with the soft update, what we do is make W 99% the old W + 1% of the new W
Same thing with the B parameter.

$$W = 0.01 W_{new} + 0.99 W$$

$$B = 0.01 B_{new} + 0.99 B$$

0.99 & 0.01 are hyperparam that can be changed

LINEAR ALGEBRA FOR MACHINE LEARNING & DATA SCIENCE

WK1 SYSTEM of LINEAR EQUATIONS: 2 VARIABLES

System of sentences:

SYSTEM 1
(i) The dog is black
(ii) The cat is orange
→ Complete ✓

SYSTEM 2
(i) The dog is black
(ii) The dog is black
→ Redundant ✗

SYSTEM 3
The dog is black
The dog is white
Contradictory ✗

- A singular system is one that is REDUNDANT or CONTRADICTORY while a Non-singular system is one that is COMPLETE

Let's look @ a system with more sentences;

SYSTEM 1	SYSTEM 2	SYSTEM 3	SYSTEM 4
The dog is BLACK	THE dog is BLACK	The dog is BLACK	The dog is BLACK
The cat is ORANGE	The dog is BLACK	The dog is BLACK	The dog is WHITE
The bird is RED	The bird is RED	The dog is BLACK	The bird is RED
... Complete ...	... Redundant ...	... Redundant ...	... Contradictory ...
∴ Non SINGULAR	SINGULAR		

P.T.O.

SYSTEM OF EQUATIONS

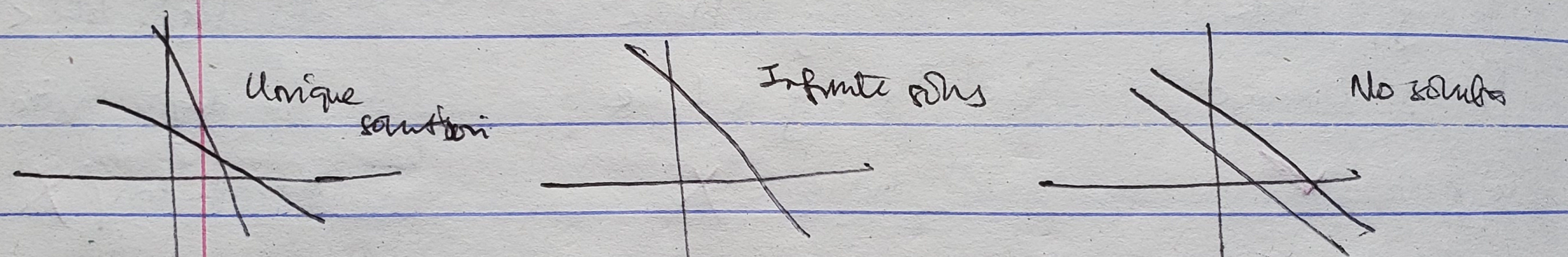
- $a + b = 10$
  $a + 2b = 12$
  $-b = -2$ → $b = 2$; $a = 8$

- $a + b = 10$
  $2a + 2b = 20$ → Not enough information <Redundant, Singular>

What is a linear equation?

LINEAR → $a + b = 10$; $2a + 3b = 15$; $3.4a + 48.99b + 2c = 1225$

Non-linear → $a^2 + b^2 = 10$; $\sin(a) + b^2 = 15$; $2^a - 3^b = 0$; $ab^2 + b/a + 3/b - \log(c) = 4^a$



Complete <Non-Singular> | Redundant <Singular> | Contradictory <Singular>

$3a + 2b = 8$
$\times 2$ $\quad 2a - b = 3$

$\rightsquigarrow + \begin{cases} 3a + 2b = 8 \\ 4a - 2b = 6 \end{cases} \rightarrow 7a = 14$
$a = 2$

$2(2) - b = 3$
$b = \cancel{3}\ 4 - 3$
$b = \cancel{4}\ 1$

∴ Non singular.

How to tell if a matrix is singular or non-singular → Using the determinant
↳ The determinant returns 0 if the matrix is singular and a non-zero number if it can't.

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$; determinant $= ad - bc$ → 0 - singular ; $\neq 0$ - non singular

W1 SYSTEM of LINEAR EQUATIONS : 3 VARIABLE

$a + b + c = 10$ → $c = 10 - a - b$
$a + 2b + c = 15$ → $a + 2b + 10 - a - b = 15$
$a + b + 2c = 12$ → $\cancel{a} - \cancel{a} + 2b - b = 15 - 10$
$b = 5$

↳ $a + b + 20 - 2a - 2b = 12$

$a - 2a + b - 2b = 12 - 20 \rightarrow a + b = 8$

$-a - b = -8$ $\quad a = 8 - b = 8 - 5 = 3$

$c = 10 - 3 - 5 = 2$; $a = 3,\ b = 5,\ c = 2$

* A non Singular has a unique solution.

LINEAR DEPENDENCE & INDEPENDENCE

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{array}{l} -a \\ -b \\ -c \end{array}$$

if row a + row b = row c, that means Row c depends on (a) & (b)

∴ these rows are linearly dependent

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & -2 \\ 2 & 4 & 10 \end{bmatrix}$$

3Row1 + 2Row2 = Row3 — Row1 - Row2 = Row3 — No Relation — 2Row1 = Row3

Dependent <Singular> — Independent <Non-singular> — Dependent <Singular>

THE DETERMINANT (3x3)

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a * \begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

OR

a	b	c
d	e	f
g	h	i

$$(a \cdot e \cdot i) + (b \cdot f \cdot g) + (c \cdot d \cdot h) - (c \cdot e \cdot g) - (b \cdot d \cdot i) - (a \cdot f \cdot h)$$

$3 \times 3 \times 3 \times 1$

$$\begin{bmatrix} 2 & 1 & 5 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \\ 15 \end{bmatrix} = \begin{matrix} 125 \\ 55 \\ 95 \end{matrix}$$

$2a + b + 5c = 20$

$a + 2b + c = 10$

$2a + b + 3c = 15$

$2c = 5$

$c = 2.5$

$\rightarrow 2a + 5b + 5(2.5) = 20$

$a + 2b + 2.5 = 10$

$2a + 5b = 7.5$

$\cancel{a + 2b = }\ 7.5$

$2a + 4b = \cancel{15}$

$b =$

$a + 2b = 10 - 2.5$

$a + 2b = 7.5$

$2a + b = 10$

$2a + 4b = 15$

$2a + b = 10$

$3b = 5$

$b =$

$2a + b + 5c = 20$

$a + 2b + c = 10$ $\sim$ $2c = 5$

$2a + b + 3c = 15$

$c = 2.5$ ✓

$a + 2b = 10 - c = 7.5$

$2a + b = 7.5$ $\to$ $2a + b = 7.5$ $\quad a = 7.5 - 2b$

$a + 2b = 7.5$ $\quad 2a + 4b = 15$ $\quad = 2.5$

$2a + b = 7.5$ $\quad -3b = -7.5$

$b = 2.5$ ✓

WK 2 Solving System of Linear Equations: Elimination

$2a + 5b = 46$ $\quad 2a + 5(32 - 8a) = 46 \Rightarrow 2a + 160 - 40a = 46$

$8a + b = 32 \to b = 32 - 8a$ $\quad -38a = -114$

$b = 8$ $\quad a = 3$

Row Echelon form:

1	*	*	*	*
0	1	*	*	*
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

$x + y = 4 \to x = 4 - y$

$-6x + 2y = 16$

$-6(4 - y) + 2y = 16$

$-24 + 6y + 2y = 16 \Rightarrow -24 + 8y = 16 \to 8y = 40 \to y = 5$

$x = 4 - \cancel{5} = \quad x = -1$

$4(-8) - (-3)7 = -32 + 21$

$$\begin{bmatrix} -3 & 8 & 1 \\ 2 & 2 & -1 \\ -5 & 6 & 2 \end{bmatrix} = (-3 \times 2 \times 2) + (8 \times -1 \times -5) + (1 \times 2 \times 6) - (1 \times 2 \times -5) - (8 \times 2 \times 2) - (-3 \times -1 \times 6)$$

