AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING
SPECIALIZED ENGINEERING PROGRAMS
JUNIOR COMMUNICATION ENGINEERING PROGRAM

Juba

SPRING 2022	Assignment #5	Total: 5 marks

PHM212s: Special Functions, Complex Analysis & Numerical Analysis

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Name: **ID:** **Deadline: Week 14**

Please, Solve each problem in its assigned place ONLY (the empty space below it)

Numericl solution of Ordinary Differential Equations

Use at least 4 decimal places in your calculations

1) $y' = x + y$, $y(0) = 0$. Find $y(0.4)$ using the following:

a) Exact Method $\frac{dy}{dx} = x + y$

let $z = x + y$ ∴ $y' = z' - 1$

∴ $\frac{dz}{dx} - 1 = z$

$\int \frac{dz}{z+1} = \int dx$

$\ln|z+1| = x + c$

$\ln(|x+y+1|) = x + c$

$y = Ae^{x} - x - 1$ @ $y(0) = 0$ ∴ $A = 1$

∴ $y = e^{x} - x - 1$

$y(0.4) = 0.0918$

b) Euler Method with h = 0.1

$n = \frac{x_n - x_0}{h} = \frac{0.4 - 0}{0.1} = 4$

n	x_n	y_n	$f(x_n, y_n)$	$\Delta y = hf$	y_{n+1}
0	0	0	0	0	0
1	0.1	0	0.1	0.01	0.01
2	0.2	0.01	0.21	0.021	0.031
3	0.3	0.031	0.3310	0.0331	0.0641
4	0.4	0.0641			

∴ $y(0.4) \simeq 0.0641$

c) Euler Method with 10 steps

$* \; h = \frac{x_n - x_0}{n} = \frac{0.4 - 0}{10} = 0.04$

n	x_n	y_n	$f(x_n, y_n)$	$\Delta y = hf$	y_{n+1}
0	0	0	0	0	0
1	0.04	0	0.04	0.0016	0.0016
2	0.08	0.0016	0.0816	0.0033	0.0049
3	0.12	0.0049	0.1249	0.005	0.0099
4	0.16	0.0099	0.1699	0.0068	0.0167
5	0.20	0.0167	0.2167	0.0087	0.0253
6	0.24	0.0253	0.2653	0.0106	0.0359
7	0.28	0.0359	0.3159	0.0126	0.0486
8	0.32	0.0486	0.3686	0.0147	0.0633
9	0.36	0.0633	0.4233	0.0169	0.0802
10	0.4	0.0802			

$\therefore \; y(0.4) \simeq 0.0802$

d) Runge-Kutta Method with h = 0.4

n	x_n	y_n	w_i	Δy
0	0	0	$w_1 = 0$	0
	0.2	0	$w_2 = 0.08$	0.16
	0.2	0.04	$w_3 = 0.096$	0.1920
	0.4	0.096	$w_4 = 0.1984$	0.1984
				0.5504

$\rightarrow y_1 = y_0 + \frac{1}{6} \Delta y$

$= 0.0917$

$\therefore y(0.4) \simeq 0.0917$

e) Runge-Kutta method with 2 steps.

$* \; h = \frac{x_n - x_0}{n} = \frac{0.4}{2} = 0.2$

n	x_n	y_n	w_i	Δy
0	0	0	$w_1 = 0$	0
	0.1	0	$w_2 = 0.02$	0.04
	0.1	0.01	$w_3 = 0.022$	0.044
	0.2	0.022	$w_4 = 0.0444$	0.0444
				0.1284

$\rightarrow y_1 = y_0 + \frac{1}{6} \Delta y$

$= 0.0214$

n	x_n	y_n	w_i	Δy
1	0.2	0.0214	0.0043	0.0043
	0.3	0.0435	0.0687	0.1374
	0.3	0.0558	0.0712	0.1423
	0.4	0.0926	0.0985	0.0985
				0.3825

$\rightarrow y_2 = y_1 + \frac{1}{6} \Delta y$

$= 0.0852$

$\therefore y(0.4) \simeq 0.0852$

2) $y' = x - y$, $y(1) = 2$. Find $y(0.5)$ using Runge-Kutta method with 2 steps.

* $h = \frac{x_n - x_0}{n} = \frac{0.5-1}{2} = -0.25$

n	x_n	y_n	w_i	Δy
0	1	2	$w_1 = 0.25$	0.25
	0.8750	2.125	$w_2 = 0.3125$	0.625
	0.8750	2.1563	$w_3 = 0.3203$	0.6406
	0.75	2.3203	$w_4 = 0.3926$	0.3926
				1.9082

$\rightarrow y_1 = y_0 + \frac{1}{6} \Delta y$

$= 2.3180$

n	x_n	y_n	w_i	Δy
1	0.75	2.3180	0.3920	0.3920
	0.625	2.5140	0.4723	0.9445
	0.625	2.5541	−1.9292	−3.8583
	0.5	0.3888	−0.0278	−0.0278
				−2.5496

