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الاســـــــــراء

ملزمة (10)

رياضـــــة

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Laplace Transform (6)

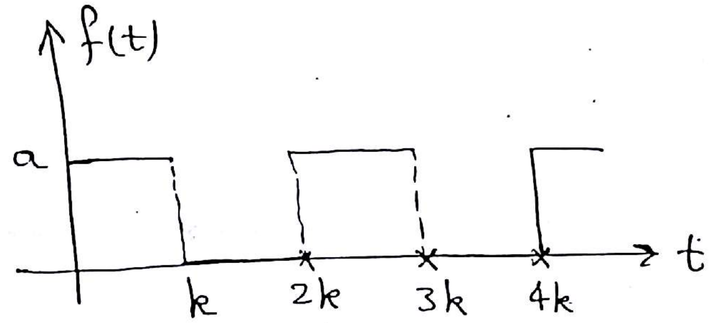
Periodic and Dirac functions

If $f(t)$ is periodic fn. such that :

$$f(t+T) = f(t) \qquad [\text{period} = T]$$

$$\Rightarrow \boxed{L[f(t)] = F(s) = \frac{1}{1-e^{-sT}} \int_0^T f(t)\, e^{-st}\, dt}$$ bop

EX① : find Laplace transform of the square wave function:



Solution

$\because$ $f(t)$ is periodic fn. with Period $T = 2k$.

$$\Rightarrow \underbrace{F(s)}_{L[f(t)]} = \frac{1}{1-e^{-2ks}} \int_0^{2k} f(t) \cdot e^{-st}\, dt \qquad \because f(t) = \begin{cases} a, & 0<t<k \\ 0, & k<t<2k \end{cases}$$

$$\because f(t+\underset{T}{2k}) = f(t)$$

$$\Rightarrow F(s) = \frac{1}{1-e^{-2ks}} \left[\int_0^k a \cdot e^{-st}\, dt + \cancel{\int_k^{2k} (0)\, e^{-st}\, dt}^{\,0} \right]$$

$$= \frac{a}{1-e^{-2ks}} \left. \frac{e^{-st}}{-s} \right|_0^k = \frac{a(1-e^{-ks})}{s(1-e^{-2ks})}$$

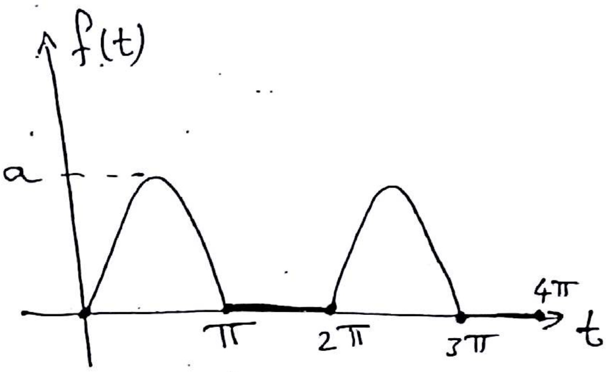
$$\Rightarrow F(s) = \frac{a\cancel{(1-e^{-ks})}}{s\cancel{(1-e^{-ks})}(1+e^{-ks})} = \boxed{\frac{a}{s(1+e^{-ks})}} \quad \#$$

EX 2: Find L.T. of:

$$f(t) = \begin{cases} a \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}, \quad f(t+2\pi) = f(t)$$

periodic with $T = 2\pi$.

Solution



$$F(s) = \frac{1}{1-e^{-2\pi s}} \int_0^{2\pi} f(t)\, e^{-st}\, dt$$

$$= \frac{1}{1-e^{-2\pi s}} \int_0^{\pi} a \sin t \cdot e^{-st}\, dt$$

$$= \frac{a}{1-e^{-2\pi s}} \operatorname{Im} \int_0^{\pi} e^{it} \cdot e^{-st}\, dt$$