$= -12 + 40 + 12 - 10 - 32 - 18$

$= -20$

$s = 2c$ $\quad b = 2$

1 1

2 3 $\quad 0.02s + 0.03c + 0.04b = 260$

$0.02(2c) + 0.03c + 0.04b = 260$

S T 4 6 $ 3 2

$S = 5, \quad C = 3, \quad b = 2$

$0.02(5000) + 0.03(3000) + 0.04(2000) = 270$

$0.02S + 0.03C + 0.04b = 270$ $\qquad b = 10000 - C$

$0.07C + 0.04b = 260$

$0.07C + 0.04(10000 - C) = 260 \rightarrow 0.07C + 400 - 0.04C = 260$

Startups = 2C $\qquad$ Bonds = 1000 − 3C

CD = C

$0.07C + 0.04(10000 - 3C) = 260 \rightarrow 0.07C + 400 - 0.12C = 260$

$-0.05C = -140 \qquad C = 2800$

$$\begin{bmatrix} -3 & 8 & 1 \\ 2 & 2 & -1 \\ -5 & 6 & 2 \end{bmatrix} \quad -3[2(2) - (-1)(6)] - 8[2(2) - (-1)(-5)] + [2(6) - (2)(-5)]$$

$= -3(10) - 8(-1) + (22) =$

$\begin{bmatrix} 3 & 2 \\ 5 & 7 \end{bmatrix}$

* The amount of information a system carries is defined as the rank

• Calculating rank of a matrix using a row echelon form

(I) $\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix}$ The idea is to get rid of the 4 here → divide each row → by the leftmost coef → (II) $\div 5$, $\div 4$ $\begin{bmatrix} 1 & 0.2 \\ 1 & -0.75 \end{bmatrix}$

III

→ In order to make the bottom left = 0, keep the first row &
→ new_2nd row = 2nd_row − 1st row

→ $\begin{bmatrix} 1 & 0.2 \\ 0 & -0.95 \end{bmatrix}$ → Divide the second row by the left most non-zero coeff. $\langle \div -0.95 \rangle$

IV

→ $\begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$ Now the matrix is in row echelon form

Let's see what we'll get if we look @ singular matrix <redundant>

$$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0.2 \\ 1 & 0.2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0.2 \\ \infty & \infty \end{bmatrix}$$

∴ Row echelon form

∴ The rank of a matrix is the number of 1's in the diagonal of its row echelon form.

Two examples of Row Echelon form

$$\begin{bmatrix} 2 & * & * & * & * \\ 0 & 1 & * & * & * \\ 0 & 0 & 3 & * & * \\ 0 & 0 & 0 & -5 & * \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 3 & * & * & * & * \\ 0 & 0 & 1 & * & * \\ 0 & 0 & 0 & 4 & * \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank 5   Rank 3

- Zero rows @ the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the row above
- Rank of the matrix is the number of pivots

< Note we can also divide each row by the leftmost non zero n° to give us our pivots = 1

Reduced Row Echelon Form

Original System

$5a + b = 17$

$4a - 3b = 6$

$\longrightarrow$

Intermediate System

$a + 0.2b = 3.4$

$b = 2$

$\longrightarrow$

Solved System

$1a + 0b = 3$

$a = 3$

$0a + 1b = 2$

$b = 2$

original matrix

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \longrightarrow$$

upper diagonal matrix

$$\begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

Row echelon form

Diagonal matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Reduced row echelon form.

How do you move from Row Echelon form to Reduced R.E. form?

$$\begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

→ Leave bottom row untouched

→ multiply bottom row by 0.2

& new top row = top row − 0.2(bottom row)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Properties of Row Echelon Form

(i) Is in row echelon form

(ii) Each pivot is a 1

(iii) Any number above a pivot is 0

Quiz

$7f + 5a + 3c = 120$

$3f + 2a + 5c = 70$

$f + 2a + c = 20$

$\rightarrow f + 2c = 25$; $2f + 4c = 50$ $\rightarrow f = 25 - 2c$

fn

$f = 20 - 2a - c$

~~$7(25 - 2c) + 5a + 3c = 120$~~

~~$+ 5a + 3c = 120$~~

$7(20 - 2a - c) + 5a + 3c = 120$

$140 - 14a - 7c + 5a + 3c = 120 \rightarrow -14a + 5a - 7c + 3c = 120 - 140$

$-9a - 4c = -20 \rightarrow \boxed{9a + 4c = 20}$

$3(20 - 2a - c) + 2a + 5c = 70$

$60 - 6a - 3c + 2a + 5c = 70 \rightarrow -6a + 2a - 3c + 5c = 70 - 60$

$-4a + 2c = 10$ — $-4a + 2c = 10 \xrightarrow{\times 2} -8a + 4c = 20$

$9a + 4c = 20 \rightarrow 9a + 4c = 20$

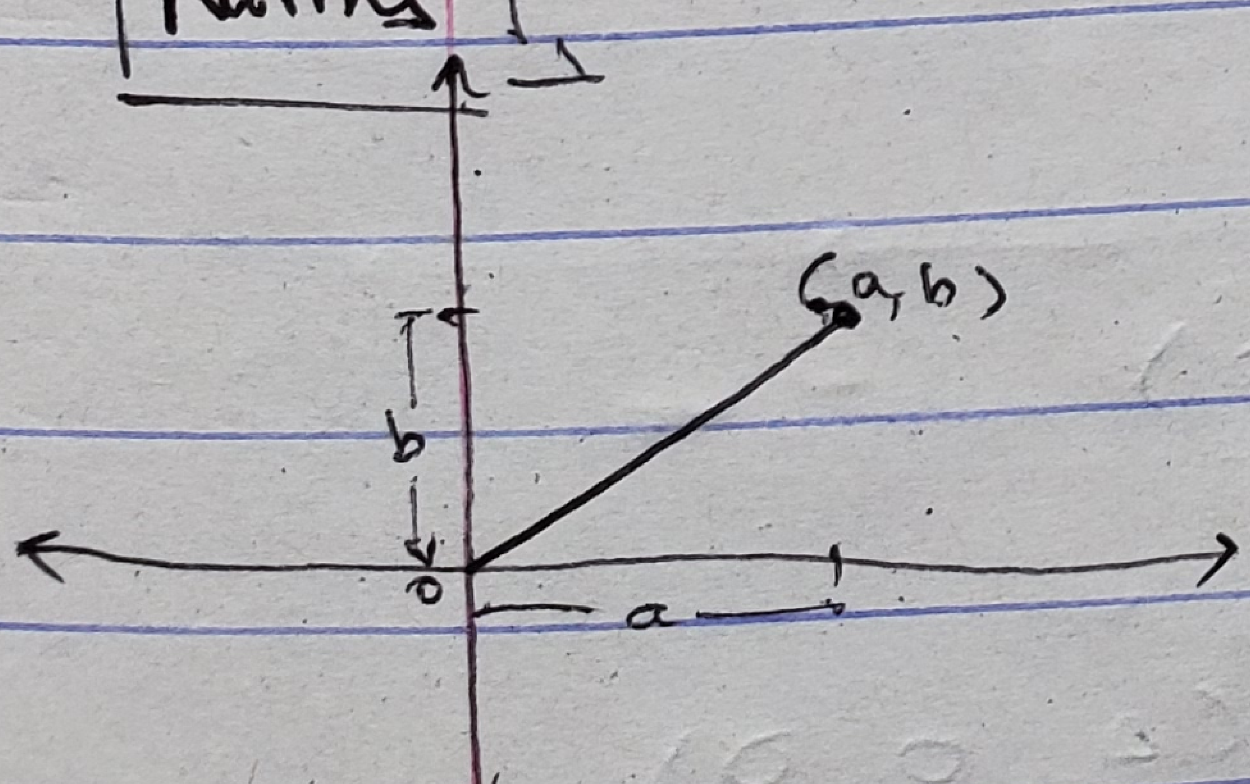
$\therefore 2c = 10 \rightarrow c = 5, f = 15 \quad a = 0 \leftarrow -17a = 0$

$$\begin{bmatrix} 7 & 5 & 3 \\ 3 & 2 & 5 \\ 1 & 2 & 1 \end{bmatrix} = 7\begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 3 & 5 \\ 1 & 1 \end{bmatrix} + 3\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix} = 7(-8) - 5(-2) + 3(4) = -34$$

