$\therefore x^2+y^2=2x$

$x=\frac{x^2+y^2}{2}$

Ain Shams University
Faculty of Engineering
Eng.Physics & Math.Departement

6/10

Total mark (70)

Engineering Math(PHM 663)
Fall 2019
Allowed Time: 3 Hrs.

Answer the following questions (The exam is composed of 6 questions in one page)

$2n\pi/\pi$ $(2n+1)\pi/4$

The mark of each question is (14) marks

1) a)Show that the function: $v(x,y)=6x^2y-2y^3-18x+15+e^{2x}\sin 2y$, is harmonic and find it's corresponding analytic function $f(z)=u+iv$. *Find* $f'^{(z)}$. 7

b)Find the image of $x^2+y^2-2x=0$ under the transformation $w=\frac{1}{z}$. Discuss the details of your work. 4

2)a) Find all values of z such that:

i) $e^{3z-3i}=9+9i$, ii) $\sin 2z=50$, iii) $z=(3-3i)^{4i}$ *and* iv) $z=ln(2-2i\sqrt{3})$ 6

b)Find all Laurent series in different domains for the function: $f(z)=\frac{16z+10}{z^2+3z-10}$ 7

3)a) If C is the circle $|z|=5$ *and* $g(z_0)=\oint_c \frac{e^{2z}+2z^2+3z+2}{(z-z_0)^3}dz$. 7

Find : i) $g(2+i)$, ii) $g(6+8i)$

b)Evaluate the following integrals:

i) $\oint_c (4z^3+9z^2-5z+4)\ \sin\frac{1}{z}dz$,; $C:|z|=\frac{1}{2}$, ii) $\oint_c \frac{e^{3z}}{(z^3-4z^2)}dz$,; $C:|z|=6$ *and* iii) $\int_{-\infty}^{\infty}\frac{dx}{(x^2+1)(x^2+4)}$ 5

4) a) Evaluate the following integral: $\int_0^1 \frac{dx}{\sqrt{x\ln\left(\frac{1}{x}\right)}}$ 7

b) Find the series solution of: $(x^2+4)y''+6xy'+4y=0$, near $x=0$.

5) a) Use Laplace transform to solve the IVP: $ty''-y'=t^2$, $y(0)=0$. 4

b) Sketch the graph of the function $f(t)=\begin{cases}2 & ; & 0<t<1\\ 3t-1 & ; & 1\le t<2\\ 5 & ; & t\ge 2\end{cases}$, and find its Laplace transform. 7

6)a)Find the following integral: $\int_0^{\pi}\frac{dx}{\sqrt{3-cos(x)}}$, put $\cos(x)=1-2\sqrt{u}$. 2

b) Solve the equation: $y''+y=\sin(t)+\int_0^t y(u)\sin(t-u)\,du$, $y(0)=1$, $y'(0)=1$ 5

GOOD LUCK

Ain Shams university
Faculty of Engineering
Eng. Physics & Math dept.

Engineering math (PHM 663)
Fall 2018 (Second term)
Time allowed: 3hours.

Answer the following questions: | The mark of each question is (14) marks

1.a) Show that the function : $u(x, y) = 3x^4 - 18x^2y^2 + 3y^4 + 8xy + 16$ is harmonic and find it's corresponding analytic function $f(z) = u + iv$. Find $f'(z)$.

b) Find all values of z for the following :

i) $e^{2z+3} = 4 - 4i$, ii) $\sin(2z) = -4\sqrt{3}\, i$ and iii) $z = (3 + 3i)^{4i}$

2.a) IF C is the circle $|z - 2| = 6$ and if: $g(z_0) = \oint_c \frac{3z^4+2z^3+6z+8}{(z-z_0)^3} dz$. Find: i) $g(2 + 2i)$, ii) $g(4 + 12i)$

b) Find all Laurent series in different domains for the function: $f(z) = \frac{10z+22}{z^2-z-20}$

