AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING
SPECIALIZED ENGINEERING PROGRAMS
JUNIOR COMMUNICATION ENGINEERING PROGRAM

SPRING 2022	Assignment #1	Total: 5 marks

PHM212s: Special Functions, Complex Analysis & Numerical Analysis

Instructors Name: Dr. Makram Roshdy, Dr. Betty Nagy

Name:	**ID:**	**Deadline: Week 3**

Please, Solve each problem in its assigned place ONLY (the empty space below it)

Gamma and Beta Functions

1. Prove that $L\left\{t^{n}\right\}=\dfrac{\Gamma(n+1)}{s^{n+1}}$ for any real number $n>-1$.

Hence, find Laplace transform for each of the following functions:

a) $t^{5/2}$

b) $t^{-1/3}$

c) $\sqrt{t}\ e^{-3t}$

---

2. Given that n is a positive integer and x is a real number, show that

$\beta(x,n)=\dfrac{(n-1)!}{x(x+1)(x+2)\ldots(x+n-1)}$. Hence, evaluate $\beta(0.1,3)$

3. For any non-negative integer 'n', show that

$$\Gamma(n+\frac{1}{2}) = \frac{(2n)!}{2^{2n}\ .\ n!}\sqrt{\pi}$$

a) Using Legendre duplication formula

b) Without using Legendre duplication formula

---

4. Show that $\int_0^\infty x^a \quad b^{-x} \quad dx = \frac{\Gamma(a+1)}{(\ln b)^{a+1}}$, where $a > -1$ and $b > 1$

---

5. Show that the area enclosed by the curve $x^4 \ + \ y^4 \ = 1$ is $\Gamma^2(\frac{1}{4}) / (2\sqrt{\pi})$

6. Use the Gamma and the Beta functions to evaluate the following integrals:

a) $\int_0^\infty x^3 \ e^{-2x^5} \ dx$

b) $\int_0^\infty 3^{-x^2} \ dx$

c) $\int_0^1 \sqrt[3]{x} \ \ln^5 x \ dx$

d) $\int_{-\infty}^{\infty} \frac{dx}{1 + x^4}$

e) $\int_0^{\pi/2} \sin^{3.04} x \ dx$

f) $\int_0^{\pi/2} \sqrt{\tan\theta} \ d\theta$

g) $\displaystyle\int_0^\infty \frac{x\ dx}{1+x^6}$

h) $\displaystyle\int_0^2 x\ (8-x^3)^{1/3}dx$

7. Show that $\beta(n\,,n+1)=\dfrac{\Gamma^2(n)}{2\,\Gamma(2n)}$.

Hence, deduce that $\displaystyle\int_0^{\pi/2}(\sin^{-3}\theta-\sin^{-2}\theta)^{1/4}\cos\theta\,d\theta=\frac{\Gamma^2(1/4)}{2\sqrt{\pi}}$

8. Show that $\beta(x\,,y)=\dfrac{y-1}{x}\beta(x+1\,,y-1)$

Best wishes,

Dr. Makram Roshdy