٣،٥٠ ج

الاســــــــــراء

ملزمة (6)

رياضـــــة

محمد عبد المنعم

اولي كهرباء

Laplace Transform (4)

solution of Differential equations

* Transforms of derivatives:

- $L[f'(t)] = sF(s) - f(0).$
- $L[f''(t)] = s^2F(s) - sf(0) - f'(0)$

وهكذا ...

- $L[f^{(n)}(t)] = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{(n-1)}(0).$

وليستخدم هنا في حل المعادلات التفاضلية على الصورة:

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \cdots + a_n y = f(x)$$

which is a linear Differential equation with constant Coefficients.

كما في الأمثلة التالية:

EX (1): solve the following initial value problem:

$$\frac{dy}{dt} - 4y = 8, \quad y(0) = 2.$$

solution

Take L.T. for both sides.

$$\Rightarrow [sY(s) - \underset{y(0)}{2}] - 4Y(s) = \frac{8}{s}$$

$$\Rightarrow (s-4)Y(s) = \frac{8}{s} + 2$$

$$= \frac{2s+8}{s}.$$

$$\Rightarrow Y(s) = \frac{2s+8}{s(s-4)}$$ ②

$$\Rightarrow y(t) = L^{-1}\left[\frac{2s+8}{s(s-4)}\right]$$

$P(s) = 2s+8,$
$Q(s) = s(s-4)$

$$= \frac{P(0)}{Q'(0)} e^{0t} + \frac{P(4)}{Q'(4)} e^{4t}.$$

$$= \frac{8}{-4} e^{0t} + \frac{16}{4} e^{4t}$$

$$\Rightarrow \boxed{y(t) = -2 + 4e^{4t}} \quad \#$$

Problems ③ ٬حل المسألة ② من تمارين الكتاب

Solve the following I.V.P. :

$3y' - 4y = \sin 2t \quad ; y(0) = \frac{1}{3}.$

$\frac{dy}{dt}$

solution

Take L.T. for both sides

$$\Rightarrow 3\left[sY(s) - \tfrac{1}{3}\right] - 4Y(s) = \frac{2}{s^2+4}$$

$$\Rightarrow (3s-4)Y(s) = \frac{2}{s^2+4} + 1 = \frac{s^2+6}{s^2+4}$$

$$\Rightarrow Y(s) = \frac{s^2+6}{3(s-\frac{4}{3})(s^2+4)}$$

$$\Rightarrow y(t) = \frac{1}{3} L^{-1}\left[\frac{s^2+6}{(s-\frac{4}{3})(s^2+4)}\right].$$

Scanned by CamScanner

the factor $(s-\frac{4}{3})$: $\quad P(s) = s^2+6$, (3)

$Q(s) = (s-\frac{4}{3})(s^2+4)$

$$\Rightarrow L^{-1} = \frac{P(\frac{4}{3})}{Q'(\frac{4}{3})} e^{\frac{4}{3}t} = \frac{\frac{16}{9}+6}{\frac{16}{9}+4} e^{4/3 t} = \boxed{\frac{35}{26} e^{4/3t}}$$

The factor (s^2+4):

$\alpha = 0, \beta = 2$

$$G(s) = \frac{s^2+6}{s-\frac{4}{3}} \Rightarrow G(2i) = \frac{-4+6}{2i-\frac{4}{3}}$$

$$\Rightarrow G(2i) = \frac{2}{-\frac{4}{3}+2i} \cdot \frac{-\frac{4}{3}-2i}{-\frac{4}{3}-2i} = \frac{-8/3-4i}{\frac{16}{9}+4}$$

$$= \frac{-24-36i}{16+36} = \frac{-24-36i}{52} = \underbrace{-\frac{6}{13}}_{R} - \underbrace{\frac{9}{13}}_{I} i$$

$$\Rightarrow L^{-1} = \frac{e^{0t}}{2}\left[-\frac{9}{13}\cos 2t - \frac{6}{13}\sin 2t\right].$$

$$= \boxed{\frac{1}{26}\left[-9\cos 2t - 6\sin 2t\right]}$$

$$\Rightarrow y(t) = \frac{1}{3}\left[\frac{35}{26} e^{4/3t} + \frac{1}{26}(-9\cos 2t - 6\sin 2t)\right]$$

#

Ex (2): solve the I.V.P.:

$y'' + k^2 y = \cos kt$, $y(0) = y'(0) = 0$.