3. Evaluate: i) $\oint_c (3z^2 + 6z + 10)e^{\left(\frac{2}{z}\right)} dz$; $c : |z| = 3$

ii) $\oint_C \frac{e^{2z}}{(z-1)^3} dz$; $c : |z| = 3$, iii) $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+9)^2}$ and $\oint_{|z|=\frac{1}{2}} (z^2 + 3z + 8) \sin\left(\frac{1}{z}\right) dz$

4.a) Evaluate : $\int_{-1}^{1} \left(\frac{1+x}{1-x}\right)^{\alpha} dx$. Hence evaluate it when $\alpha = \frac{1}{2}$.

b) Find the series solution in powers of x of: $(1 + x^2)\, y'' + xy' - y = 0$.

5. a)Use $L[f''(t)] = s^2 L[f(t)] - s\, f(0) - f'(0)$ to find $L[\cos(kt)]$.

b) Sketch the graph of the function : $f(t) = \begin{cases} 3 \ ; & 0 < t < 4 \\ 2t - 5 \ ; & t > 4 \end{cases}$ and find it's L.T.

c) Solve the IVP : $y'' + y = f(t)$, where $f(t)$ in 5(b) and $y(0) = 0$, $y'(0) = 0$.

6.a) Use L.T. to solve : $y' = t + \int_0^t y(u) \cos(t - u)\, du$; $y(0) = 4$.

b) By two different methods evaluate $L^{-1}\left(\frac{s}{(s^2+k^2)^2}\right)$.

GOOD LUCK

Ain Shams University
Faculty of Engineering
Eng.Physics & Math.Depaṛtement

Engineering Math(PHM 663)
January 2018
Allowed Time: 3 Hrs.

Total mark (70)

Answer the following questions — **The mark of each question is (14)**

1)a)Show that an analytic function $f(z) = U + iV$ is constant if $f(z)$ = constant.

b)Show that $U = e^{x}\cos(y) + 8xy + 6$ is harmonic and find the corresponding analytic function.

2)a)Find all values of z such that:

$i)\ e^{z-2} = 6 + 6i$, $ii)\ \cos(2z) = -10$, $iii)\ z = \ln(2 + i2\sqrt{3})$, $iv)\ z = (5 + 5i)^{2-i}$

b)If C is the circle $|z| = 6$ and if $g(Z_o) = \oint_C \frac{3z^4 + 2z^3 + 7z + 10}{(z - z_0)^3}\,dz$, find: (i) $g(1+2i)$ (ii) $g(6+8i)$

3)a)Find all the Laurent series in different domains for $f(Z) = \frac{7z-4}{z^2-2z-8}$.

b)Evaluate $i)\ \oint_C (3z^4 + 2z^3 + 6z^2 - 5)\, e^{\frac{2}{z}}dz$, where $C: |z| = 1$

$ii)\ \oint_C \frac{e^{3z+2i}}{(z-2)^2}\,dz$, where $C: |z| = 4$, $iii)\ \int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$

4)a)Evaluate: $i)\ \int_3^{\infty} e^{(6x-x^2)}dx$, $ii)\ \int_0^{\infty} \frac{x^{a-1}}{1+x^b}\,dx$

b) Find the series solution in powers of x of: $2y'' + xy' - 4y = 0$, near $x = 0$.

5)a) Find and sketch the graph of the function : $f(t) = L^{-1}\left[\frac{(1-e^{-2s})(1+2e^{-3s})}{s^2}\right]$

b)Evaluate $L^{-1}\left(\frac{10s^2+10}{(s-2)(s+1)^2}\right)$ and hence solve the integral equation

$$x(t) + 2\int_0^t x(u)\cos(t-u)\,du = 10e^{2t}$$

6)a)Show that $\beta(x,n) = \frac{(n-1)!}{x(x+1)(x+2)\ldots(x+n-1)}$, where n is a positive integer, then find $\beta(0.1, 3)$.

b) Use L.T. solve the initial value problem : $3y' - 4y = \sin(2t)$, $y(0) = \frac{1}{3}$.

