Math. (3) Midterm Exam.

2nd Year Electrical Eng.

November 11th , 2019. Allowed Time: 75 Minutes.

MODEL ANSWER		Model A

The exam consists of TWO questions in TWO pages.

Question 1 (10 Marks)

(A) Evaluate in terms of the Gamma function [4 Marks]

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt[3]{1 + x^6}} .$$

$$I = 2\int_0^{\infty} \frac{dx}{\sqrt[3]{1 + x^6}}$$

$$let\ x^6 = u \ , x = u^{1/6} \ , \ dx = \frac{1}{6} u^{-5/6} \ du ....\boxed{2}$$

$$I = \frac{1}{3}\int_0^{\infty} \frac{u^{-5/6}\ du}{(1+u)^{1/3}} \ , \quad x-1 = -5/6 \to x = 1/6$$

$$x + y = 1/3 \to y = 1/6................\boxed{1}$$

$$I = \frac{1}{3}\beta(\frac{1}{6},\frac{1}{6}) = \frac{1}{3}\frac{\Gamma^2(1/6)}{\Gamma(1/3)} ....\boxed{1}$$

(B) Find the general solution in powers of $(x-1)$ for the following differential equation:

$$y'' + 2(x-1)y' = 0$$ **[6 Marks]**

$p(x) = 2(x-1), q(x) = 0$

$\Rightarrow p(x)$ & $q(x)$ are both analytic at $x_0 = 1$

$\Rightarrow x_0 = 1$ is an ordinary point $\Rightarrow$ Power series Method ....$\boxed{1}$

$let\ \ t = x-1\ ,\ y' = \dot{y}\ ,\ y'' = \ddot{y}$...................$\boxed{1}$

$let\ \ y = \sum_{n=0}^{\infty} a_n t^n \Rightarrow \dot{y} = \sum_{n=1}^{\infty} n a_n t^{n-1} \Rightarrow \ddot{y} = \sum_{n=2}^{\infty} n(n-1) a_n t^{n-2}$

$\Rightarrow \ddot{y} + 2t\dot{y} = 0$

$\Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n t^{n-2} + 2\sum_{n=1}^{\infty} n a_n t^n = 0$

$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} t^n + 2\sum_{n=1}^{\infty} n a_n t^n = 0$

$2a_2 + \sum_{n=1}^{\infty} \begin{Bmatrix} (n+2)(n+1)a_{n+2} \\ +2n a_n \end{Bmatrix} t^n = 0$...................$\boxed{1}$

$a_2 = 0$.................$\boxed{1}$

$\Rightarrow (n+2)(n+1)a_{n+2} + 2n a_n = 0$

$\Rightarrow a_{n+2} = \dfrac{-2n}{(n+2)(n+1)} a_n, n \geq 1$..................$\boxed{1}$

$a_2 = a_4 = a_6 = ... = 0$

$n = 1 \Rightarrow a_3 = \dfrac{-2.1}{3.2} a_1,$

$n = 3 \Rightarrow a_5 = \dfrac{-2.3}{5.4} a_3 = \dfrac{(-1)^2 2^2 (1.3)}{(5.4.3.2.1)} a_1,$

$\Rightarrow a_{2n+1} = \dfrac{(-1)^n 2^n (1.3........(2n-1))}{(2n+1)!} a_1$

$\Rightarrow y = a_0 + a_1 (t + \sum_{1} \dfrac{(-1)^n 2^n (1.3........(2n-1))}{(2n+1)!} t^{2n+1})$

$y = a_0 + a_1 \left[(x-1) + \sum_{n=1}^{\infty} \dfrac{(-2)^n (1.3........(2n-1))}{(2n+1)!} (x-1)^{2n+1} \right]$..................$\boxed{1}$

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		Model A

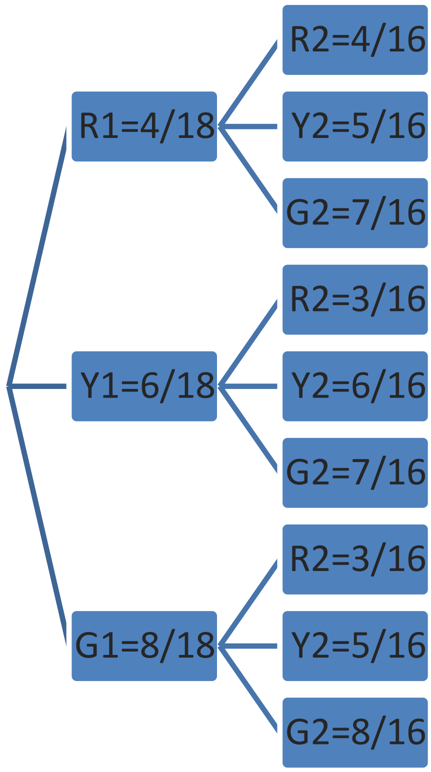
Question 2 (10 + 1 Marks)

The distribution of colored balls in two boxes I and II are as follows:

