Logistic Regression Introduction

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Statistics with Python Course Developer

Cartwheel Data

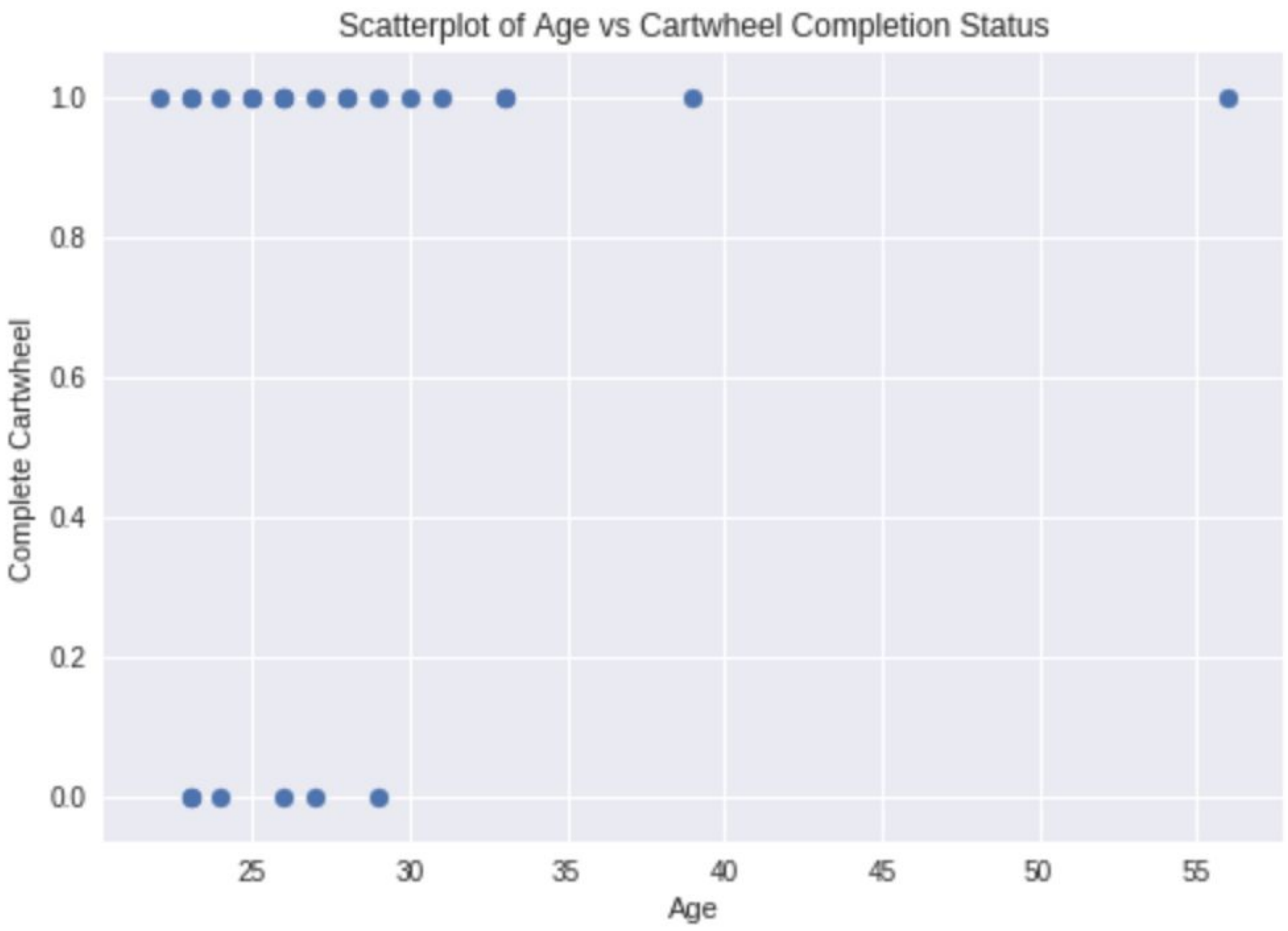
Random sample of 25 adults attempted cartwheels