GOOD LUCK

Ain Shams University
Faculty of Engineering
Eng.Physics & Math.Departement

Total (70) marks

Engineering Math(PHM 663)
First Term Jan. 2017
Allowed Time: 3 Hrs.

Answer the following questions :

The mark of each question is (14) marks

1) a)If $G(x,y)$ and $H(x,y)$ are harmonic in a region D show that $w = \left(\frac{\partial G}{\partial y} - \frac{\partial H}{\partial x}\right) + i\left(\frac{\partial G}{\partial x} + \frac{\partial H}{\partial y}\right)$ is an analytic function in D.

b) Discuss (a) for $H(x,y) = x^3 - 3xy^2 + 4y + 5$ and $G(x,y) = e^{2x}\cos(2y)$

and find: $f(z) = u(x,y) + iv(x,y)$, where $u(x,y) = G(x,y) = e^{2x}\cos(2y)$.

2)a)Discuss the analyticity of $f(z) = e^{3z}$ and evaluate $\int_{1-i}^{4-4i} e^{3z}\,dz$ by two different methods.

b)Find all values of z such that :

$i)\ e^{2z+i} = 5 + 5i$, $ii)\ z = \left(\sqrt{3} - i\right)^{3+3i}$, $iii)\ \sin(2z) = 10i$

3) a)Find Laurent series for the function $f(z) = \frac{7z+14}{z^2+3z-10}$ in different domains.

b)Evaluate the following integrals:

$i)\ \oint_c \frac{e^{2z}}{z^3 - iz^2}\,dz$ where C is $|z| = 3$

$ii)\ \oint_c (5z^4 - 2z^3 + 4z^2)\,e^{\frac{3}{z}}\,dz$ where C is $|z| = \frac{1}{2}$

$iii)\ \int_{-\infty}^{\infty} \frac{dx}{(x^2+4)(x^2+16)}$

$iv)\ \int_0^{2\pi} \frac{\cos(\theta)}{5+4\cos(\theta)}\,d\theta$

4) a) Evaluate : $\int_0^{\infty} x^{\frac{-3}{2}}(1 - e^{-x})\,dx$. Hint: the first step use the integration by parts.

b) Find the series solution of: $(x^2+4)y'' + 6xy' + 4y = 0$, near $x = 0$.

5) a) Find the arc length of the curve: $r^2 = a^2\cos(2\theta)$, where the length $L = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + r'^2}\,d\theta$, $r' = \frac{dr}{d\theta}$

b) Solve the equation: $y'' + y = \sin(t) + \int_0^t y(t-u)\sin(u)\,du$, $y(0) = 1$, $y'(0) = 1$

6)a) Sketch the graph of the function $f(t) = \begin{cases} 2 & ; \quad 0 < t < 1 \\ 5 - 3t & ; \quad 1 \le t < 2 \\ 2t - 5 & ; \quad t \ge 2 \end{cases}$, and find its Laplace transform.

b) Use Laplace transform to solve the IVP: $ty'' - y' = t^2$, $y(0) = 0$.

GOOD LUCK

Ain Shams university
Faculty of Engineering
Eng. Physics & Math dept.

Engineering math (PHM 663)
Fall 2016
Time allowed: 3hours.

Answer the following questions:

The mark of each question is (14) marks

1.a) Find the value of A such that the function : $u(x,y) = 3x^4 - 2Ax^2y^2 + 3y^4 + 5xy + 5$ is harmonic and find it's corresponding analytic function $f(z) = u + iv$. Find $f'(z)$.

b) Show that if **v** is a harmonic conjugate of **u** in a domain **D**, then uv is harmonic in **D**.