$\hookrightarrow -34$

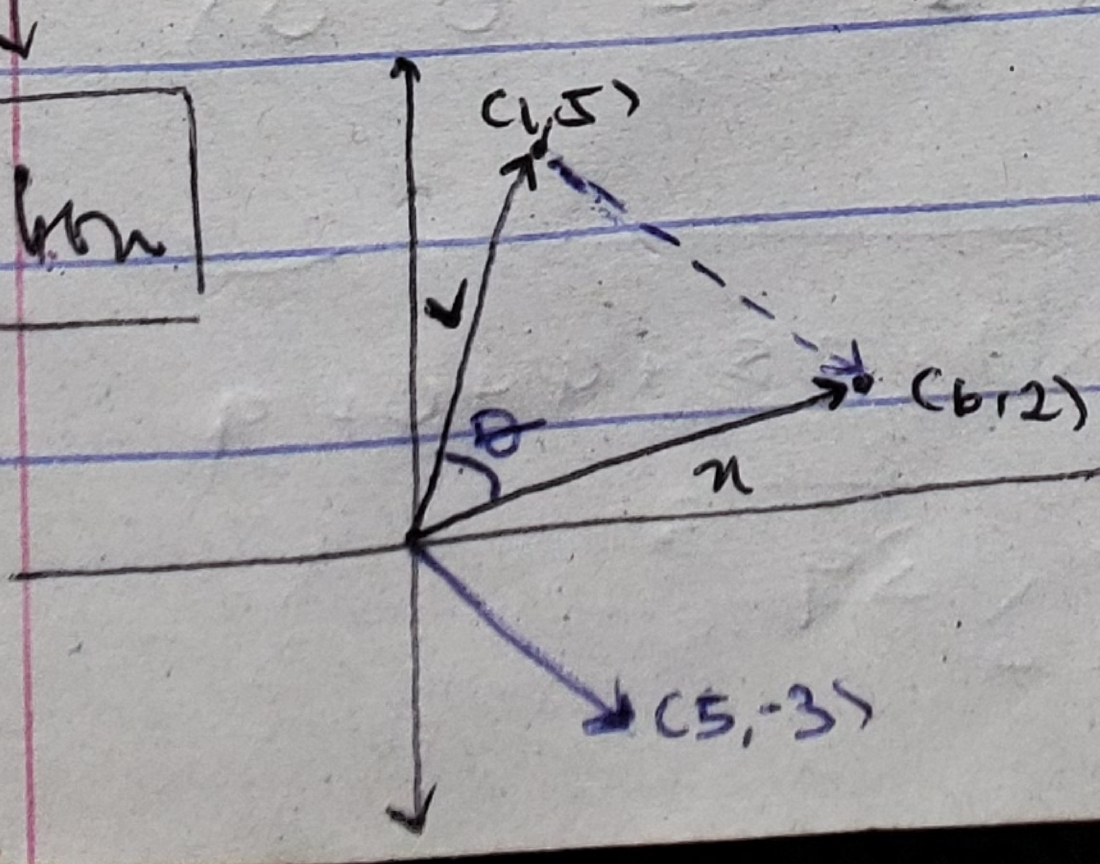
WK3 VECTOR ALGEBRA

Norms



- L1-norm $= |(a,b)|_1 = |a| + |b|$
- L2-norm $= |(a,b)|_2 = \sqrt{a^2 + b^2}$

Distances from vectors

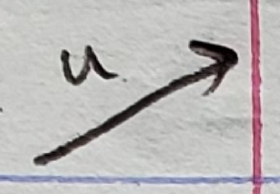


L1-distance $|u - v|_1 = |5| + |-3| = 8$

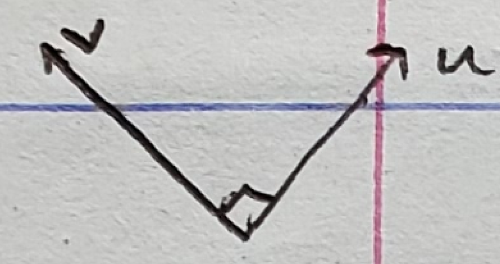
L2-distance $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$

Cosine distance $= \cos(\theta)$

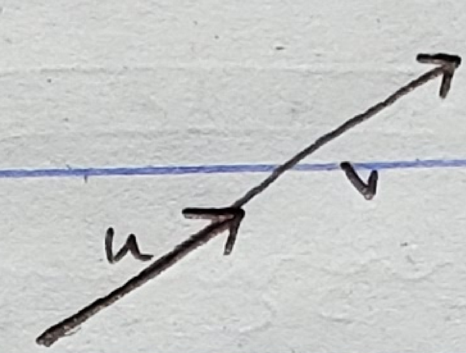
The Dot Product



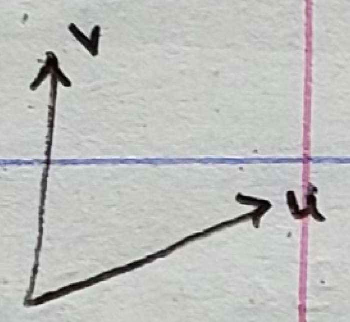
$\langle u,u\rangle = |u|^2 = (L2\text{-norm})^2$



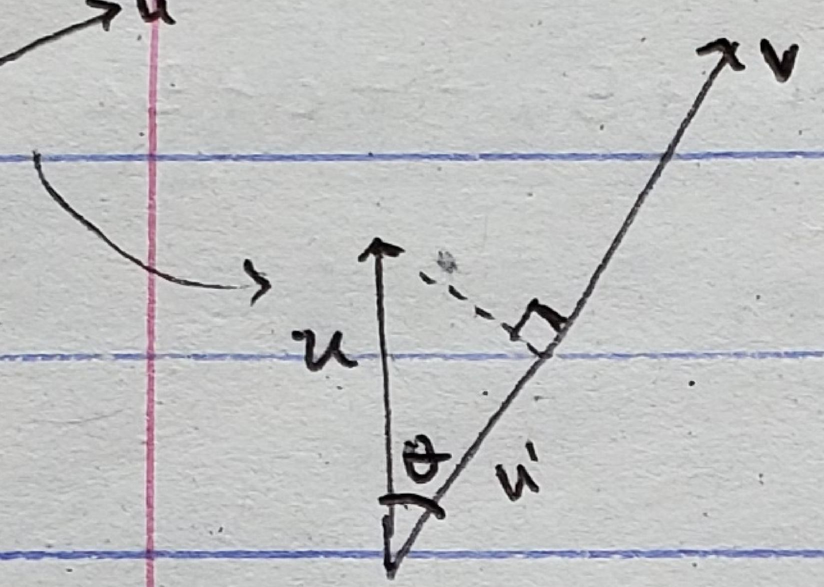
$\langle u,v\rangle = 0$



$\langle u,v\rangle = |u|\cdot|v|$



$\langle v,u\rangle = ?$



$\langle u,v\rangle = |u'|\cdot|v|$

$= |u||v|\cos\theta$

Note: The expression "$v \in \mathbb{R}^2$" means that the vector v belongs to a set of real numbers $\mathbb{R}$ and is in a 2-dimensional space (2D)

LINEAR TRANSFORMATIONS

$$A = \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \qquad \det A = 5(2) - (2)(1) = 10 - 2 = 8$$

$$A^{-1} = \frac{1}{8}\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 2/8 & -2/8 \\ -1/8 & 5/8 \end{bmatrix} = \begin{bmatrix} 1/4 & -1/4 \\ -1/8 & 5/8 \end{bmatrix}$$