Primary Variable of interest: Cartwheel completion

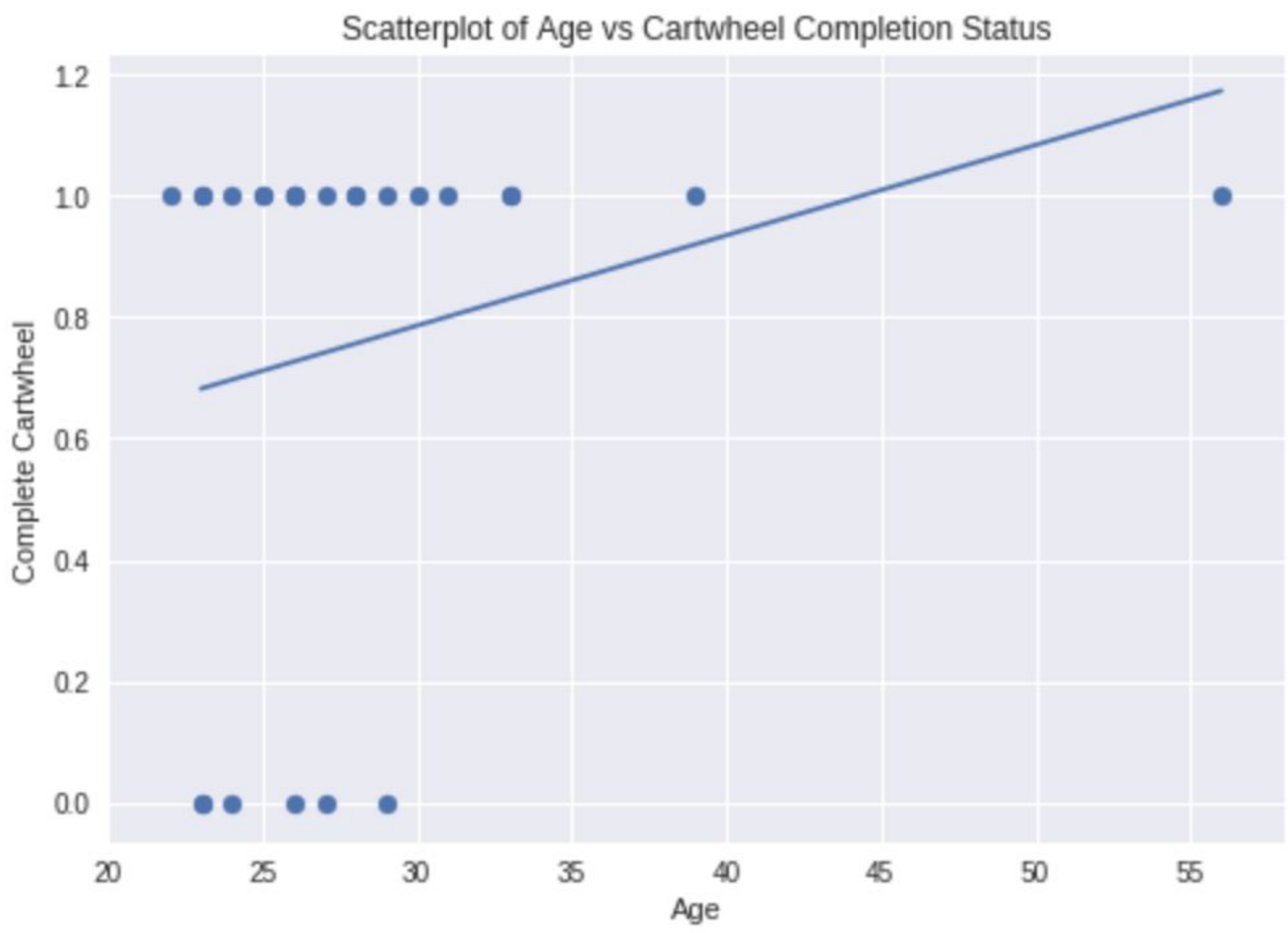
Research Question

Based on age, can we predict whether a cartwheel is completed?

Let's Look at the Data



Linear Model



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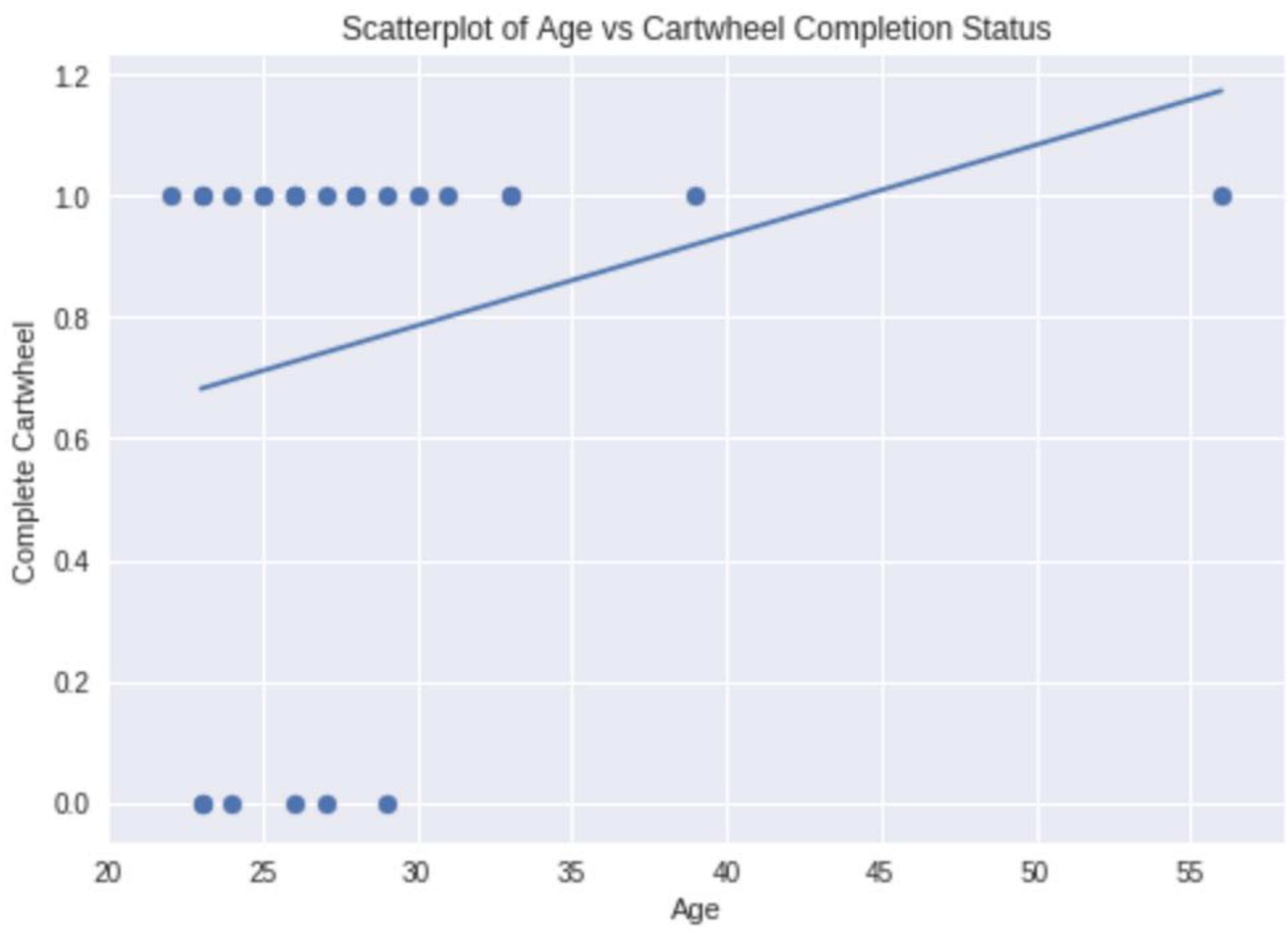