Scanned by CamScanner

Solution

④

Take L.T. for both sides

$\Rightarrow s^2 Y(s) - \cancel{s(0)} - \cancel{0} + k^2 Y(s) = \dfrac{s}{s^2+k^2}$

$\Rightarrow (s^2+k^2) Y(s) = \dfrac{s}{s^2+k^2}$

$\Rightarrow Y(s) = \dfrac{s}{(s^2+k^2)^2}$

$\Rightarrow y(t) = L^{-1}\left[\dfrac{s}{(s^2+k^2)^2}\right] = L^{-1}\left[\dfrac{s}{s^2+k^2} \cdot \dfrac{1}{s^2+k^2}\right]$

$\because L^{-1}\left[\dfrac{s}{s^2+k^2}\right] = \cos kt, \quad \dfrac{1}{k}L^{-1}\left[\dfrac{k}{s^2+k^2}\right] = \dfrac{1}{k}\sin kt$

$\Rightarrow y(t) = \cos kt * \dfrac{1}{k}\sin kt$

$= \dfrac{1}{k}\displaystyle\int_0^t \cos ku \cdot \sin k(t-u)\, du$

$= \dfrac{1}{2k}\displaystyle\int_0^t [\sin kt + \sin k(t-2u)]\, du$

$= \dfrac{1}{2k}\left[u \sin kt - \dfrac{\cos k(t-2u)}{-2}\right]_0^t$

$= \dfrac{1}{2k}\left[t \sin kt + \dfrac{1}{2}(\cancel{\cos kt - \cos kt})\right]$

$\boxed{y(t) = \dfrac{1}{2k}\, t \sin kt}$ #

⑤ حل المسألة (١٠) من تمارين الكتاب

solve: $y'' - 4y' + 4y = \sin 3t$, $y(0) = 0$, $y'(0) = 4$.

solution

Take L.T. for both sides

$$\Rightarrow s^2 Y(s) - \cancel{s(0)} - 4 - 4[sY(s) - \cancel{0}] + 4Y(s) = \frac{3}{s^2+9}.$$

$$\Rightarrow \underbrace{(s^2 - 4s + 4)}_{(s-2)^2} Y(s) = \frac{3}{s^2+9} + 4 = \frac{4s^2+39}{s^2+9}$$

$$\Rightarrow Y(s) = \frac{4s^2+39}{(s-2)^2(s^2+9)}.$$

$$\Rightarrow y(t) = L^{-1}\left[\frac{4s^2+39}{(s-2)^2(s^2+9)}\right].$$

the factor $(s-2)^2$:

$$\Phi(s) = \frac{4s^2+39}{s^2+9}$$

$$\Rightarrow \Phi'(s) = \frac{8s(s^2+9) - 2s(4s^2+39)}{(s^2+9)^2} = \frac{-6s}{(s^2+9)^2}$$

$$\Rightarrow L^{-1} = [\Phi'(2) + \Phi(2)t]\, e^{2t} = \boxed{\left[\frac{-12}{169} + \frac{55}{13}t\right] e^{2t}}$$

, the factor (s^2+9): $\alpha = 0$, $\beta = 3$

$$G(s) = \frac{4s^2+39}{(s-2)^2} \Rightarrow G(3i) = \frac{-36+39}{(3i-2)^2} = \frac{3}{-9-12i+4}$$

$$\therefore G(3i) = \frac{3}{-5-12i} \cdot \frac{(-5+12i)}{(-5+12i)} = \frac{-15+36i}{169}$$

$$= \underbrace{\frac{-15}{169}}_{\leftarrow R} + \underbrace{\frac{36}{169}}_{\leftarrow I} i$$

$$\Rightarrow L^{-1} = \frac{e^{0t}}{3}\left[\frac{36}{169}\cos 3t - \frac{15}{169}\sin 3t\right] = \frac{1}{3}\left[\frac{36}{169}\cos 3t - \frac{15}{169}\sin 3t\right]$$

$$\Rightarrow y(t) = \left[\frac{-12}{169} + \frac{55}{13}t\right] e^{2t} + \frac{1}{3}\left[\frac{36}{169}\cos 3t - \frac{15}{169}\sin 3t\right] \#$$

EX③ : solve the following system of D.E.'s :

$(D+2)x - 2y = 0$, $-x + (D+1)y = 2e^{t}$, $x(0) = 0$, $y(0) = 1$.