Quiz

1) Distance between $\vec{v} = (1,0,7)$ & $\vec{w} = (0,-1,2)$

$v - w = (1,1,5) = \sqrt{1^2+1^2+5^2} = \sqrt{27}$

2) $P = (1,0,-3)$, $Q = (-1,0,-3)$ $\Rightarrow P - Q = (2,0,0)$

3) $\vec{v} = (1,-5,2,0,-3)$ || $\|v\| = \sqrt{1+25+4+0+9}$

$= \sqrt{39}$

$0, \sqrt{14}, \sqrt{6}, 4 ; \sqrt{29}$ ✓

$\begin{bmatrix} -1 & 5 & 2 \end{bmatrix}\begin{bmatrix} -3 \\ 6 \\ -4 \end{bmatrix} = 3 + 30 - 8 = 25$

$\begin{bmatrix} -9 & -1 \end{bmatrix}\begin{bmatrix} -3 \\ -5 \end{bmatrix} = 32$

$\begin{bmatrix} 2 & -1 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & -5 \\ 15 & -9 \end{bmatrix}$

~~Determ~~ Determinant of a product : $\det(AB) = \det(A) \cdot \det(B)$

Determinant of a non-singular matrix & a singular matrix $= 0$

$\begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} = 0.4(0.6) - (-0.2)(-0.2)$
$= 0.24 - 0.04 = 0.2$

$\begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix} = 0.25(0.625) - (-0.25)(-0.125)$
$= 0.15625 - 0.03125 = 0.125$

- Determinant of Inverse : $\det(A^{-1}) = \dfrac{1}{\det(A)}$

Quiz

$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = 1\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - 2\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} - 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= 1[0-1] - 2(0) - 1(1) = -1-1 = -2$

$W^{-1}\begin{bmatrix} 1 & & \\ & & \\ & & \end{bmatrix}$

$3 \times 3 \quad 3 \times 1$

$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$

~~$\begin{bmatrix} 3 & 4 \\ 6 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 3$~~

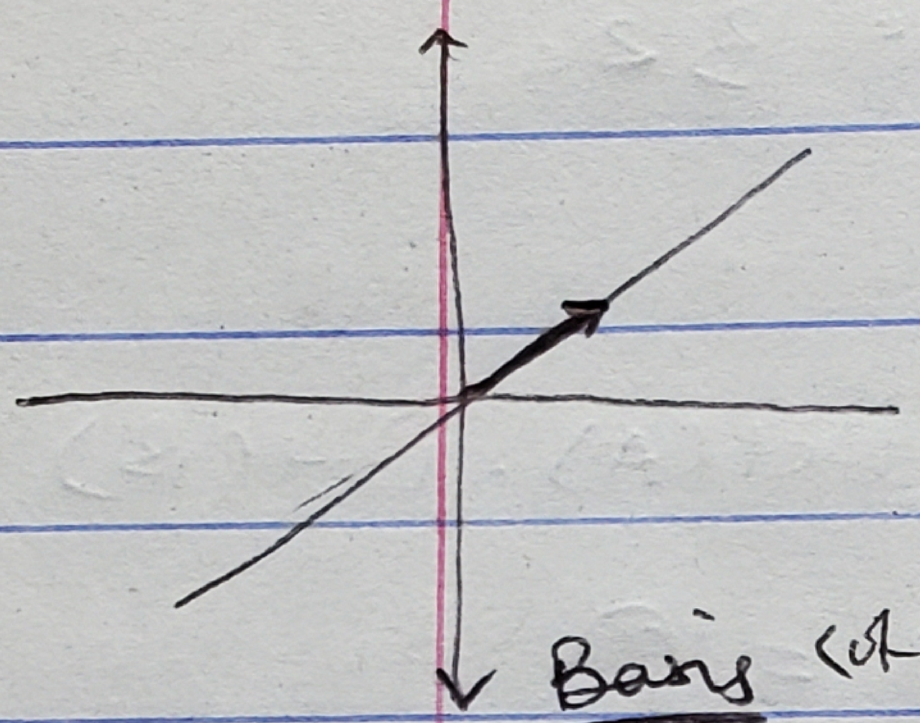
$4\begin{bmatrix} 3 & 1 & -7 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = 6 + 2 = 8$

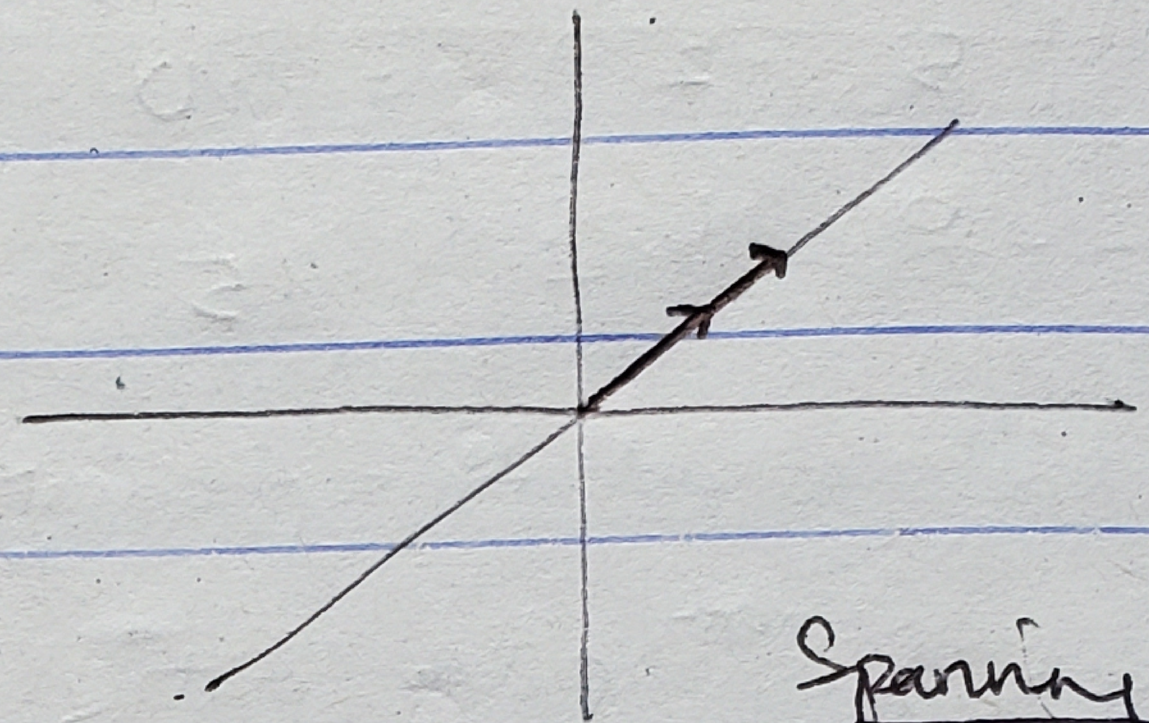
$\det(A \cdot B)^{-1} = \dfrac{1}{\det}$

$\begin{bmatrix} 5 & 2 & 3 \\ -1 & -3 & 2 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -4 \\ 2 & 1 & 0 \\ 8 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 33 \\ 9 \end{bmatrix}$