$\rightarrow y_2 = y_1 + \frac{1}{6} \Delta y$

$= 1.8931$

$\therefore y(0.5) \simeq 1.8931$

3) $x' = x - y - t, y' = 4x - 2y, x(0) = 1 \;\&\; y(0) = 0$

Find $x(0.2) \& y(0.2)$ using Runge-Kutta method with h=0.1

n	t_n	x_n	y_n	W_i	V_i	ΔX	Δy
0	0	1	0	0.1	0.4	0.1	0.4
	0.05	1.05	0.2	0.08	0.38	0.16	0.7600
	0.05	1.04	0.19	0.08	0.3780	0.16	0.7560
	0.1	1.08	0.189	0.0791	0.3942	0.0791	0.3942
						0.4991	2.3102

$\rightarrow x(0.1) = x(0) + \frac{1}{6}\Delta x$

$= 1.0832$

$\rightarrow y(0.1) = y(0) + \frac{1}{6}\Delta y$

$= 0.3850$

n	t_n	x_n	y_n	W_i	V_i	ΔX	Δy
1	0.1	1.0832	0.3850	0.0598	0.3563	0.0598	0.3563
	0.15	1.1131	0.5631	0.04	0.3326	0.08	0.6652
	0.15	1.1032	0.5513	0.0402	0.3310	0.0804	0.6620
	0.2	1.1234	0.5502	0.0373	0.3393	0.0373	0.3393
						0.2575	2.0228

$\rightarrow x(0.2) = x(0.1) + \frac{1}{6}\Delta x$

$= 1.1261$

$\rightarrow y(0.2) = y(0.1) + \frac{1}{6}\Delta y$

$= 0.7221$

4) $x'' + t^2x' + 3x = t,\ x(0) = 1$ & $x'(0) = 2$

Find $x(0.2)$ using Runge-Kutta method with h=0.1

→ let $x' = y \rightarrow (1)$ $\therefore x'' = y'$ ⇒ $y' + t^2 y + 3x = t$

$y' = t - 3x - t^2 y \rightarrow (2)$

n	t_n	x_n	y_n	w_i	v_i	Δx	Δy
0	0	1	2	0.2	-0.3	0.2	-0.3
	0.05	1.1	1.85	0.1850	-0.3255	0.37	-0.6509
	0.05	1.0925	1.8373	0.1837	-0.3232	0.3675	-0.6464
	0.1	1.1837	1.8384	0.1838	-0.3470	0.1838	-0.3470
						1.1213	-1.9443

→ $x(0.1) = x(0) + \frac{1}{6}\Delta x$

$= 1.1869$

→ $y(0.1) = y(0) + \frac{1}{6}\Delta y$

$= 1.6759$

n	t_n	x_n	y_n	w_i	v_i	Δx	Δy
1	0.1	1.1869	1.6759	0.1676	-0.3477	0.1676	-0.3477
	0.15	1.2707	1.5020	0.1502	-0.3696	0.3004	-0.7392
	0.15	1.2620	1.4911	0.1491	-0.3670	0.2982	-0.7339
	0.2	1.3360	1.4924	0.1492	-0.3868	0.1492	-0.3868
						0.9155	-2.2076

→ $x(0.2) = x(0.1) + \frac{1}{6}\Delta x$

$= 1.3395$

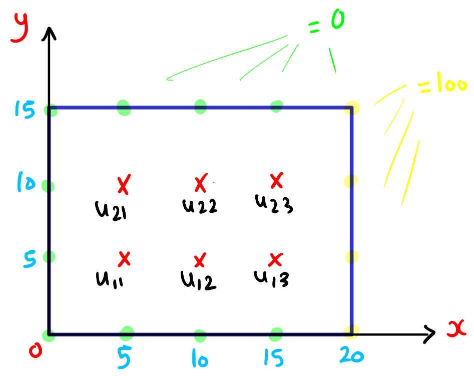
→ $y(0.2) = y(0.1) + \frac{1}{6}\Delta y$

$= 1.3080$

Numericl solution of Partial Differential Equations

5) Find U (x, y) such that $\nabla^2 U(x, y) = 0$ over a rectangle 20 x 15 cm using a grid with step size h = 5, and the boundary conditions: U (x, 0) = 0, U (x, 15) = 0, U (0, y) = 0, U (20, y) = 100. Use Gauss-Seidel method to solve the resulting linear system. **Accurate to 2D**

$$\rightarrow \nabla^2 U_{(x,y)} = \begin{vmatrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{vmatrix} U = 0$$



