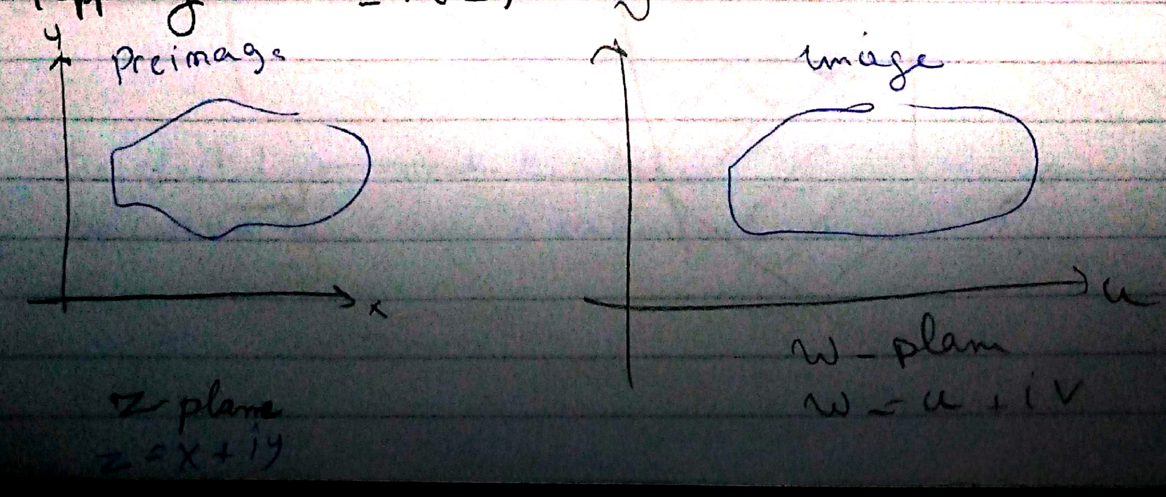
$$= \frac{z+iy+i}{(x+iy)^2+1} = \frac{x+i(y+1)}{(x^2-y^2+1)+i(2xy)} * \frac{(x^2-y^2+1)-i2xy}{(x^2-y^2+1)-i2xy}$$

*Conjugate

$$= \frac{(x^3 - xy^2 + x + 2xy^2 + 2xy) + i\,(-2x^2y + (y+1)(x^2-y^2+1))}{(x^2-y^2+1)^2 + (2xy)^2}$$

$$= \underbrace{\frac{x^3 + xy^2 + x + 2y^2 + 2xy}{(x^2-y^2+1)^2 + (2xy)^2}}_{Re\{z\}} + i\underbrace{\left[\frac{-2x^2y + (y+1)(x^2-y^2+1)}{}\right]}_{Img\{z\}}$$

Mapping: $w = f(z)$



3 in As.

exp'r Pre image $y > 0$

$\omega = (1+i)z$? Image ?

