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#### **PHM212s**:

# Complex, Special Functions and Numerical Analysis Series Solutions for Linear DEs





#### \* Series Solutions:

-> 1t's a method used to solve second order linear differential equation in Form of:

The <u>Solution</u> is in the torm of infinite series:  $\sum_{n=0}^{\infty} a_n (x-x_n)^n$ This solution is called solution around the point  $x_n$ a series solution in powers of  $(x-x_n)$ 

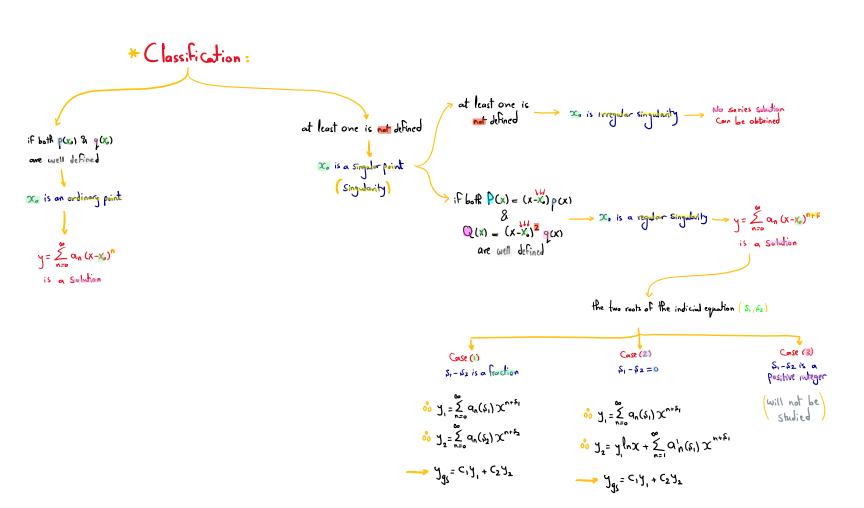
we will Solve for x = 0

Otherwise, we have to do some substitutions

let 
$$t = x - x$$
.  $\Rightarrow x = t + x$ .  
 $\frac{dt}{dx} = 1$   
 $y' = \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \dot{y}$ 

$$y'' = \frac{d}{dx} (y') = \frac{d}{dx} (\dot{y}) = \frac{d}{dt} (\dot{y}) \cdot \frac{dt}{dx} = \ddot{y}$$

-> then substitute in DE & the solution will be \$\frac{5}{2} ant^n



## \* Steps of Solving around an ordinary point:

(1) Compare with y" + p(x)y' + q(x)y = 0

2 Let 
$$y = \sum_{n=0}^{\infty} a_n x^n$$
 is a solution 
$$y^n = \sum_{n=1}^{\infty} (n) a_n x^{n-1}$$
$$y^n = \sum_{n=2}^{\infty} (n) (n-1) a_n x^{n-2}$$

3 Substitute in the DE ماط تخلی که Replace each n wift n 22

Coefficients of  $\chi^n = 0$   $\longrightarrow$  find a relation between  $a_{n+2}$  &  $a_n$  (Recourrence Relation)

5 Try to find a general form for any an interms of a. & a, X Subs. with 0,2,4 to get a general form for even values of n X Subs. with 1,2,3 to get a general form for odd values of n Careful: recourrence may be defined for no 2 the general form's defined for loss

the Solution is  $y = \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} a_{2n} x^{2n} + \sum_{n=0}^{\infty} a_{2n+1} x^{2n+1}$  $y = a_0 \left[ 1 + \sum_{k=1}^{\infty} a_{2k} x^{2k} \right] + a_1 \left[ x + \sum_{k=1}^{\infty} a_{2k+1} x^{2k+1} \right]$ 

### \* Steps of Solving around a Regular Singularity:



- Compare with y" + p(x)y' + q(x)y = 0
- 2 Let  $y = \sum_{n=0}^{\infty} a_n x^{n+s}$  is a solution  $y'' = \sum_{n=0}^{\infty} (n+s) a_n x^{n+s-1}$   $y''' = \sum_{n=0}^{\infty} (n+s) (n+s-1) a_n x^{n+s-2}$

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- 5 Solve the indicial equation to specify the Case 2
- Try to find a general form for an in 185 in terms of a.
- The Solution is ygs = C, y, + C2 y2

#### \* Case ! :

→ if Si - Sz is a fraction

$$\therefore y_1 = \sum_{n=0}^{\infty} Q_n(\S_1) \chi^{n+\S_2}$$

$$y_2 = \sum_{n=0}^{\infty} Q_n(\S_2) \chi^{n+\S_2}$$

\* Case 2 :

$$\rightarrow$$
 if  $S_1 - S_2 = 0$ 

$$\therefore y_1 = \sum_{n=0}^{\infty} Q_n(S_1) \chi^{n+S_1}$$

$$Q_n'(S_1) = \frac{d}{dS} \left[ Q_n(S) \right]$$

$$S = S_1$$

$$A_n(S_1) = \frac{d}{dS} \left[ A_n(S) \right]$$

$$S = S_1$$

$$A_n(S_1) = \frac{d}{dS} \left[ A_n(S) \right]$$

$$S = S_1$$

$$A_n(S_1) = \frac{d}{dS} \left[ A_n(S_1) \right]$$

$$\therefore \alpha_n'(s) = [\dots] \cdot \alpha_n(s)$$