Technical University of Denmark

31792 Advanced Optimization and Game Theory for Energy Systems

Report

Strategic energy planning

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1. Benders Descomposition

The chosen problem under uncertainty can be formulated as a two-stage SP model. One chooses the first-stage investment decision variables or complicated variables ($\mathbf{F}$, $\%\mathbf{DHN}$) before the realization of uncertain parameters minimizing the associated investment cost plus the expected second-stage cost that depends on the recourse operation variables. The second-stage variables adapt optimally to the revealed uncertainty.

The single-cut version of the bender decomposition formulates the master problem in the following way:

$$
\begin{aligned}
&\min \quad \sum_{j \in T} (\tau_j c_j^{inv} + c_j^{maint}) \mathbf{F}_j + \theta \\
&\text{s.t.} \\
&\mathbf{F}_{DHN} = \sum_{j \in T - EUT(Heat_{lowTdhn})} \mathbf{F}_j \\
&\mathbf{F}_{Grid} = 1 + \frac{9400}{c_{Grid}^{inv}} \frac{\mathbf{F}_{Wind} + \mathbf{F}_{PV}}{\bar{f}_{Wind} + \bar{f}_{PV}} \\
&\mathbf{F}_{EFFICIENCY} = \frac{1}{1 + i_{rate}} \\
&\mathbf{F}_{PowerToGas^*} \geq \mathbf{F}_{PowerToGas} \\
&\mathbf{F}_{PowerToGas^*} \geq \mathbf{F}_{GasToPower} \\
&\mathbf{F}_{NUCLEAR} = 0 \\
&\%dhn_{min} \leq \%\mathbf{DHN} \leq \%dhn_{max} \\
&\mathbf{F}_{StoHydro} \leq \bar{f}_{StoHydro} \frac{\mathbf{F}_{NewHydroDam} - \underline{f}_{NewHydroDam}}{\bar{f}_{NewHydroDam} - \underline{f}_{NewHydroDam}} \\
&\underline{f}_j \leq \mathbf{F}_j && \forall j \in T \\
&\mathbf{F}_j \leq \bar{f}_j && \forall j \in T \\
&\theta >= -M \\
&\theta >= \sum_{\xi \in \Omega} p_\xi Q(\bar{F}^k, \overline{\%DHN}^k, \xi) + \sum_{\xi \in \Omega} p_\xi \sum_{j \in T} \alpha_{j,\xi}^k (\mathbf{F}_j - \bar{F}_j^k) + \sum_{\xi \in \Omega} p_\xi \beta_\xi^k (\%\mathbf{DHN} - \overline{\%DHN}^k) \forall k \in K
\end{aligned}
\tag{1}
$$

while the multi-cut version of bender is represented as follows

$$
\begin{aligned}
&\min \quad \sum_{j \in T} (\tau_j c_j^{inv} + c_j^{maint}) \mathbf{F}_j + \sum_{\xi} p_\xi \theta_\xi \\
&\text{s.t.} \\
&\mathbf{F}_{DHN} = \sum_{j \in T - EUT(Heat_{lowTdhn})} \mathbf{F}_j \\
&\mathbf{F}_{Grid} = 1 + \frac{9400}{c_{Grid}^{inv}} \frac{\mathbf{F}_{Wind} + \mathbf{F}_{PV}}{\bar{f}_{Wind} + \bar{f}_{PV}} \\
&\mathbf{F}_{EFFICIENCY} = \frac{1}{1 + i_{rate}} \\
&\mathbf{F}_{PowerToGas^*} \geq \mathbf{F}_{PowerToGas} \\
&\mathbf{F}_{PowerToGas^*} \geq \mathbf{F}_{GasToPower} \\
&\mathbf{F}_{NUCLEAR} = 0 \\
&\%dhn_{min} \leq \%\mathbf{DHN} \leq \%dhn_{max} \\
&\mathbf{F}_{StoHydro} \leq \bar{f}_{StoHydro} \frac{\mathbf{F}_{NewHydroDam} - \underline{f}_{NewHydroDam}}{\bar{f}_{NewHydroDam} - \underline{f}_{NewHydroDam}} \\
&\underline{f}_j \leq \mathbf{F}_j && \forall j \in T \\
&\mathbf{F}_j \leq \bar{f}_j && \forall j \in T \\
&\theta_\xi >= -M && \forall \xi \in \Omega \\
&\theta_\xi >= p_\xi Q(\bar{F}^k, \overline{\%DHN}^k, \xi) + p_\xi \sum_{j \in T} \alpha_{\xi,j}^k (\mathbf{F}_j - \bar{F}_j^k) + p_\xi \beta_\xi^k (\%\mathbf{DHN} - \overline{\%DHN}^k) \forall k \in K, \forall \xi \in \Omega
\end{aligned}
\tag{2}
$$

