Linear Regression Introduction

Brenda Gunderson

Cartwheel Study

- 25 team members/colleagues (all adults) asked to perform a cartwheel
- **Many Variables recorded**:
  Primary outcome of interest = Cartwheel Distance (inches)

Cartwheel Study Data

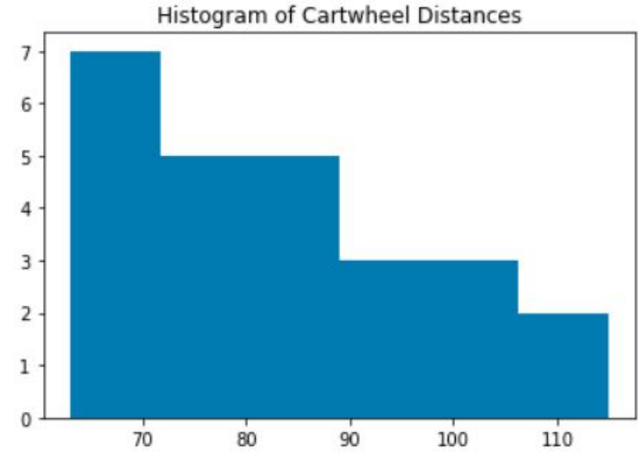
	ID	Age	Gender	GenderGroup	Glasses	GlassesGroup	Height	Wingspar	CWDistance	Complete	CompleteGroup	Score
0	1	56	F	1	Y	1	62.0	61.0	79	Y	1	7
1	2	26	F	1	Y	1	62.0	60.0	70	Y	1	8
2	3	33	F	1	Y	1	66.0	64.0	85	Y	1	7
3	4	39	F	1	N	0	64.0	63.0	87	Y	1	10
4	5	27	M	2	N	0	73.0	75.0	72	N	0	4

Possible Research Goals/Questions

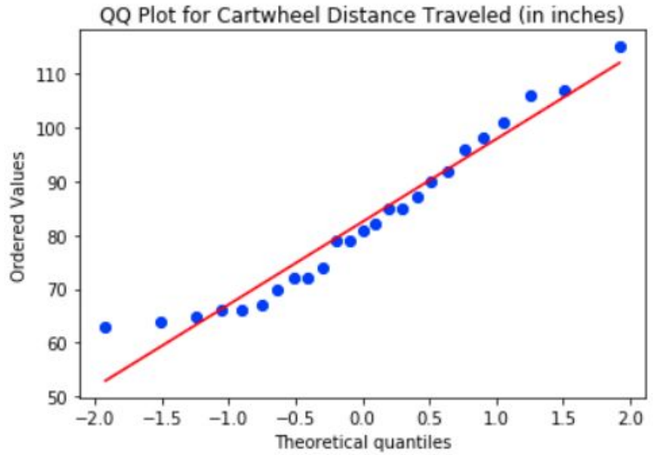
Develop a model to predict the (mean) cartwheel distance for the population of all such adults...

- Is a person's height a useful predictor for cartwheel distance?
- Does knowing if they actually *completed* the cartwheel make a difference in terms of cartwheel distance?

Cartwheel Distance Summary



Cartwheel Distance Traveled (in inches)