$\omega = u + iv = f(z)$

$Re^{i\phi}$ $x+iy$ $re^{i\theta}$

$y = g(u, v)$

$u + iv = (1+i)(x+iy)$

$x + iy = \frac{u+iv}{1+i} * \frac{1-i}{1-i}$

$= \frac{(u+v) + i(-u+v)}{1+1}$

$x + iy = \frac{1}{2}(u+v) + i\,\frac{1}{2}(v-u)$

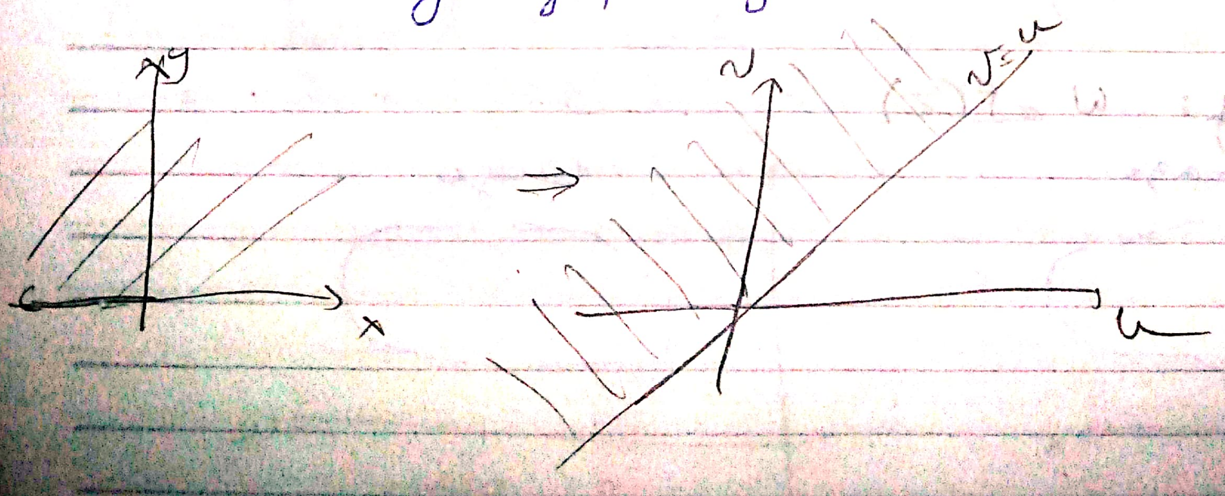
$x = \frac{u+v}{2}$, $y = \frac{v-u}{2}$

$y > 0$ $\frac{v-u}{2} > 0 \Rightarrow v - u > 0$

$\therefore \boxed{v > u}$ Image

Show the region graphically.

$v = u$



Preimage

Exp: $\omega = F(z) = z^3 \quad , |z| < 2 \quad , 0 \leq \theta \leq \frac{\pi}{3}$

$$\omega = z^3 = (re^{i\theta})^3 = r^3 e^{i3\theta}$$

$\Rightarrow R = |\omega| = r^3 = \mathrm{Mag}(\omega)$

$\Phi = 3\theta \quad \mathrm{Arg}(\omega)$

Image

$|z| < 2 \Rightarrow r < 2 \Rightarrow R = r^3 < 8 \Rightarrow \boxed{R < 8}$

$0 \leq \theta \leq \frac{\pi}{3} \Longrightarrow 0 \leq 3\theta \leq \pi \rightarrow \boxed{0 \leq \Phi \leq \pi}$

