Section 10 : Numerical analysis part :

exact method (مش علينا في الفاينال)

$y' = x\sqrt{y}$

$y(2) = 1$

$y(3) = ?$

$y = \frac{1}{16} x^4 \Rightarrow y(3) = 5.06$

Euler method

$y_{n+1} = y_n + \Delta y$

$\Delta y = hf(x,y)$

① $x_0 = ✓$ $y_0 = ✓$, $y_n = ?$ $x_n = ✓$

② $y' = f(x,y)$

③ h: step size $\Rightarrow h = \frac{x_n - x_0}{n}$

④ n: no of steps $\Rightarrow n = \frac{x_n - x_0}{h}$

x = 2 ... x = 3 step = h

n	x_n	y_n	$\Delta y = hf = 0.2 \times \sqrt{3}$	y_{n+1}
0	2	1	$0.2 \times 2\sqrt{1} = 0.4$	1.4 = (0.4 + 1)
1	+h 2.2	1.4	$0.2(2.2)\sqrt{1.4} = 0.5206$	0.5206 + 1.4
2	+h 2.4	✓	✓	✓✓
3	2.6	✓✓	✓✓✓	✓✓✓
4	2.8			
5	3	(—)	don't need	

↳ the answer

$y(3) \simeq y_5 = 4.4579$

$\text{error} = |y(3) - y_5| = \left|\frac{1}{16}(3)^4 - 4.4579\right|$

* If we need to calculate the error she will give us the exact solution
* easy approximation, less accuracy.
* you pick your error depending on your application

* we can increase accuracy by more ~~higher~~ steps or lower step size

exp : h = 0.2 ; error ≃ 0.61) ÷2
h = 0.1) ÷ ; error ≃ 0.33

error = O(h) i.e proportional to h

* or increase accuracy using modified euler / Improve or Runge-Kutta 4 ∝ h^4