2.a) Find all values of z for the following :

i) $e^{2z+3} = 5 + 5i$, ii) $\sin(2z) = 10i$ and iii) $z = (2-2i)^{4i}$

b) IF C is the circle $|z-2| = 4$ and if: $g(z_0) = \oint_c \frac{3z^4+2z^3+10z+5}{(z-z_0)^3} dz$ find: i) $g(2+i)$, ii) $g(8+2i)$

3.a) Find all Laurent series in different domains for the function: $f(z) = \frac{7z+7}{z^2-z-12}$

b) Evaluate: i) $\oint_c (3z^2 - 5z + 10)e^{\left(\frac{2}{z}\right)} dz$; $c: |z| = 2$

ii) $\oint_C \frac{e^{3z+5}}{z^2+4} dz$; $c: |z| = 4$ and iii) $\int_{-\infty}^{\infty} \frac{dx}{(x^2+4)^3}$

4.a) Evaluate : $\int_0^{\infty} x^{\frac{-3}{2}} (1 - e^{-x})\, dx$. Hint: the first step use the integration by parts.

b) Solve the initial value problem: $y'' + 6y' + 9y = 6t^2e^{-3t}$; $y(0) = 0$, $y'(0) = 0$

5.a) Find the arc length of the curve : $r^2 = a^2\cos(2\theta)$, where the length $L = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + r'^2}\, d\theta$, $r' = \frac{dr}{d\theta}$

b)i)Use $L[f''(t)] = s^2L[f(t)] - s f(0) - f'(0)$ to find $L[\cos(kt)]$.

ii) Evaluate : $L\left[\frac{\cos(t)-\cos(3t)}{t}\right]$

6.a) Use L.T. to solve : $y' = t + \int_0^t y(t-u)\cos(u)\, du$; $y(0) = 4$.

b) Find the series solution in powers of x of: $(1+x^2)y'' + xy' - y = 0$.

GOOD LUCK

FACULTY OF ENGINEERING

ENG.PHYSICS &MATH.DEPT.

Engineering Math.(PHM 663)

Final Exam: May 2015

TIME ALLOWED:3HRS

Answer the following questions

The mark of each question is (14)marks

1)a)Show that the function $U = e^{2x}\cos(2y) + 5x - 3y + 7$ is harmonic and find a corresponding analytic function $f(z) = U + iV$ in terms of Z.

b)Find the image of: (i) the unit circle $|z| = 1$ (ii)the line $y = x$, under the mapping $w = \frac{1}{z}$, show the regions graphically.

2)a)Find all values of z such that:

i) $e^{2z+2i} = 5 + 5i$, ii) $\cos(z) = 10$, iii) $z = \ln(1 - i\sqrt{3})$, iv) $z = (3 - 3i)^{2+i}$

b)If C is the circle $|z| = 4$ and if $g(Z_o) = \oint_C \frac{2z^4+3z^3+5z-6}{(z-z_o)^3}\, dz$, find: (i) $g(1+i)$ (ii) $g(3-4i)$

3)a)Find all the Laurent series in different domains for $f(Z) = \frac{14z+12}{z^2+z-6}$.

b)Evaluate i) $\oint_C (6z^4 - 2z^3 + 5z^2 + 6)\sin\left(\frac{2}{z}\right) dz$, where $C: |z| = \frac{1}{2}$

ii) $\oint_C \frac{e^{3z-2i}}{(z+i)^2}\, dz$, where $C: |z| = 3$, iii) $\int_0^{\infty} \frac{dx}{(x^2+1)(x^2+9)}$

4)a)Evaluate the following integrals: i) $\int_0^1 \sqrt[n]{1 - x^n}\, dx$, put $x^n = \sin^2(\theta)$, ii) $\int_0^1 \frac{dx}{\sqrt{x \ln\left(\frac{1}{x}\right)}}$

b)Find the series solution of: $(x^2 + 4)y'' + 6xy' + 4y = 0$, near $x = 0$.

