Series solution Sec 2

[1] ordinary point

$\because x_0=0$

exp1: Find a series solutions in powers of (x). $(x_0=0)$

① $y'' - 2xy' - 8y = 0$ Compare with $y'' + p(x)y' + q(x)y = 0$

$p(x) = -2x$, $q(x) = -8$

at $x_0 \Rightarrow p, q$ are defined

① $\therefore x_0 = 0$ is ordinary point

② let $y = \sum_{n=0}^{\infty} a_n x^n$ is a solution

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

③ Sub in equation:

$$\sum_{n=2} n(n-1) a_n x^{n-2} - \cancel{2x} \sum_{n=1} n a_n x^{n-1} - 8 \sum_{n=0} a_n x^n = 0$$

($+2$; $x^n \cancel{x^{-1}}$)

$$\sum_{\substack{n+2=2 \\ n=0}} (n+2)(n+1) a_{n+2} x^n - \sum_{n=0} 2n a_n x^n - \sum_{n=0} 8 a_n x^n = 0$$

at $n=0$ the coeff $=0$ اكني مزودتش حاجة

$$\Rightarrow \sum_{n=0}^{\infty} [(n+2)(n+1) a_{n+2} - 2n a_n - 8 a_n] x^n = 0$$

$\rightarrow$ Coef$(x^n) = 0$

thus $(n+2)(n+1) a_{n+2} - (2n+8) a_n = 0$

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$$a_{n+2} = \frac{2(n+4)}{(n+2)(n+1)} a_n \quad , n \geq 0$$

reccurence relation

NB: you need to put the Rec. relation in the simplest form possible so that it can be easily used in the general form later.

$K=1$ $\qquad\qquad\qquad\qquad$ $K=3$

(5) $n=0$; $a_2 = \frac{2(4)}{2(1)} a_0$ $\qquad$ $n=4 \Rightarrow a_6 = \frac{2\times 8}{6\times 5} a_4$

$n=2$; $a_4 = \frac{2(6)}{(4)(3)} a_2$ $\qquad$ $a_6 = \frac{2^3 (4,6,8)}{(2,4,6)(1,3,5)}$

$K=2$

$= \frac{2(6)\times 2(4)}{(4)(3)(2)} a_0$

→ we need 3 terms ∴ Sub with $n = 4, 1, 3, 5$.

for even values of n. $\qquad$ $(0K + 0)$

الفرق

$$a_{2K} = \frac{2^K [4.6.8\ldots(2K+2)]}{(2K)!} a_0$$

for odd values of n.

$$a_{2K+1} = \frac{2^K [5,7.9\ldots(2K+3)]}{(2K+1)!} a_1 \quad K \geq 1$$

the trick is to be able to accomplish all the steps

(6) $y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$= a_0 + \sum_{k=1} a_{2k} x^{2k}$

$+ a_1 x + \sum_{k=1} a_{2k+1} x^{2k+1}$

$y = a_0 \left[1 + \sum_{k=1}^{\infty} \frac{2^k [4.6.8..(2k+2)}{(2k)!} x^{2k} \right]$

$+ a_1 \left[1 + \sum_{k=1}^{\infty} \frac{2^k [5.7.9.(2k+3)}{(2k+1)!} x^{2k+1} \right.$

$= c_1 y_1 + c_2 y_2$

Exp 2: $(1 - x^2) y'' - 2xy' + 6y = 0$

$p(x) = \frac{-2x}{1-x^2}$, $q(x) = \frac{6}{1-x^2}$

(1) $x_0 = 0$, p, q are defined $\therefore$ x_0 is O.P

(2) let $y = \sum_{n=0}^{\infty} a_n x^n$ is a solution

$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$

$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$

same steps for OP.

(3) sub in equation

NB: don't leave the original equation with a denominator

$(1-x^2) \sum_{n=2} n(n-1) a_n x^{n-2} - 2x$

$\sum_{n=1} n a_n x^n + 6 \sum_{n=0} a_n x^n = 0$

at $n=1$ sum $=0$ ∴ you can start
$n=0$ sum $=0$ from zero without error

$$\Rightarrow \sum_{n=2} n(n-1)a_n x^{n-2} - \sum_{n=2} n(n-1)a_n X^n + \sum_{n=0} 6a_n X^n = 0$$

$$\Rightarrow \sum_{n=0} (n+2)(n+1)a_{n+2} X^n - \sum_{n=0} n(n-1)a_n X^n - \sum_{n=0} 2n\, a_n X^n$$

$$+ \sum_{n=0} 6a_n X^n = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} \left[(n+2)(n+1)a_{n+2} - n(n-1)a_n - 2na_n + 6a_n\right] X^n = 0$$

(4) $\Rightarrow$ Cof $(X^n) = 0$

$$(n+2)(n+1)a_{n+2} - [n(n-1) + 2n - 6]a_n = 0$$

$$a_{n+2} = \frac{n^2+n-6}{(n+2)(n+1)} a_n$$

$$a_{n+2} = \frac{(n+3)(n-2)}{(n+2)(n+1)} a_n \quad , n \geq 0$$

(5) $n=1$; $a_3 = \frac{(4)(-1)}{(3)(2)} a_1$

$n=3$; $a_5 = \frac{(6)(1)}{(5)(4)} a_3 = \frac{(4)(6)(-1)(1)}{(3.5)(2.4)} a_1$

$n=5 \Rightarrow a_7 = \frac{(8)(3)}{(7)(6)} a_5$

$[-1,1,3\ldots[2K-3]]$

$$a_7 = \frac{4.6.8(-1,1,3)}{(3,5,7)(2.4.6)} a_1 \qquad a_{2K+1} = \frac{[4.6.8\ldots(2K+2)]}{(2K+1)!} a_1$$

$K \geq 1$

for even values

$n=0 . \quad a_2 = \frac{3 \times -2}{2 \times 1} a_0$

$n=2 \Rightarrow a_4 = 0 \; a_2$

$n=4 \Rightarrow a_6 = 0 \; a_4 = 0 =$ zero

$a_4 = a_6 = a_8 = a_{2k} = 0$

$y = \sum_{n=0}^{\infty} a_n x^n$

$= a_0 + a_2 x^2$

$+ a_1 x + \sum_{k=1}^{\infty} a_{2k+1} x^{2k+1}$

don't forget these terms; they'll always exist.

$y = a_0 [1 - 3x^2]$