Ain Shams University
Faculty of Engineering
Eng.Physics & Math.Departement

Total mark (70)

Engineering Math (PHM 663)
Fall 2023-2024
Allowed Time: 3 Hrs.

The exam is composed of 6 questions in one page.

Part (1): Answer TWO questions only (Each question of 20 marks)

1) a) Show that the function: $u(x, y) = 2x + 3y + \sin xchy$ is harmonic and find the function $v(x, y)$, such that $f(z) = u + iv$, is an analytic function. Find $f'(z)$. 10

b) Find all values of z such that: 5 $i)\ e^{3z-3} = 6 + 6i$, $ii)\ z = (4 - 4i)^{3i}$ 5

2) a) Find all Laurent series that represent the function $f(z) = \frac{13z-44}{z^2-3z-10}$ in different domains. 10

b) Evaluate the following integrals:

$i)\ \oint \frac{z+1}{z\,(z-i)^2}\, dz$; where C is a. $|z| = \frac{1}{2}$ $2\pi i$ b. $|z - i| = \frac{1}{2}$ $2\pi i$?! c. $|z| = 3$

$ii)\ \oint_c (2z^3 + 3z^2 + 12)\, e^{\frac{1}{z}}\, dz$ where c is $|z| = \frac{1}{2}$

$iii)\ \int_0^\infty \frac{dx}{(x^2+1)\,(x^2+4)}$ $\frac{-12\pi}{25}$

3) a) Under the mapping $W = \frac{1}{z}$, Find and Sketch the image of $i)\ x^2 + y^2 - 2y = 0$, $ii)\ y = 2x$

Discuss how the point (0,0) exchange with $\pm\infty$.

b) $i)$ If $G(z_o) = \oint_C \frac{z^4+3z^2+5}{(z-z_o)^2}\, dz$ where c is the circle $|z| = 4$ Find a) $G(1 + i)$, b) $G(1 - 5i)$

$ii)\ \int_{-\infty}^{\infty} \frac{dx}{(x^2+1)\,(x^2+9)}$ $x^4 + 10x^2 + 9$ $x = -x$ $x^4 + 10x^2 + 9$ $F(-x) = -F(x)$

$iii)\ \oint_C (z^4 + 3z^2 + 6z + 10)\sin\frac{1}{z}\, dz$

Part (2): Answer TWO questions only (Each question of 20 marks)

4) a) Evaluate the following integral $\int_0^\infty x^{\frac{-5}{2}}\,(2 - e^{-3x})dx$. (Hint: In the first step use integration by parts)

b) Sketch the function: $f(t) = \begin{cases} \cos(4t) & ; 0 \le t \le \pi \\ 0 & t \ge \pi \end{cases}$ and find it's **LaPlace Transform**.

c) Solve the integral equation: $y' = 1 - \sin(t) - \int_0^t y(u)du$, given that: $y(0) = 1$.

5) a) Find the arc length of the curve: $r = a[1 + \cos(\theta)]$, where $dL = \sqrt{r^2+r'^2}\, d\theta$, $r' = \frac{dr}{d\theta}$ (Hint: use Beta function) 20

b) Find and sketch the function: $f(t) = L^{-1}\left[\frac{3}{s} - \frac{4e^{-s}}{s^2} + \frac{4e^{-3s}}{s^2}\right]$.

c) Sketch and find LaPlace Transform of the periodic function $f(t) = \begin{cases} \sin(t) & ; \quad 0 < t < \pi \\ 0 & ; \quad \pi < t < 2\pi \end{cases}$, $f(t + 2\pi) = f(t)$

6) a) Find the value of $\Gamma\left(\frac{5}{3}\right)\Gamma\left(\frac{-5}{3}\right)$. ?

b) Solve the IVP: $y'' - 2y' + 4y = \cos(t)$, given that: $y(0) = 0$ & $y'(0) = 0$, using **LaPlace Transform.**

c) Find the series solution of: $(x^2 + 1)y'' + xy' - y = 0$, near the ordinary point $x = 0$.

GOOD LUCK