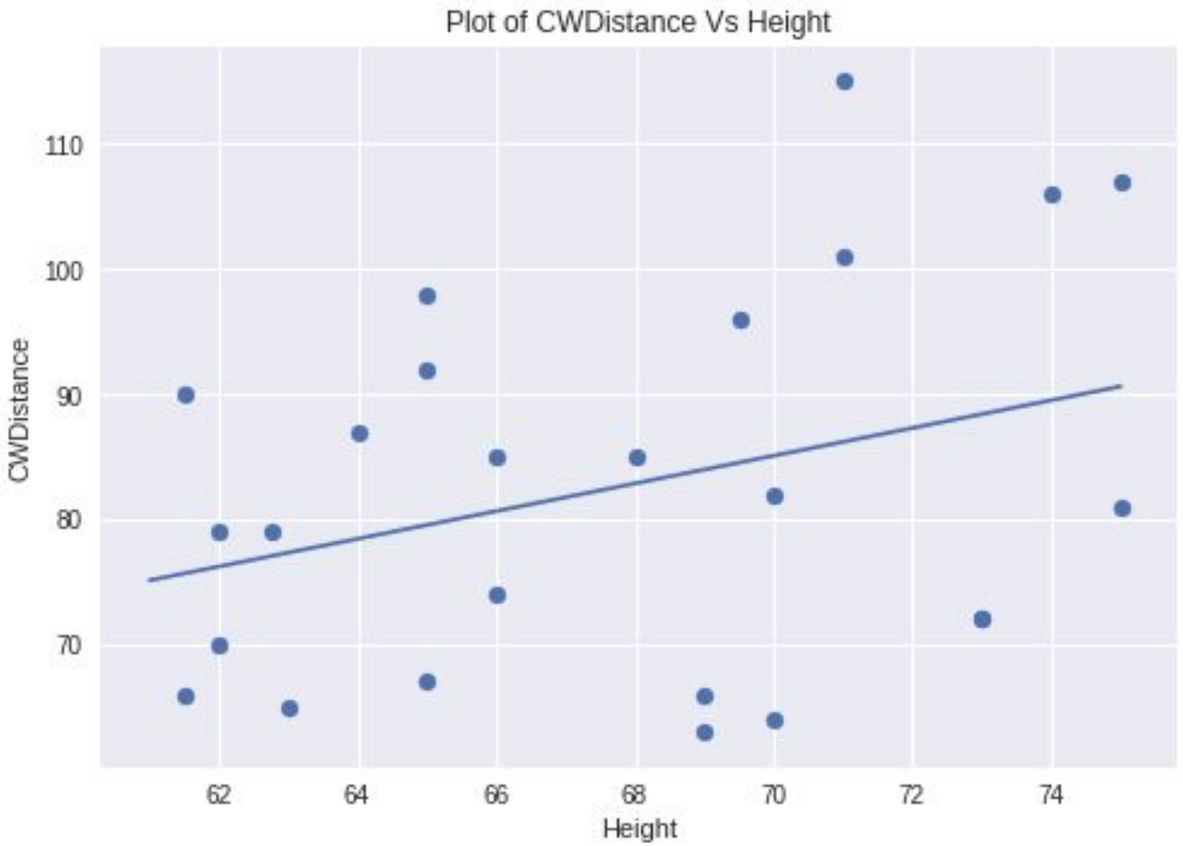

```
df.describe()["CWDistance"]
```

```
count     25.000000
mean      82.480000
std       15.058552
min       63.000000
25%       70.000000
50%       81.000000
75%       92.000000
max      115.000000
Name: CWDistance, dtype: float64
```

Is there a Relationship?

- **Is HEIGHT a useful predictor for cartwheel distance?**
- **Do taller people generally have larger cartwheel distances?**
- **Is there a significant (positive) relationship between the height and cartwheel distance?**