$e^{i\theta} = \cos\theta + i \sin\theta$

$\therefore \cos\theta = \operatorname{Re}.\, e^{i\theta}$

$, \sin\theta = \operatorname{Im}.\, e^{i\theta}$

$$= \frac{a}{1-e^{-2\pi s}} \operatorname{Im}. \int_0^{\pi} e^{-(s-i)t}\, dt.$$

$$= \frac{a}{1-e^{-2\pi s}} \operatorname{Im}. \left. \frac{e^{-(s-i)t}}{-(s-i)} \right|_0^{\pi}$$

$$= \frac{a}{1-e^{-2\pi s}} \operatorname{Im}. \frac{\left(1 - e^{-(s-i)\pi}\right)}{(s-i)} \qquad e^{-s\pi + i\pi}$$

$$= \frac{a}{1-e^{-2\pi s}} \operatorname{Im}. \frac{1 - e^{-\pi s} \cdot e^{i\pi}}{(s-i)} \qquad e^{i\pi} = -1$$

$e^{i\pi} = \cos\pi + i \sin\pi$ ($\sin\pi = 0$)

$= -1$

$$= \frac{a}{1-e^{-2\pi s}} \operatorname{Im}. \left[\frac{1 + e^{-\pi s}}{(s-i)} \cdot \frac{(s+i)}{(s+i)} \right]$$

$$= \frac{a}{1-e^{-2\pi s}} \operatorname{Im}. \left[\frac{(1+e^{-\pi s})(s+i)}{s^2+1} \right] = \frac{a}{(1-e^{-2\pi s})} \cdot \frac{(1+e^{-\pi s})}{(s^2+1)}$$

$(1-e^{-2\pi s}) = (1-e^{-\pi s})(1+e^{-\pi s})$

$$\Rightarrow F(s) = \boxed{\frac{a}{(1-e^{-\pi s})\,(s^2+1)}} \#$$

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Ex 3 : Find : $L^{-1}\left[\dfrac{1-3e^{-2s}+2e^{-3s}}{s(1-e^{-3s})}\right]$

sketch the wave form and express the result in function form.

Solution

$$F(s) = \frac{1-3e^{-2s}+2e^{-3s}}{s(1-e^{-3s})} = \frac{(1-3e^{-2s}+2e^{-3s})}{s}\left[1+e^{-3s}+e^{-6s}+\cdots\right]$$

$$= \frac{1}{s}\left[1-3e^{-2s}+2e^{-3s}+e^{-3s}-3e^{-5s}+2e^{-6s}+e^{-6s}-3e^{-8s}+2e^{-9s}+\cdots\right]$$

$$\Rightarrow F(s) = \frac{1}{s}\left[1-3e^{-2s}+3e^{-3s}-3e^{-5s}+3e^{-6s}-3e^{-8s}+3e^{-9s}-\cdots\right]$$

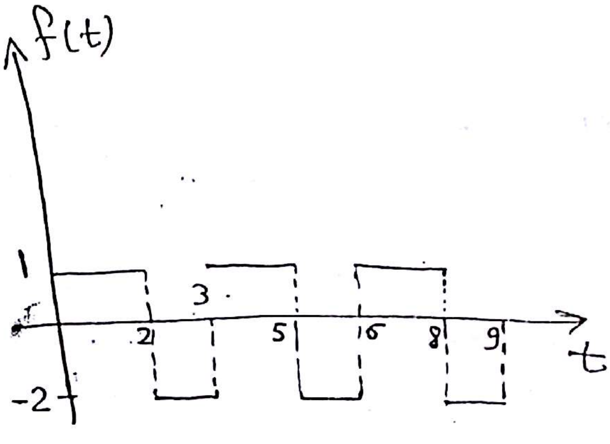
$$= \frac{1}{s}e^{0s} - \frac{3}{s}e^{-2s} + \frac{3}{s}e^{-3s} - \frac{3}{s}e^{-5s} + \frac{3}{s}e^{-6s} - \frac{3}{s}e^{-8s} + \frac{3}{s}e^{-9s} - \cdots$$

$$\Rightarrow f(t) = H(t) - 3H(t-2) + 3H(t-3) - 3H(t-5) + 3H(t-6) - 3H(t-8) + 3H(t-9) - \cdots \quad \#$$

$$\boxed{L^{-1}\left[\frac{e^{-ks}}{s}\right] = H(t-k)}$$

$$f(t) = \begin{cases} 1 & 0<t<2 \\ -2 & 2<t<3 \\ 1 & 3<t<5 \\ -2 & 5<t<6 \\ \vdots & \end{cases}$$

periodic fn.