linear transformation

$\omega = az + b = r_a e^{i\theta_a} z + (b_1 + ib_2)$

1: Shape preImage & Image are the same

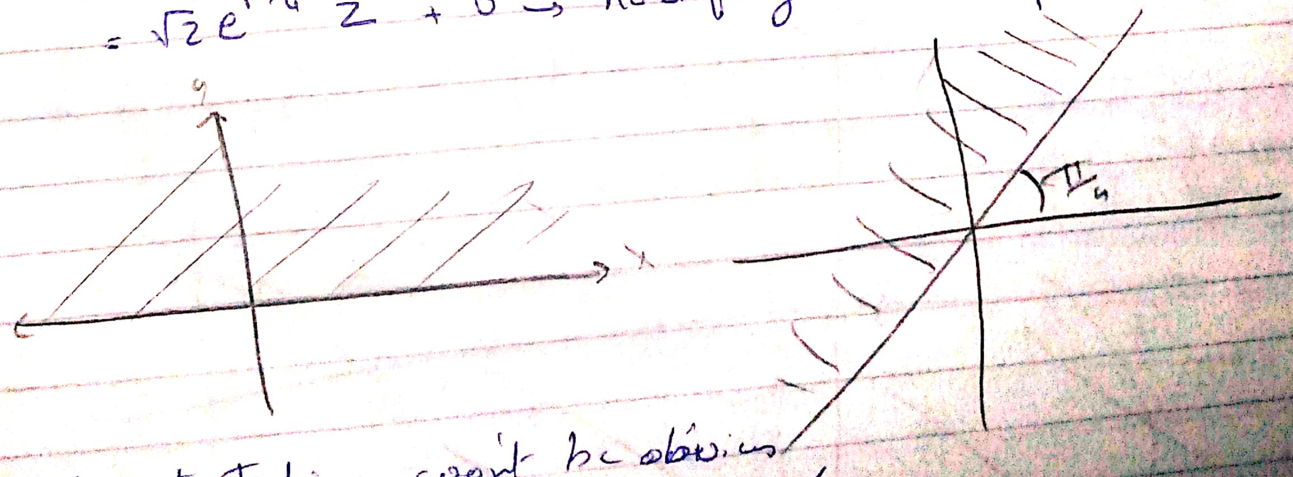
2- Rotation with θ_a

3 - stretching/shrinking with r_a

4- shifting with $b_1 \rightarrow$, $b_2 \uparrow$

$\omega = (1+i)z$

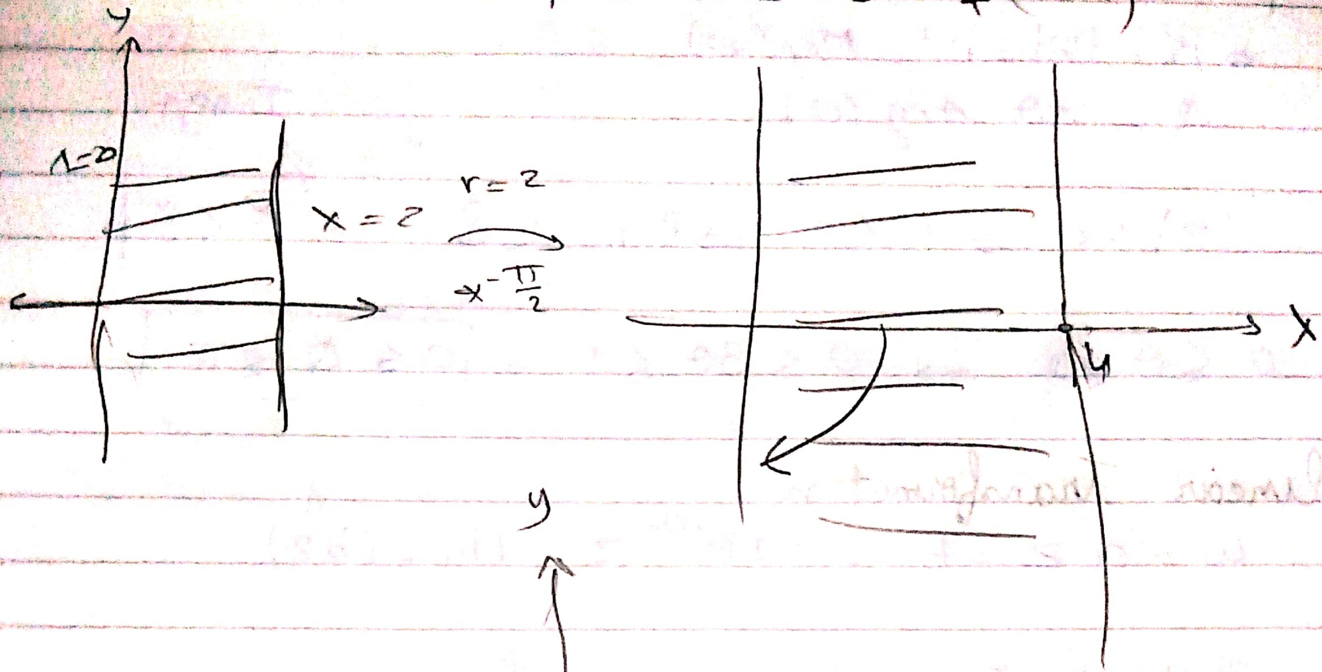
$= \sqrt{2}e^{i\pi/4} z + 0 \rightarrow$ no shifting $\quad \theta = \frac{\pi}{4}$