5)a)Find $L^{-1}\left(\frac{s}{(s^2+1)^2}\right)$ by two different methods and hence find $L^{-1}\left(\frac{s(e^{-s}+e^{-2s})}{(s^2+1)^2}\right)$.

b)Solve the equation: $y'' + y = \sin(t) + \int_0^t y(t-u)\sin(u)\, du$, $y(0) = 1$, $y'(0) = 1$

6)a)Sketch the graph of the function $f(t) = \begin{cases} 2 & ; \quad 0 < t < 1 \\ 5 - 3t & ; \quad 1 \le t < 2 \\ 2t - 5 & ; \quad t \ge 2 \end{cases}$, and find its Laplace transform.

b)Solve the IVP: $y'' + 2y' + y = t$, $y(0) = -3$, $y(1) = -1$ use Laplace transform.

GOOD LUCK

Ain Shams University
Faculty of engineering
Eng.Physics & Math.Dept.

Engineering math.(PHM 663)
December 2014
Time allowed:3 hours

Answer the following questions: **The mark of each question is (14)marks**

1)a) Prove that an analytic function $f(z) = u + iv$ must be constant if u =constant.

b) Show that the function $u = 5x^3 - 15xy^2 + 6y + 25$ is harmonic and find the corresponding analytic function $f(z)=u+iv$, find $f'(z)$.

2)a) Find all roots of the functions: i) $e^{2z-3} = 6i$, ii) $sin(z)= 5$, iii) $(1 + \sqrt{3}\, i)^i$

b) Evaluate : $\oint_c \frac{e^{4z+8}}{z^2-4iz} dz$ where c is: a) $|z| = \frac{1}{2}$, b) $|z - 4i| = 1$, c) $|z| = 5$

3)a) Find all **Laurant series** of the function $f(z) = \frac{5z-8}{z^2-2z-8}$ in different domains.

b) Evaluate the following integrals:

i) $\oint_{|z|=1/2} (4z^3 + 8z)\cos\left(2/z\right) dz$, ii) $\int_{-\infty}^{\infty} \frac{3x^2 dx}{(x^2+1)(x^2+4)}$, iii) $\int_0^{2\pi} \frac{d\theta}{2-\cos(\theta)}$

4)a) Evaluate: i) $\int_{-1}^{1} (1+x)^{p-1}(1-x)^{q-1}\, dx$, ii) $\int_0^{\infty} \frac{dx}{\sqrt{-\ln x}}$

b)Evaluate: i) $L\left[\frac{e^{2t}-e^{3t}}{t}\right]$, ii) $L^{-1}\left(\frac{1}{(s-1)(s+2)^2}\right)$, use **convolution method.**

5)a)Evaluate: $L^{-1}\left(\frac{3s+4}{(s^2+s)(s^2+1)}\right)$ and hence solve the IVP: $y'''+y''+y'+y=4$,

$y(0)=y'(0)=y''(0)=0.$

b)Find the series solution in powers of x of the differential equation:

$$(1 - x^2)y'' - 4xy' + 4y = 0$$

6)a)Evaluate $L^{-1}\left(\frac{9s^2+9}{(s-2)(s+1)^2}\right)$, and hence solve the integral equation:

$$x(t) + 2\int_0^t x(u)\cos(t-u)\, du = 9e^{2t}$$

b)Find and sketch the graph of the function: $L^{-1}\left[\frac{3}{s} - \frac{4e^{-s}}{s^2} + \frac{4e^{-3s}}{s^2}\right]$.

GOOD LUCK

Ain Shams University
Faculty of Engineering
Eng.Physics & Math.Departement

(1)

Engineering Math(PHM 663)
June – second term 2014
Allowed Time: 3 Hrs.

The mark of each question is (14)

Answer the following questions :

1) a) Find A such that the function $4x^3 - 4axy^2 + 4y + 6x + 8$ is harmonic and find it's corresponding analytic function $f(z) = u + iv$.

b) Find the image of : the semi – infinite strip $x > 0$, $0 < y < 3$, under the mapping $w = iz + 2$, show the regions graphically.