$$\therefore f(t) = \begin{cases} 1 & 0<t<2 \\ -2 & 2<t<3 \end{cases}, \quad f(t+\underset{T}{3}) = f(t)$$

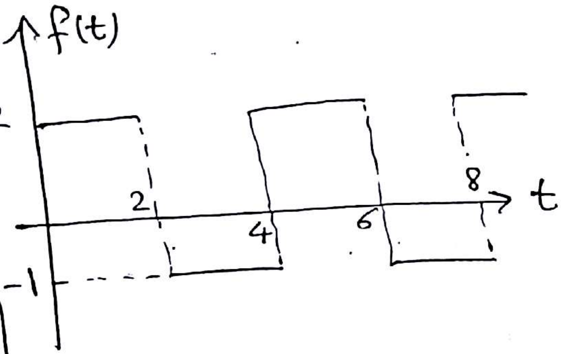
The function form

Problems (5) حل بعض مسائل من تمارين الكتاب

(1) Find L.T. of the following periodic fn.:

$$f(t) = \begin{cases} 2, & 0<t<2 \\ -1, & 2<t<4 \end{cases}, \quad f(t+4) = f(t)$$

solution



$T = 4$

$$F(s) = \frac{1}{1-e^{-4s}} \int_0^4 f(t)\, e^{-st}\, dt$$

$$= \frac{1}{1-e^{-4s}} \left[\int_0^2 2e^{-st}\, dt + \int_2^4 -e^{-st}\, dt \right]$$

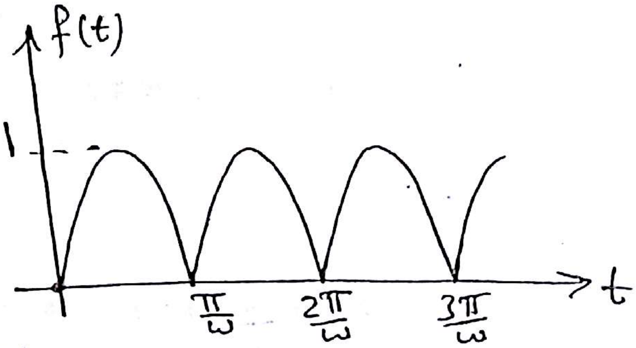
$$= \frac{1}{1-e^{-4s}} \left[\frac{2e^{-st}}{-s} \Big|_0^2 - \frac{e^{-st}}{-s} \Big|_2^4 \right]$$

$$\therefore F(s) = \frac{1}{1-e^{-4s}} \left[\frac{2}{s}\left(1-e^{-2s}\right) + \frac{1}{s}\left(e^{-4s}-e^{-2s}\right) \right] \#$$

[3] Find L.T. of: $f(t) = \sin\omega t, \quad 0<t<\frac{\pi}{\omega}$

$$, f\left(t+\frac{\pi}{\omega}\right) = f(t)$$

solution



$T = \frac{\pi}{\omega}$

$$F(s) = \frac{1}{1-e^{-\frac{\pi}{\omega}s}} \int_0^{\frac{\pi}{\omega}} \sin\omega t \cdot e^{-st}\, dt$$

$$= \frac{1}{1-e^{-\frac{\pi}{\omega}s}} \, \text{Im.} \int_0^{\frac{\pi}{\omega}} e^{i\omega t} \cdot e^{-st}\, dt$$

$$\Rightarrow F(s) = \frac{1}{1-e^{-\frac{\pi}{\omega}s}} \operatorname{Im} \int_0^{\frac{\pi}{\omega}} e^{-(s-i\omega)t}\, dt$$

$$= \frac{1}{1-e^{-\frac{\pi}{\omega}s}} \operatorname{Im} \left. \frac{e^{-(s-i\omega)t}}{-(s-i\omega)} \right|_0^{\frac{\pi}{\omega}}$$

