AIN SHAMS UNIVERSITY FACULTY OF ENGINEERING

SPECIALIZED ENGINEERING PROGRAMS
JUNIOR COMMUNICATION ENGINEERING PROGRAM



SPRING 2022 Assignment #3 Total: 5 marks

PHM212s: Special Functions, Complex Analysis & Numerical Analysis

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Name:

Deadline: Week 9

Please, Solve each problem in its assigned place ONLY (the empty space below it)

ID:

Bessel Functions

1. Solve in terms of Bessel functions the following differential equations:

a)
$$x^2 y'' + x y' + (x^2 - 9)y = 0$$

b)
$$x^2 y'' + x y' + (x^2 - 8)y = 0$$

c)
$$x^2 y'' + x y' + (3x^2 - 4)y = 0$$

d)
$$x^2 y'' + x y' + 4(x^4 - n^2) y = 0$$
 , $n \in I$

e)
$$x y'' + 3 y' + x y = 0$$

f)
$$4 x y'' + 4 y' + y = 0$$

- 2. Find the solution of x^2 y'' + x $y' + (4x^2 1)y = 0$ which is bounded at x = 0 and y(2) = 5
- 3. Show that:

a)
$$J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$$

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b)
$$Y_{1/2}(x) = -\sqrt{\frac{2}{\pi x}} \cos x$$

c)
$$J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right)$$

d)
$$J_{-3/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\cos x}{x} + \sin x \right)$$

4. Show that $y = x^n J_n(x)$ is a solution for the differential equation x y'' + (1-2n) y' + x y = 0 using two different methods. <u>1st Method:</u> 2nd Method:

5. Show that

a)
$$J_n''(x) = \frac{1}{4} (J_{n-2}(x) - 2J_n(x) + J_{n+2}(x))$$

b)
$$\frac{d}{dx} (J_n^2(x)) = \frac{x}{2n} (J_{n-1}^2(x) - J_{n+1}^2(x))$$

c)
$$\frac{d}{dx} (x J_n(x) J_{n+1}(x)) = x (J_n^2(x) - J_{n+1}^2(x))$$

6. Solve the following integrals in terms of Bessel Functions:

a)
$$\int x^3 J_2(x) dx$$

b)
$$\int x^{-4} J_5(x) dx$$

c)
$$\int x^4 J_1(x) dx$$

d)
$$\int \sqrt{x} J_{1/2}(x) dx$$

e)
$$\int x^{-2} J_2(x) dx$$

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7. Solve in terms of Bessel functions the following differential equations: y'' + x y = 0

$$y'' + x y = 0$$

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8. Solve in terms of Bessel functions the following differential equations: $x \ y'' + y = 0$

$$x y'' + y = 0$$

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Extra Problems

9. Use the generating function of Bessel functions to show that:

a)
$$J_0(x) + 2J_2(x) + 2J_4(x) + \dots = 1$$

b)
$$\cos(x \sin \theta) = J_0(x) + 2J_2(x) \cos 2\theta + 2J_4(x) \cos 4\theta + \dots$$

c)
$$\sin(x\sin\theta) = 2J_1(x)\sin\theta + 2J_3(x)\sin3\theta + \dots$$

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10. Prove that $J_n(x) = \frac{1}{2\pi} \int_0^{2\pi} \cos(n\theta - x \sin\theta) d\theta$ for n is a non-negative integer.

11. Show that $\int x^m J_n(x) dx$ can be evaluated in a closed form if m+n is odd and in terms of $\int J_0(x) dx$ (which can't be integrated in closed form) if m+n is even where n and m are integers.

Best wishes,
Dr. Makram Roshdy