	Red	Yellow	Green
Box I	4	6	8
Box II	3	5	7

A ball (B_1) is selected at random from Box I and then transferred unseen to Box II. A ball (B_2) is now selected from the new Box II. Find

1. **P (B_1 was Red), P (B_1 was Yellow), and P (B_1 was Green).**
2. **P (B_2 is red), P (B_2 is Yellow), and P (B_2 is Green).**
3. **P (B_1 and B_2 are both Red), P (B_1 and B_2 are both Yellow), and P (B_1 and B_2 are both Green).**
4. **P (B_1 and B_2 both have the same color).**
5. **The probability that the first ball was Red, given that the second ball was found Green.**



2 marks

$$1.\ P(R1)=4/18,\ \ P(Y1)=6/18,$$

$$P(G1)=8/18\ \ ....\boxed{1}$$

$$2.\ P(R2)=\frac{4}{18}.\frac{4}{16}+\frac{6}{18}.\frac{3}{16}+\frac{8}{18}.\frac{3}{16}=\frac{29}{144}....\boxed{1}$$

$$P(Y2)=\frac{4}{18}.\frac{5}{16}+\frac{6}{18}.\frac{6}{16}+\frac{8}{18}.\frac{5}{16}=\frac{1}{3}....\boxed{1}$$

$$P(G2)=\frac{4}{18}.\frac{7}{16}+\frac{6}{18}.\frac{7}{16}+\frac{8}{18}.\frac{8}{16}=\frac{67}{144}\ \ ....\boxed{1}$$

$$3.\ P(R1\cap R2)=\frac{4}{18}.\frac{4}{16}=\frac{1}{18}....\boxed{1}$$

$$P(Y1\cap Y2)=\frac{6}{18}.\frac{6}{16}=\frac{1}{8}....\boxed{1}$$

$$P(G1\cap G2)=\frac{8}{18}.\frac{8}{16}=\frac{2}{9}\ ....\boxed{1}$$

$$4.\ P(R1\cap R2)+P(Y1\cap Y2)+P(G1\cap G2)=\frac{29}{72}....\boxed{1}$$

$$5.P(R1|G2)=\frac{P(G2|R1).P(R1)}{P(G2)}=\frac{\frac{7}{16}.\frac{4}{18}}{\frac{67}{144}}=\frac{14}{67}....\boxed{1}.$$

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MODEL ANSWER		Model B

The exam consists of TWO questions in TWO pages.

Question 1 (10 Marks)

(A) Evaluate in terms of the Gamma function

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{1 + x^6}}$$

[4 Marks]

$$I = 2\int_0^{\infty} \frac{dx}{\sqrt{1 + x^6}}$$

$$let\ x^6 = u\ ,\ x = u^{1/6}\ ,\ dx = \frac{1}{6}u^{-5/6}\,du\ldots\ldots\ldots\ldots\ldots\boxed{2}$$

$$I = \frac{1}{3}\int_0^{\infty} \frac{u^{-5/6}\,du}{(1+u)^{1/2}}\ ,\quad x-1=-5/6 \rightarrow x = 1/6$$

$$x + y = 1/2 \rightarrow y = 1/3\ldots\ldots\ldots\ldots\ldots\boxed{1}$$

$$I = \frac{1}{3}\beta\left(\frac{1}{6},\frac{1}{3}\right) = \frac{1}{3}\frac{\Gamma(1/6)\Gamma(1/3)}{\Gamma(1/2)}\ldots\ldots\ldots\ldots\ldots\boxed{1}$$

(B) Find the general solution in powers of $(x-2)$ for the following differential equation:

$$\boldsymbol{y'' + 3(x-2)\,y' = 0}.$$

[6 Marks]

$p(x) = 3(x-2), q(x) = 0$

$\Rightarrow p(x)\ \&\ q(x)$ are both analytic at $x_0 = 2$

$\Rightarrow x_0 = 2$ is an ordinary point $\Rightarrow$ Power series Method .... $\boxed{1}$

$let\ \ t = x-2\ \ , y' = \dot{y}\ \ ,\ \ y'' = \ddot{y}$.................... $\boxed{1}$

$$let\ \ y=\sum_{n=0}^{\infty} a_n t^n \Rightarrow y' = \sum_{n=1}^{\infty} n a_n t^{n-1} \Rightarrow y'' = \sum_{n=2}^{\infty} n(n-1) a_n t^{n-2}$$

$$\Rightarrow \ddot{y} + 3t\dot{y} = 0$$

$$\Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n t^{n-2} + 3\sum_{n=1}^{\infty} n a_n t^n = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} t^n + 3\sum_{n=1}^{\infty} n a_n t^n = 0$$

$$2a_2 + \sum_{n=1}^{\infty} \begin{Bmatrix} (n+2)(n+1)a_{n+2} \\ +3na_n \end{Bmatrix} t^n = 0 \text{....................} \boxed{1}$$