$$= \frac{1}{1-e^{-\frac{\pi}{\omega}s}} \operatorname{Im} \frac{\left(1-e^{-(s-i\omega)\frac{\pi}{\omega}}\right)}{(s-i\omega)}$$

$$= \frac{1}{1-e^{-\frac{\pi}{\omega}s}} \operatorname{Im} \frac{\left(1-e^{-\frac{\pi}{\omega}s} \cdot \overbrace{e^{i\pi}}^{-1}\right)}{(s-i\omega)}$$

$$= \frac{1}{1-e^{-\frac{\pi}{\omega}s}} \operatorname{Im} \frac{\left(1+e^{-\frac{\pi}{\omega}s}\right)}{(s-i\omega)} \frac{(s+i\omega)}{(s+i\omega)}$$

$$F(s) = \frac{1}{1-e^{-\frac{\pi}{\omega}s}} \frac{\omega\left(1+e^{-\frac{\pi}{\omega}s}\right)}{(s^2+\omega^2)} = \boxed{\frac{\omega\left(1+e^{-\frac{\pi}{\omega}s}\right)}{\left(1-e^{-\frac{\pi}{\omega}s}\right)(s^2+\omega^2)}}$$

#

5 Find : $L^{-1}\left[\frac{2(1-e^{-s})}{s(1-e^{-4s})}\right]$

sketch the wave form and express the result in function form.

solution

$$F(s) = \frac{2(1-e^{-s})}{s(1-e^{-4s})} = \frac{2(1-e^{-s})}{s}\left[1+e^{-4s}+e^{-8s}+\cdots\right]$$

$$= \frac{2}{s}\left[1-e^{-s}+e^{-4s}-e^{-5s}+e^{-8s}-e^{-9s}+\cdots\right]$$

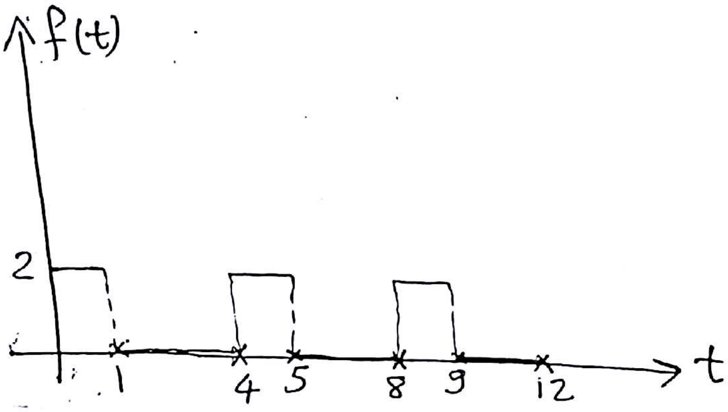
$$= \frac{2}{s}e^{0s} - \frac{2}{s}e^{-s} + \frac{2}{s}e^{-4s} - \frac{2}{s}e^{-5s} + \frac{2}{s}e^{-8s} - \frac{2}{s}e^{-9s} + \cdots$$

$$\Rightarrow f(t) = 2H(t) - 2H(t-1) + 2H(t-4) - 2H(t-5) + 2H(t-8) - 2H(t-9) + \cdots \quad \#$$

$$f(t) = \begin{cases} 2 & 0<t<1 \\ 0 & 1<t<4 \\ 2 & 4<t<5 \\ 0 & 5<t<8 \\ \vdots & \end{cases} \longrightarrow \text{periodic fn.}$$

$$\therefore f(t) = \begin{cases} 2 & 0<t<1 \\ 0 & 1<t<4 \end{cases}$$

$$, f(t+\underbrace{4}_{T}) = f(t) \quad \#$$



⑦ (i) If: $f(t) = \begin{cases} a, & 0<t<T \\ 0, & T<t<2T \end{cases}$; $f(t+2T) = f(t)$

Find $F(s)$.

ii) solve: $\ddot{y} + k^2 y = f(t)$; $y(0) = \dot{y}(0) = 0$.