solution

$$\frac{dx}{dt} + 2x - 2y = 0 \longrightarrow ①$$

$$\boxed{D = \frac{d}{dt}}$$

$$, \quad -x + \frac{dy}{dt} + y = 2e^{t} \longrightarrow ②$$

Take L.T. for ①

$$\Rightarrow sX(s) - 0 + 2X(s) - 2Y(s) = 0$$

$$\Rightarrow (s+2)X(s) - 2Y(s) = 0 \longrightarrow ①'$$

, Take L.T. for ②

$$\Rightarrow -X(s) + sY(s) - 1 + Y(s) = \frac{2}{s-1}$$

$$\Rightarrow -X(s) + (s+1)Y(s) = 1 + \frac{2}{s-1}$$

$$= \frac{s+1}{s-1} \longrightarrow ②'$$

$①' + ②' * (s+2)$ نحذف $X(s)$ كمايلي

$$\Rightarrow [(s+1)(s+2) - 2]\, Y(s) = \frac{(s+1)(s+2)}{(s-1)}$$

$$\Rightarrow [s^2 + 3s]\, Y(s) = \frac{(s+1)(s+2)}{(s-1)}$$

$$\Rightarrow Y(s) = \frac{(s+1)(s+2)}{s(s+3)(s-1)} \Rightarrow y(t) = L^{-1}\left[\frac{(s+1)(s+2)}{s(s+3)(s-1)}\right]$$

$P(s) = (s+1)(s+2)$, $Q(s) = s(s+3)(s-1)$.

$$\Rightarrow y(t) = \frac{P(0)}{Q'(0)} e^{0t} + \frac{P(-3)}{Q'(-3)} e^{-3t} + \frac{P(1)}{Q'(1)} e^{t}$$

$$= \frac{2}{-3} e^{0t} + \frac{2}{12} e^{-3t} + \frac{6}{4} e^{t}$$

$$\boxed{y(t) = -\frac{2}{3} + \frac{1}{6} e^{-3t} + \frac{3}{2} e^{t}} \quad \#$$

⑦ ولإيجاد $x(t)$ نعوض في المعادلة التي بها x فقط وليس $\frac{dx}{dt}$

$$\text{in } ② \Rightarrow -x + \frac{d}{dt}\left[-\frac{2}{3} + \frac{1}{6}e^{-3t} + \frac{3}{2}e^{t}\right] + \frac{-2}{3} + \frac{1}{6}e^{-3t} + \frac{3}{2}e^{t} = 2e^{t}.$$

$$\Rightarrow -x - \frac{3}{6}e^{-3t} + \frac{3}{2}e^{t} - \frac{2}{3} + \frac{1}{6}e^{-3t} + \frac{3}{2}e^{t} = 2e^{t}.$$

$$\Rightarrow \boxed{x(t) = e^{t} - \frac{1}{3}e^{-3t} - \frac{2}{3}} \quad \#$$

حل المسألة رقم (15) من تمارين الكتاب

solve the system:

$$4\dot{y} - 2\dot{x} + 10y - 5x = 0 \longrightarrow ① \quad , \quad \dot{x} - 18y + 15x = 10, \longrightarrow ②$$

$\dot{y} = \frac{dy}{dt}$, $\dot{x} = \frac{dx}{dt}$

$$x(0) = 4, \; y(0) = 2.$$

solution

Take L.T. for ①

$$\Rightarrow 4[sY(s) - 2] - 2[sX(s) - 4] + 10Y(s) - 5X(s) = 0$$

$$\Rightarrow -(2s+5)X(s) + (4s+10)Y(s) = 0 \longrightarrow ①'$$

, Take L.T. for ②

$$\Rightarrow sX(s) - 4 - 18Y(s) + 15X(s) = \frac{10}{s}$$

$$\Rightarrow (s+15)X(s) - 18Y(s) = \frac{10}{s} + 4$$

$$= \frac{4s+10}{s} \longrightarrow ②'$$

$①' * 18 + ②' * (4s+10)$ نحذف $Y(s)$:

$$\Rightarrow \left[(s+15)(4s+10)-18(2s+5)\right]X(s) = \frac{(4s+10)^2}{s}$$

$$\Rightarrow \left[\cancel{4}^{2}s^2 + \cancel{34}^{17}s + \cancel{60}^{30}\right]X(s) = {}^{2}\cancel{4}\,\frac{(2s+5)^2}{s}$$

$$\Rightarrow X(s) = \frac{2(2s+5)^2}{s(2s^2+17s+30)} = \frac{2(2s+5)^{\cancel{2}}}{s\,\cancel{(2s+5)}(s+6)}$$

$$X(s) = \frac{2(2s+5)}{s(s+6)} \Rightarrow x(t) = 2L^{-1}\left[\frac{2s+5}{s(s+6)}\right]$$

$$P(s) = 2s+5,\ Q(s) = s(s+6)$$

$$\Rightarrow x(t) = 2\left[\frac{P(0)}{Q'(0)}e^{0t} + \frac{P(-6)}{Q'(-6)}e^{-6t}\right]$$