The recourse function $Q(\bar{F}^k, \overline{\%DHN}^k, \xi)$ dependson the first-stage decision $\bar{F}^k, \overline{\%DHN}^k$ and the parameters $\xi \in \Omega$.

$$
\begin{aligned}
Q(\cdot) = \ \min \quad & \sum_{j\in R}\sum_{m\in M} {c^{op}}_{j,m,\xi}\mathbf{Ft}_{j,m,\xi}h_m + 5000\sum_{j\in L}\sum_{m\in M}\mathbf{w}_{j,m,\xi} \\
\text{s.t.} \quad & \\
& \mathbf{Ft}_{j,m,\xi} \le \mathbf{F}_{j,\xi}\widehat{k}_{j,m,\xi} \quad \forall j\in T, m\in M \\
& \sum_{m\in M}\mathbf{Ft}_{j,m,\xi}h_m \le \mathbf{F}_{j,\xi}k_{j,\xi}\sum_{m\in M}h_m \quad \forall j\in T \\
& \mathbf{Ft}_{j,m,\xi} = \mathbf{Ft}_{j,m-1,\xi} + h_m\left(\sum_{\substack{l\in L\\ \eta^-_{j,l}>0}}\mathbf{Sto}^-_{j,l,m,\xi}\eta^-_{j,l} - \sum_{\substack{l\in L\\ \eta^+_{j,l}>0}}\mathbf{Sto}^+_{j,l,m,\xi}/\eta^+_{j,l}\right) \forall j\in Sto, m\in M \\
& \sum_{j'\in T-EUT(eut)} f^{min\,\%}_j\sum_{m\in M}\mathbf{Ft}_{j',m,\xi}h_m \le \sum_{m\in M}\mathbf{Ft}_{j,m,\xi}h_m \forall eut\in EUT, j\in T\text{-}EUT(eut) \\
& \sum_{j'\in T-EUT(eut)} f^{max\,\%}_j\sum_{m\in M}\mathbf{Ft}_{j',m,\xi}h_m \ge \sum_{m\in M}\mathbf{Ft}_{j,m,\xi}h_m \forall eut\in EUT, j\in T\text{-}EUT(eut) \\
& \mathbf{Sto}^-_{StoHydro,Elec,m,\xi} \le \mathbf{Ft}_{HydroDam,m,\xi} + \mathbf{Ft}_{NewHydroDam,m,\xi} \quad \forall m\in M \\
& \mathbf{MAX}^{DHN}_\xi \ge \mathbf{EndUses}_{Heat_{lowTdhn},m,\xi} + \mathbf{Loss}_{Heat_{lowTdhn},m,\xi} - \mathbf{w}_{Heat_{lowTdhn},m,\xi} \quad \forall m\in M \\
& \sum_{j\in T-EUT(Heat_{lowTdhn})}\mathbf{F}^k_{j,\xi} \ge \%peak_{DHN}\mathbf{MAX}^{DHN}_\xi \\
& \sum_{i\in R\cup T\setminus Sto} f_{i,l}\mathbf{Ft}_{i,m,\xi} + \sum_{\substack{j\in Sto\\ \eta^+_{j,l}>0}}(\mathbf{Sto}^+_{j,l,m,\xi} - \mathbf{Sto}^-_{j,l,m,\xi}) - \mathbf{EndUses}_{l,m,\xi} - a_l\mathbf{Loss}_{l,m,\xi} + \mathbf{w}_{l,t,\xi} = 0 \forall l,m \\
& \sum_{m\in M}\mathbf{Ft}_{i,m,\xi}h_m \le avail_{i,\xi} \quad \forall i\in R \\
& \mathbf{w}_{l,m,\xi} = 0 \quad \forall l\in L\setminus EUT, \forall m\in M \\
& \mathbf{F}_{j,\xi} = \bar{F}^k_j \quad \forall j\in T \quad : \alpha^k_{j,\xi} \\
& \%\mathbf{DHN}_\xi = \overline{\%DHN}^k \quad : \beta^k_\xi
\end{aligned} \tag{3}
$$