Find the value of y at $t = 2T$.

solution

i) $f(t)$ is periodic fn. with period $= \boxed{2T}$

$$\therefore F(s) = \frac{1}{1-e^{-2sT}} \int_0^{2T} f(t)\, e^{-st}\, dt = \frac{1}{1-e^{-2sT}} \int_0^{T} a\, e^{-st}\, dt$$

$$= \frac{a}{1-e^{-2sT}} \left. \frac{e^{-st}}{-s} \right|_0^T = \frac{a}{s\,(1-e^{-2sT})} (\cancel{1-e^{-sT}})$$

$$\to (\cancel{1-e^{-sT}})(1+e^{-sT})$$

$$\Rightarrow F(s) = \boxed{\frac{a}{s(1+e^{-sT})}} \quad \#.$$

ii) $\ddot{y} + k^2 y = f(t) \quad ; y(0) = \dot{y}(0) = 0.$

Take L.T. for both sides

$$\Rightarrow s^2 Y(s) - \cancel{s(0)} - \cancel{0} + k^2 Y(s) = \frac{a}{s(1+e^{-sT})}.$$

$$\Rightarrow (s^2+k^2) Y(s) = \frac{a}{s(1+e^{-sT})}$$

$$\Rightarrow Y(s) = \frac{a}{s(s^2+k^2)(1+e^{-sT})}$$

$$= \frac{a}{s(s^2+k^2)}\left[1 - e^{-sT} + e^{-2sT} - \cdots\right]$$

$$\Rightarrow Y(s) = \frac{a}{s(s^2+k^2)} - \frac{a}{s(s^2+k^2)} e^{-sT} + \frac{a}{s(s^2+k^2)} e^{-2sT} - \cdots$$

$$\Rightarrow y(t) = L^{-1}\left[\frac{a}{s(s^2+k^2)} e^{0s} - \frac{a}{s(s^2+k^2)} e^{-sT} + \frac{a}{s(s^2+k^2)} e^{-2sT} - \cdots\right]$$

$\because L^{-1}\left[\frac{a}{s(s^2+k^2)}\right]$:

the factor s: $P(s) = a,\ Q(s) = s(s^2+k^2)$

$$\Rightarrow L^{-1} = \frac{P(0)}{Q'(0)} e^{0t} = \boxed{\frac{a}{k^2}}$$

, the factor (s^2+k^2): $\alpha = 0,\ \beta = k$

$$G(s) = \frac{a}{s} \Rightarrow G(ik) = \frac{a}{ik} = -\frac{a}{k} i$$

$$\Rightarrow L^{-1} = \frac{e^{0t}}{k}\left[-\frac{a}{k}\cos kt\right] = \boxed{-\frac{a}{k^2}\cos kt}$$

$$\Rightarrow L^{-1}\left[\frac{a}{s(s^2+k^2)}\right] = \frac{a}{k^2} - \frac{a}{k^2}\cos kt$$

$$= \boxed{\frac{a}{k^2}\left(1-\cos kt\right)}$$

$$\Rightarrow y(t) = \frac{a}{k^2}\left(1-\cos kt\right)H(t) - \frac{a}{k^2}\left(1-\cos k(t-T)\right)H(t-T)$$

$$+ \frac{a}{k^2}\left(1-\cos k(t-2T)\right)H(t-2T) - \cdots \quad \#$$

$$\boxed{L^{-1}\left[F(s)\,e^{-ks}\right] = f(t-k)\,H(t-k)}$$

at $t = 2T$

$$\Rightarrow H(t) = 1,\ H(t-T) = 1,$$

$$H(t-2T) = 1$$

$$,\ H(t-3T) = H(t-4T) = \cdots = \boxed{0}$$

$$\Rightarrow y(2T) = \frac{a}{k^2}\left(1-\cos 2kT\right) - \frac{a}{k^2}\left(1-\cos kT\right)$$

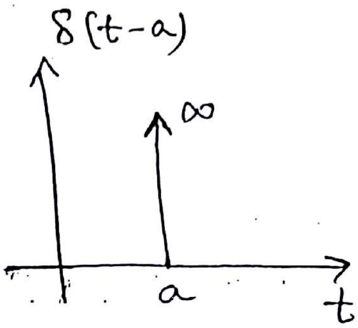
$$+ \frac{a}{k^2}\cancel{\left(1-1\right)}^{\,0}$$

$$= \boxed{\frac{a}{k^2}\left(\cos kT - \cos 2kT\right)} \quad \#$$

* Dirac delta impulse function :

$$\delta(t-a) = \begin{cases} \infty & , t=a \\ 0 & \text{othewise } (t \neq a) \end{cases}$$

