Ain Shams University
Faculty of Engineering
Eng. Physics & Math. Department

Total mark (70)

Engineering Math (PHM 663)
Fall 2022-2023
Allowed Time: 2 Hrs.

The exam is composed of 6 questions in one page (The mark of each question is (20) marks)

Part (1): Answer two questions only

1) a) Show that the function: $u(x,y) = \sin 2x \cosh 2y + 6x - 4y + 5$ is harmonic and find it's corresponding analytic function $f(z) = u + iv$. Find $f'(z)$. 12

b) Find all values of z such that: i) $e^{z-1} = ie^3$ 4 and ii) $z = (1+i)^{(1+i)}$ 4

2) a) Find all Laurent series for the function: $f(z) = \frac{5z+6}{z^2+z-12}$ in the domains. 9

i) $|z| < 3$ ii) $3 < |z| < 4$ iii) $|z| > 4$

b) Evaluate the following integrals: i) $\oint_C \frac{1}{(z-2)^2(z-4)}\,dz$, where C: 3 2

First. The rectangle defined by $x = 1, x = 6, y = -2, y = 2$. Second. The circle $|z| = 3$.

ii) $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+9)}$ 6

3)a) Find all values of z such that (i) $\cos 2z = 20i$ and (ii) $(1-i)^{2i}$. 4 4

b) Evaluate the following integrals: i) $\oint_c (2z^3 + 5z^2 + 4)\, e^{\frac{2}{z}} dz$ $c: |z| = 1$,ii) $\oint_c \frac{e^{2z}}{z^3+2z^2} dz$ $c: |z| = 3$ 4 4

iii) If c is the circle $|z| = 5$ and $g(z_0) = \oint \frac{z^3 + 2z^2 + 6}{(z-z_0)^2} dz$ Find a) $g(2i)$ b) $g(4+5i)$ 4

Part (2) : Answer two questions only

4)a) Evaluate the following integral: $\int_0^1 \frac{dx}{\sqrt{-\ln(x)}}$

b) Find the series solution of : $2y'' + xy' - 4y = 0$, near $x = 0$.

c) Evaluate the following integral : $\int_0^\infty \frac{dx}{\sqrt{x} + x^{3/2}}$, use substitution $t = \frac{1}{1+x}$

5)a) Use LaPlace transform to solve the IVP : $ty'' - y' = t^2$, $y(0) = 0$

b) Evaluate the following integral : $\int_0^\infty \frac{\sin(5t)}{te^{5t}}\, dt$, use LaPlace transform.

c) Sketch the graph of the function: $f(t) = \begin{cases} \sin(t) & ; 0 < t < 2\pi \\ 0 & ; \pi < t < 2\pi \end{cases}$, period 2π.

6)a) Find $f(t) = L^{-1}\left(\frac{1-e^{-2s}+2e^{-3s}-2e^{-5s}}{s^2}\right)$ and sketch the function $f(t)$.

b) Solve the integral equation: $\int_0^t \frac{y(u)}{\sqrt{t-u}}\, du = 1 + t + t^2$.

End of Exam

Good Luck

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<u>Part (1): Answer two questions only</u>

1) a)Show that the function: $u(x,y) = e^{2x}\cos 2y + 6x - 3y + 15$ is harmonic and find it's corresponding analytic function $f(z) = u + iv.$ $Find\ f'(z).$

b) Find all values of z such that: i) $e^{2z+3i} = 10i$ *and* ii) $z = (3-3i)^{2i}$

2)a) Find all Laurent series for the function : $f(z) = \frac{8z-6}{z^2-4z-21}$ in different domains.

b) Evaluate the following integrals: $(i)\ \oint_C \frac{\cosh(2z)}{z^3-2z^2}\,dz, where\ C: |z| = 3$ *and* $(ii)\ \int_{-\infty}^{\infty}\frac{dx}{(x^2+9)(x^2+4)}$.

c) If $(z_0) = \oint_C \frac{z^2+3z}{(z-z_0)^2}\,dz$, find $H(2i) and\ H(5+i)$, where $C: |z| = 4.$

3)a)Find the image of $(i)\ y = 2x$ *and* $(ii)\ x^2 + y^2 + 2x = 0$ under the transformation $w = \frac{1}{z}$.

Discuss the details of your work.

b)Evaluate the following integrals: i)$\oint_{|z|=1}(2z^3 + 3z^2 + 1)\,e^{\frac{3}{z}}dz$, ii) $\int_0^{\infty}\frac{dx}{(x^2+4)^2}$ *and* iii) $\int_0^{2\pi}\frac{d\theta}{5+4\cos(\theta)}$

<u>Part (2) : Answer two questions only</u>

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c) Evaluate the following integral : $\int_0^{\infty}\frac{dx}{\sqrt{x} + x^{3/2}}$, use substitution $t = \frac{1}{1+x}$

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