



Mid Term Examination

Spring 2022

Exam Time: 60 minutes.

PHM212s: Complex, Special Functions and Numerical Analysis

The Exam Consists of THREE Questions in THREE Pages. Answer All Questions Total Marks: 20 Marks

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General Instructions:

- Please read the examination paper carefully.
- Be sure to solve each question in its paper (you can use the back).
- Programmable & Graphical Calculators are NOT Allowed.

Question no. 1 (6 marks)

a) Use the Gamma function to evaluate the following integral:

$$\int_0^1 x^{4/3} \ln^3 x \, dx$$

[3 Marks]

b) Evaluate  $\int_{-\infty}^{\infty} \frac{x^4}{1+x^6} \, dx$  using the Gamma function.

[3 Marks]

Question no. 2 (7 marks)

a) Find and classify the singularities of the following differential equation:

$$(x - x^2)^2 y'' + 3x y' + (1 - x^2)y = 0$$

[3 Marks]

B) Solve in terms of Bessel functions the following differential equation:

$$x^2 y'' + x y' + (x^3 - 4)y = 0$$

[4 Marks]

**Question no. 3 (7 marks)**

Find two linearly independent solutions in powers of “  $x$  ” for the following differential equations:

$$(3 - x^2) y'' - x y' + 9 y = 0$$

*Best Wishes,  
Dr. Makram Roshdy, Dr. Betty Nagy.*