2) a) Find all values of : i) $e^{2z+7} = 4 - 4i$, ii) $\sin 2z = 6i$, iii) $z = \ln(3 + 3i)$ and iv) $z = (1 + i)^{-3i}$

b) If C is the circle $|z - 2i| = 4$ and if : $g(z_0) = \oint \frac{3z^4+2z^2+5z+3}{(z-z_0)^3}\, dz$ find : i) $g(3i)$, ii) $g(2 + 6i)$

3) a) Find all the Laurent series in different domains for : $f(z) = \frac{5z+6}{z^2+z-12}$.

b) Evaluate i) $\oint_c (3z^4 + 6z^2 + 4z + 5)e^{3/z}\, dz$, where $c : |z| = 1$, ii) $\int_0^{2\pi} \frac{d\theta}{5-4\cos\theta}$

iii) $\oint_c \frac{e^{2z+5}}{(z-2i)^2}\, dz$, where $c : |z| = 3$ and iv) $\int_0^{\infty} \frac{dx}{(x^2+4)^2}$

4) a) Evaluate: $\int_1^{\infty} e^{(8x-x^2)}\, dx$, ii) $\int_{-1}^{1} \left(\frac{1+x}{1-x}\right)^{1/2} dx$

b) Find the area enclosed by the curve $x^{2/3} + y^{2/3} = a^{2/3}$

5) a) Find L.T. of : i) $f(t) = \frac{\cos 3t - \cos 5t}{t}$, ii) $f(t) = \begin{cases} t & ; \ 0 \le t \le 1 \\ 1 & ; \ t > 1 \end{cases}$ by using two different methods (for ii).

$\int^{\infty} F(s)\, ds$

b) Find the series solution in powers of x of : $2x^2y'' + 3xy' - (1 + x^2)y = 0$

6) a) i) Find and sketch the graph of the function $f(t) = L^{-1}\left[\frac{2}{s^3} - \frac{2e^{-2s}}{s^3} - \frac{4e^{-2s}}{s^2} - \frac{4e^{-5s}}{s}\right]$

ii) Sketch the graph of the periodic function $f(t) = \begin{cases} a\cos(t) & ; \ 0 < t < \pi \\ 0 & ; \ \pi < t < 2\pi \end{cases}$, $f(t + 2\pi) = f(t)$ and find L.T. of it

b) solve the equation : $y'' + y = \sin(t) + \int_0^t \sin(t - u)\, y(u)\, du$; $y(0) = 0$, $y'(0) = 1$

$1 + z + z^2 + z^3$ $\quad$ $1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!}$

Good luck

$e^{i\theta} = z$ $\quad$ $e^{i\theta} = \cos\theta + i\sin\theta$

AIN SHAMS UNIVERSITY — Engineering math.(PHM 663)

FACULTY OF ENGINEERING — Final Exam: Jan. 2014

ENG.PHYSICS &MATH.DEPT. — TIME ALLOWED:3HRS

Answer the following questions:

1)a) Find Laplace Transform of the following functions:

i) $f(t) = \frac{e^{-2t} - \cos(3t)}{t}$, ii) $f(t) = \left(e^{-t} ch\,(5t)\sin(4t)\right)$, iii) Evaluate: $L^{-1}\left(\ln\left(\frac{s+2}{s+3}\right)\right)$

b) Solve the equation: $x'' + x = \sin(t) + \int_0^t x(u)\sin(t-u)du$; $x(0) = 0$, $x'(0) = -1$

2)a) Find A such that $u = 5x^4 + 2Ax^2y^2 + 3xy + 5y^4 + 10$ is harmonic and find it's corresponding analytic function $f(z) = u + iv$.

b) Show that: if v is a harmonic conjugate of u in a domain D, then $(u^2 - v^2)$ is harmonic in D.

