Exercise Sheet (5) Numerical Analysis

[1] use Runge-Kutta method to:

i. $y' = x^2 - y$, from $x=0$ to $x=0.5$, $h=0.5$

$y(0)=1$, and from $x=0.5$ to $x=1.5$, $h=1$

n	x_n	y_n	w_i
	0	1	-0.5
0	0.25	0.75	-0.3437
	0.25	0.8281	-0.3828
	0.5	0.6172	-0.18360

$\Delta y = \frac{1}{6}(w_1 + 2w_2 + 2w_3 + w_4)$

$= \frac{1}{6}(0.5 + 0.6874 + 0.7656 + 0.1836)$

$= -0.3561$

$y_1 = y_0 + \Delta y = 1 - 0.3561 = 0.6439$

n	x_n	y_n	w_i
	0.5	0.6439	-0.3939
	1	0.4470	0.5531
0	1	0.9204	0.0796
	1.5	0.7235	1.5265

$\Delta y = \frac{1}{6}(w_1 + 2w_2 + 2w_3 + w_4)$

$= 0.3996$

$y_1 = y_0 + \Delta y = 1.0435$

ii. $y' = x + y^2$ to find $y(1)$; $y(0.5)=1$ & $h=0.25$

n	x_n	y_n	w_i
	0.5	1	0.375
	0.625	1.1875	0.5080
0	0.625	1.2544	0.5496
	0.75	1.5496	0.7878

$\Delta y = \frac{1}{6}(w_1 + 2w_2 + 2w_3 + w_4)$

$= 0.5466$

$y_1 = y_0 + \Delta y = 1.5466$

n	x_n	y_n	w_i
	0.75	1.5466	0.7855
	0.875	1.9393	1.1590
1	0.875	2.1261	1.3488
	1	2.8954	2.3459

$\Delta y = \frac{1}{6}(w_1 + 2w_2 + 2w_3 + w_4)$

$= 1.3578$

$y_2 = y_1 + \Delta y = 2.9044$

$n=2$; $h=0.1$

iv. Using two steps, solve the following system ODEs to find $x(0.2)$ & $y(0.2)$

$x' = x - y - t$, $y' = 4x + 2y$ $\quad x(0)=1$, $y(0)=0$

n	t	x	y	v	w	Δx	Δy
0	0	1	0	0.1	0.4	$=\frac{1}{6}(v_1+2v_2+2v_3+v_4)$	$\frac{1}{6}(w_1+2w_2+2w_3+w_4)$
	0.05	1.05	0.2	0.08	0.38		
	0.05	1.04	0.19	0.08	0.378		
	0.1	1.08	0.189	0.0791	0.394	$=0.0832$	$=0.385$

$x_1 = x_0 + \Delta x = 1.0832$

$y_1 = y_0 + \Delta y = 0.385$

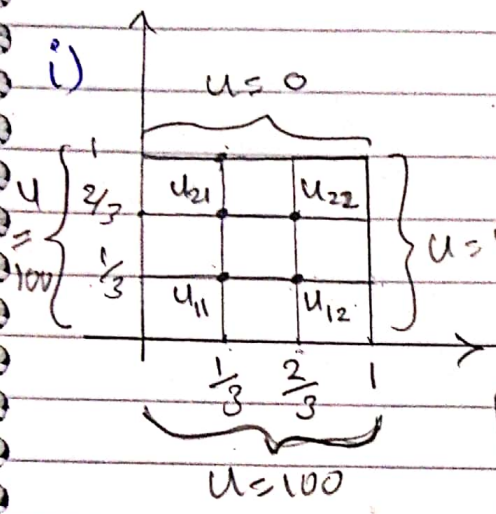
n	t	x	y	v	w	$\Delta x = \frac{1}{6}(v_1+2v_2+2v_3+v_4)$
1	0.1	1.0832	0.385	0.0598	0.3563	
	0.15	1.1131	0.5631	0.04	0.3326	$=0.0429$
	0.15	1.1632	0.5513	0.0462	0.3310	$\Delta y = \frac{1}{6}(w_1+2w_2+2w_3+w_4)$
	0.2	1.1234	0.5502	0.0373	0.3393	
						$=0.3371$

$x_2 = x_1 + \Delta x = 1.1261$

$y_2 = y_1 + \Delta y = 0.7221$

[2] Consider the Dirichlet Problem $\nabla^2 u(x,y) = 18(x^2+y^2)$ $0 \le x \le 1$ and $0 \le y \le 1$: $u(x,y) = 0$ on the top of the square and $u(x,y) = 100$ on the sides and bottom of the square

i)



ii) Zero initial values, the first three Successive approximation $= 1/3$

$$\nabla^2 u = \frac{1}{h^2}\begin{bmatrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{bmatrix} u = 18(x^2+y^2)$$

$$\begin{bmatrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{bmatrix} u = 2(x^2+y^2)$$

@ $u_{11} \Rightarrow u_{12} + u_{21} + 200 - 4u_{11} = 2\left(\frac{1}{9}+\frac{1}{9}\right) = \frac{4}{9}$