$$= 2\left[\frac{5}{6}e^{0t} + \frac{-7}{-6}e^{-6t}\right]$$

$$\Rightarrow \boxed{x(t) = \frac{1}{3}\left[5+7e^{-6t}\right]} \ \#$$

ولإيجاد y(t) نعوض في المعادلة التي بها y فقط وليس ẏ

$$\text{in } (2) \Rightarrow \frac{1}{3}(7)(-6)e^{-6t} - 18y + 5(5+7e^{-6t}) = 10$$

$$\Rightarrow 18y = 21e^{-6t} + 15$$

$$\Rightarrow \boxed{y(t) = \frac{7}{6}e^{-6t} + \frac{5}{6}} \ \#$$

انظر حل المسألة 14 في حلول التمارين Problems③

EX 4 : solve the following integral equation:

$$x(t) = t + \int_0^t x(u)\,\sin(t-u)\,du.$$

solution

يمكن كتابة المعادلة المعطاه على الصورة :

$$x(t) = t + x(t) \overset{\text{Convolution}}{*} \sin t$$

Take L.T. for both sides

$$X(s) = \frac{1}{s^2} + X(s)\cdot\frac{1}{s^2+1}$$

$$\boxed{\begin{aligned} &L^{-1}[F(s).G(s)] = f(t) * g(t) \\ &\therefore L[f(t) * g(t)] = F(s).G(s) \end{aligned}}$$

$$\Rightarrow \left(1 - \frac{1}{s^2+1}\right) X(s) = \frac{1}{s^2}$$

$$\Rightarrow \left(\frac{s^2 + \cancel{1} - \cancel{1}}{s^2+1}\right) X(s) = \frac{1}{s^2} \Rightarrow X(s) = \frac{s^2+1}{s^4}$$

$$\Rightarrow X(s) = \frac{1}{s^2} + \frac{1}{s^4}$$

$$\Rightarrow x(t) = L^{-1}\left[\frac{1}{s^2} + \frac{1}{3!}\frac{3!}{s^4}\right] = \boxed{t + \frac{1}{6}t^3} \quad \#$$

حل المسألة (17) من تمارين الكتاب :

solve the following integral equation:

$$x(t) + 2\int_0^t x(u)\,\cos(t-u)\,du = 9e^{2t}.$$

solution

$$x(t) + 2\left(x(t) * \cos t\right) = 9e^{2t}$$

Take L.T. for both sides

$$\Rightarrow X(s) + 2\,X(s) \cdot \frac{s}{s^2+1} = \frac{9}{s-2}$$

$$\Rightarrow \left(1 + \frac{2s}{s^2+1}\right) X(s) = \frac{9}{s-2}$$

$$\Rightarrow \left(\frac{s^2+2s+1}{s^2+1}\right) X(s) = \frac{9}{s-2}$$

$$\Rightarrow X(s) = \frac{9\,(s^2+1)}{(s-2)(s^2+2s+1)} \Rightarrow X(s) = \frac{9\,(s^2+1)}{(s-2)(s+1)^2}$$

$$\Rightarrow x(t) = 9\,L^{-1}\left[\frac{s^2+1}{(s-2)(s+1)^2}\right]$$

<u>the factor $(s-2)$:</u>

$$P(s) = s^2+1, \quad Q(s) = (s-2)(s+1)^2$$

$$L^{-1} = \frac{P(2)}{Q'(2)}\,e^{2t} = \boxed{\frac{5}{9}\,e^{2t}}$$

<u>the factor $(s+1)^2$:</u>

$$\phi(s) = \frac{s^2+1}{s-2}$$

$$\therefore \phi'(s) = \frac{2s(s-2)-(s^2+1)}{(s-2)^2} = \frac{s^2-4s-1}{(s-2)^2}$$

$$\therefore L^{-1} = [\phi'(-1) + \phi(-1)\,t]\,e^{-t} = \boxed{\left[\frac{4}{9} - \frac{2}{3}t\right] e^{-t}}$$

$$\therefore x(t) = 9\left[\frac{5}{9}\,e^{2t} + \left(\frac{4}{9} - \frac{2}{3}t\right) e^{-t}\right]$$

$$\therefore \boxed{x(t) = 5e^{2t} + (4-6t)\,e^{-t}}\ \#$$

حل المسألة 8 في الدليل:

Solve the following integro-differential equation:

$$y'(t) = 1 - \sin t - \int_0^t y(u)\,du \quad , \; y(0) = 0 .$$

$$\int_0^t 1 \cdot y(u)\,du$$

Solution

$$y'(t) = 1 - \sin t - [1 * y(t)]$$

Take L.T.