$\hat{y} = 0.34 + 0.015 \text{ age}$

Logit Transformation

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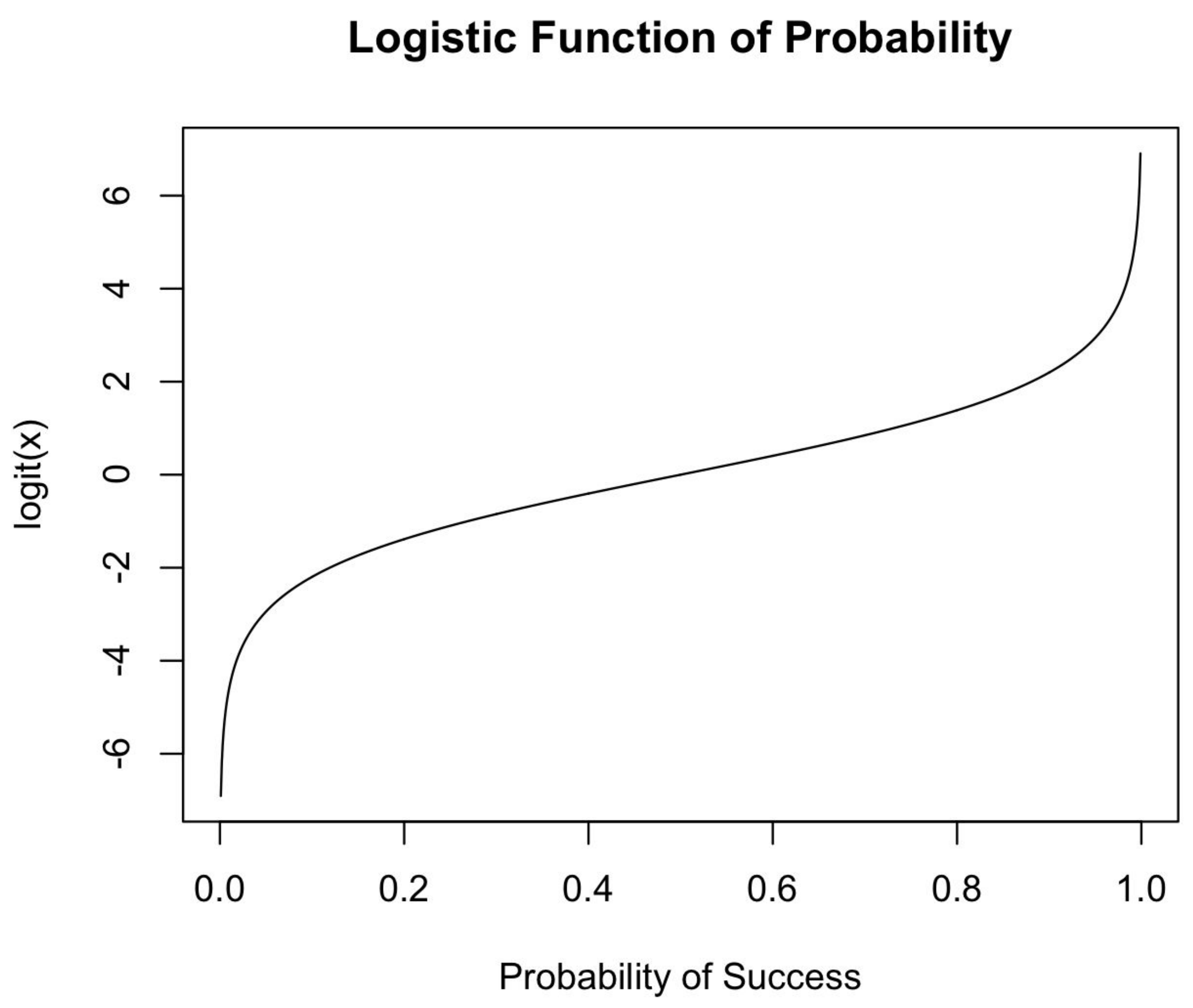