@ $u_{12} \Rightarrow u_{11} + u_{22} + 200 - 4u_{12} = 2\left(\frac{4}{9}+\frac{1}{9}\right) = \frac{10}{9}$

@ $u_{21} \Rightarrow u_{11} + u_{22} + 100 - 4u_{21} = 2\left(\frac{1}{9}+\frac{4}{9}\right) = \frac{10}{9}$

@ $u_{22} \Rightarrow u_{21} + u_{12} + 100 - 4u_{22} = 2\left(\frac{4}{9}+\frac{4}{9}\right) = \frac{16}{9}$

$\therefore u_{11} = \frac{1}{4}\left(u_{12} + u_{21} + 200 - \frac{4}{9}\right)$

$u_{12} = \frac{1}{4}\left(u_{11} + u_{22} + 200 - \frac{10}{9}\right)$

$u_{21} = \frac{1}{4}\left(u_{11} + u_{22} + 100 - \frac{10}{9}\right)$

$u_{22} = \frac{1}{4}\left(u_{21} + u_{12} + 100 - \frac{16}{9}\right)$

n	u_{11}	u_{12}	u_{21}	u_{22}
0	0	0	0	0
1	49.888	62.8944	37.1944	49.4628
2	74.7361	80.7569	55.7569	58.6840
3	84.0174	85.3976	60.397	61.0043

$u_{11} \simeq 84.0174$

$u_{12} \simeq 85.3976$

$u_{21} \simeq 60.3976$

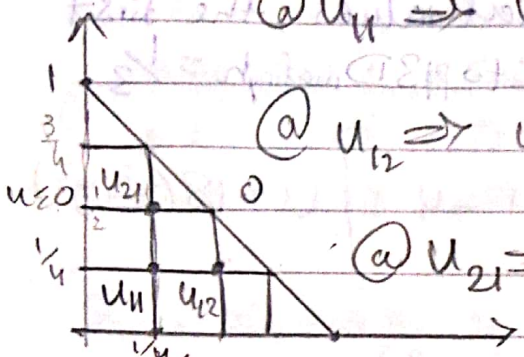
$u_{22} \simeq 61.0043$

[3] $h = \frac{1}{4}$ $\nabla^2 u = 0$ $x>0$, $y>0$, $x+y<1$

$u(0,y)=0$, $u(x,1-x)=0$ $u_x(x,0) = x(1-x)$

starting with zero values at interior Points.

Perform three iterations



@ $u_{11} \Rightarrow u_{12} + u_{21} + \frac{3}{16} - 4u_{11} = 0$

@ $u_{12} \Rightarrow u_{11} + \frac{1}{4} - 4u_{12} = 0$

@ $u_{21} \Rightarrow u_{11} - 4u_{21} = 0$

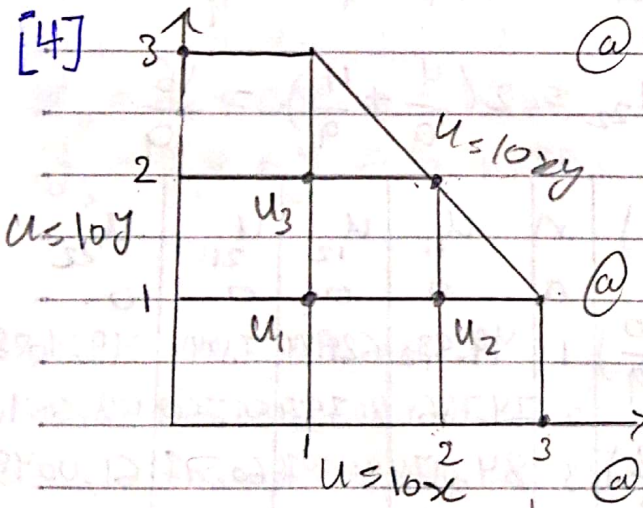
$u_{11} = \frac{1}{4}(u_{12} + u_{21} + \frac{3}{16})$

$u_{12} = \frac{1}{4}(u_{11} + \frac{1}{4})$

$u_{21} = \frac{1}{4}u_{11}$

n	u_{11}	u_{12}	u_{21}
0	0	0	0
1	0.0469	0.0742	0.0117
2	0.0684	0.0796	0.0171
3	0.0710	0.0803	0.0178

[4]



@ $u_1 \Rightarrow u_2 + u_3 + 20 - 4u_1 = 20$

$u_1 = \frac{1}{4}(u_2 + u_3)$

@ $u_2 \Rightarrow u_1 + 20 + 40 + 30 - 4u_2 = 50$

$u_2 = \frac{1}{4}(u_1 + 46)$

@ $u_3 \Rightarrow u_1 + 46 + 30 + 20 - 4u_3 = 50$

$u_3 = \frac{1}{4}(u_1 + 40)$

n	u_1	u_2	u_3
0	0	0	0
1	0	10	10
2	5	11.25	11.25
3	5.625	11.406	11.406
4	5.703	11.425	11.425
5	5.712	11.428	11.428

⇒

n	u_1	u_2	u_3
6	5.712	11.428	11.428

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