→ when we added the conjugate to our complex variable we got Real value which means we rotated our variable to a

→ when we add a constant it is shifted right
~ ~ sub ~ ~ ~~ ~ left

→ default is anticlockwise.

Functions of Complex variables
Lecture 7

Complex n. in polar form

If $z_1 = \sqrt{3} + i \therefore r = 2 \quad \theta = \frac{\pi}{6}$

let $z_2 = z_1 \therefore r_1 = r_2$ $\boxed{But}$

$\theta_{z_2} = 2\pi \, \theta_{z_1} n \quad n = 0, \pm 1, \pm 2, \ldots$

→ Complex numbers

Multiplication :

$(r_1, \theta_1)(r_2, \theta_2) = r_1 r_2 (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$

$(r_1, \theta_1)(\ldots)(r_n, \theta_n) = (r_1 \ldots r_n, \theta_1 \ldots + \theta_n)$

$(r_1, \theta_1)^n = (r_1^n, n\theta_1)$

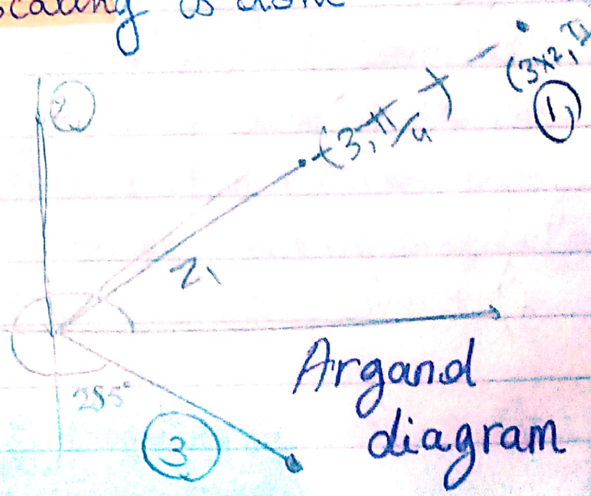
$(z)^n = (r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta_1 + i\sin n\theta_1)$

De Moivre theorem
↳ helps us get the roots

geometry of complex numbers let $z_1 = 3e^{i\frac{\pi}{4}}$ & $z_2 = 2e^{i\frac{\pi}{3}}$

① $z_1 \cdot 2 = z_1 : 2e^{i0}$ ⇒ Same argument of z
↳ when we multiply by a constant a scaling is done

② $z_1 \cdot i = z_1 e^{i\frac{\pi}{2}}$
↳ Same norm/magnitude but different angle
∴ rotation



Argand diagram

③ $z_1 (e^{i\frac{4\pi}{3}}) = z_1 e^{i\frac{19}{12}\pi}$
rotation

④ $z_1 z_2 = 6 e^{\frac{7}{12}\pi}$
different norm and argument (a Scaling & rotation)

$\tan x$ selected domain for it to be a function
one to one

→ remember:
→ why is $(-\pi, \pi)$ $-\pi$ doesn't belong to the interval of principal argument

enlarging happens when we multiply by const
shrinking happens when we divide
rotation anticlockwise happens when we multiply by +angle

remember: adding → Cartesian
mult. → polar

Mapping: Transformations:

* when we perform transformation it's like we transfered point from z plane to w plane

let $w = f(z)$; $z = x + iy$ (z: complex variable)

$w = \underbrace{u(x,y)}_{Re\{w\}} + i\,\underbrace{v(x,y)}_{Im\{w\}}$

u   v

Exp: $w = z^2 = (x+iy)^2 = (x^2 - y^2) + 2xy\,i$ ∴ $u(x,y) = x^2 - y^2$ & $v(x,y) = 2xy$

exp: $w = e^z = e^{x+iy} = e^x(\cos y + i \sin y)$ $u(x,y) = e^x \cos y$; $v(x,y) = e^x \sin y$

Translation: when we add to $f(z) = z + b$
b: constant complex number
Rotation: $F(z) = e^{i\alpha} z$
Scaling: $f(z) = 2 . z$

→ when we perform transformation the new plane is called image of the first

Recall when we had $z = F(x,y)$ $D \in R^2$ z was a third axis and drawing was 3D here we have real and imaginary and the 2 outputs we will need 4D

* Famous tranformations:

Linear transformation

$w = az + b$

a, b are general constant complex

↳ it doesn't change figure shapes

↳ It shifts the origin to $\underline{b}$

It scales with a

shrinking if $|a| < 1$ enlarging $|a| > 1$

It rotates with arg (a)

anticlockwise $\arg(a) > 0$ clockwise $\arg(a) < 0$

Receprocal transformation

$w = \frac{1}{z}$

recall $z\bar{z} = |z|^2$

$\frac{1}{z} = \left(\frac{\bar{z}}{|z|^2}\right)$

→ Cartesian

$\bar{z} = \frac{z}{|z|^2}$, $w = \bar{z}$

Scalling shrinking or magnifying

mirror about x-axis

$\overline{\left(\frac{1}{z}\right)} = \frac{1}{\bar{z}}$

→ Polar: $w = \frac{1}{z} = \frac{1}{re^{i\theta}} = \frac{1}{r} e^{-i\theta}$

thus $|w| = \frac{1}{|z|}$

& $\arg(w) = -\arg(z)$

* inversion: $f(z) = \frac{1}{z}$

Scaling &

rotation Counter clockwise