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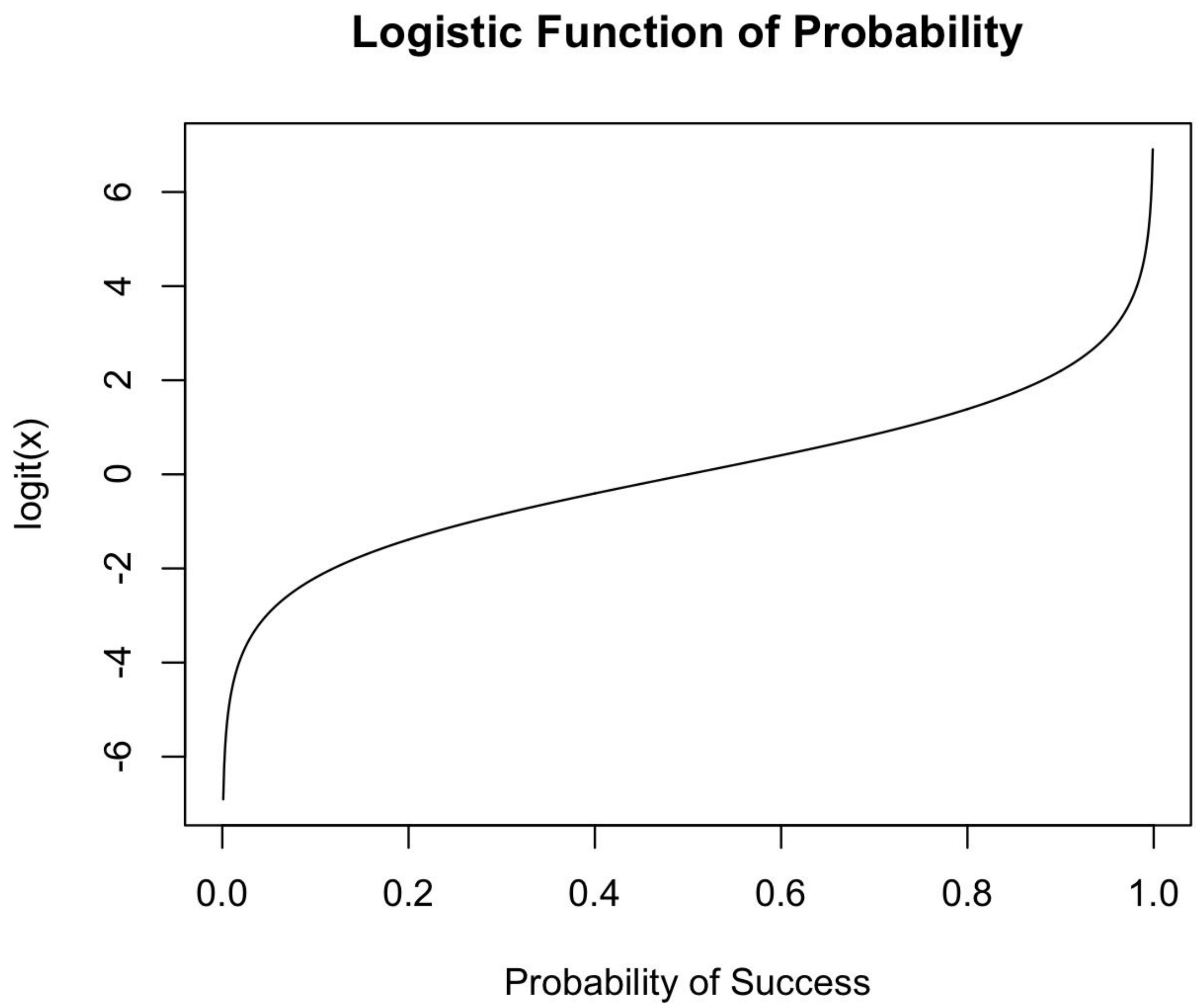
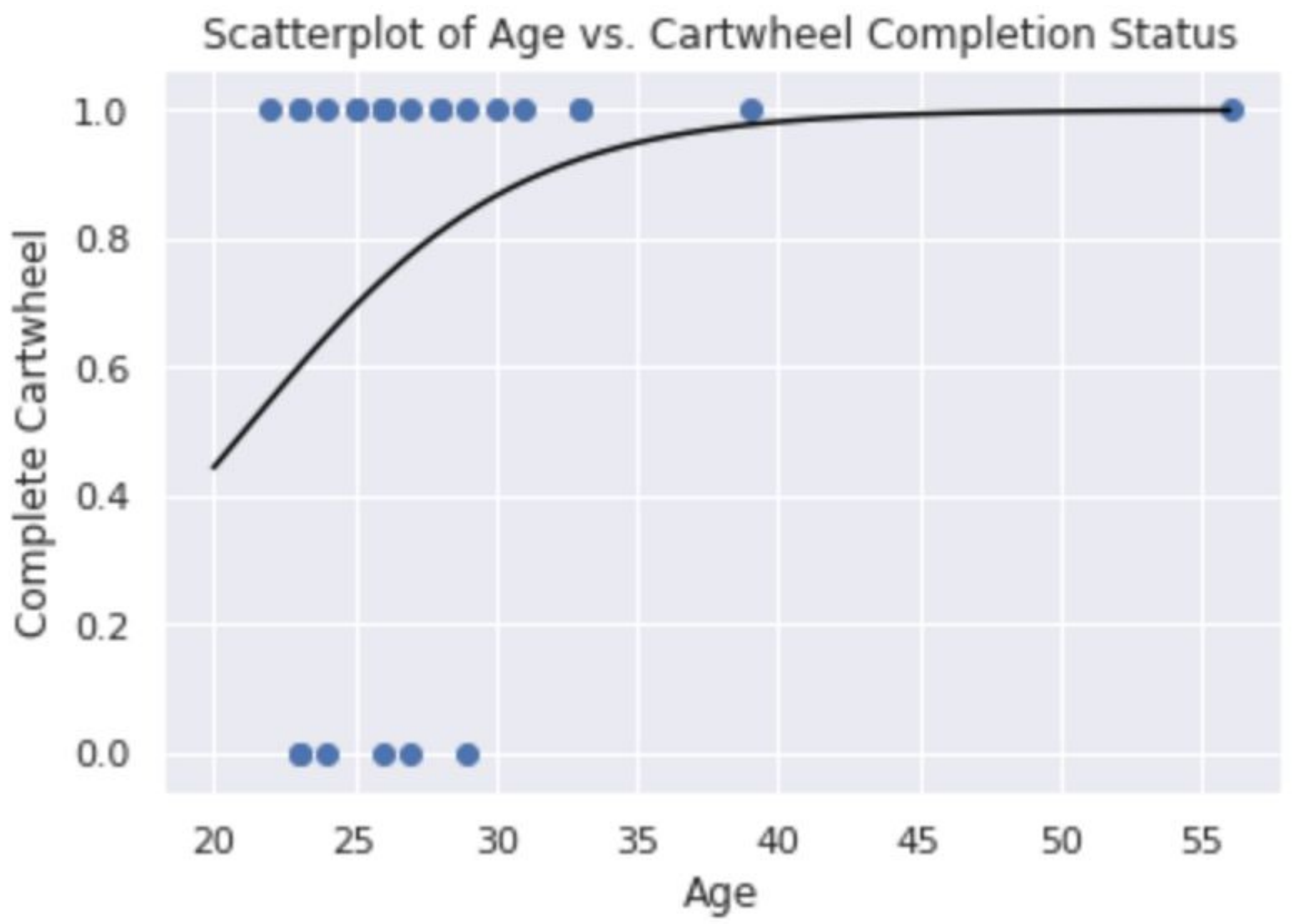
- Instead of predicting completion status, we predict a ***transformed version*** of the probability of a success
- Uses the logit function: $\ln(\frac{p}{1-p})$
- $\text{logit}(\hat{y}) = b_0 + b_1 x$



