Finite differena

PDE : method → System of equations

we will solve using Gauss-Siedel method :

$u_1 = F_1(u_2, u_3, \dots, c)$
$u_2 = F_2(u_1, u_3, \dots, c)$
$u_3 = F_3(u_1, u_2, \dots, c)$

$a_{n1}x_1 + a_{12}x_2 + \dots = b_n$

* to take :

$x_1 = \frac{1}{a_{11}}(-a_{12}x_2 - a_{13}x_3 - \dots + b_n)$

* Cond :

$|a_{11}| > |a_{12}| + |a_{13}| + \dots$

to converge to the exact sol.

suitable for u_3

eg: $2u_1 + u_2 - 4u_3 = 10$ ✓
$3u_2 + u_3 - 4u_1 = 5$
$u_1 + 2u_3 - 4u_2 = -6$ ✓

suitable for u_2

$u_2^{(n+1)} = \frac{3}{2} + 0.25u_1^{(n+1)} + 0.5u_3^{(n)}$ not yet updated

$u_3^{(n+1)} = -2.5 + 0.5u_1^{(n+1)} + 0.25u_2^{(n+1)}$

reorder $u_1 =$
$u_2 =$
$u_3 =$

$u_1^{(n+1)} = -1.25 + 0.75u_2^{(n)} + 0.25u_3^{(n)}$

n	u_1	u_2	u_3
0	-1.25	1.5	-2.5
1	substitute	step	
2	by	step	

start your initial guess / approximation by using the constants from the equatio unless given in the proble and we go on making successive approximat

* when do we stop ? "Changing the first guess doesn't affect the outcome too much"

perform n steps
$0 \to n$
(initial guess usually given)

Accurate to 2 Decimal places

اول لما اتنين 2D نثبتوا على
نفس الأرقام

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* we work with 2 extra decimal places then our given accuracy

يعني لازم نثبتوا كل الـ unknowns

* Pros : making a mistake in one of the iteration, it will be self corre after some iteration, the accuracy may be affected tho.

⚠ Don't get the equations wrong ⚠

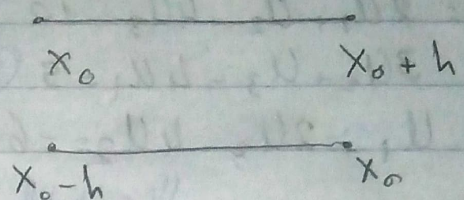
PDE: $f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \dots$

replace $x - x_0 = h$

$$\Rightarrow f(x_0 + h) = f(x_0) + f'(x_0)\frac{h}{1!} + \underbrace{\frac{f''(x_0)}{2!}h^2 + \dots}_{\text{neglect}}$$

$$\Rightarrow f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} \quad \text{(Forward approximation)}$$

$$\Rightarrow f'(x_0) = \frac{f(x_0) - f(x_0 - h)}{h} \quad \text{(Backwards)}$$



* (getting f' as function of f at same point and either $x_0 + h$ or $x_0 - h$)
* taking average:

$$\Rightarrow f'(x_0) = \frac{f(x_0+h) - f(x_0 - h)}{2h} \quad \text{(Central)}$$

* From this we can also get a function for f''

$$\Rightarrow f''(x_0) = \frac{f(x_0+h) + f(x_0-h) - 2f(x_0)}{h^2}$$

* Our goal is to solve: $u_{xx} + u_{yy} = 0$ (Laplace eq)

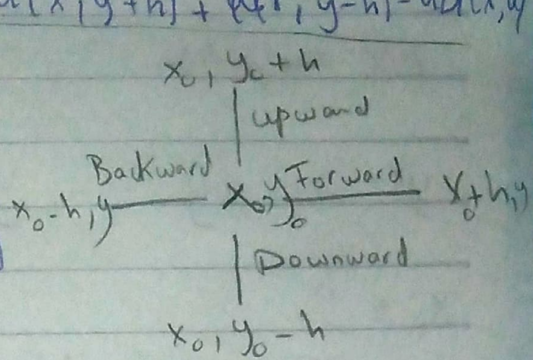
$$\hookrightarrow u_{xx} = \frac{u(x+h, y) + u(x-h, y) - 2u(x,y)}{h^2}$$

$$\hookrightarrow u_{yy} = \frac{u(x, y+h) + u(x, y-h) - 2u(x,y)}{h^2}$$

Poisson eq: $u_{xx} + u_{yy} = f(x,y)$

$$\nabla^2 u = u_{xx} + u_{yy} = \frac{u(x+h,y) + u(x-h,y) + u(x,y+h) + u(x,y-h) - 4u(x,y)}{h^2}$$

$$\Longrightarrow \nabla^2 u = \frac{1}{h^2}\begin{bmatrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{bmatrix}$$

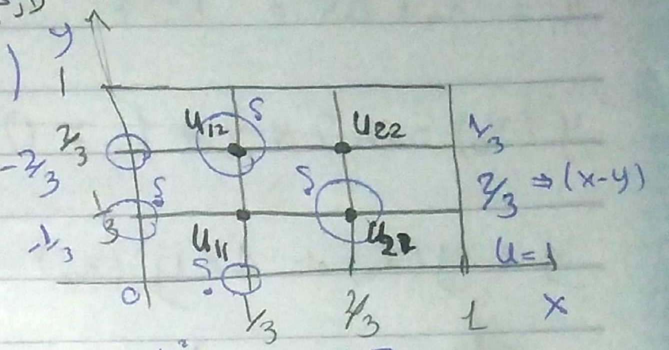


exp: $\nabla^2 u = 18(x^2+y^2)$, $h = \frac{1}{3}$ (step)

sol:

لازم يشرح شكل ال region

$$\frac{1}{h^2}\begin{bmatrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{bmatrix} u = 18(x^2+y^2)$$



If we want to solve @ u_{11}

∂R: boundary of R

$u(x,y) = x - y$ @ ∂R

$h^2 \quad \frac{1}{3} \quad \frac{2}{3}$

at u_{11} : $-\frac{1}{3} + u_{21} + \frac{1}{3} + u_{12} - 4u_{11} = 18(\frac{1}{3})^2(\frac{1}{3}^2 + \frac{1}{3}^2)$

thus: $u_{21} + u_{12} - 4u_{11} = \frac{4}{9}$ — ①

at $u_{1,2}$ $(\frac{1}{3}, \frac{2}{3})$: $-\frac{2}{3} + u_{22} + u_{11} + (-\frac{2}{3}) - 4u_{12} = 18(\frac{1}{3})^2((\frac{1}{3})^2 + (\frac{2}{3})^2)$

$\Rightarrow u_{22} + u_{11} - 4u_{12} = \frac{44}{18}$ — ②

$\Rightarrow u_{11} + u_{22} - 4u_{21} = -\frac{4}{18}$ — ③

$\Rightarrow u_{12} + u_{21} - 4u_{22} = \frac{32}{18}$ — ④

Using Gauss siedel to solve: $u_{11} = \frac{1}{4}(u_{12} + u_{21} - \frac{4}{9})$

$u_{12} = \frac{1}{4}(u_{11} + u_{22} - \frac{44}{18})$ $\quad u_{21} = \frac{1}{4}(u_{11} + u_{22} + \frac{4}{18})$

$u_{22} = \frac{1}{4}(u_{12} + u_{21} - \frac{32}{18})$

given initial condition or using constant of the equation.. Solve