AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING
SPECIALIZED ENGINEERING PROGRAMS
JUNIOR COMMUNICATION ENGINEERING PROGRAM

SPRING 2022	Assignment #4	Total: 5 marks

PHM212s: Special Functions, Complex Analysis & Numerical Analysis

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Name:	**ID:**	**Deadline: Week 11**

Please, Solve each problem in its assigned place ONLY (the empty space below it)

Functions of the complex variable

1. Find the real and imaginary parts of each of the following functions and state its domain
   a) $f(z) = z^2$

   b) $f(z) = e^{z^2}$

   c) $f(z) = \sin z$

   d) $f(z) = 2z^3 - 3z$

   e) $f(z) = \dfrac{(z+i)}{(z^2+1)}$

2. Describe the image of the following regions under the specified function (Show the regions graphically)

   a) $\mathrm{Re}(z) \geq 0$ for $f(z) = z + 5$

   b) $|\arg(z)| \leq \pi/4$, $|z| \leq 2$ for $f(z) = z^3$

   c) $|z| \geq 3$ for $f(z) = z^2$

3. Find the region into which the half plane $y > 0$ is mapped by the function $w = (1+i)\,z$. Show the regions graphically.

4. Find the image of the semi – infinite strip $x > 0$, $0 < y < 2$ under the function $w = i\,z + 1$. Show the regions graphically.

5. Find a linear transformation that maps the half plane $\mathrm{Im}(z) > 0$ into the region $\mathrm{Re}(w) > 1$.

6. Show that under the transformation $w = \frac{1}{z}$ circles or straight lines are mapped into circles or straight lines.

7. Under the Reciprocal transformation, find the image of the following regions & show the regions (The pre-image & The image) graphically.
   a) The infinite strip $0 < y < 1/2c$

b) The circle $|z-2|=1$

c) The circle $|z-2|=2$

d) The line $x + y = 0$

e) The line $x + y = 1$

8. Find the image of the region $\alpha < \arg(z) < \alpha + 2\pi/3$ under the function $w = z^3$.

9. Show that the imaginary axis $x = 0$ is mapped into the imaginary axis $u = 0$ under the mapping $w = \sin z$.

10. Find a transformation that maps the region $0 < \arg(z) < \pi/4$ into the region $\mathrm{Re}(w) > 1$.

11. Using Cauchy – Riemann's equations, determine where the following functions are differentiable and where they are analytic? If the function is differentiable, find its derivatives.

a) $w = \overline{z}$

b) $w = \cos z$

c) $w = \ln z$

d) $w = 2y - i\ x$

e) $w = e^{x}(\cos y + i \sin y)$

f) $w = z^{2}\ \bar{z}$

g) $w = e^{y}(\cos x + i \sin x)$

h) $w = 1/z$

i) $w = x^3 + 3xy^2 - 3x + i\,(y^3 + 3x^2 y - 3y)$

12. Prove that the function $f(z) = 3x^2 + 2x - 3y^2 - 1 + i\,(6xy + 2y)$ is entire. Hence, write this function in terms of z.

13. Prove that the function $f(z) = e^{x^2 - y^2}(\cos 2xy + i \sin 2xy)$ is entire and find its derivative in terms of z.

14. Prove that an analytic function $f(z)$ must be constant if any one of the following conditions hold:

a) $\mathbf{Re}(f(z)) =$ Constant.

b) $\mathbf{Im}(f(z)) =$ Constant.

c) $|f(z)| =$ Constant.

d) $\overline{f(z)}$ is analytic.

e) $|f(z)|$ is analytic.

15. Show that each of the following functions is harmonic and find a corresponding analytics function $f(z) = u + i\,v$:

a) $u = 2\,x\,(1 - y)$

b) $u = \cos x \ \cosh y$

c) $v = y/(x^2 + y^2)$

d) $v = x^3 - 3\ x\ y^2 + \ y$

e) $u = \arg(z)$

16. Prove that if $f(z) = u + i\,v$ is analytic in some domain **D**, then both ***u*** and ***v*** are harmonic functions in the same domain **D**.

17. Prove that if ***v*** is a harmonic conjugate of ***u*** in some domain **D**, then ***uv*** is harmonic in the same domain **D** .

18. Let $f(z)$ be analytic and non – zero in a domain **D**. Prove that $\ln|f(z)|$ is harmonic in **D**.

19. Find all the roots of the following equations:

    a) $\cos z = 2$

b) $\sin z = \cosh 4$

c) $\cosh z = 0$

20. Find all the values of

a) $\ln 1$

b) $\ln(-1)$

c) $\ln i$

d) $\ln(-e\,i)$

e) $\ln(1-i)$

f) $\ln(-3+i\sqrt{27})$

g) $(1+i)^{i}$

h) $(-1)^{1/\pi}$

i) $\left(\frac{e}{2}\left(-1-i\sqrt{3}\right)\right)^{3\pi i}$

Best wishes,

Dr. Makram Roshdy, Dr. Betty Nagy