Logistic Regression Line



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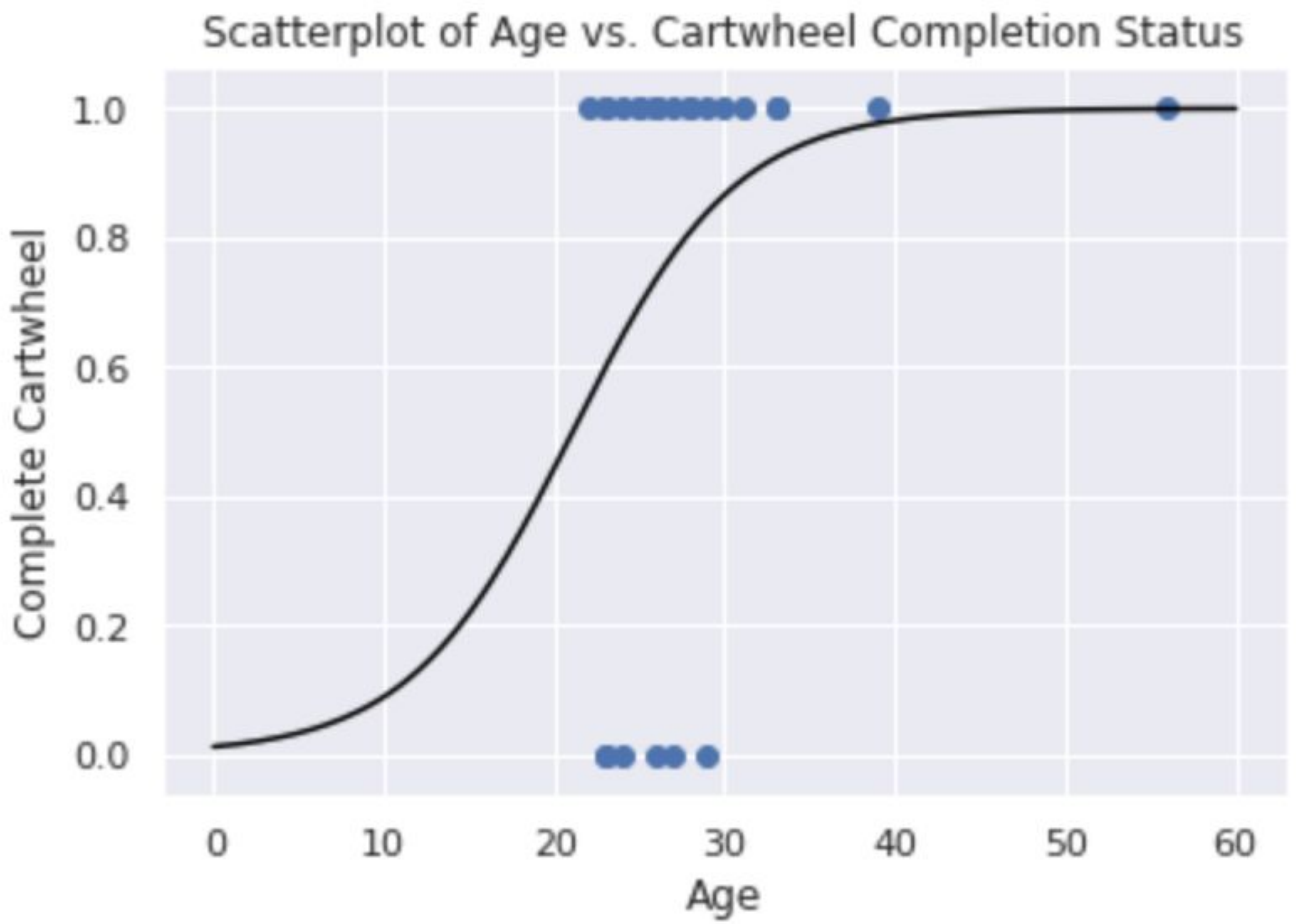

Extrapolation IVQ

Would you feel comfortable using this model to estimate the probability that a teenager who is 15 can complete a cartwheel?

Logistic Regression Equation

Generalized Linear Model Regression Results

Dep. Variable:	CompleteGroup	**No. Observations:**	25
Model:	GLM	**Df Residuals:**	23
Model Family:	Binomial	**Df Model:**	1
Link Function:	logit	**Scale:**	1.0

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
Intercept	-4.4213	4.429	-0.998	0.318	-13.101	4.259
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Slope interpretation:
For each increase in age by 1 year, the log odds of a successful cartwheel increases by about 0.2096, on average.

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Logistic Regression Equation

$$\text{logit}(\hat{y}) = -4.42 + 0.2096 \text{ age}$$

Slope interpretation: For each year increase in age, the odds of a successful cartwheel increases by about 1.23 ($e^{0.2096}$) times that of the younger age, on average.