End-uses demand, Loss calculation constraints
$\forall l \in L, m \in M, \xi \in \Omega$ $\mathbf{EndUses}_{l,m\xi} =$ if l = *Electricity* then $\dfrac{eUI_{Electricity,\xi}}{\sum\limits_{m' \in M} h_{m'}} + eUI_{Lighting,\xi}\dfrac{\%lighting_m}{h_m} + \mathbf{Loss}_{Electricity,m,\xi}$ elseif l = $Heat_{lowTdhn}$ (centralized) then $\left(\dfrac{eUI_{Heat_{lowThw},\xi}}{\sum\limits_{m' \in M} h_{m'}} + \dfrac{eUI_{Heat_{lowTsh},\xi}\%sh_m}{h_m}\right)\%\mathbf{DHN}_\xi$ elseif l = $Heat_{lowTdecen}$ (decentralized) then $\left(\dfrac{eUI_{Heat_{lowThw},\xi}}{\sum\limits_{m' \in M} h_{m'}} + \dfrac{eUI_{Heat_{lowTsh},\xi}\%sh_m}{h_m}\right)(1 - \%\mathbf{DHN}_\xi)$ elseif l = $Heat_{highT}$ (industrial) then $\left(\dfrac{eUI_{Heat_{highT},\xi}}{\sum\limits_{m' \in M} h_{m'}}\right)$ else then   0   end
$\mathbf{Loss}_{eut,m,\xi} = \sum\limits_{i \in R \cup T \setminus Sto: f_{i,eut}>0} f_{i,eut}\mathbf{Ft}_{i,m,\xi}\,\%loss_{eut,\xi} \quad \forall eut \in EUT, m \in M, \xi \in \Omega$
$\forall \xi \in \Omega$ $eUI_{Electricity,\xi} = eUYear_{Electricity,HOUSEHOLDS,\xi} + eUYear_{Electricity,SERVICES,\xi} + eUYear_{Electricity,INDUSTRY,\xi}$ $eUI_{Lighting,\xi} = eUYear_{Lighting,HOUSEHOLDS,\xi} + eUYear_{Lighting,SERVICES,\xi} + eUYear_{Lighting,INDUSTRY,\xi}$ $eUI_{Heat_{lowThw},\xi} = eUYear_{Heat_{lowThw},HOUSEHOLDS,\xi} + eUYear_{Heat_{lowThw},SERVICES,\xi} + eUYear_{Heat_{lowThw},INDUSTRY,\xi}$ $eUI_{Heat_{lowTsh},\xi} = eUYear_{Heat_{lowTsh},HOUSEHOLDS,\xi} + eUYear_{Heat_{lowTsh},SERVICES,\xi} + eUYear_{Heat_{lowTsh},INDUSTRY,\xi}$ $eUI_{Heat_{highT},\xi} = eUYear_{Heat_{highT},HOUSEHOLDS,\xi} + eUYear_{Heat_{highT},SERVICES,\xi} + eUYear_{Heat_{highT},INDUSTRY,\xi}$

In the (3) subproblems, the parameter $a_l = 1$ if $l = Heat_{lowTdhn}$ otherwise 0, the other parameters can be found in Anexo (A). An auxiliary variable with a penalty value in the objective function is also added so that all subproblems are feasible for any realization of the random variable and for any value of $\bar{F}^k$ and $\overline{\%DHN}^k$. The algorithm is developed in Julia version 1.4.0, where the subproblems are solved serially. The pseudo-codes for both versions of Benders can be found in Anexo (B).

