Ain Shams University
Faculty of Engineering
Eng.Physics & Math.Departement

Engineering Math (PHM 663)
Spring 2022-2023
Allowed Time: 3 Hrs.

Total mark (70)

The exam is composed of 6 questions in one page. The mark for each question is (15)

1) a) Show that the function: $u(x,y) = x^3 - 3x\,y^2 + 5x + e^x \cos y$ is harmonic and find it's corresponding analytic function $f(z) = u + iv.$ $Find\ f'(z).$

b) Find all values of z such that:

i) $e^{z-2i} = 4 - 4i$, ii) $z = \left(2 + 2\sqrt{2}\,i\right)^i$

2) a) Find the image of the unit circle $|z| = 1$, under the mapping $f(z) = \frac{1}{z}$, then discuss how the point (0,0) exchange with $\pm\infty$, for the line $y = 2x.$ *sketch both the line* and its image,

b) Evaluate the following integrals:

i) $\oint_C \frac{\cos 2z}{z^3 - iz^2}\,dz$; where C: A) $|z - i| = \frac{1}{2}$ B) $|Z| = 2$, ii) $\int_0^\infty \frac{dx}{(x^2+1)\,(x^2+4)}$

3) a) Find all Laurent series that represent the function $f(z) = \frac{10z-12}{z^2-z-12}$ in different domains.

b) Evaluate the following integrals:

i) $\oint_C (8z^3 + 4z + 6)e^{\frac{3}{z}}\,dz$; where C: $|z| = \frac{1}{2}$

ii) *Given that* $G(z_0) = \oint_C \frac{z^3+4z^2}{(z-z_0)^3}\,dz$; *find* : A) $G(2+i)$, B) $G(3+4i)$ *where* C: $|z| = 3$

iii) $\int_{-\infty}^{\infty} \frac{dx}{(x^2+9)(x^2+1)^2}$

4) a) Evaluate the following integral $\int_a^\infty e^{(2ax-x^2)}\,dx.$

b) Find the series solution of: $y'' + xy = 0$, near the ordinary point $x = 0.$

5) a) Find the area enclosed by the equation of the curve $x^{2/3} + y^{2/3} = a^{2/3}.$

b) Evaluate: $\int_0^\infty \frac{\sin(5t)}{te^{5t}}\,dt$ (Hint: use Laplace Transform)

c) Solve the initial value problem: $y'' + 3y' + 2y = e^t$, given that: $y(0) = 1$ & $y'(0) = 0$, using **Laplace Transform.**

6) (a) Solve the equation: $y(t) = \cosh(t) + \int_0^t y(u)\cosh(t-u)\,du$

b) Find $f(t) = L^{-1}\left[\frac{3}{s} + \frac{2e^{-2s}}{s^2} - \frac{2e^{-5s}}{s^2}\right]$ and sketch the graph of this function

GOOD LUCK

Prof. Dr. R.A. El-Barkouky

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