$$\Rightarrow sY(s) - 0 = \frac{1}{s} - \frac{1}{s^2+1} - \frac{1}{s} \cdot Y(s) .$$

$$\Rightarrow \left(s + \frac{1}{s}\right) Y(s) = \frac{1}{s} - \frac{1}{s^2+1}$$

$$\Rightarrow \left(\frac{s^2+1}{s}\right) Y(s) = \frac{1}{s} - \frac{1}{s^2+1} .$$

$$\Rightarrow Y(s) = \frac{1}{s^2+1} - \frac{s}{(s^2+1)^2}$$

$$\Rightarrow y(t) = \sin t - L^{-1}\left[\frac{s}{s^2+1} \cdot \frac{1}{s^2+1}\right]$$

$$, \because L^{-1}\left(\frac{s}{s^2+1}\right) = \cos t , \; L^{-1}\left(\frac{1}{s^2+1}\right) = \sin t$$

$$\therefore L^{-1}\left[\frac{s}{s^2+1} \cdot \frac{1}{s^2+1}\right] = \cos t * \sin t = \int_0^t \cos(t-u) \sin u \, du$$

$$= \frac{1}{2} \int_0^t [\sin t + \sin(2u - t)]\, du$$

$$= \frac{1}{2} \left[u \sin t - \frac{\cos(2u-t)}{2} \right]_0^t$$

$$= \frac{1}{2} \left[t \sin t - \frac{1}{2} \cancel{(\cos t - \cos t)} \right]$$

$$= \frac{1}{2} t \sin t$$

$$\Rightarrow \boxed{y(t) = \sin t - \frac{1}{2} t \sin t} \quad \#$$

رياضيات كهربائية (حلول تمارين الكتاب) Problems (3) (1)

د/ احمد ابراهيم 9

(1) $2y' + 3y = e^{4t} \quad ; \; y(0) = 5.$

$$2\left[sY(s) - 5\right] + 3Y(s) = \frac{1}{s-4}$$

$$(2s+3)\,Y(s) = \frac{1}{s-4} + 10.$$

$$\Rightarrow Y(s) = \frac{1}{(s-4)(2s+3)} + \frac{10}{(2s+3)}$$

$$= \frac{1}{2(s-4)(s+3/2)} + \frac{10}{2(s+3/2)}$$

$$\Rightarrow y(t) = \frac{1}{2}\left[\frac{\overset{P(4)}{1}}{\underset{Q'(4)}{\left(\frac{11}{2}\right)}} e^{4t} + \frac{\overset{P(-3/2)}{1}}{\underset{Q'(-3/2)}{\left(\frac{-11}{2}\right)}} e^{-3/2\,t}\right] + 5 \cdot e^{-3/2\,t}$$

$$\Rightarrow y(t) = \frac{1}{11}\left[e^{4t} - e^{-3/2\,t}\right] + 5e^{-3/2\,t} \quad \#$$

(2) $3y' - 4y = \sin 2t \quad , \; y(0) = 1/3.$

$$\Rightarrow 3\left[sY(s) - \frac{1}{3}\right] - 4Y(s) = \frac{2}{s^2+4}$$

$$\Rightarrow (3s-4)\,Y(s) = \frac{2}{s^2+4} + 1$$

$$\Rightarrow Y(s) = \frac{2}{(s^2+4)(3s-4)} + \frac{1}{(3s-4)}$$

$$= \frac{2}{3(s-\frac{4}{3})(s^2+4)} + \frac{1}{3(s-4/3)}.$$

$$\Rightarrow y(t) = \frac{2}{3} L^{-1}\left[\frac{1}{(s-\frac{4}{3})(s^2+4)}\right] + \frac{1}{3} e^{4/3\,t}.$$

for the factor $\left(s-\frac{4}{3}\right)$: $\frac{1}{\left(\frac{4}{3}\right)^2+4} e^{4/3 t}$

$$= \frac{9}{52} e^{4/3 t}.$$

for the factor s^2+4:

$\alpha = 0, \beta = 2.$

$$G(s) = \frac{1}{s-4/3} \Rightarrow G(2i) = \frac{1}{2i-4/3} \cdot \frac{\left(-\frac{4}{3}-2i\right)}{\left(-\frac{4}{3}-2i\right)}$$

$$= \frac{\left(-4/3-2i\right)}{16/9+4} = \frac{-\frac{4}{3}-2i}{52/9}$$

$$= -\frac{3}{13} - \frac{9}{26} i.$$

$$\therefore \frac{1}{2} e^{0t}\left[\frac{-9}{2\cancel{6}} \cos 2t - \frac{3}{13} \sin 2t\right].$$

$$\Rightarrow y(t) = \frac{2}{3}\left[\frac{9}{52} e^{4/3 t} + \frac{1}{26}\left(-\frac{9}{2} \cos 2t - 3 \sin 2t\right)\right.$$
$$\left. + \frac{1}{3} e^{4/3 t}\right].$$