2. Results

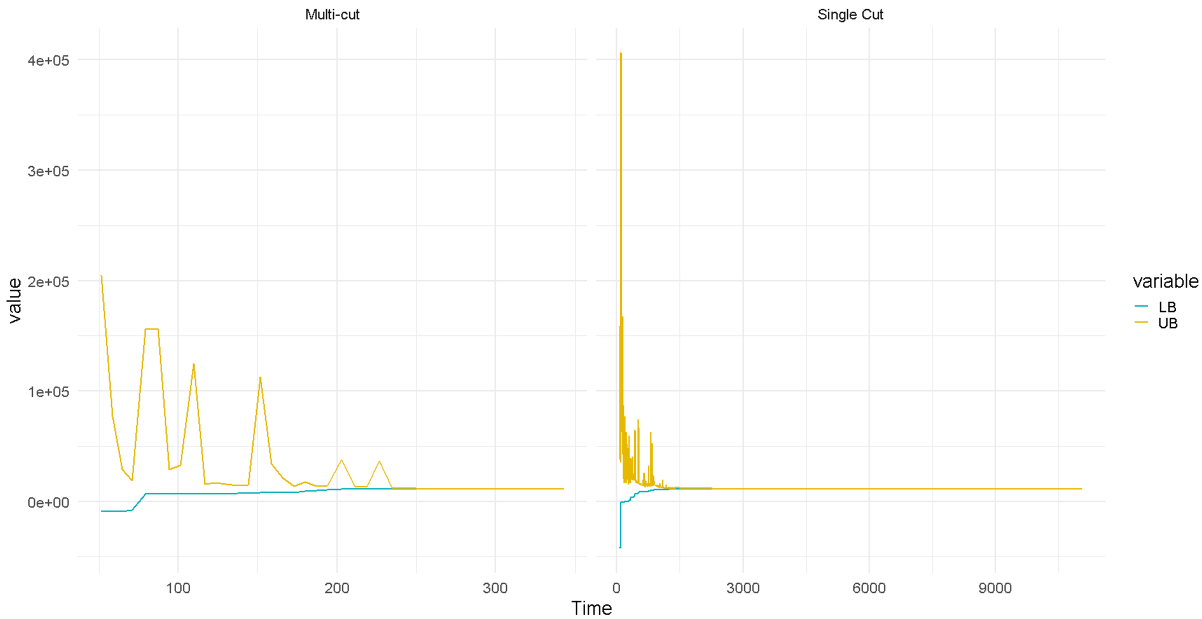

Figura 1: Benders Descomposicion version Multi-cut vs single cut. Both plots converge to the optimal value, using a sample size of $|\Omega| = 100$.

Tabla 1: Benders Descomposicion version Multi-cut vs single cut.

	Single Cut	Multi-cut			
Time	11066.07	343.12	1078.70	1574.71	2106.93
UB	11410.41	11410.50	11501.27	11482.93	11510.37
LB	11410.39	11410.35	11501.07	11482.89	11510.18
k	1025	47	50	43	43
$\|\Omega\|$	100	100	300	500	700

In figure (1) we can see that the multi-cut BD version converges faster than the single-cut BD version. We can also observe that the upper bound (UB) does not decrease monotonically. In the table (1), we show in the last row the number of scenarios or subproblems solved in each iteration of the benders method. As can be seen, we only run the single-cut BD algorithm once, since it takes a long time to reach a gap of less than 0.01. The run time of the single-cut BD algorithm was 3 hr with a number of iterations (or aggregate cuts) of 1025, while the run time with 700 subproblems in the multi-cut BD version is less than 1 hr, but with a total number of cuts $k \cdot |\Omega|$ =30100. For our case, the multi-cut BD method turns out to be more efficient than the single-cut BD method, even though the solution of the subproblems was not parallelized and the Master problem grew rapidly. The multi-cut Benders descomposition is unstable for certain $|\Omega|$, e.g $|\Omega| = 2$,$|\Omega| = 6$.