$\delta(t-a)$ → Dirac delta impulse fn.



$$\left.\begin{aligned} & L[\delta(t-a)] = \boxed{e^{-as}} \\ & , L[f(t)\,\delta(t-a)] = \boxed{f(a)\, e^{-as}} \end{aligned}\right\} \text{ فقط}$$

ex: $L[\delta(t)] = 1 \quad , L[\delta(t-3)] = e^{-3s}$

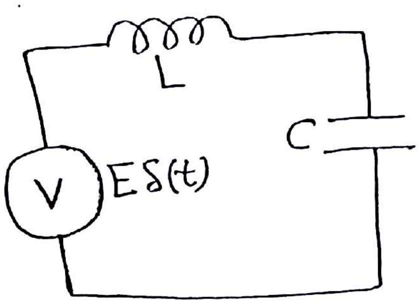
$, L[e^{-t}\delta(t-2)] = e^{-2} \cdot e^{-2s} = e^{-2(1+s)}$.

$, L[\cos 2t \; \delta(t-\pi)] = \underset{=1}{\cos 2\pi} \cdot e^{-\pi s} = e^{-\pi s}$.

لاحظ ان :

$$L^{-1}[1] = \delta(t) \; , \; L^{-1}[a] = a\,\delta(t).$$

EX : An impulsive voltage $E\delta(t)$ is applied to a circuit containing inductance L and capacitance C in series. If the current and charge are initially zero. Find an expression for the current as a function of time for this circuit.



Solution

we have 2 diff. eqn's. :

$$\because \; L\frac{di}{dt} + \frac{q}{C} = E\,\delta(t) \rightarrow (1) \quad , \quad i = \frac{dq}{dt} \rightarrow (2)$$

$$; \; i(0) = q(0) = 0.$$

Take L.T. for both sides of (1)

$$\Rightarrow L\left[s\,I(s) - \underset{i(0)}{0}\right] + \frac{Q(s)}{C} = E\,(\underset{L[\delta(t)]}{1})$$

$$\Rightarrow Ls\,I(s) + \frac{Q(s)}{C} = E \rightarrow (1)'$$

& Take L.T. for both sides of (2)

$$\Rightarrow I(s) = s\,Q(s) - \overset{q(0)}{0} \Rightarrow I(s) = s\,Q(s) \rightarrow (2)'$$

$$(2)' \Rightarrow Q(s) = \frac{I(s)}{s}$$

$$\text{in } (1)' \Rightarrow Ls\,I(s) + \frac{I(s)}{sC} = E.$$

$$\Rightarrow \left(Ls + \frac{1}{sC}\right) I(s) = E.$$

$$\Rightarrow \left(\frac{Lcs^2+1}{sC}\right) I(s) = E$$

$$\Rightarrow I(s) = \frac{ECs}{Lcs^2+1} = \frac{E\cancel{C}s}{L\cancel{C}\left(s^2+\frac{1}{LC}\right)}$$

$$\Rightarrow I(s) = \frac{Es}{L\left(s^2+\frac{1}{LC}\right)}$$

$$\Rightarrow i(t) = L^{-1}\left[\frac{Es}{L\left(s^2+\frac{1}{LC}\right)}\right] = \boxed{\frac{E}{L}\cdot\cos\left(\frac{t}{\sqrt{LC}}\right)} \;\#$$

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Problems ⑤ حل بعض مسائل من تمارين الكتاب (11)

(9) solve : $\ddot{x} + 4\dot{x} + 3x = 2\delta(t-6)$

, at $t=0$; $x=0$, $\dot{x}=2$.

solution

Take L.T. for both sides

$\Rightarrow s^2X(s) - s\cancel{x(0)} - 2 + 4[sX(s) - \cancel{0}] + 3X(s) = 2e^{-6s}$.

