AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING
ENG. PHYSICS & MATH. DEPARTMENT
Electronics and Communication Engineering Program
Final Exam

Spring 2021 | **Course Code: PHM 212s** | ***Time: 2.00 Hrs.***

Complex Variables, Special functions and Numerical Analysis

The Exam Consists of FOUR Questions in TWO Pages. | ***Total Marks: 60 Marks*** | **1/2**

Important Rules:
- Having a mobile, Smart Watch or earphones inside the examination hall is forbidden and is considered as a cheating behavior.
- It is forbidden to have any references, notes, books, or any other materials even if it is not related to the exam content with you in the examination hall
- It is not allowable to use programable or graphical calculators.

تعليمات هامة
- حيازة (المحمول- الساعات الذكية - سماعة الأذن) داخل لجنة الامتحان يعتبر حالة غش تستوجب العقاب .
- لايسمح بدخول أي كتب أو ملازم أو أوراق داخل اللجنة والمخالفة تعتبر حالة غش.
- ممنوع استخدام الآلات الحاسبة المبرمجة و التى تستطيع الرسم.

Question (1): (12 Marks)

(A) Solve in terms of the Gamma function $\int_0^1 \sqrt[n]{1-x^m}\ dx$ where n and m are positive integers.

Hence, use it to evaluate $\int_0^1 \sqrt{1-x^2}\ dx$.

[6 Marks]

(B) By two different methods (one of them is by using the Gamma function) evaluate

$$\int_0^{\pi/2} \sin^4 x\ \cos^5 x\ dx$$

[6 Marks]

Question (2): (16 Marks)

(A) Find two linearly independent solutions in powers of " x " for the following differential equation:

$$2x^2 y'' + 3x\, y' - (1+x^2)\, y = 0$$

[10 Marks]

(C) Solve in terms of Bessel function the following differential equation

$$x\, y'' - 7\, y' + x\, y = 0$$

[6 Marks]

Question (3): (12 Marks)

(A) Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. Hence, use it to evaluate $\int x^{3/2}\ J_{-1/2}(x)\ dx$.

[6 Marks]

(B) Starting from $\frac{d}{dx}\left(x^n J_n\right) = x^n J_{n-1}$ *and* $\frac{d}{dx}\left(x^{-n} J_n\right) = -x^{-n} J_{n+1}$. Show that

$\int J_{n+1}\, dx = \int J_{n-1}\, dx - 2J_n$. Hence, evaluate $\int J_5(x)\ dx$.

[6 Marks]

P. T. O.

Question (4): (20 Marks)

(A) Solve (using h=1/4) the Dirichlet boundary value problem for the Laplace equation $\nabla^2 u = 0$ in the region $x > 0,\quad y > 0,\quad x + y < 1.$ With the boundary conditions defined by $u(0,y) = 0,\quad u(x,1-x) = 0,\quad u(x,0) = x(1-x)$. Use Gauss-Seidel method to solve the resulting system of linear equations starting with zero values at interior points. Perform three iterations using 3 decimal places in your calculations.

[12 Marks]

(B) Given $y' = x^2 - y \quad , \quad y(1) = 0.5$. Use Runge – Kutta method to find $y(1.2)$ with $h = 0.1$.

Use 3 decimal places in your calculations.

[8 Marks]

Best Wishes.