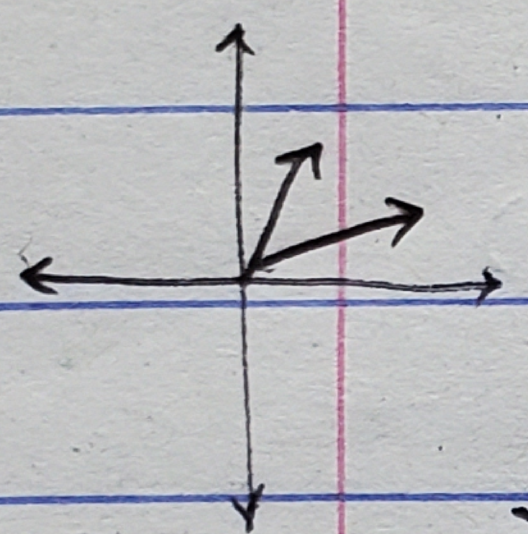
Wk 4 EIGENVALUES AND EIGENVECTORS

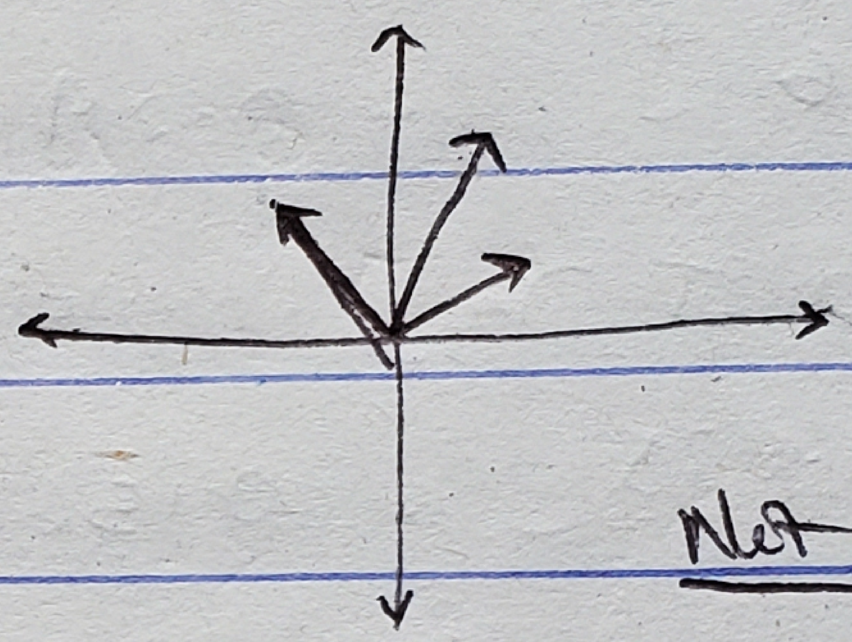
- A basis is a ~~[illegible]~~ minimal spanning set <2 vectors 2 many>





- Number of elements in the basis = the dimension



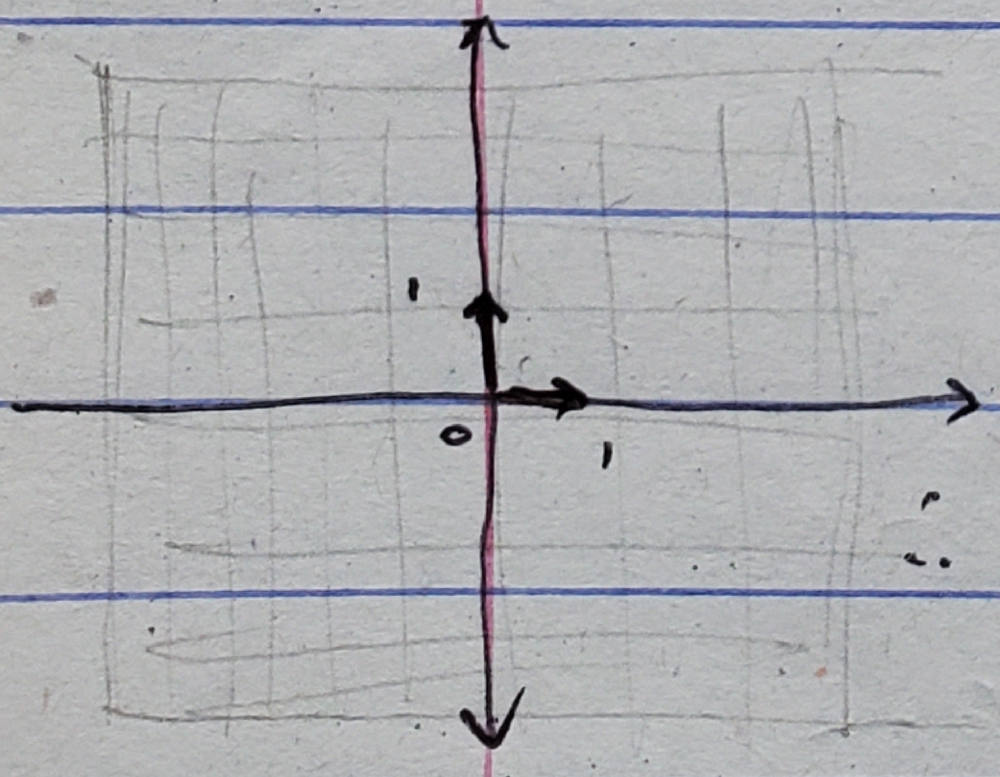


EIGENBASES, EIGENVALUES & EIGENVECTOR

- Let's look at the linear transformation corresponding to matrix;

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

First, we'll use the fundamental basis [0, 1] & [1, 0]

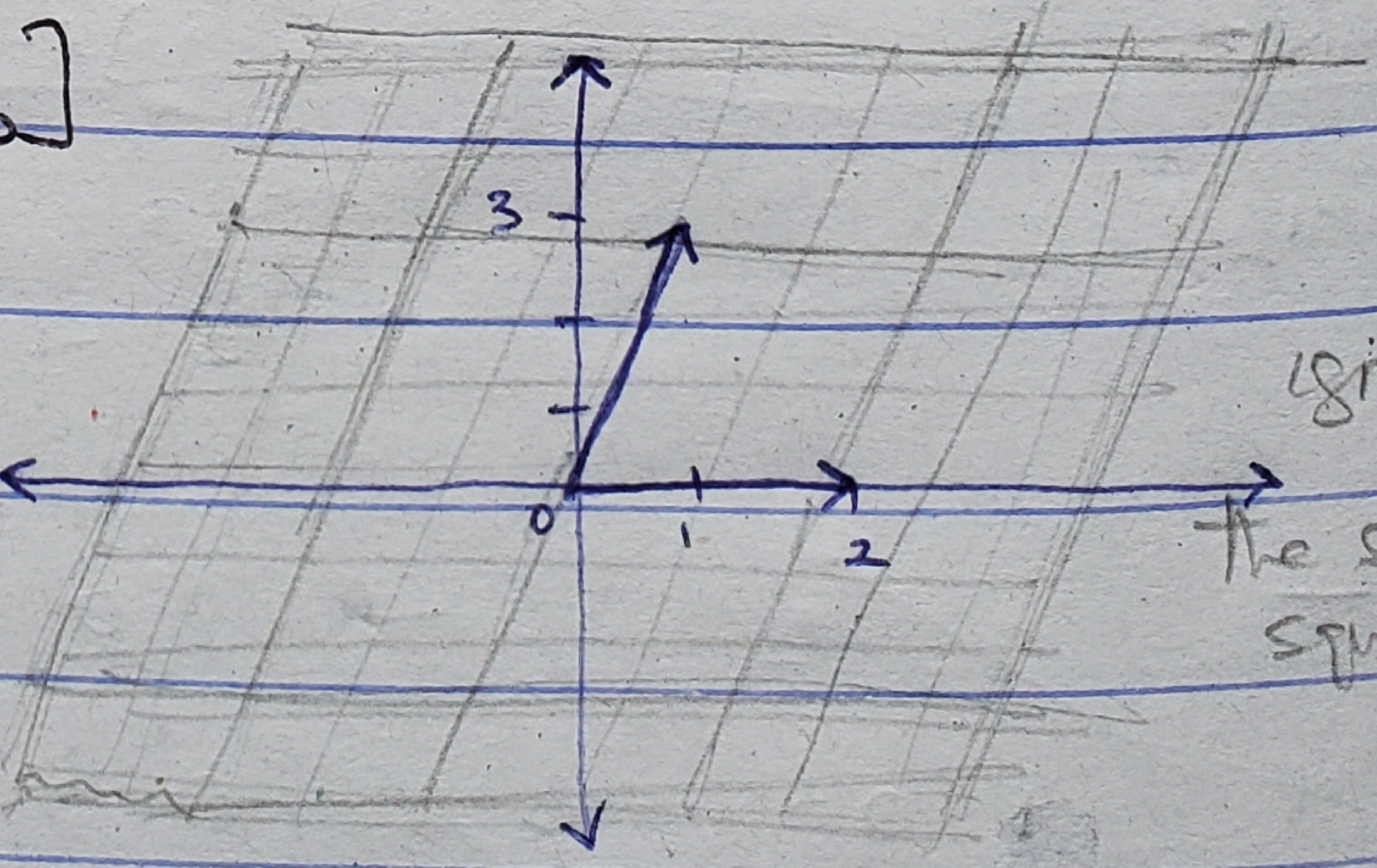


$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$\therefore$ $(1, 0) \rightarrow (2, 0)$