3)a) i) Find and sketch the graph of the function: $f(t) = L^{-1}\left[\frac{3}{s} - \frac{4e^{-s}}{s^2} + \frac{4e^{-3s}}{s^2}\right]$

ii) Show that: $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

b) Sketch the graph of the periodic function: $f(t) = \begin{cases} 1 & ; \ 0 < t < 2 \\ -2 & ; \ 2 < t < 3 \end{cases}$, $f(t+3) = f(t)$ and find its L.T.

4) a) Find all values of z : i) $e^{2z-2i} = 3 + 3i$ ii) $\cos 2z = 5i$

iii) $z = \ln(4 - 4i)$ iv) $z = (1 - i)^{3i}$

b) If c is the circle $|z - 1| = 4$ and if $g(z_0) = \oint_c \frac{2z^4 + 4z^3 + 5z}{(z - z_0)^3} dz$ find:

i) g (1+2i) ii) g (1- 6i).

5)a) Evaluate the following integrals: i) $\int_0^{\infty} \frac{dx}{\sqrt{-\ln(x)}}$, ii) $\int_0^{\infty} \frac{dx}{\sqrt{x}(1+x)}$.

b) Find the series solution of : $(1 - x^2)y'' - 4xy' + 4y = 0$, near $x = 0$.

6)a) Find all the Laurent series in different domains for $f(z) = \frac{7z+4}{z^2+2z-8}$.

partial fraction

b) Evaluate i) $\oint_C (4z^5 + 2z^3 + 5z^2)\cos\left(\frac{2}{z}\right) dz$, $c: |z| = 1$

ii) $\oint_c \frac{e^{3z+4}}{(z-i)^2} dz$, $c: |z| = 2$ iii) $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)}$

AIN SHAMS UNIVERSITY

ا.ه. هندسة السيارات

Engineering math.(PHM 663)

ماجستير ودكتوراه

FACULTY OF ENGINEERING

رياضيات هندسية متقدم

May 2013

(4)

ENG.PHYSICS &MATH.DEPT.

ا فو الطلاب مايو ١٣ .٢

TIME ALLOWED:3HRS

Answer the following questions:

1.a) Show that the function : $u = 3x^4 - 18x^2y^2 + 3y^4 - 6xy + 5$ is harmonic and find it's Corresponding analytic function $f(z) = u + iv$

b) Under the transformation $w = \frac{1}{z}$, find the image of: $i)\ |z-1| = 1$, $ii) y + x = 0$

2.a) Find the Laurent series of the function: $f(z) = \frac{7z-4}{z^2+z-6}$

b) IF C is the circle $|z| = 4$ and if: $g(z_0) = \oint_c \frac{3z^3+4z^2+9}{(z-z_0)^3} dz$ find: $i) g(1+i)$, $ii) g(-6i)$

3.a) Find the values of z for the following:

$i) e^{2z+i} = 3+3i$, $ii) z = (1+i)^i$, $iii) z = ln(4-4i)$ and $iv) sin(2z) = 5i$

b) Evaluate: $i) \oint_c (z^3 + 4z^2 - 3z + 8)e^{\left(\frac{-2}{z}\right)} dz$; $c: |z| = 1$

$ii) \oint_c \frac{sin(2\bar{z}+3)}{(z-2)^3} dz$; $c: |z| = 3$ and $iii) \int_{-\infty}^{\infty} \frac{5x^2+2}{(x^2+4)^2} dx$

4.a) Find L. T. of

$i) f(t) = \frac{e^{2t}-e^{3t}}{t}$, $ii) f(t) = \left(e^{-t} ch\,(3t) cos(4t)\right)$, $iii) Evaluate: L^{-1}\left(tan^{-1}\left(\frac{1}{s}\right)\right)$

b) Solve the equation: $y'' + y = sin(t) + \int_0^t y(u) sin(t-u) du$; $y(0) = 0$, $y'(0) = -1$