$a_2 = 0$.................. $\boxed{1}$

$$\Rightarrow (n+2)(n+1)a_{n+2} + 3na_n = 0$$

$$\Rightarrow a_{n+2} = \frac{-3n}{(n+2)(n+1)} a_n \qquad , n \ge 1 \text{..................} \boxed{1}$$

$$a_2 = a_4 = a_6 = \ldots\ldots = 0$$

$$n = 1 \Rightarrow a_3 = \frac{-3.1}{3.2} a_1,$$

$$n = 3 \Rightarrow a_5 = \frac{-3.3}{5.4} a_3 = \frac{(-1)^2 3^2 (1.3)}{(5.4.3.2.1)} a_1,$$

$$\Rightarrow a_{2n+1} = \frac{(-1)^n 3^n (1.3\ldots\ldots(2n-1))}{(2n+1)!} a_1$$

$$\Rightarrow y = a_0 + a_1\Big(t + \sum_{1} \frac{(-1)^n 3^n (1.3\ldots\ldots(2n-1))}{(2n+1)!} t^{2n+1}\Big)$$

$$y = a_0 + a_1\left[(x-2) + \sum_{n=1}^{\infty} \frac{(-3)^n (1.3\ldots\ldots(2n-1))}{(2n+1)!} (x-2)^{2n+1}\right] \text{..................} \boxed{1}$$

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		Model B

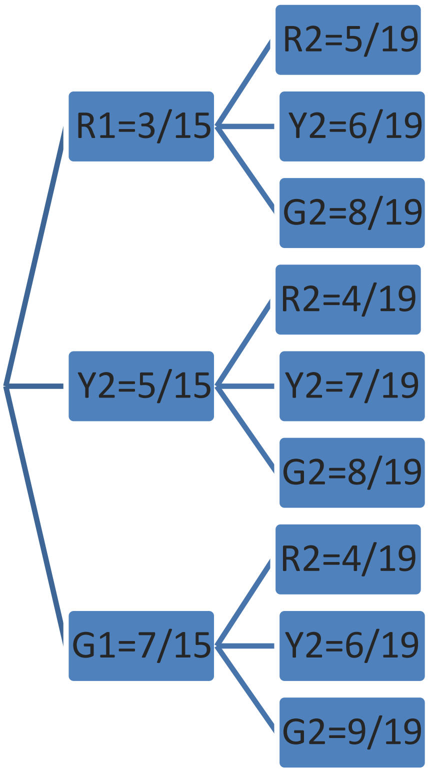
Question 2 (10+1 Marks)

The distribution of colored balls in two boxes I and II are as follows:

	Red	Yellow	Green
Box I	3	5	7
Box II	4	6	8

A ball (B_1) is selected at random from Box I and then transferred unseen to Box II. A ball (B_2) is now selected from the new Box II. Find

1. **P (B_1 was Red), P (B_1 was Yellow), and P (B_1 was Green).**
2. **P (B_2 is red), P (B_2 is Yellow), and P (B_2 is Green).**
3. **P (B_1 and B_2 are both Red), P (B_1 and B_2 are both Yellow), and P (B_1 and B_2 are both Green).**
4. **P (B_1 and B_2 both have not the same color).**
5. **The probability that the first ball was Yellow, given that the second ball was found Red.**



$$1.\ P(R1) = 3/15,\ \ P(Y1) = 5/15,$$

$$P(G1) = 7/15 \ \ldots\ldots\ldots\ldots\ldots \boxed{1}$$

$$2.\ P(R2) = \frac{3}{15}.\frac{5}{19} + \frac{5}{15}.\frac{4}{19} + \frac{7}{15}.\frac{4}{19} = \frac{21}{95} \ldots\boxed{1}$$

$$P(Y2) = \frac{3}{15}.\frac{6}{19} + \frac{5}{15}.\frac{7}{19} + \frac{7}{15}.\frac{6}{19} = \frac{1}{3} \ldots\boxed{1}$$

$$P(G2) = \frac{3}{15}.\frac{8}{19} + \frac{5}{15}.\frac{8}{19} + \frac{7}{15}.\frac{9}{19} = \frac{127}{285} \ldots\boxed{1}$$

$$3.\ P(R1 \cap R2) = \frac{3}{15}.\frac{5}{19} = \frac{1}{19} \ldots\boxed{1}$$

$$P(Y1 \cap Y2) = \frac{5}{15}.\frac{7}{19} = \frac{7}{57} \ldots\boxed{1}$$

$$P(G1 \cap G2) = \frac{7}{15}.\frac{9}{19} = \frac{21}{95} \ldots\boxed{1}$$

2 marks

$$4.\ 1 - \left\{ P(R1 \cap R2) + P(Y1 \cap Y2) + P(G1 \cap G2) \right\} = \frac{172}{285} \ldots\boxed{1}$$

$$5.\ P(Y1 | R2) = \frac{P(R2 | Y1).P(Y1)}{P(R2)} = \frac{\frac{4}{19}.\frac{5}{15}}{\frac{21}{95}} = \frac{20}{63} \ldots\boxed{1}$$