$(0, 1) \rightarrow (1, 3)$

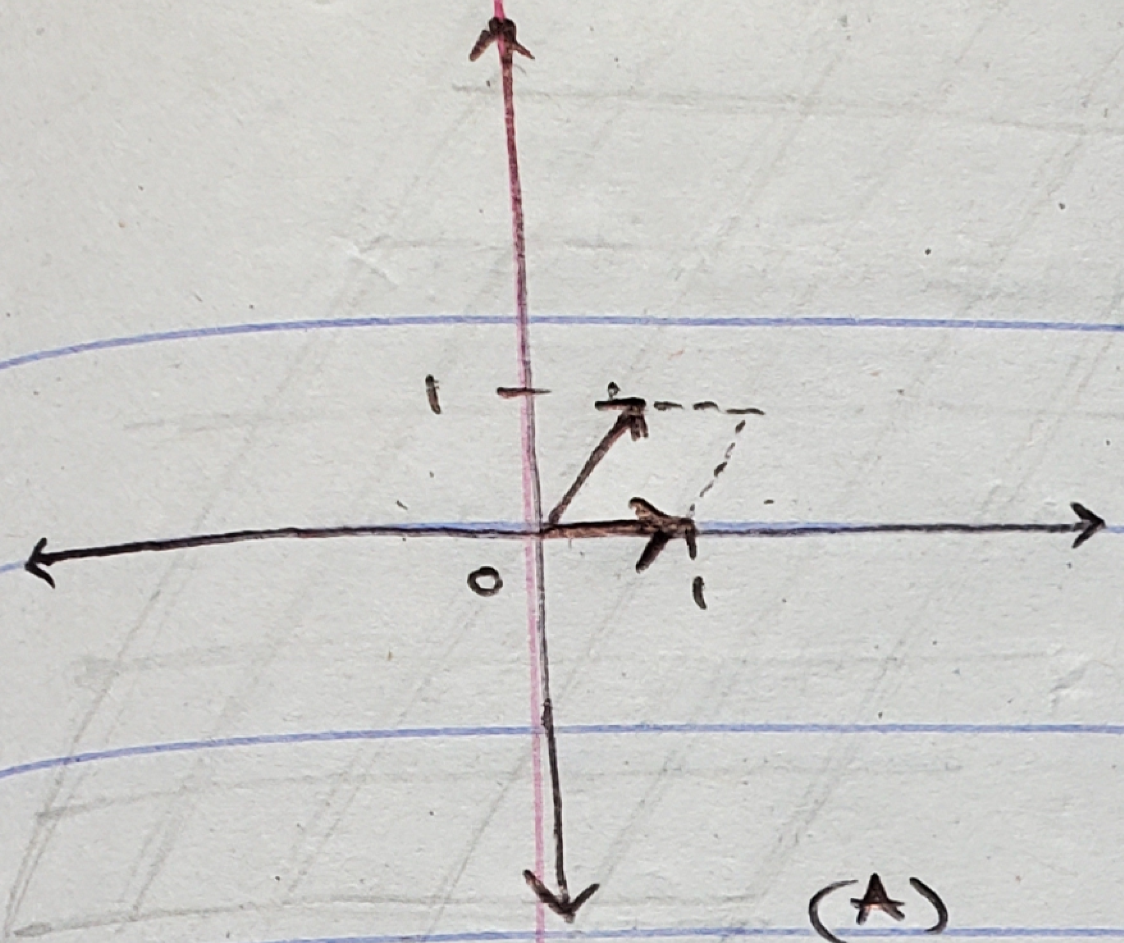


ignore the smaller squares

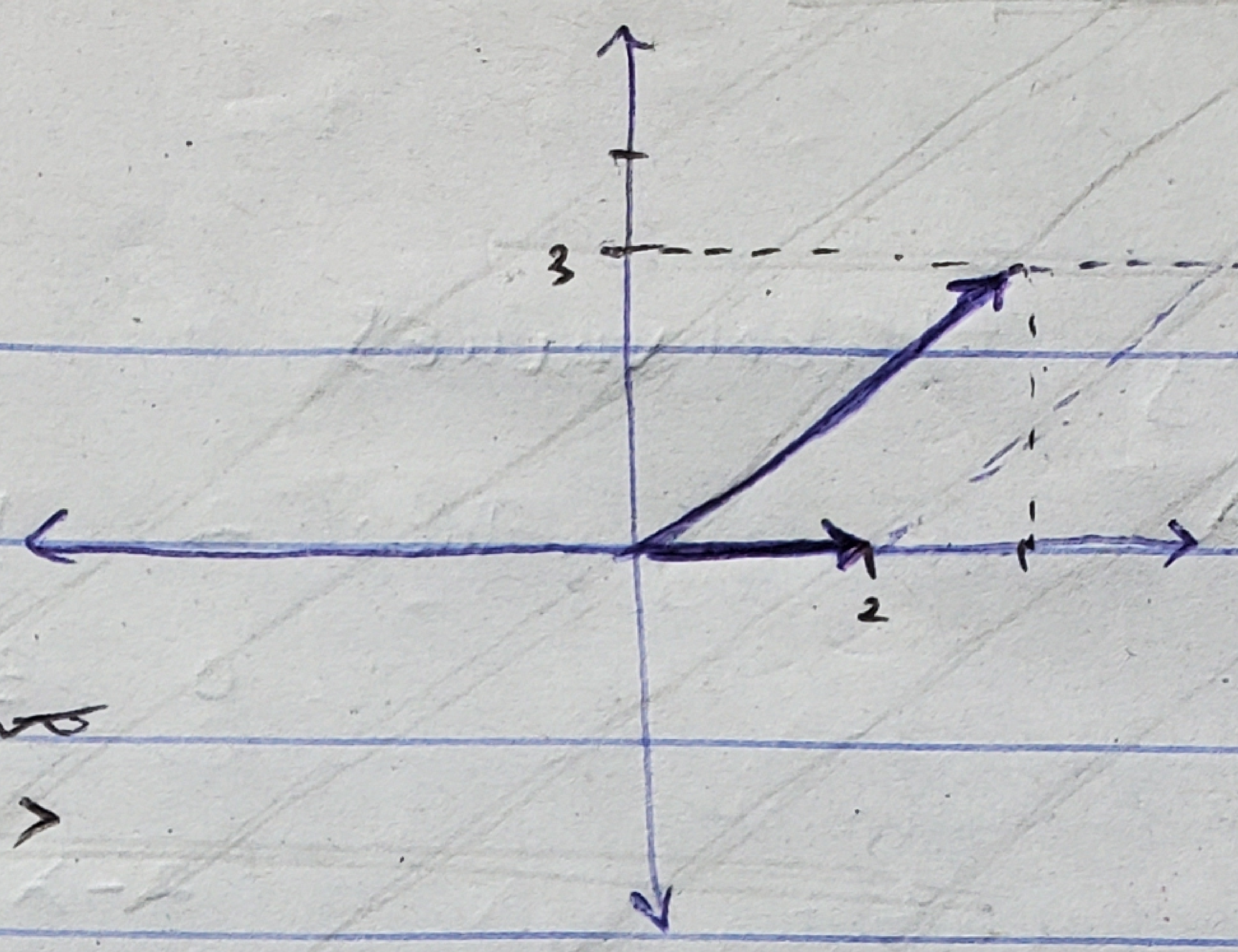
Now let's see what will happen when we choose basis (1, 0) & (1, 1)

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} ; \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

(1,0) → (2,0)
(1,1) → (3,3)



(A)



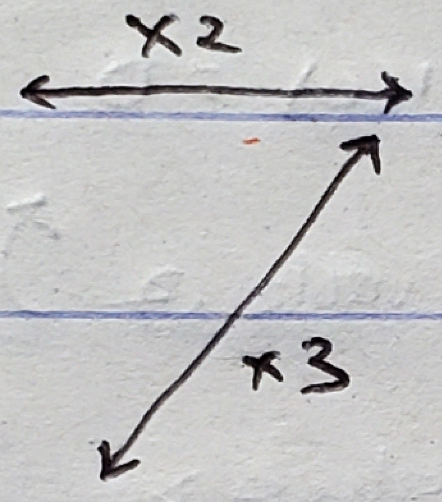
(B)