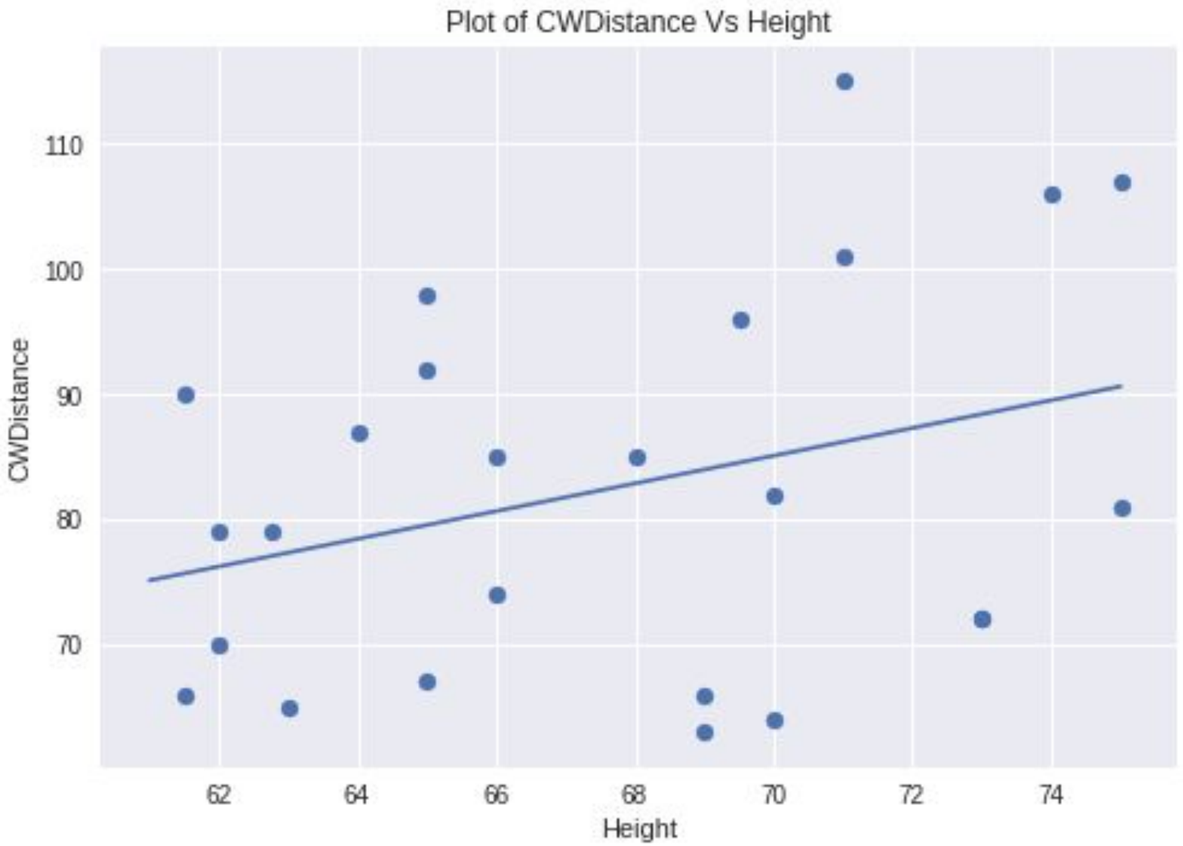
Visualizing the Relationship



Dependent Variable (DV)
= CWDistance

Independent Variable (IV)
= Height

Visualizing the Relationship