---

(3) $y' - 3y = te^{2t}, \quad y(0) = 0.$

$$\Rightarrow sY(s) - 3Y(s) = \frac{1}{(s-2)^2}$$

$$\Rightarrow (s-3)Y(s) = \frac{1}{(s-2)^2} \Rightarrow Y(s) = \frac{1}{(s-3)(s-2)^2}$$

~~ylt~~ $(s-3)$: $\frac{1}{1} e^{3t} = \boxed{e^{3t}}$

$(s-2)^2$: $\Phi(s) = \frac{1}{s-3} \Rightarrow \Phi'(s) = -\frac{1}{(s-3)^2}$

$$\Rightarrow \left[\Phi'(2) + \Phi(2)t\right] e^{2t} = (-1-t)e^{2t}$$

$$\Rightarrow y(t) = e^{3t} - (1+t)e^{2t}. \quad \#$$

4. $y'' - 3y' + 2y = 0 \; ; y(0) = 0, \; y'(0) = 3.$

$$\Rightarrow \left[s^2 Y(s) - \cancel{s y(0)} - y'(0)\right] - 3\left[sY(s) - y(0)\right] + 2Y(s) = 0$$

$$s^2 Y(s) - 3 - 3(sY(s)) + 2Y(s) = 0.$$

$$\Rightarrow (s^2 - 3s + 2)\,Y(s) = 3.$$

$$\Rightarrow Y(s) = \frac{3}{s^2 - 3s + 2} = \frac{3}{(s-2)(s-1)}$$

$$\Rightarrow y(t) = \frac{3}{1} e^{2t} + \frac{3}{-1} e^{t} = 3e^{2t} - 3e^{t}.$$

#

5. $y'' + 36y = 0 \; , \; y(0) = -1, \; y'(0) = 2.$

$$\Rightarrow s^2 Y(s) - s(-1) - 2 + 36\,Y(s) = 0.$$

$$\Rightarrow (s^2 + 36)\,Y(s) = 2 - s \Rightarrow Y(s) = \frac{2-s}{s^2+36}$$

$$\Rightarrow Y(s) = \frac{-(s-2)}{s^2+36} = -\left[\frac{s}{s^2+36} - \frac{2}{s^2+36}\right]$$

$$\Rightarrow y(t) = -\left[\cos 6t - \frac{2}{6}\sin 6t\right] = \frac{1}{3}\sin 6t - \cos 6t$$

#

6. $y'' - 7y' + 12y = 2 \; ; \; y(0) = 1, \; y'(0) = 5.$

$$\Rightarrow s^2 Y(s) - s(1) - 5 - 7\left[sY(s) - 1\right] + 12Y(s) = \frac{2}{s}$$

$$\Rightarrow (s^2 - 7s + 12)Y(s) = \frac{2}{s} + s - 2 = \frac{2 + s^2 - 2s}{s}$$

$$\Rightarrow Y(s) = \frac{s^2 - 2s + 2}{s(s^2 - 7s + 12)} = \frac{s^2 - 2s + 2}{s(s-4)(s-3)}$$

$$\Rightarrow y(t) = \frac{2}{12} e^{0t} + \frac{10}{4} e^{4t} + \frac{5}{-3} e^{3t}$$

$$= \frac{1}{6} + \frac{5}{2} e^{4t} - \frac{5}{3} e^{3t}$$

#

⑦ $y'' - 6y' + 8y = e^{3t}$, $y(0) = 0$, $y'(0) = 2$.

$\Rightarrow s^2 Y(s) - \cancel{s(0)} - 2 - 6[sY(s) - 0] + 8Y(s) = \frac{1}{s-3}$

$\Rightarrow (s^2 - 6s + 8) Y(s) = \frac{1}{s-3} + 2 = \frac{2s-5}{s-3}$

$Y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{2s-5}{(s-3)(s-4)(s-2)}$

$\Rightarrow y(t) = \frac{1}{(-1)(1)} e^{3t} + \frac{3}{(1)(2)} e^{4t} + \frac{-1}{(-1)(-2)} e^{2t}$

$= -e^{3t} + \frac{3}{2} e^{4t} - \frac{1}{2} e^{2t}$ #

⑧ $y'' + y' = 6\cos 2t$, $y(0) = 3$, $y'(0) = 1$.