<ignore my representation ~~both~~ planes are parallel>

• Notice that the sides of the two parallelograms are parallel to each other & therefore the rest of the plane also follows

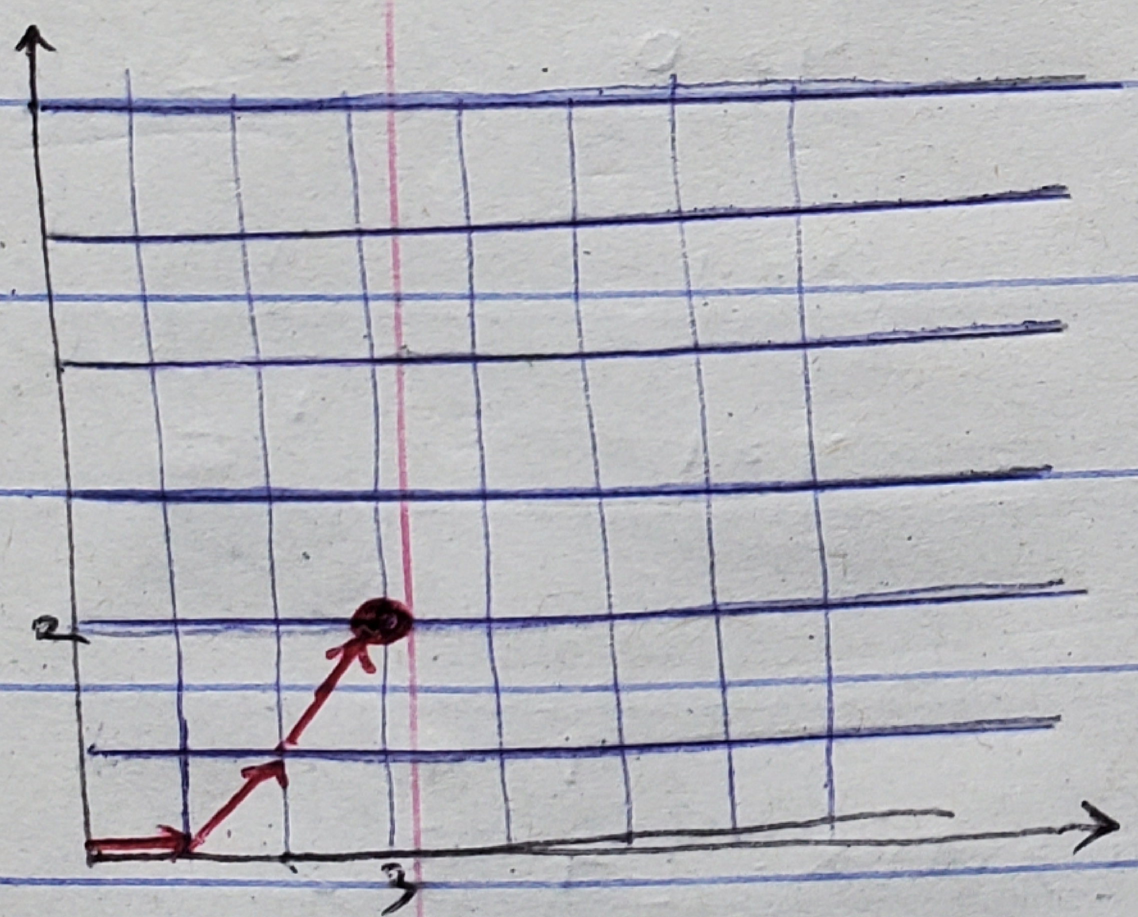
Therefore to get plane B, we're stretching plane A by 2 & by 3

Plane A ⇒

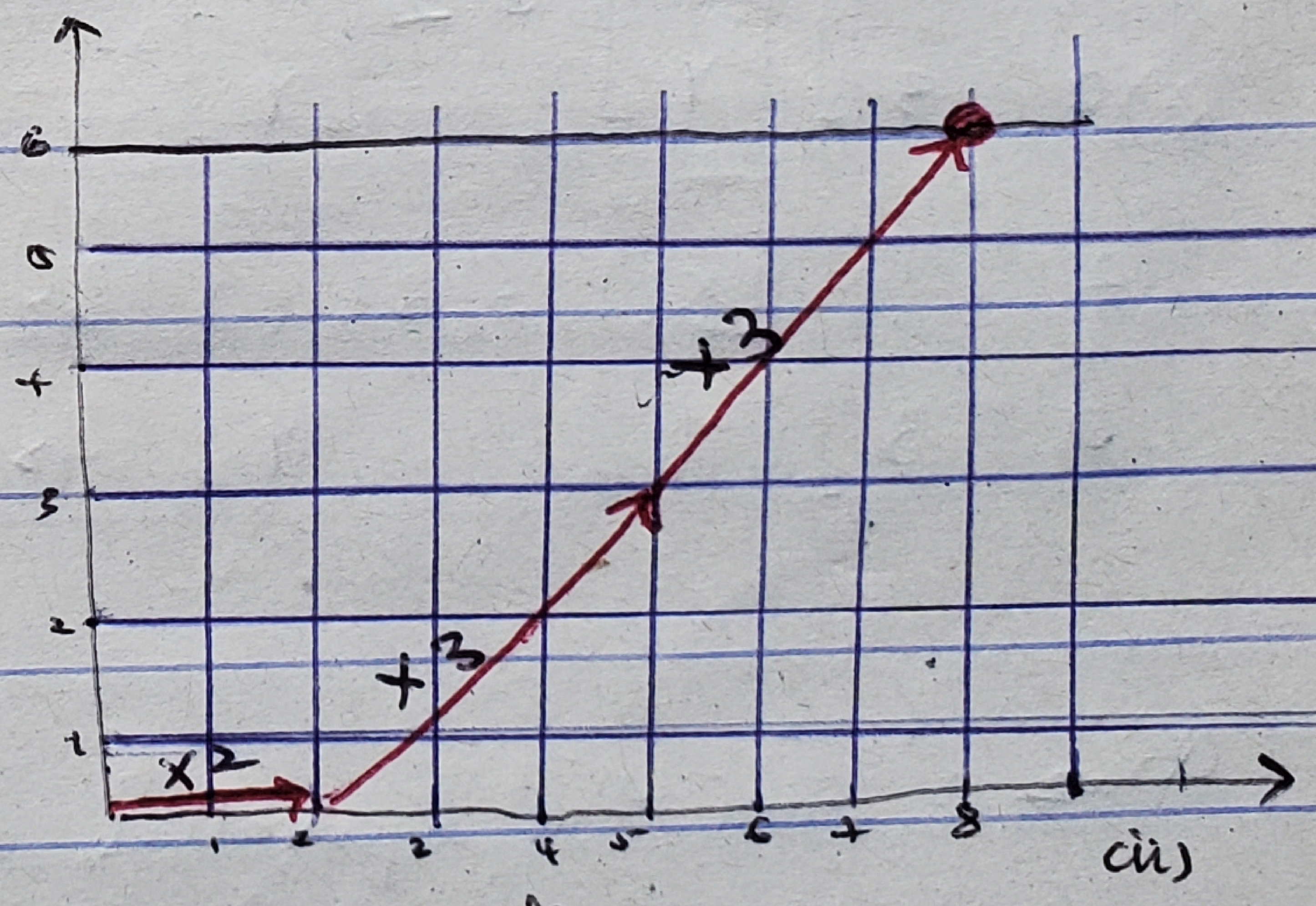


= Plane (B)

How is the above useful? Let's say you want to find the image of pt (3,2) on plane B ↦ $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$



(3,2) → (8,6)



These points can also be expressed as combinations of the ~~bases~~ elements of the bases

i.e. finding a path to get to that point using the directions that we have. So we just stretch the vectors in (i) plane A by (2,3) respectively.