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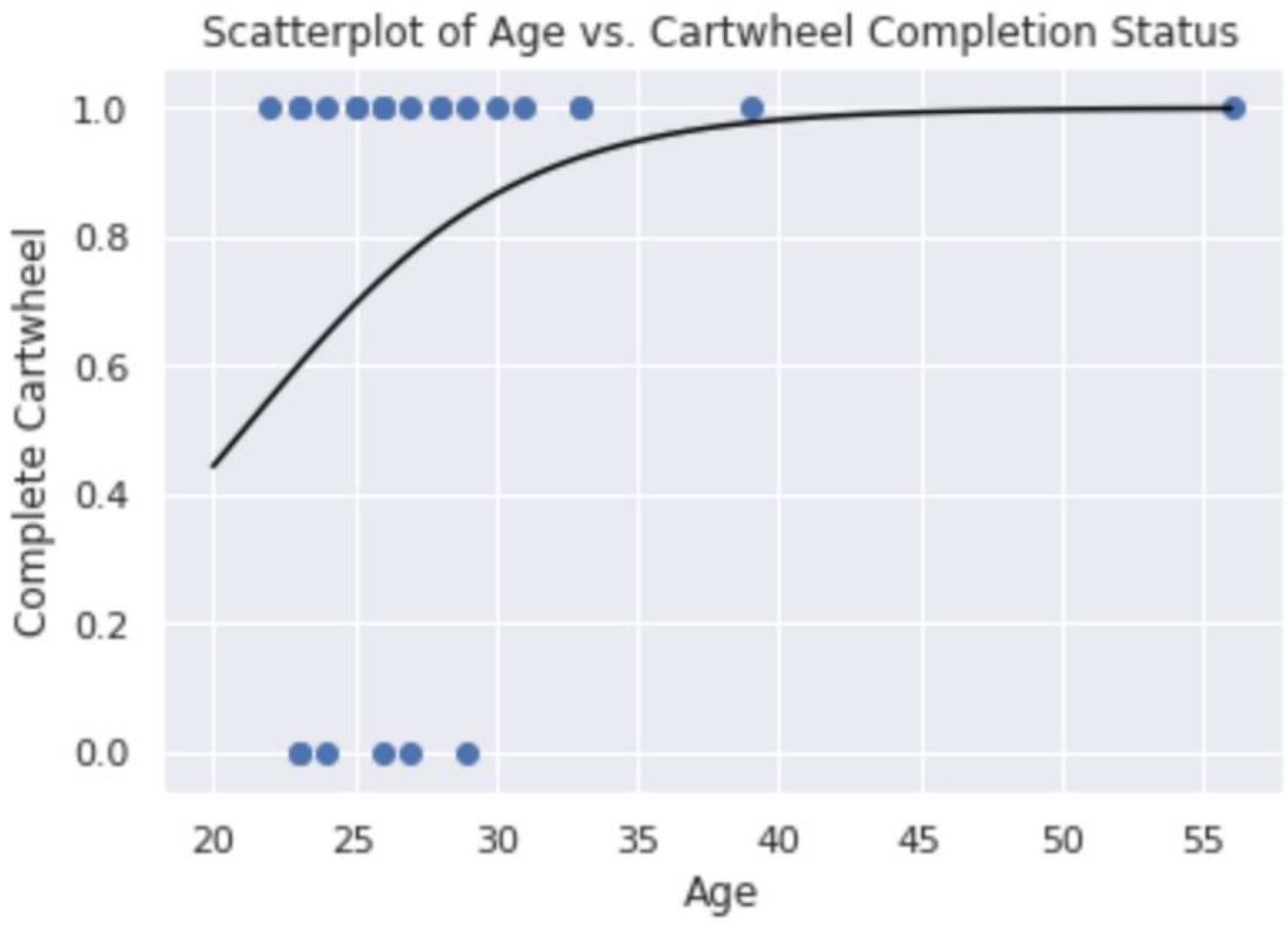
$$\text{logit}(\hat{y}) = -4.42 + 0.2096 \text{ age}$$
$$= -4.42 + 0.2096\ (36)$$
$$= 3.13$$

Predicted Probability of Success