$\Rightarrow s^2 Y(s) - s(3) - 1 + sY(s) - 3 = 6 \cdot \frac{s}{s^2+4}$

$\Rightarrow (s^2 + s) Y(s) = \frac{6s}{s^2+4} + 3s + 4$

$= \frac{6s + 3s^3 + 12s + 4s^2 + 16}{s^2+4}$

$\Rightarrow Y(s) = \frac{3s^3 + 4s^2 + 18s + 16}{s(s+1)(s^2+4)}$

for $s(s+1)$: $\frac{16}{(1)(4)} e^{0t} + \frac{(-3+4-18+16)}{(-1)(5)} e^{-t}$

$= 4 + \frac{1}{5} e^{-t}$.

for s^2+4: $\alpha = 0$, $\beta = 2$

$G(s) = \frac{3s^3 + 4s^2 + 18s + 16}{s(s+1)}$

$\Rightarrow G(2i) = \frac{-24i + 16(-1) + 36i + \cancel{16}}{(2i)(2i+1)} = \frac{12i}{-4+2i}$

$= \frac{6i}{-2+i} \cdot \frac{-2-i}{-2-i} = \frac{6-12i}{5}$

$$\Rightarrow G(2i) = \underset{R}{\frac{6}{5}} - \underset{I}{\frac{12}{5}}\, i$$

$$\Rightarrow \boxed{\frac{1}{2}\left[-\frac{12}{5}\cos 2t + \frac{6}{5}\sin 2t\right]}$$

$$\Rightarrow y(t) = 4 + \frac{1}{5}e^{-t} - \frac{6}{5}\cos 2t + \frac{3}{5}\sin 2t. \quad \#$$

(11) $y'' - y' + 2y = 12e^{-2t}(4\sin t - \cos t)$, $y(0) = 0$
$y'(0) = 3.$

$$\Rightarrow s^2 Y(s) - \cancel{s(0)} - 3 - [sY(s) - 0] + 2Y(s).$$

$$= 48 \cdot \frac{1}{(s+2)^2+1} - 12 \cdot \frac{(s+2)}{(s+2)^2+1}$$

$$\Rightarrow (s^2 - s + 2)Y(s) = \frac{48 - 12(s+2)}{(s+2)^2+1} + 3 = \frac{-12s + 24 + 3[(s+2)^2+1]}{(s+2)^2+1}$$

$$\Rightarrow Y(s) = \frac{3s^2 + 39}{(s^2 - s + 2)[(s+2)^2+1]}$$

$$= \frac{3s^2 + 39}{[(s-\frac{1}{2})^2 + \frac{7}{4}][(s+2)^2+1]}$$

For $(s-\frac{1}{2})^2 + 7/4$: $\quad \alpha = 1/2$, $\beta = \frac{\sqrt{7}}{2}$.

$$G(s) = \frac{3s^2 + 39}{(s+2)^2 + 1}$$

ثم أكمل

For $(s+2)^2 + 1$: $\quad \alpha = -2$, $\beta = 1$

$$G(s) = \frac{3s^2 + 39}{(s-1/2)^2 + 7/4}$$

ثم أكمل

$\Rightarrow y(t) = \ldots$

(13) $y''' - 2y'' - y' + 2y = t + 2.$

$y(0) = 0,\ y'(0) = 1,\ y''(0) = 0.$

$$\Rightarrow \left[s^3 Y(s) - \cancel{s^2(0)} - s(1) - 0\right] - 2\left[s^2 Y(s) - \cancel{s(0)} - 1\right]$$

$$- \left[s Y(s) - 0\right] + 2Y(s) = \frac{1}{s^2} + \frac{2}{s}.$$

$$\Rightarrow (s^3 - 2s^2 - s + 2)\, Y(s) = \frac{1}{s^2} + \frac{2}{s} + s - 2.$$

$$= \frac{1 + 2s + s^3 - 2s^2}{s^2}$$

$$\Rightarrow Y(s) = \frac{s^3 - 2s^2 + 2s + 1}{s^2(s^3 - 2s^2 - s + 2)}$$

تكونه الجذور مابين الاعداد

$\pm 1, \pm 2$

بالتجربة نجد أنه 1 جذر

$$\Rightarrow Y(s) = \frac{s^3 - 2s^2 + 2s + 1}{s^2(s-1)(s^2 - s - 2)}$$

$s^2 - s - 2 \rightarrow (s-2)(s+1)$

1	1	-2	-1	2
		1	-1	-2
	1	-1	-2	0

$$= \frac{s^3 - 2s^2 + 2s + 1}{s^2(s-1)(s-2)(s+1)}$$

for $(s-1)(s-2)(s+1)$:

$$\frac{P(1)}{Q'(1)} e^{t} + \frac{P(2)}{Q'(2)} e^{2t} + \frac{P(-1)}{Q'(-1)} e^{-t}$$