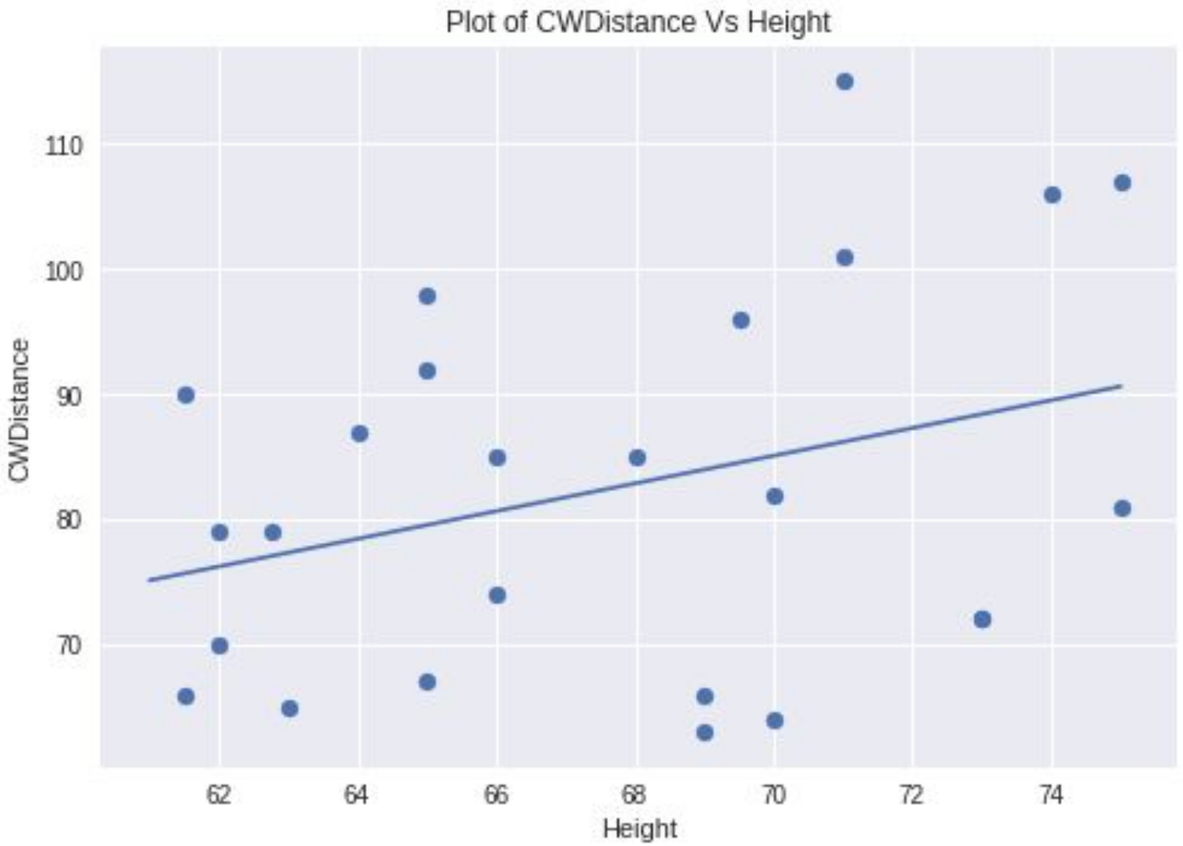
Dependent Variable (DV) = CWDistance

Independent Variable (IV) = Height

- **Form**: _____
- **Direction**: _____
- **Strength**: _____
- **Outliers**: _____