∴ Eigen basis = the two vectors in bases i.e (1,0) & (1,1)

∴ Eigen values = the stretching factors the 2 & 3 (eg,3)

Finding EIGENVALUES

<bro, just take it like that!>

if λ is an eigen value: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ for infinitely many (x, y)

[as far as the outputs are $[0,0]$, it has infinitely many solutions]

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \text{has infinitely many solns}$$

$\therefore$ so det is 0 $\Rightarrow$ $\det\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$

Characteristic polynomial $(2-\lambda)(3-\lambda) - 1(0) = 0$

$\therefore$ To find the eigen values, all we look @ are the roots of

$\therefore$ here; $\lambda = 2$ & 3 <Eigen values>

Now to find the Eigen vector;

$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} x \\ y \end{bmatrix}$ $\rightsquigarrow$ $2x + y = 2x$; $\rightsquigarrow$ $0x + 3y = 2y$; $x = 1$, $y = 0$; $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 3\begin{bmatrix} x \\ y \end{bmatrix}$ $\rightsquigarrow$ $2x + y = 3x$; $\rightsquigarrow$ $0x + 3y = 3y$; $x = 1$, $y = 1$; $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Quiz: Find the eigenvalues and eigenvectors of this matrix: $\begin{bmatrix} 9 & 4 \\ 4 & 3 \end{bmatrix}$

$$\begin{bmatrix} 9 & 4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det\begin{bmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{bmatrix} = 0$$

$$(9-\lambda)(3-\lambda) - 4(4) = 0$$

$9(3) - 9\lambda - 3\lambda + \lambda^2 - 16 = 0$

$27 - 12\lambda + \lambda^2 - 16 = 0$

$\lambda^2 - 12\lambda + 11 = 0 \rightsquigarrow \lambda^2 - \lambda - 11\lambda + 11 = 0$

$\lambda(\lambda-1) - 11(\lambda-1) = 0 \rightsquigarrow (\lambda-11)(\lambda-1) = 0$

Eigenvalues : $\lambda = 1$; and $\lambda = 11$

$\begin{bmatrix} 9 & 4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \rightsquigarrow 9x + 4y = x \rightarrow \frac{8x}{-4} = \frac{-4y}{\cancel{-4}}$

$y = -2x$

$4x + 3y = y$

$\frac{\overset{-2}{\cancel{4x}}}{\cancel{-2x}} + \frac{3(-2x)}{\cancel{-2x}} = \frac{\overset{1}{-2x}}{\cancel{-2x}}$ $\quad \begin{cases} 8x + 4y = 0 \\ 4x + 2y = 0 \end{cases} \rightsquigarrow x = 1,\ y = -2$

~~$-2x+3$~~ $= 4$ possible solution $\equiv \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 9-11 & 4 \\ 4 & 3-11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$= \begin{bmatrix} -2 & 4 \\ 4 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightsquigarrow \begin{matrix} -2x + 4y = 0 \\ 4x - 8y = 0 \end{matrix}$

$\hookrightarrow \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ for $\lambda_2 = 11$ $\quad$ possible solut: $x = 2$; $y = 1$

Grade ~~Lesson~~ Quiz :

(1) Find characteristic polynomial of $\begin{bmatrix} 2 & 1 \\ -3 & 6 \end{bmatrix}$

$\det \begin{bmatrix} 2-\lambda & 1 \\ -3 & 6-\lambda \end{bmatrix} = 0 \rightsquigarrow (2-\lambda)(6-\lambda) - (1)(-3) = 0$

$12 - 2\lambda - 6\lambda + \lambda^2 + 3 = 0$

$\lambda^2 - 8\lambda + 15 = 0 \rightsquigarrow \lambda^2 - 3\lambda - 5\lambda + 15 = 0$

$\lambda(\lambda-3) - 5(\lambda-3) = 0 \rightsquigarrow (\lambda-5)(\lambda-3) = 0$

$\lambda_1 = 5$; $\lambda_2 = 3$

(i) $\begin{bmatrix} 2-5 & 1 \\ -3 & 6-5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad$ | $\quad$ $\begin{bmatrix} 2-3 & 1 \\ -3 & 6-3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} -3 & 1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad$ | $\quad$ $\begin{bmatrix} 1 & 1 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ for $\lambda_1 = 5$ $\quad$ | $\quad$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ for $\lambda_2 = 3$

$\begin{pmatrix} 1 \\ 3 \end{pmatrix} ; \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Find eigenvalues

$$\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 1-\lambda & 2 \\ 0 & 4-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightsquigarrow (1-\lambda)(4-\lambda) = 0$$

$$\therefore \det \begin{bmatrix} 1-\lambda & 2 \\ 0 & 4-\lambda \end{bmatrix} = 0 \qquad \lambda_1 = 1 \ ; \ \lambda_2 = 4$$

(i) $\begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightsquigarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ for $\lambda_1 = 1$

(ii) $\begin{bmatrix} -3 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightsquigarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ for $\lambda_2 = 4$

Q: Which of the vectors span the matrix: $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 5 \\ 3 & -2 & -1 \end{bmatrix}$

~~(A)~~ $\cancel{\begin{matrix} 4 & 3 & 0 \\ 3 & 4 & -10 \\ 0 & -10 & 1 \end{matrix}}$ ~~(B)~~

$v_1 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} ; v_2 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} ; v_3 = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$

Q) Eigenbasis of $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 0 & 0 \\ 1 & 2-\lambda & 1 \\ -1 & 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det \begin{bmatrix} 2-\lambda & 0 & 0 \\ 1 & 2-\lambda & 1 \\ -1 & 0 & 1-\lambda \end{bmatrix} = 0 \rightsquigarrow$$

$$\rightsquigarrow (2-\lambda) \begin{vmatrix} 2-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} - 0 + 0 = 0 \rightsquigarrow (2-\lambda)(2-\lambda)(1-\lambda)$$

$$\lambda_1 = 2, \ \lambda_2 = 2, \ \lambda_3 = 1$$

for $\lambda_1 = 2$ $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightsquigarrow$ $x + z = 0$

$-x - z = 0$

$\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

for $\lambda_2 = 2$ $\rightsquigarrow \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

for $\lambda_3 = 1$ $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow$ $\cancel{x = 0}$, $\cancel{x + y + z = 0}$, $\cancel{-x = 0}$

$x + 0y + 0z = 0$

$x + y + z = 0$ $\approx \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

$-x + 0y + 0z = 0$

Q: Find characteristic polynomial for $\begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

$\det \begin{bmatrix} 3-\lambda & 1 & -2 \\ 4 & -\lambda & 1 \\ 2 & 1 & -1-\lambda \end{bmatrix} = (3-\lambda) \begin{vmatrix} -\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} - 1 \begin{vmatrix} 4 & 1 \\ 2 & -1-\lambda \end{vmatrix} - 2 \begin{vmatrix} 4 & -\lambda \\ 2 & 1 \end{vmatrix} = 0$

$\cancel{0 = (3-\lambda)(-1) - [4(-1-\lambda) - 2] - 2(4) = \lambda - 3 - (-4 - 4\lambda - 2) - 8}$

$\cancel{0 = \lambda - 3 + 4 + 4\lambda + 2 - 8 \Rightarrow \lambda + 4\lambda - 3 + 4 + 2 - 8}$

$\cancel{0 = 5\lambda + 5}$ $\approx -\lambda^3 + 2\lambda^2 + 4\lambda - 5$

CALCULUS FOR MACHINE LEARNING & DATA SCIENCE

WK 1 DERIVATIVES

- Derivatives are important in Machine Learning because they are used to maximise & minimize (i.e optimize) functions.

- Instantaneous velocity: an example of a derivative
  - A derivative is the instantaneous rate of change of a function. Here the function is distance and its derivative is velocity

12 155 * The derivative of a function @ a point = the slope of the tangent @ that particular point.
13 170

- The maxima or minima of a function occur @ one of the points where the derivative is 0

Approximation of instant velocity $= \frac{\Delta x}{\Delta t}$; Exact instantaneous velocity $= \frac{dx}{dt}$