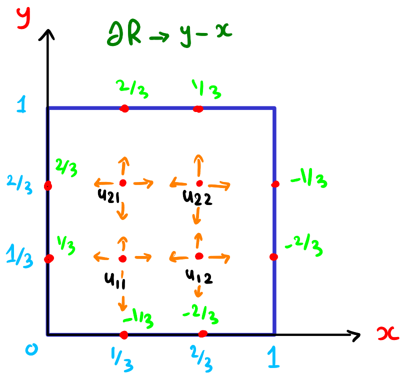
* At U_{11}: $u_{12} + u_{21} - 4u_{11} = 0$ ∴ $u_{11} = \frac{1}{4}[u_{12} + u_{21}]$
* At U_{12}: $u_{11} + u_{13} + u_{22} - 4u_{12} = 0$ ∴ $u_{12} = \frac{1}{4}[u_{11} + u_{13} + u_{22}]$
* At U_{13}: $u_{12} + u_{23} + 100 - 4u_{13} = 0$ ∴ $u_{13} = \frac{1}{4}[u_{12} + u_{23} + 100]$
* At U_{21}: $u_{11} + u_{22} - 4u_{21} = 0$ ∴ $u_{21} = \frac{1}{4}[u_{11} + u_{22}]$
* At U_{22}: $u_{21} + u_{12} + u_{23} - 4u_{22} = 0$ ∴ $u_{22} = \frac{1}{4}[u_{12} + u_{21} + u_{23}]$
* At U_{23}: $u_{13} + u_{22} + 100 - 4u_{23} = 0$ ∴ $u_{23} = \frac{1}{4}[u_{13} + u_{22} + 100]$

n	u_{11}	u_{12}	u_{13}	u_{21}	u_{22}	u_{23}
0	0	0	100	0	0	100
1	0	25	56.25	0	31.25	46.875
2	6.25	23.4375	42.5781	9.375	19.9219	40.625
3	8.2031	17.6758	39.5752	7.0313	16.333	38.9771
4	6.1768	15.5212	38.6246	5.6274	15.0314	38.4140
5	5.2872	14.7358	38.2874	5.0797	14.5574	38.2112
6	4.9539	14.4497	38.1652	4.8778	14.3847	38.1375
7	4.8319	14.3454	38.1207	4.8041	14.3218	38.1106
8	4.7874	14.3075	38.1045	4.7773	14.2988	38.1008
9	4.7712	14.2936	38.0986	4.7676	14.2905	38.0973
10	4.7653	14.2886	38.0965	4.7639	14.2875	38.0960
11	4.7631	14.2868	38.0957	4.7626	14.2863	38.0955

6) Solve (using **h = 1/3**) the Dirichlet problem

$$\nabla^2 u(x, y) = 3(x^2 + y^2) \quad in \quad R$$
$$and\ u(x, y) = y - x \quad on \quad \partial R$$

Here ∂R is the boundary of R and R is the region in the unit square $0 \le x \le 1$ and $0 \le y \le 1$. Perform 5 steps of Gauss-Seidel method with the initial approximation $u_{11}^{(0)} = u_{12}^{(0)} = u_{21}^{(0)} = u_{22}^{(0)} = 0$.



$$\rightarrow \nabla^2 U_{(x,y)} = \begin{vmatrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{vmatrix} U = \cancel{h^2}^{\,1/9}\left(3(x^2+y^2)\right) = \frac{1}{3}(x^2+y^2)$$

$*$ At U_{11} : $\frac{1}{3} - \frac{1}{3} + u_{21} + u_{12} - 4u_{11} = \frac{1}{3}\left(\frac{1}{9} + \frac{1}{9}\right)$ $\therefore u_{11} = \frac{1}{4}\left(u_{12} + u_{21} - \frac{2}{27}\right)$

$*$ At U_{12} : $\frac{-2}{3} - \frac{2}{3} + u_{11} + u_{22} - 4u_{12} = \frac{1}{3}\left(\frac{4}{9} + \frac{1}{9}\right)$ $\therefore u_{12} = \frac{1}{4}\left(u_{11} + u_{22} - \frac{41}{27}\right)$

$*$ At U_{21} : $\frac{2}{3} + \frac{2}{3} + u_{22} + u_{11} - 4u_{21} = \frac{1}{3}\left(\frac{4}{9} + \frac{1}{9}\right)$ $\therefore u_{21} = \frac{1}{4}\left(u_{11} + u_{22} + \frac{31}{27}\right)$

$*$ At U_{22} : $\frac{1}{3} - \frac{1}{3} + u_{21} + u_{12} - 4u_{22} = \frac{1}{3}\left(\frac{4}{9} + \frac{4}{9}\right)$ $\therefore u_{22} = \frac{1}{4}\left(u_{12} + u_{21} - \frac{8}{27}\right)$

n	U_{11}	U_{12}	U_{21}	U_{22}
0	0	0	0	0
1	-0.0185	-0.3843	0.2824	-0.0995
2	-0.0440	-0.4155	0.2512	-0.1152
3	-0.0596	-0.4233	0.2433	-0.1191
4	-0.0635	-0.4253	0.2414	-0.12
5	-0.0645	-0.4258	0.2409	-0.1203

Best wishes,

Dr. Makram Roshdy, Dr. Betty Nagy