PAUSE HERE to provide time for IVQ

Visualizing the Relationship

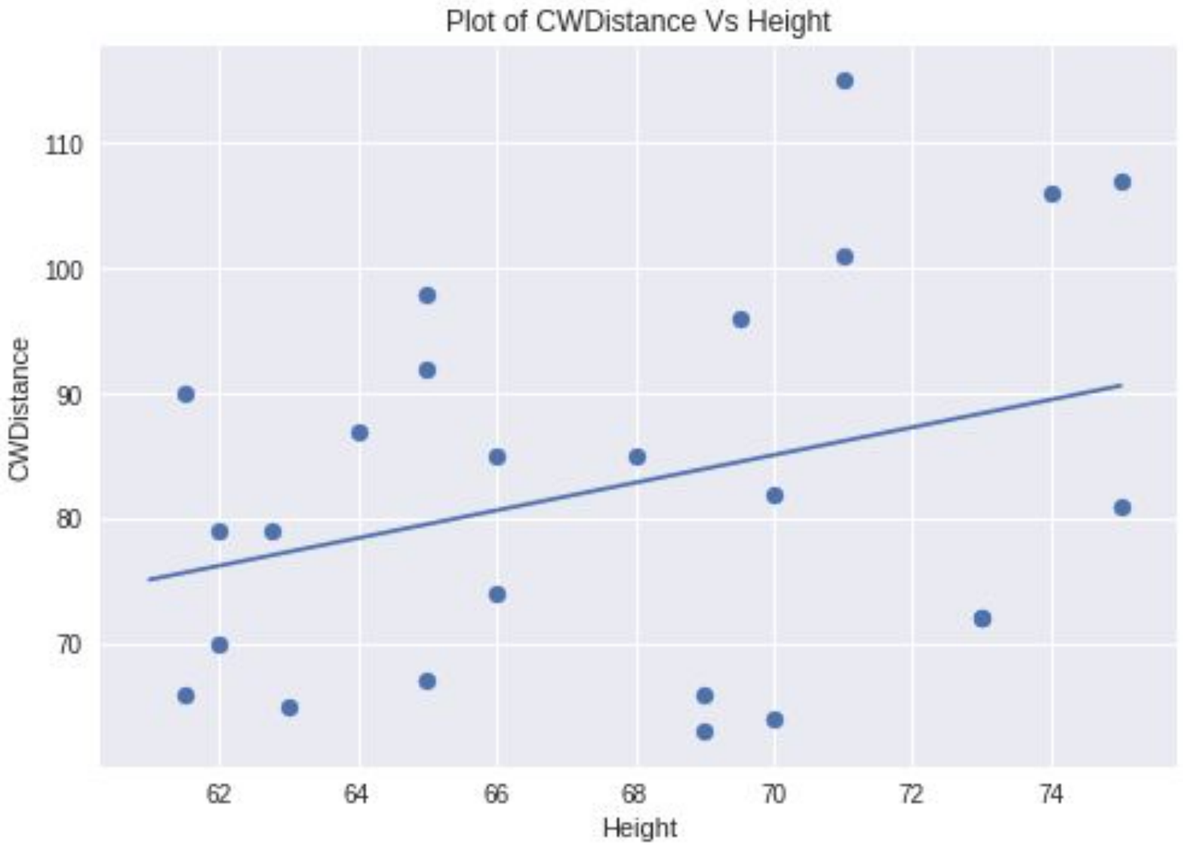


Dependent Variable (DV) = CWDistance

Independent Variable (IV) = Height

- **Form**: approximately linear
- **Direction**: positive
- **Strength**: weak to moderate
- **Outliers**: none apparent