$= \ldots$

for s^2:

$$\Phi(s) = \frac{s^3 - 2s^2 + 2s + 1}{(s-1)(s-2)(s+1)}$$

$$\Rightarrow \Phi'(s) = \ldots$$

$$\Rightarrow \left[\Phi'(0) + \Phi(0)\, t\right] e^{0t}$$

$= \ldots$

(14) $2\dot{x} - x + \dot{y} + y = \sin t \longrightarrow (1)$ (7)

$3\dot{x} - x + 2\dot{y} + y = e^{t} \longrightarrow (2)$ $x(0) = y(0) = 0$

$(1) \Rightarrow 2[sX(s) - 0] - X(s) + sY(s) - 0 + Y(s) = \frac{1}{s^2+1}$

$(2s-1)X(s) + (s+1)Y(s) = \frac{1}{s^2+1} \rightarrow (1)'$

$(2) \Rightarrow 3sX(s) - X(s) + 2sY(s) + Y(s) = \frac{1}{s-1}$

$\Rightarrow (3s-1)X(s) + (2s+1)Y(s) = \frac{1}{s-1} \rightarrow (2)'$

$(1)' * (2s+1) - (2)' * (s+1)$

$\Rightarrow [(2s-1)(2s+1) - (3s-1)(s+1)]X(s) = \frac{2s+1}{s^2+1} - \frac{s+1}{s-1}$

$\Rightarrow [4s^2 - 1 - (3s^2 + 2s - 1)]X(s) = \frac{(2s+1)(s-1) - (s^2+1)(s+1)}{(s^2+1)(s-1)}$

$\Rightarrow (s^2 - 2s)X(s) = \frac{s^2 - 2s - s^3 - 2}{(s^2+1)(s-1)}$

$\Rightarrow X(s) = \frac{s^2 - 2s - s^3 - 2}{s(s-2)(s-1)(s^2+1)}$

in $(1)' \Rightarrow \frac{(2s-1)(s^2 - 2s - s^3 - 2)}{s(s-2)(s-1)(s^2+1)} + (s+1)Y(s) = \frac{1}{s^2+1}$

$\Rightarrow (s+1)Y(s) = \frac{s(s-1)(s-2) - (2s-1)(s^2 - 2s - s^3 - 2)}{s(s^2+1)(s-1)(s-2)}$

$\Rightarrow Y(s) = \frac{s(s-1)(s-2) - (2s-1)(s^2 - 2s - s^3 - 2)}{s(s+1)(s-1)(s-2)(s^2+1)}$

أكمل لا دينا مشي

$x(t), y(t) = \_\_\_$

(16) $x(t) - \int_0^t e^{t-u}\, x(u)\, du = 2.$ (8)

$\Rightarrow x(t) - e^{t} * x(t) = 2.$

by taking L.T. of both sides.

$$X(s) - L(e^t).L(x(t)) = \frac{2}{s}.$$

$$X(s) - \frac{1}{s-1} X(s) = \frac{2}{s}$$

$$X(s)\left[1 - \frac{1}{s-1}\right] = \frac{2}{s}$$

$$X(s).\left[\frac{s-2}{s-1}\right] = \frac{2}{s} \Rightarrow X(s) = \frac{2(s-1)}{s(s-2)}$$

$$\Rightarrow x(t) = \frac{-2}{-2} e^{0t} + \frac{2}{2} e^{2t}$$

$$= 1 + e^{2t} \quad \#$$

(18) $\dot{x} + \int_0^t x(u)\, ch(t-u)\, du = 0, \quad x(0) = 1.$

$\Rightarrow \dot{x} + x(t) * ch t = 0.$

$\Rightarrow s X(s) - 1 + X(s).\frac{s}{s^2-1} = 0.$

$\Rightarrow X(s)\left[s + \frac{s}{s^2-1}\right] = 1 \Rightarrow X(s).\left[\frac{s^3 - \cancel{s} + \cancel{s}}{s^2-1}\right] = 1$

$\Rightarrow X(s) = \frac{s^2-1}{s^3} = \frac{1}{s} - \frac{1}{s^3}.$

$\Rightarrow x(t) = 1 - \frac{1}{2!} t^2 = \boxed{1 - \frac{1}{2} t^2} \quad \#$