5.a) i)Find and sketch the graph of the function: $f(t) = L^{-1}\left[\frac{3}{s} - \frac{4e^{-s}}{s^2} + \frac{4e^{-3s}}{s^2}\right]$

ii) Using Legendre's formula, show that : $\Gamma\left(n + \frac{1}{2}\right) = \frac{(2n)!\sqrt{\pi}}{2^{2n}\, n!}$

b) By two different methods find L.T. of : $f(t) = \begin{cases} 4t & ; 0 \le t \le 1 \\ 4 & ; t > 1 \end{cases}$

6.a) Evaluate: $i) \int_0^{\infty} x^{m-1} sin(ax) dx$, $ii) \int_0^{\infty} \frac{dt}{\sqrt{t}(1+t)}$ use $u = \frac{1}{1+t}$

b) find the series solution in power of x of : $2y'' + xy' - 4y = 0$

Ain Shams University
Faculty of Engineering
Eng.Physics & Math.Departement

Engineering Math(PHM 663)
Second -term 2012
Allowed Time: 3 Hrs.

Answer the following questions :

1)a) Find A such that the function $u = x^3 - Axy^2 + 5y + 6$ is harmonic and find it's corresponding analytic function $w = u + iv$.

b)Find all values of z such that :

$(i)\ e^{2z-3} = 1$, $(ii)\ \cos(2z) = 2i$, $(iii)\ \ln(3z) = 3+3i$, $(iv)\ z = (2-2i)^i$

2)a)Find Laurent series of the function $f(z) = \dfrac{7z+4}{z^2 - z - 6}$ in different domains.

b)If C is the circle $|z| = 3$ and if: $g(z_\circ) = \oint_C \dfrac{z^3 + 2z^2 + 3}{(z - z_\circ)^3} dz$ find: $(i)\ g(2i)$, $(ii)\ g(3-4i)$

3)Evaluate the following integrals:

$(i)\ \oint_C (4z^3 + 2z^2 + 5z + 3)\sin\left(\dfrac{2}{z}\right) dz \quad ; C : |z| = \dfrac{1}{2}$

$(ii)\ \oint_C \dfrac{e^{3z+2} dz}{z^2 - 2iz}$, where C is : $(a)\ |z| = \dfrac{1}{2}$, $(b)\ |z - 2i| = \dfrac{1}{2}$, $(c)\ |z| = 6$

$(iii)\ \int_0^\infty \dfrac{x^2 + 1}{(x^2+4)^2} dx$, $(iv)\ \int_0^{2\pi} \dfrac{d\theta}{2 - \cos(\theta)}$.

4)a)Find L.T. of :(i) $f(t) = \dfrac{e^t - e^{-t}}{t}$. $(ii)\ f(t) = \begin{cases} t & ; \ 0 \le t \le 1 \\ 1 & ; \ t > 1 \end{cases}$

b)solve the equation :- $y'' + y = \sin(t) + \int_0^t \sin(t-u)\, y(u)\, du \ ; y(0) = 0 \ , y'(0) = 1$

5)a)i)Find and sketch the graph of the function $f(t) = L^{-1}\left[\dfrac{2}{s^3} - \dfrac{2e^{-2s}}{s^3} - \dfrac{4e^{-2s}}{s^2} - \dfrac{4e^{-5s}}{s}\right]$

ii) Sketch the graph of the periodic function $f(t) = \begin{cases} a\cos(t) & ; \ 0 < t < \pi \\ 0 & ; \ \pi < t < 2\pi \end{cases}$, $f(t+2\pi) = f(t)$ and find L.T. of it.

b)Find the series solution in powers of x of : $2x^2 y'' + 3xy' - (1+x^2)y = 0$

6)a)Evaluate: $(i)\ \int_0^\infty t^{-3/2}(1 - e^{-t})\, dt$, $(ii)\ \int_0^\infty x^{3/2} 3^{-x} dx$

b)Find the area enclosed by the curve $x^4 + y^4 = 1$

Good luck

30
70

max