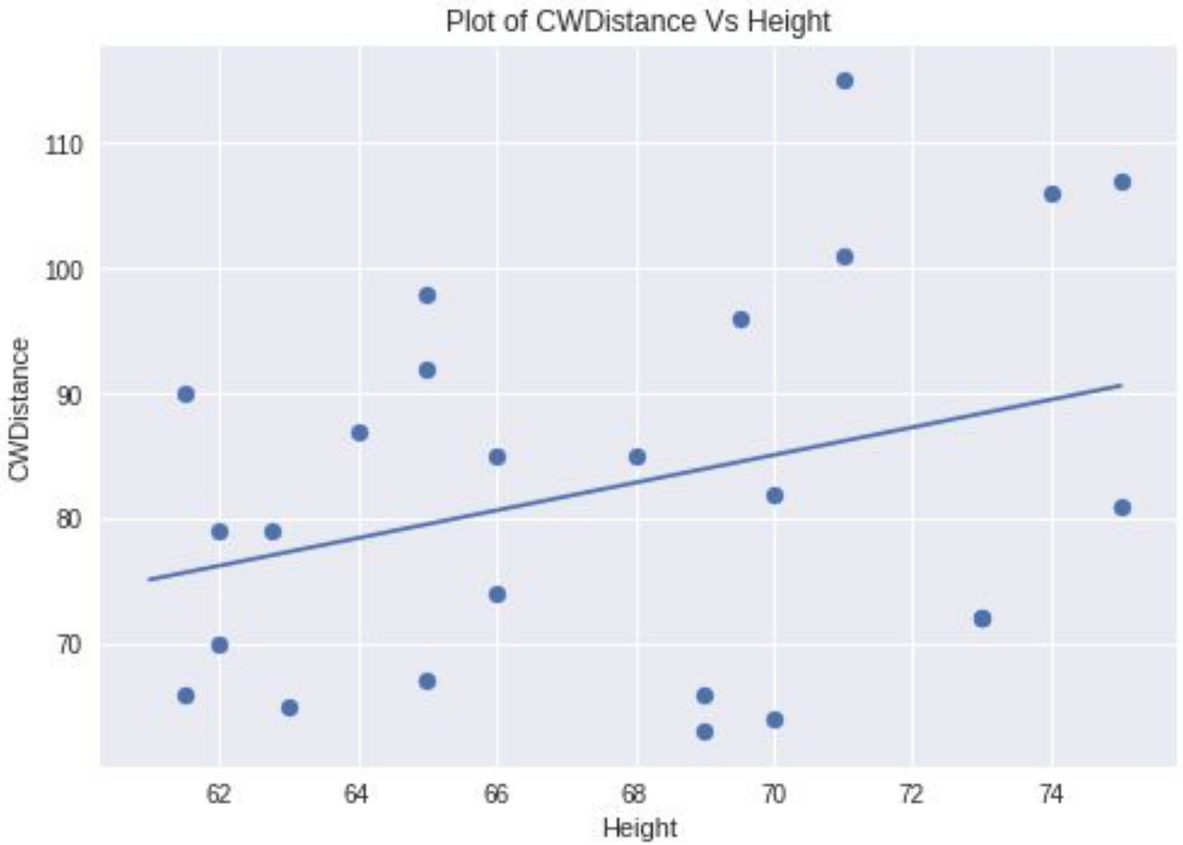
Visualizing the Relationship



- Strength:

  $r = 0.33$

Visualizing the Relationship



- Strength:

