

SPRING 2022

Assignment #5

Total: 5 marks

**PHM212s: Special Functions, Complex Analysis & Numerical Analysis**

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**Name:**

**ID:**

**Deadline: Week 14**

*Please, Solve each problem in its assigned place ONLY (the empty space below it)*

**Numerical solution of Ordinary Differential Equations**

**Use at least 4 decimal places in your calculations**

1)  $y' = x + y$ ,  $y(0) = 0$ . Find  $y(0.4)$  using the following:

a) Exact Method

b) Euler Method with  $h = 0.1$

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c) Euler Method with 10 steps

d) Runge-Kutta Method with  $h = 0.4$

e) Runge-Kutta method with 2 steps.

2)  $y' = x - y$ ,  $y(1) = 2$ . Find  $y(0.5)$  using Runge-Kutta method with 2 steps.

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3)  $x' = x - y - t, y' = 4x - 2y, x(0) = 1$  &  $y(0) = 0$

Find  $x(0.2)$  &  $y(0.2)$  using Runge-Kutta method with  $h=0.1$

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4)  $x'' + t^2 x' + 3x = t$ ,  $x(0) = 1$  &  $x'(0) = 2$

Find  $x(0.2)$  using Runge-Kutta method with  $h=0.1$

**Numerical solution of Partial Differential Equations**

5) Find  $U(x, y)$  such that  $\nabla^2 U(x, y) = 0$  over a rectangle  $20 \times 15$  cm using a grid with step size  $h = 5$ , and the boundary conditions:  $U(x, 0) = 0$ ,  $U(x, 15) = 0$ ,  $U(0, y) = 0$ ,  $U(20, y) = 100$ . Use Gauss-Seidel method to solve the resulting linear system. **Accurate to 2D**

6) Solve (using  $h = 1/3$ ) the Dirichlet problem

$$\nabla^2 u(x, y) = 3(x^2 + y^2) \quad \text{in } R$$

$$\text{and } u(x, y) = y - x \quad \text{on } \partial R$$

Here  $\partial R$  is the boundary of  $R$  and  $R$  is the region in the unit square  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$ . Perform 5 steps of Gauss-Seidel method with the initial approximation  $u_{11}^{(0)} = u_{12}^{(0)} = u_{21}^{(0)} = u_{22}^{(0)} = 0$ .