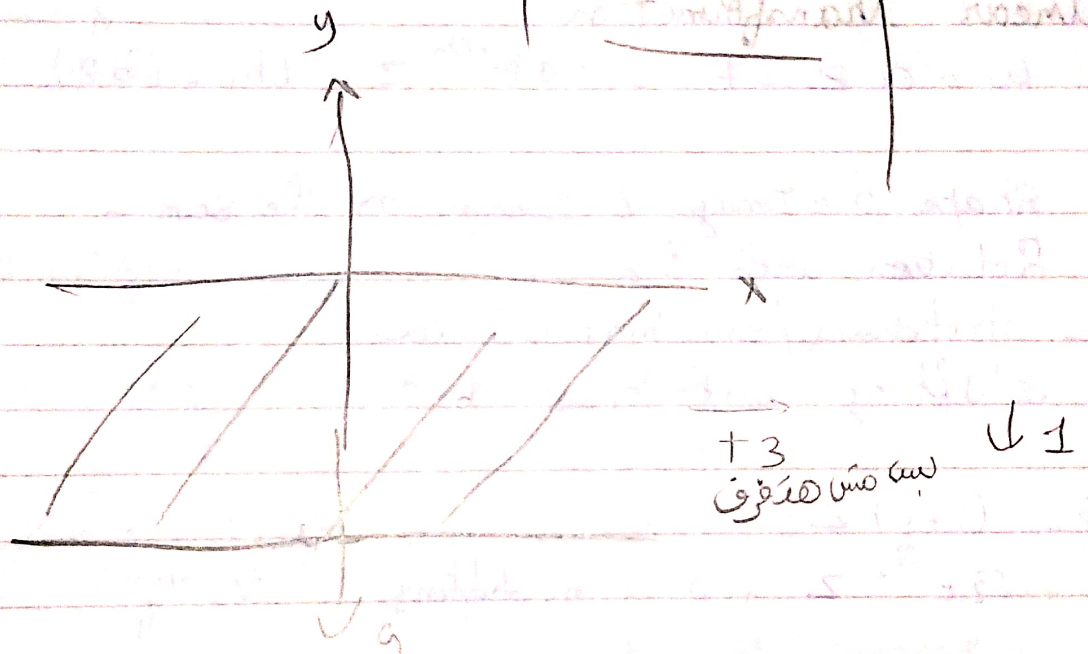


the stretching won't be obvious because the x axis is unlimited graphical tranformation in linear transformation only

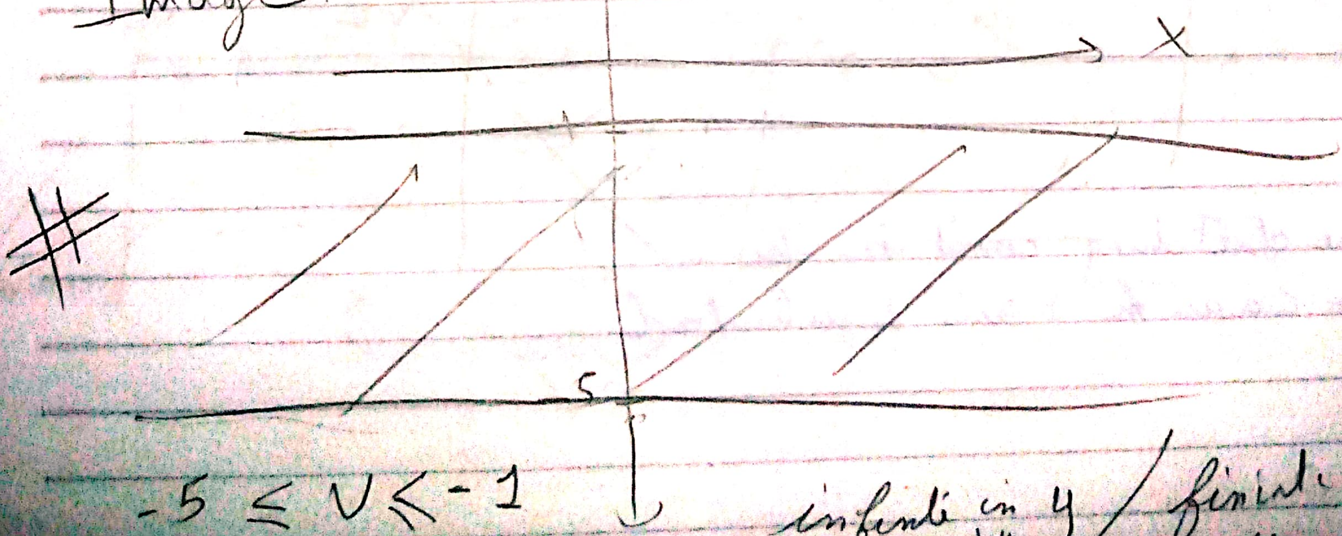
Pre-image = semi infinite strip : $0 \leq x \leq 2$

$$\omega = -2iz + 3 - i = 2e^{i\frac{-\pi}{2}} + (3-i)$$





Image



$-5 \leq v \leq -1$

infinite in y / finite in x
infinite x / finite y