$\Rightarrow (s^2+4s+3)X(s) = 2 + 2e^{-6s}$.

$\Rightarrow X(s) = \dfrac{2}{s^2+4s+3} + \dfrac{2}{s^2+4s+3}e^{-6s}$.

$\Rightarrow x(t) = L^{-1}\left[\dfrac{2}{s^2+4s+3} + \dfrac{2}{s^2+4s+3}e^{-6s}\right]$

$\because L^{-1}\left[\dfrac{2}{s^2+4s+3}\right] = 2L^{-1}\left[\dfrac{1}{(s+2)^2-1}\right] = 2e^{-2t}\,sh\,t$

$\Rightarrow x(t) = 2e^{-2t}\,sh\,t + 2e^{-2(t-6)}\cdot sh(t-6)\cdot H(t-6)$. #

(10) solve : $3\dot{x} + 4\dot{y} + x - 12y = 2\,\delta(t)$, $x(0)=1, y(0)=2$.

, $2\dot{x} + 3\dot{y} + x - 6y = 0$

solution

$3\dot{x} + 4\dot{y} + x - 12y = 2\delta(t)$

Take L.T. for both sides

$\Rightarrow 3[sX(s) - 1] + 4[sY(s) - 2] + X(s) - 12Y(s) = 2$

$\Rightarrow (3s+1)X(s) + (4s-12)Y(s) = 13 \longrightarrow (1)$

& $2\dot{x} + 3\dot{y} + x - 6y = 0$

Take L.T. for both sides:

$\Rightarrow 2[sX(s) - 1] + 3[sY(s) - 2] + X(s) - 6Y(s) = 0$.

$\Rightarrow (2s+1)X(s) + (3s-6)Y(s) = 8 \rightarrow (2)$

نحذف $Y(s)$ كمايلي :

$① * (3s-6) - ② * (4s-12)$

$$\Rightarrow \left[(3s+1)(3s-6)-(2s+1)(4s-12)\right] X(s) = 13(3s-6)-8(4s-12)$$

$$\Rightarrow \left[s^2+5s+6\right] X(s) = 7s+18$$

$$\Rightarrow X(s) = \frac{7s+18}{s^2+5s+6} = \boxed{\frac{7s+18}{(s+3)(s+2)}}$$

in ②

$$\Rightarrow \frac{(2s+1)(7s+18)}{(s+3)(s+2)} + (3s-6)Y(s) = 8$$

$$\Rightarrow (3s-6)Y(s) = \frac{8(s+3)(s+2)-(2s+1)(7s+18)}{(s+3)(s+2)}$$

$$\Rightarrow Y(s) = \frac{-6s^2-3s+30}{3(s-2)(s+3)(s+2)} = \frac{-\cancel{3}(2s^2+s-10)}{\cancel{3}(s-2)(s+3)(s+2)}$$

$$= \frac{-(2s+5)\cancel{(s-2)}}{\cancel{(s-2)}(s+3)(s+2)} = \boxed{\frac{-(2s+5)}{(s+3)(s+2)}}$$

$$\therefore x(t) = L^{-1}\left[\frac{7s+18}{(s+3)(s+2)}\right] = \frac{P(-3)}{Q'(-3)}e^{-3t} + \frac{P(-2)}{Q'(-2)}e^{-2t}$$

$$= \frac{-3}{-1}e^{-3t} + \frac{4}{1}e^{-2t} = \boxed{3e^{-3t}+4e^{-2t}} \#$$

$$\& \ y(t) = L^{-1}\left[\frac{-(2s+5)}{(s+3)(s+2)}\right]$$

$$= -\left[\frac{P(-3)}{Q'(-3)}e^{-3t} + \frac{P(-2)}{Q'(-2)}e^{-2t}\right]$$

$$= -\left[\frac{-1}{-1}e^{-3t} + \frac{1}{1}e^{-2t}\right] = \boxed{-e^{-3t}-e^{-2t}} \#$$

حل المسألة (7) في دليل الطالب

The differential equation for the instantaneous charge $q(t)$ on the Capacitor in an L-R-C series Circuit is given by:

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{c}q=E(t).$$

Determine $q(t)$ when $L=1$ henry, $R=20$ ohms, $C=0.005$ farad, $E(t)=150$ volts, $t>0$ and $q(0)=0$, $i(0)=0$.