  $r = 0.33$

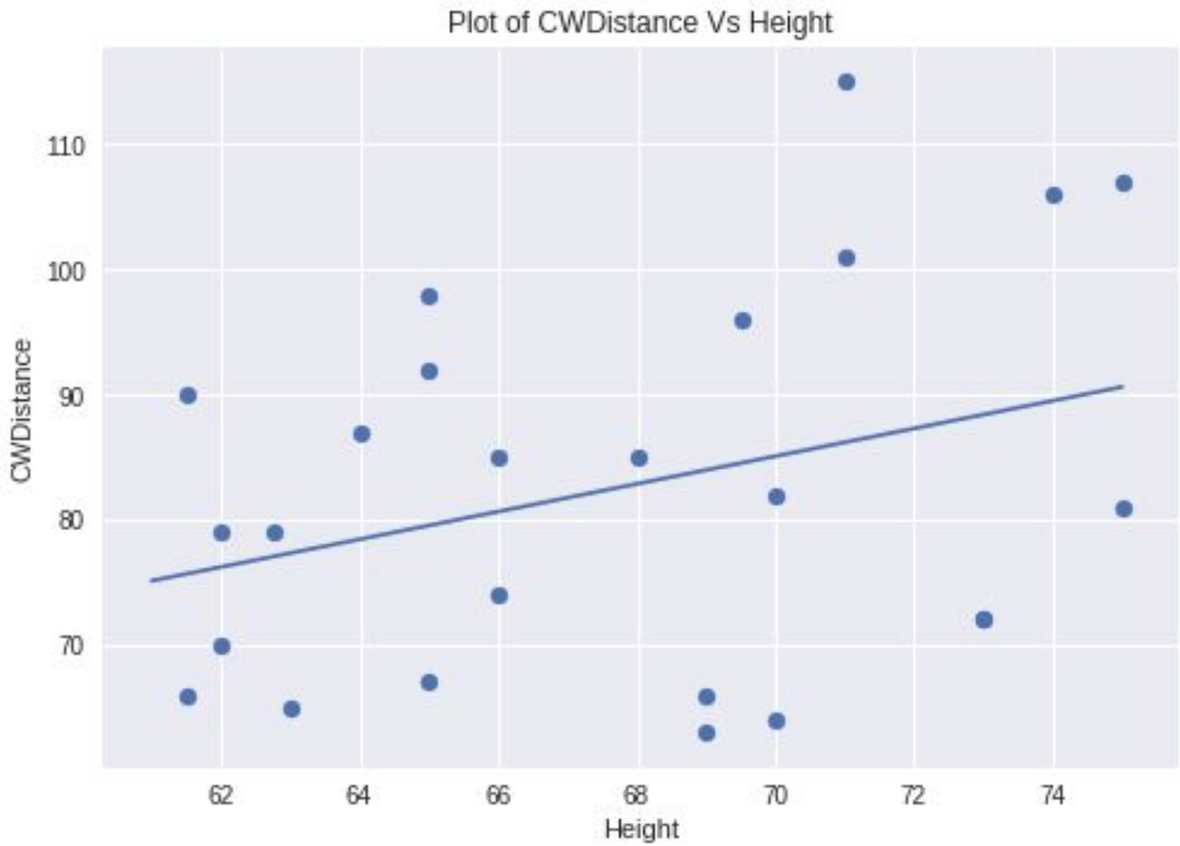
  $r^2 = 0.107$

Only about 11% of the variation in CW Distance is explained by the linear relationship with height

Best Fitting Line

- General Line:

$$y = mx + b$$



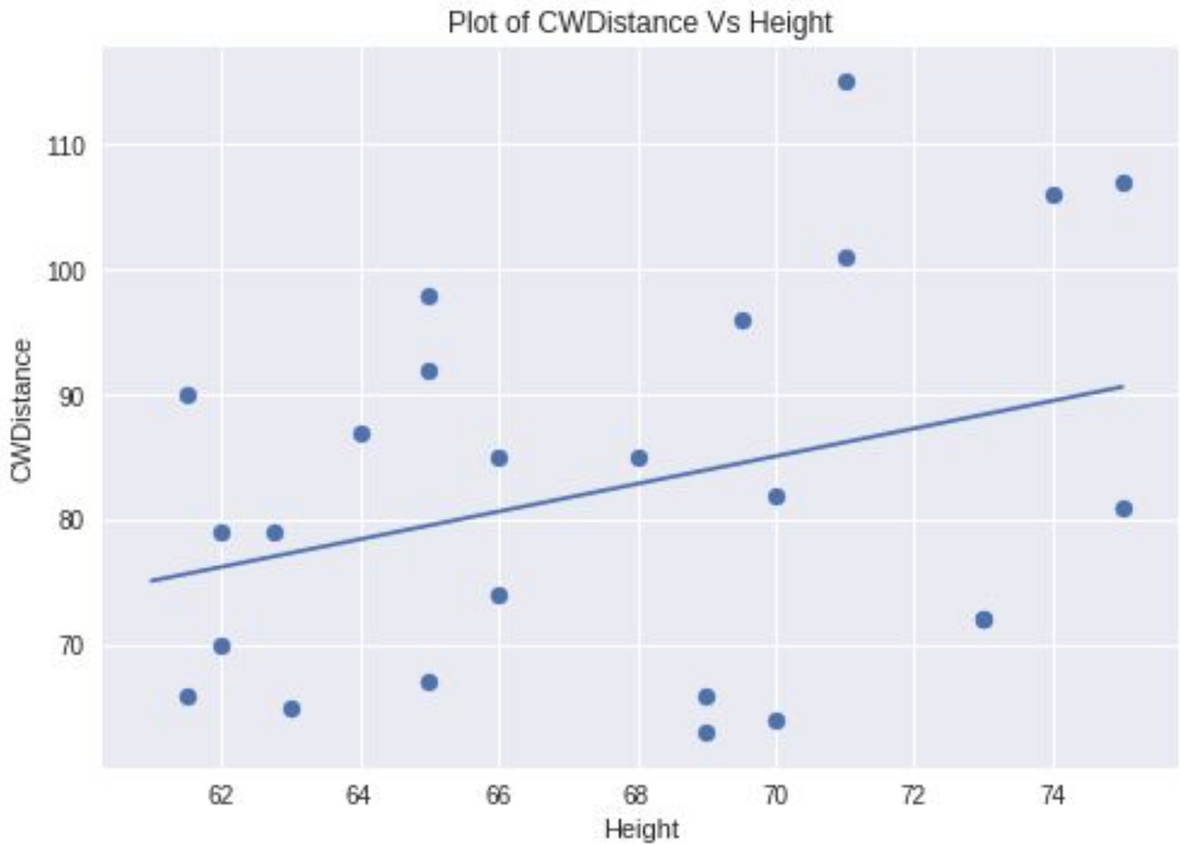
Best Fitting Line

- General Line:

