Ain Shams University
Faculty of Engineering
Credit Hours Engineering Programs

جامعة عين شمس
كلية الهندسة
برامج الساعات المعتمدة

PHM 116 & EMAT 232 - Complex & Special Functions
Final Exam, summer 2015
Dr. Makram Roshdy.

General:
1- Examination's duration: 180 minutes.
2- The Exam consists of Five Questions in Two pages – Attempt all questions. ***1/2***

Question (1): (8 Marks)

(A) **Evaluate** $\int_0^1 \sqrt[4]{1-x^4}\, dx$ **in terms of the Gamma function.**

(B) **Find two linearly independent series solutions in powers of** x **for the following differential equation:**

$$(x-x^2)\, y'' + (1-5x)\, y' - 4\, y = 0$$

(C) **Solve in terms of Bessel's function the differential equation** $x\, y'' - 7\, y' + x\, y = 0$.

Question (2): (6 Marks)

(A) **Show that** $J_{-1/2}(x) = \sqrt{\dfrac{2}{\pi x}} \cos x$. **Then use it to evaluate** $\int x^{3/2}\, J_{-1/2}(x)\, dx$.

(B) **Evaluate** $\int J_0(x)\, \sin x\, dx$.

(C) **Use the generating function for Legendre's polynomials to obtain an expression for** $P_n(0)$.

Question (3): (12 Marks)

(A) **Show that** $\int_{-1}^{1} P_n(x)\, P_m(x)\, dx = 0 \;\; , \;\; n \neq m$.

(B) **Show that** $(n+1) P_{n+1} = (2n+1)\, x\, P_n - n\, P_{n-1}$. **Hence, discuss** $\int_{-1}^{1} x\, P_n(x)\, P_5(x)\, dx$ **for all the values of** n.

(C) Find the first three non – zero terms in Legendre's series for $f(x)=\begin{cases}0 & -1<x\le 0\\ x & 0<x\le 1\end{cases}$.

Question (4): (8 Marks)

(A) Find the image of the semi – infinite strip $x\ge 0$, $0\le y\le 1/2$ under the transformation $w=\dfrac{1}{z}$

(Hint: Show the regions graphically).

(B) Show that Laplace's equation in the polar form is given by $u_{rr}+\dfrac{1}{r}u_r+\dfrac{1}{r^2}u_{\theta\theta}=0$.

(C) Show that $u=\dfrac{x}{x^2+y^2}$ is a harmonic function and find its conjugate "v" then find the analytic function $f(z)=u+iv$ in terms of z.

Question (5): (8 Marks)

(A) Find all the roots of.

(1) $\sin z = 2$ (2) $e^{3z-1}=1+i$

(B) Find all the values of $\left(\sqrt{3}+\mathrm{i}\right)^{5\pi i}$. **Hence, find its principal value.**

(C) Evaluate $\oint_C \dfrac{\cosh z}{z^4-4z^2}\,dz$, **where** C **is:**

i) $|z-2|=1$ ii) $|z-2|=3$

Best Wishes.