What is the current $i(t)$? what is the charge $q(t)$ if the same constant voltage is turned off for $t\geq 2$?

solution

at $L=1$, $R=20$, $C=0.005$, $E(t)=150$

$$\Rightarrow \frac{d^2q}{dt^2}+20\frac{dq}{dt}+200q=150$$

Take L.T. for both sides

$$\Rightarrow s^2Q(s)-\cancel{s(0)}-\cancel{0}+20\left(sQ(s)-0\right)+200Q(s)=\frac{150}{s}$$

(Note that: $i=\frac{dq}{dt}=q' \Rightarrow q'(0)=i(0)=0$).

$$\Rightarrow (s^2+20s+200)\,Q(s)=\frac{150}{s}$$

$$\Rightarrow Q(s)=\frac{150}{s(s^2+20s+200)} \Rightarrow q(t)=150\,L^{-1}\left[\frac{1}{s(s^2+20s+200)}\right]$$

$(s+10)^2+100$

The factor s: $P(s)=1$, $Q(s)=s(s^2+20s+200)$

$$\Rightarrow L^{-1}=\frac{P(0)}{Q'(0)}e^{0t}=\boxed{\frac{1}{200}}$$

The factor $(s^2+20s+200)$: $(s+10)^2+100 \Rightarrow \alpha=-10,\ \beta=10$

$$G(s)=\frac{1}{s}\Rightarrow G(-10+10i)=\frac{1}{-10+10i}=\frac{1}{10(-1+i)}\cdot\frac{(-1-i)}{(-1-i)}$$

$$=\frac{1}{10}\frac{(-1-i)}{(2)}=-\frac{1}{20}-\frac{1}{20}i$$

(14)

$$\Rightarrow \mathcal{L}^{-1} = \frac{e^{-10t}}{10}\left[\frac{-1}{20}\cos 10t - \frac{1}{20}\sin 10t\right]$$

$$\Rightarrow q(t) = 150\left[\frac{1}{200} + \frac{e^{-10t}}{10}\left(-\frac{1}{20}\cos 10t - \frac{1}{20}\sin 10t\right)\right]$$

$$q(t) = \frac{3}{4}\left[1 - e^{-10t}\left(\cos 10t + \sin 10t\right)\right] \#$$

$$\therefore i(t) = \frac{dq}{dt} = q'(t) = -\frac{3}{4}\left[-10e^{-10t}(\cancel{\cos 10t} + \sin 10t) + e^{-10t}(-10\sin 10t + \cancel{10\cos 10t})\right]$$

$$= \boxed{15e^{-10t}\sin 10t} \#$$

When $E(t) = \begin{cases} 150 & 0 < t < 2 \\ 0, & t \geq 2 \end{cases}$

turned off for $t \geq 2$

$$= 150\left[H(t) - H(t-2)\right]$$

$$= 150H(t) - 150H(t-2).$$

$\Rightarrow$ The differential eqn. becomes:

$$\frac{d^2q}{dt^2} + 20\frac{dq}{dt} + 200q = 150H(t) - 150H(t-2)$$

Take L.T. for both sides

$$\Rightarrow s^2Q(s) - s(0) - 0 + 20\left(sQ(s) - 0\right) + 200Q(s) = \frac{150}{s} - 150\frac{e^{-2s}}{s}$$

$$\Rightarrow (s^2 + 20s + 200)Q(s) = \frac{150}{s} - \frac{150}{s}e^{-2s}$$

$$\Rightarrow Q(s) = \frac{150}{s(s^2+20s+200)} - \frac{150\,e^{-2s}}{s(s^2+20s+200)}$$

كما سبق

$$\Rightarrow q(t) = \frac{3}{4}\left[1 - e^{-10t}(\cos 10t + \sin 10t)\right] - \frac{3}{4}\left[1 - e^{-10(t-2)}\left(\cos 10(t-2) + \sin 10(t-2)\right)\right]H(t-2). \#$$