Referencias

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[2] Moret, S. Strategic energy planning under uncertainty. PhD thesis, Ecole Polytechnique Fédérale de Lausanne, Switzerland. (2017)
https://github.com/energyscope/EnergyScope/tree/v1.0

[3] Moret, S., Babonneau, F., Bierlaire, M., and Marechal, F. Decision support for strategic energy planning: A robust optimization framework. European Journal of Operational Research, 280(2):539–554. (2020)

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[5] S. J. Kazempour, A. J. Conejo and C. Ruiz, “Strategic Generation Investment Using a Complementarity Approach,” in IEEE Transactions on Power Systems, vol. 26, no. 2, pp. 940-948, May 2011, doi: 10.1109/TPWRS.2010.2069573.

Anexo A. Nomenclature

For interested readers, we report in this section the complete MILP model formulation as described in [3]. For the sake of simpler notations, we shorten the name of some variables.

In the following, we use the indicator function of a subset A of a set X as a function $\mathbf{1}_A : X \to \{0, 1\}$ defined as:

$$\mathbf{1}_A = \begin{cases} 1 & if \quad x \in A \\ \\ 0 & if \quad x \notin A \end{cases}$$

(I) Definition of sets.

T	:Set of technologies	Sto	:Set of storage units
R	:Set of resources	EUC	:Set of end-uses categories
M	:Set of periods	S	:Set of sectors
$BioFuels$	:Set of biofuels import ($\subset R$)	L	:Set of layers
$Export$	:Set of exported resources ($\subset R$)	EUI	:Set of end-uses Input
I	:Set of infrastructure	EUT	:Set of end-uses types
$T\text{-}EUT\{eut\}$	:Set of technologies $\forall eut \in EUT$	$T\text{-}EUC\{euc\}$	:Set of technologies $\forall euc \in EUC$

(II) Definition of variables

Tabla A.1: Variables for the first-stage problem

Name		Description	Units
$\%\mathbf{DHN}$	:	Ratio [0; 1] centralized over total low-temperature heat	
$\mathbf{F}_i$	:	Installed capacity with respect to main output i, $\forall i \in T$	[GW]
$\mathbf{Ft}_{i,m}$	:	Operation the i in each period t, $\forall i \in T \cup R$, $\forall\ m \in M$	[GW]
$\mathbf{Sto}^{-}_{j,l,m}$	:	Input to storage units $j \in Sto$ the $l \in L$ in period $m \in M$	[GW]
$\mathbf{Sto}^{+}_{j,l,m}$	:	Output from storage units $j \in Sto$ the $l \in L$ in period $m \in M$	[GW]
Auxiliary var			
$\mathbf{EndUses}_{l,m}$	:	End-uses demand. Set to 0 if l $\notin$ EUT, $\forall l \in L$, $\forall m \in M$	[GW]
$\mathbf{Loss}_{eut,m}$	:	Losses in the networks (grid and DHN), $\forall eut \in EUT$, $\forall m \in M$	[GW]
$\mathbf{MAX}^{DHN}$	:		[GW]