- For someone who is 36, what is their predicted log odds of a successful cartwheel using the model?
- Using the graph on the right, estimate what the probability of success might be?



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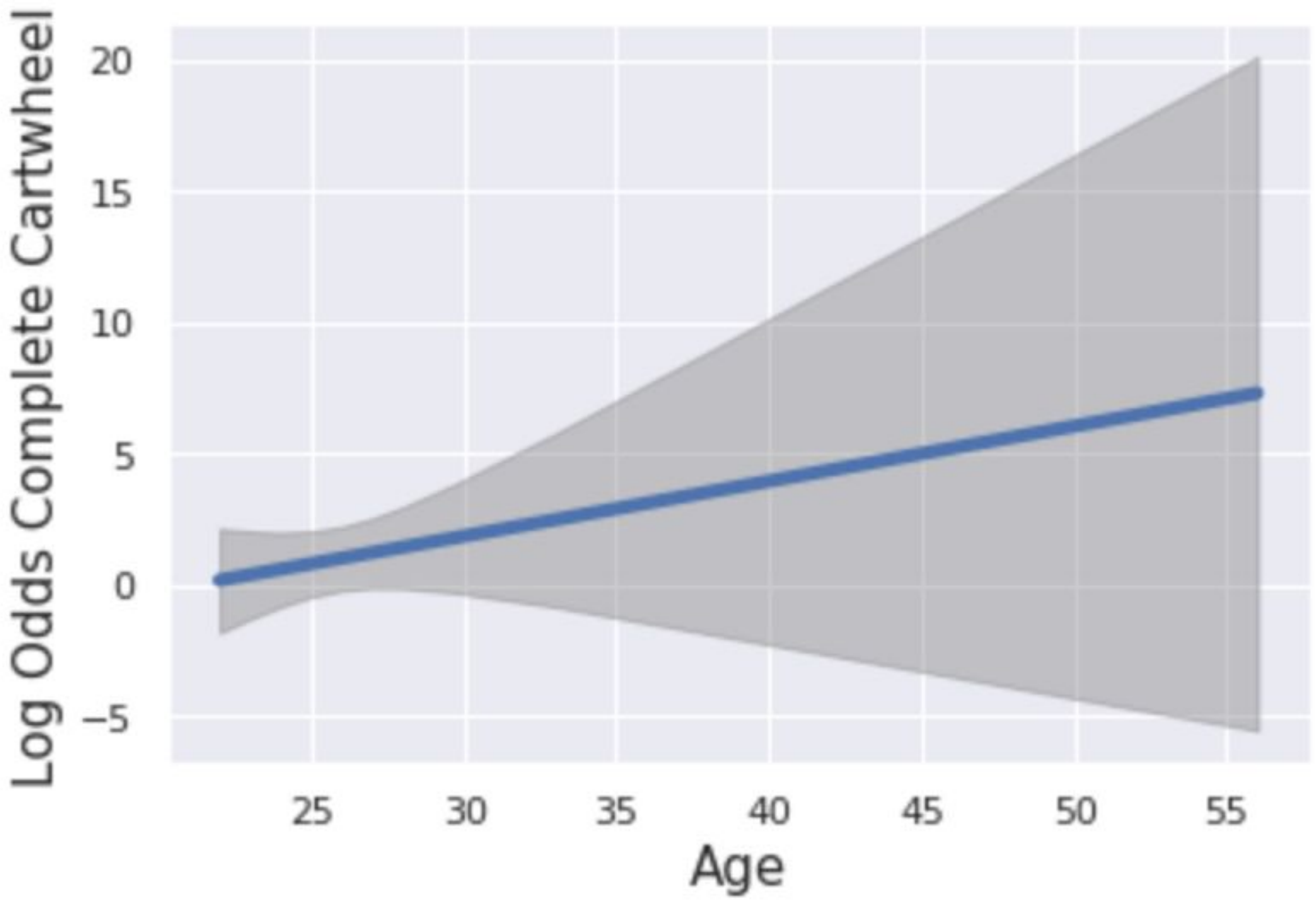

