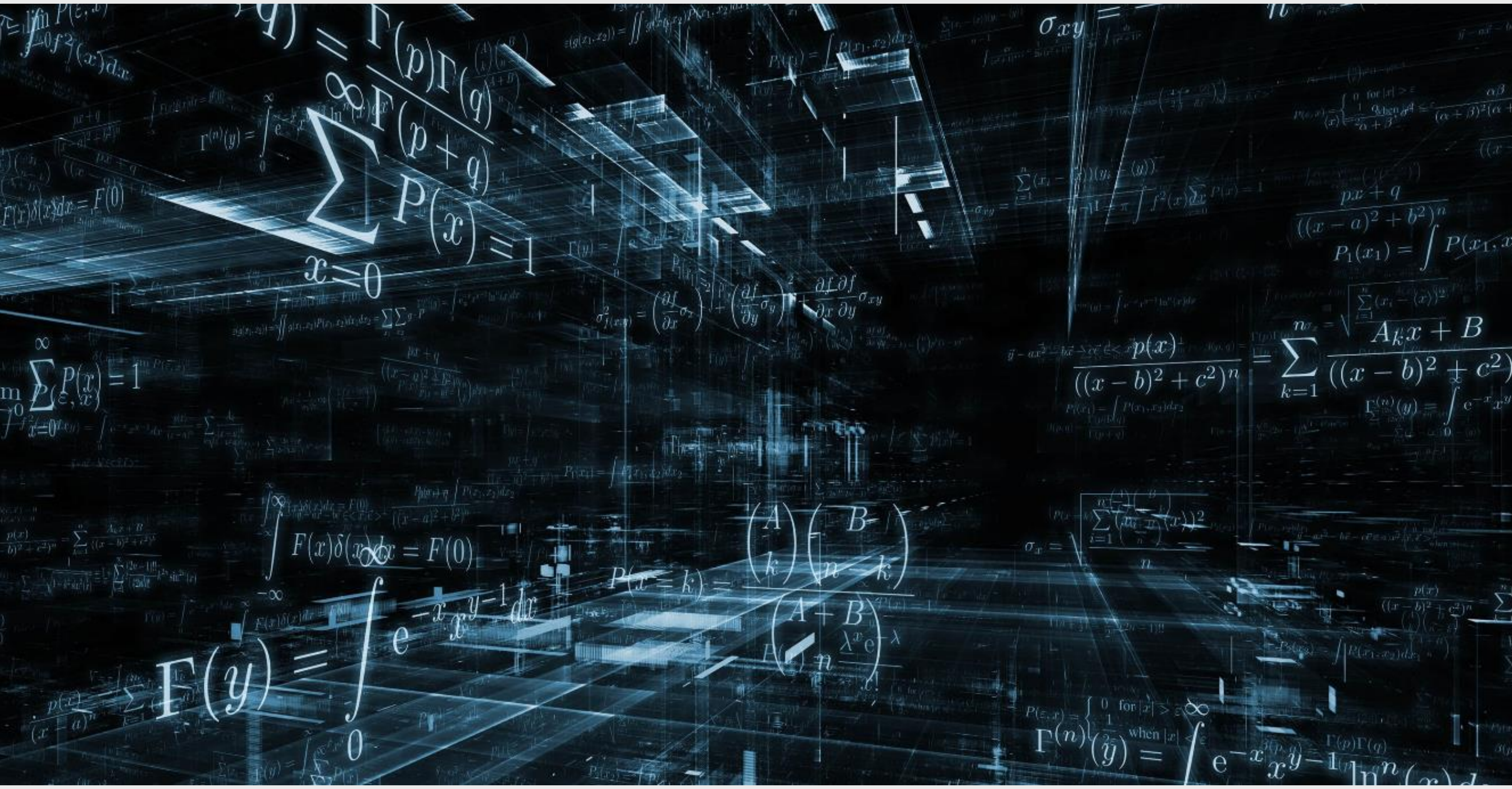
PHM212s: Complex, Special Functions and Numerical Analysis

Exercise Sheet

Special Functions



"All the world's a differential equation, and

the men and women are merely variables."

-Ben Orlin

[1] Evaluate in terms of the Gamma function:

(a) $\int_{0}^{\pi/2} (csc^3\theta - csc^2\theta)^{1/5} cos\,\theta\,\mathrm{d}\theta$ *[Midterm 2017]*

(b) $\int_{0}^{\pi/2} (\sec^3\theta - \sec^2\theta)^{1/4} \sin\theta\,\mathrm{d}\theta$ *[Midterm 2017]*

(c) $\int_{0}^{\pi/2} (tan^5 x + tan^7 x)\, e^{-\tan^2 x}\,\mathrm{d}x$ *[Midterm 2017]*

(d) $\int_{0}^{\sqrt{2}} \frac{\mathrm{d}x}{\sqrt{4+x^4}}$ *[Midterm 2018]*

(e) $\int_{0}^{\sqrt{3}} \frac{\mathrm{d}x}{\sqrt{9+x^4}}$ *[Midterm 2018]*

(f) $\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{\sqrt[3]{1+x^6}}$ *[Midterm 2019]*

(g) $\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{\sqrt{1+x^6}}$ *[Midterm 2019]*

(h) $\int_{0}^{1} \sqrt[n]{1-x^n}\,\mathrm{d}x$ put $x^n = sin^2(\theta)$. *[Final Fall 2015]*

(i) $\int_{a}^{\infty} e^{(2ax-x^2)}\,\mathrm{d}x$. *[Final Fall 2015]*

(j) $\int_{0}^{1} \left(\sqrt{x}\right)^3 \left(ln\frac{1}{x}\right)^4 \mathrm{d}x$

(k) $\int_0^1 x^{m-1} \ln\left(\frac{1}{x}\right) \mathrm{d}x$. [Final Spring 2017]

(l) $\int_0^1 \sqrt[4]{1-x^4}\, \mathrm{d}x$. [Final Summer 2015]

[2] Evaluate in terms of the Gamma function $\int_0^\infty x^a\, b^{-x}\, \mathrm{d}x$. hence State the conditions on the constants **a** & **b** such that the integral converges. [Midterm 2016]

[3] Evaluate in terms of the Gamma function $\int_0^\infty \frac{x^c}{c^x}\, \mathrm{d}x$. hence State the conditions on the constants **C** such that the integral converges. [Midterm 2016] [Midterm Spring 2021]

[4] By using Gamma and Beta functions evaluate the following integrals: [Midterm Spring 2017]

i) $\int_0^2 \frac{x^2}{\sqrt{2-x}} \mathrm{d}x$

ii) $\int_0^1 \frac{1}{\sqrt{-\ln x}} \mathrm{d}x$

iii) $\int_0^\infty \frac{1}{1+x^4} \mathrm{d}x$

[5] Find the area enclosed by:

(a) asteroid $x^{2/3} + y^{2/3} = 1$

(b) the curve $x^{2/5} + y^{2/5} = 1$

[6] By two different methods obtain a closed form for $\Gamma(n + 3/2)$ where n is any positive integer. [Midterm Spring 2021]