(III) Definition of parameters

Name		Description	Units
$eUYear_{eui,s}$	:	Annual end-uses in energy services per sector s, $\forall s \in S$,$\forall eui \in EUI$ short name of $endUses_{year}$	[GWh/y]
eUI_{eui}	:	Total annual end-uses in energy services eui, $\forall eui \in EUI$ $eUI_{eui} = \sum_{s \in S} eUYear_{eui,s}$ short name of endUsesInput	[GWh/y]
τ_i	:	Investment i cost annualization factor, $\forall i \in T$; $\tau_i = \frac{i_{rate}(i_{rate}+1)^{n_i}}{(i_{rate}+1)^{n_i}-1}$	
i_{rate}	:	Real discount rate	
$\underline{g}_k$, $\overline{g}_k$	:	Upper and lower limit to G_k, $\forall k \in \{\%DHN\}$	
h_m	:	Time periods m duration, $\forall m \in M$	[h]
$\%lighting_m$	:	Yearly share (adding up to 1) of lighting end-uses, $\forall m \in M$	
$\%sh_m$	:	Yearly share (adding up to 1) of SH end-uses, $\forall m \in M$	
$f_{i,l}$	:	Input from (< 0) or output to (> 0) layers, $\forall i \in R \cup T \setminus Sto, \forall l \in L$	[GW]
c_i^{Inv}	:	Technology i specific investment cost, $\forall i \in T$	[MCHF/GW]
c_i^{Maint}	:	Technology i specific yearly O&M cost, $\forall i \in T$	[MCHF/GW/y]
n_i	:	Technology i lifetime, $\forall i \in T$	[y]
$\underline{f}_i$, $\bar{f}_i$	:	Min./max. installed size of the technology i, $\forall i \in T$	[GW]
$f_i^{min\%}$, $f_i^{max\%}$	:	Min./max. relative share of a technology in a layer i, $\forall i \in T$	
$avail_r$	:	Resource r yearly total availability, $\forall r \in R$	[GWh/y]
$\widehat{k}_{i,m}$	:	Period capacity factor of technology i in period m, $\forall i \in T, \forall m \in M$ (default 1)	
k_i	:	Yearly capacity i factor, $\forall i \in T$	
$c_{r,m}^{op}$	:	Specific cost of resources r in periods m, $\forall r \in R, m \in M$	[MCHF/GWh]
$\eta_{j,l}^{-}$, $\eta_{j,l}^{+}$	:	Efficiency [0;1] of storage j input from/output to layer l. $\forall j \in Sto, \forall l \in L$	
$\%loss_{eut}$	:	Losses [0;1] in the networks (grid and DHN), $\forall eut \in EUT$	
$\%Peak_{DHN}$	:	Ratio peak/max. average DHN heat demand	

Anexo B. Algorithms

Algorithm 1: Benders Descomposition: version Single cut

Initialization: k = 0, LB = -∞, UB= ∞, $\delta = \delta^*$, $K = \emptyset$, $k = 0$
while $|UB - LB| \geq \delta^*$ *or* $k \leq maxIter$ **do**
  Get $(\bar{F}^k, \overline{\%DHN}^k, \theta^k)$ solving Master problem 1
  $LB \leftarrow f(\bar{F}^k) + \theta^k$
  for $\xi \in \Omega$ **do**
    Solve LP $Q(\bar{F}^k, \overline{\%DHN}^k, \xi)$, (subproblem 3)
    $y_\xi^k \leftarrow$ optimal solution value
    $\alpha_{j,\xi}^k \leftarrow$ dual vulue of $F_{j,\xi} = \bar{F}_j^k$
    $\beta_\xi^k \leftarrow$ dual value of $\%DHN_\xi = \overline{\%DHN}^k$
  $UB \leftarrow f(\bar{F}^k) + \sum_{\xi \in \Omega} p_\xi y_\xi^k$
  $K = K \cup \{k\}$
  $k \leftarrow k + 1$
return LB

Algorithm 2: Benders Descomposition: version Multi-cut

Initialization: k = 0, LB = -∞, UB= ∞, $\delta = \delta^*$, $K = \emptyset$, $k = 0$

while $|UB - LB| \geq \delta^*$ *or* $k \leq$ *maxIter* **do**

  Get $(\bar{F}^k, \overline{\%DHN}^k, \theta_\xi^k)$ solving Master problem 2

  $LB \leftarrow f(\bar{F}^k) + \sum_{\xi \in \Omega} \theta_\xi^k$

  for $\xi \in \Omega$ **do**

    Solve LP $Q(\bar{F}^k, \overline{\%DHN}^k, \xi)$, (subproblem 3)

    $y_\xi^k \leftarrow$ optimal solution value

    $\alpha_{j,\xi}^k \leftarrow$ dual vulue of $F_{j,\xi} = \bar{F}_j^k$

    $\beta_\xi^k \leftarrow$ dual value of $\%DHN_\xi = \overline{\%DHN}^k$

  $UB \leftarrow f(\bar{F}^k) + \sum_{\xi \in \Omega} p_\xi y_\xi^k$

  $K = K \cup \{k\}$

  $k \leftarrow k + 1$

return LB