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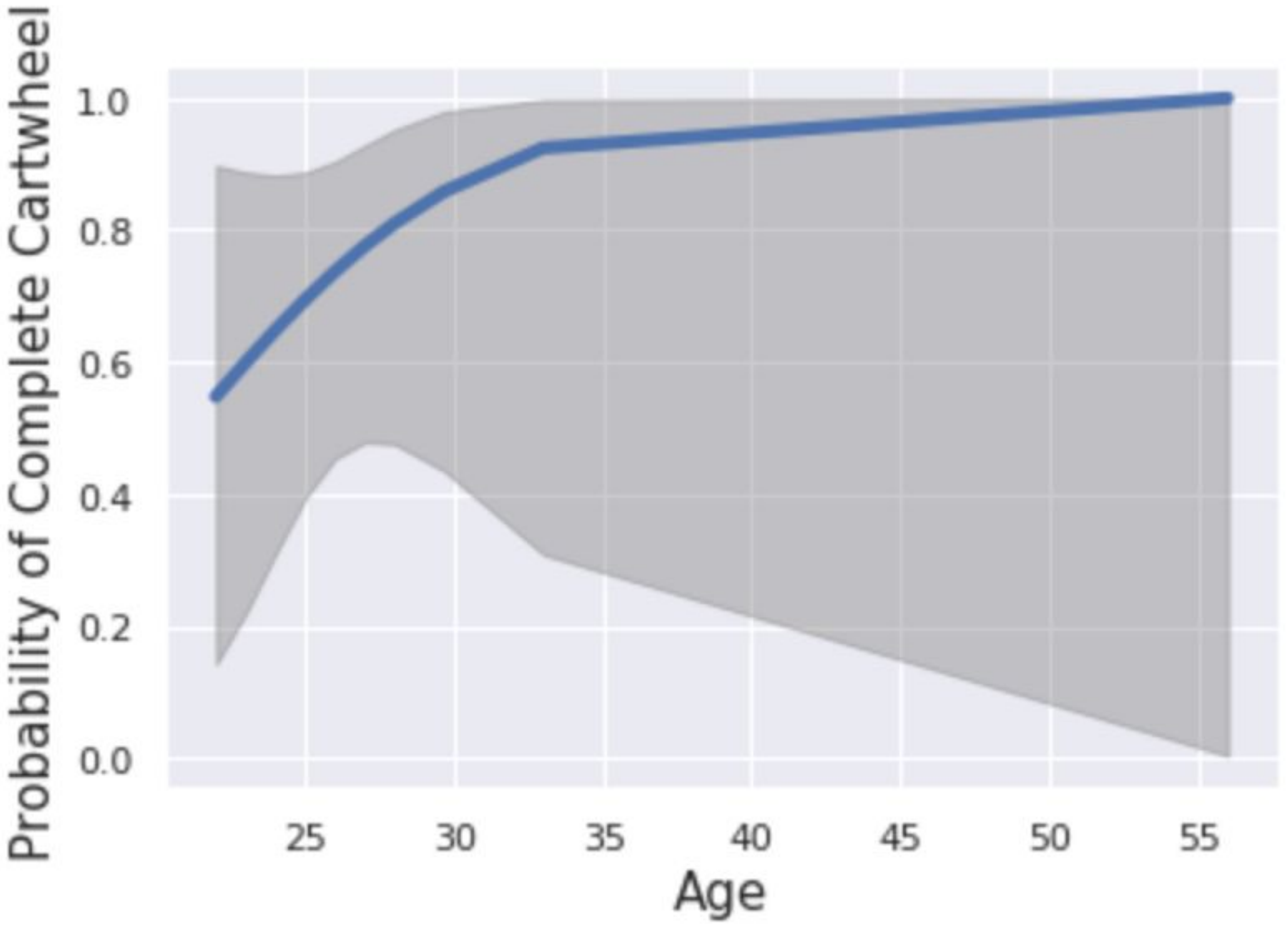

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- ~with a large enough sample size, you can identify discrepancies with residual plots
- ~y only takes two values, so residuals can be limited
- ~to create informative residual plots, it helps if x takes a wide range of values and to have additional covariates