$$y = mx + b$$

- Estimate Regression Line:

$$\hat{y} = b_0 + b_1 x$$



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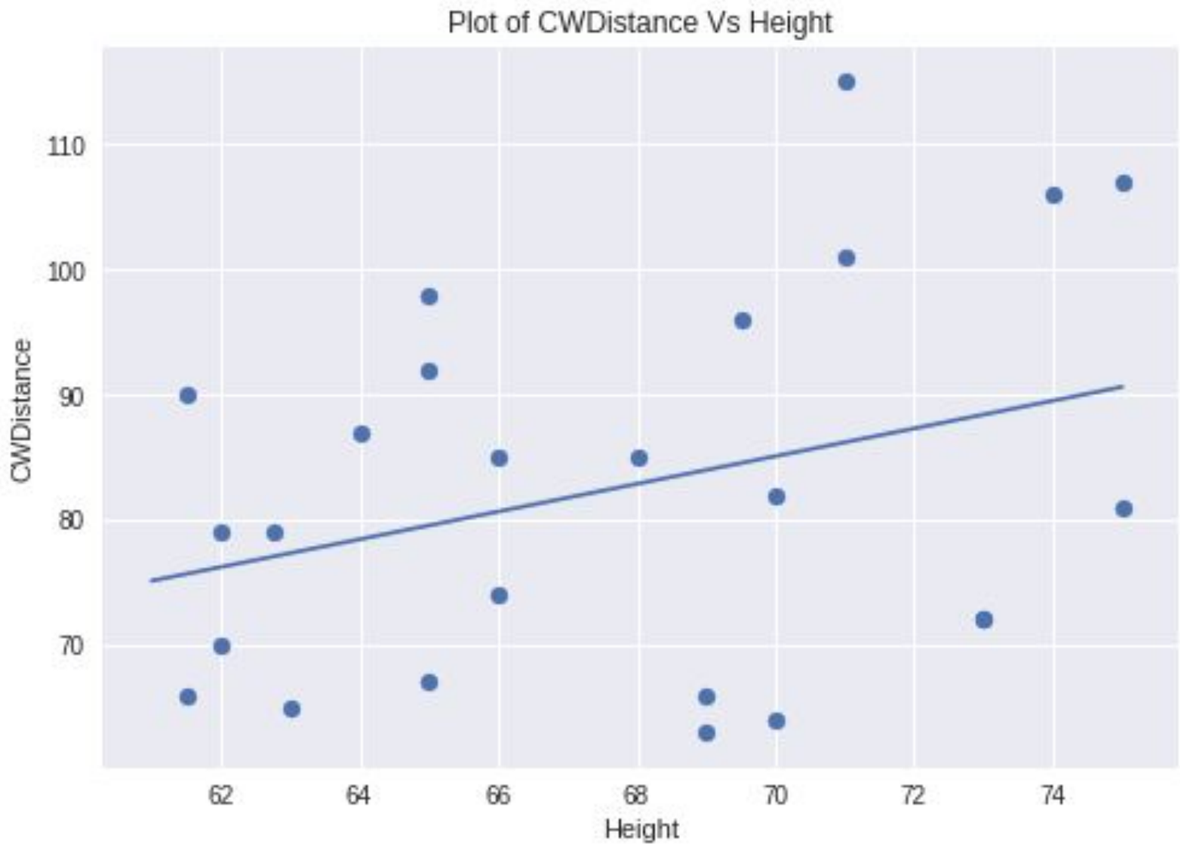

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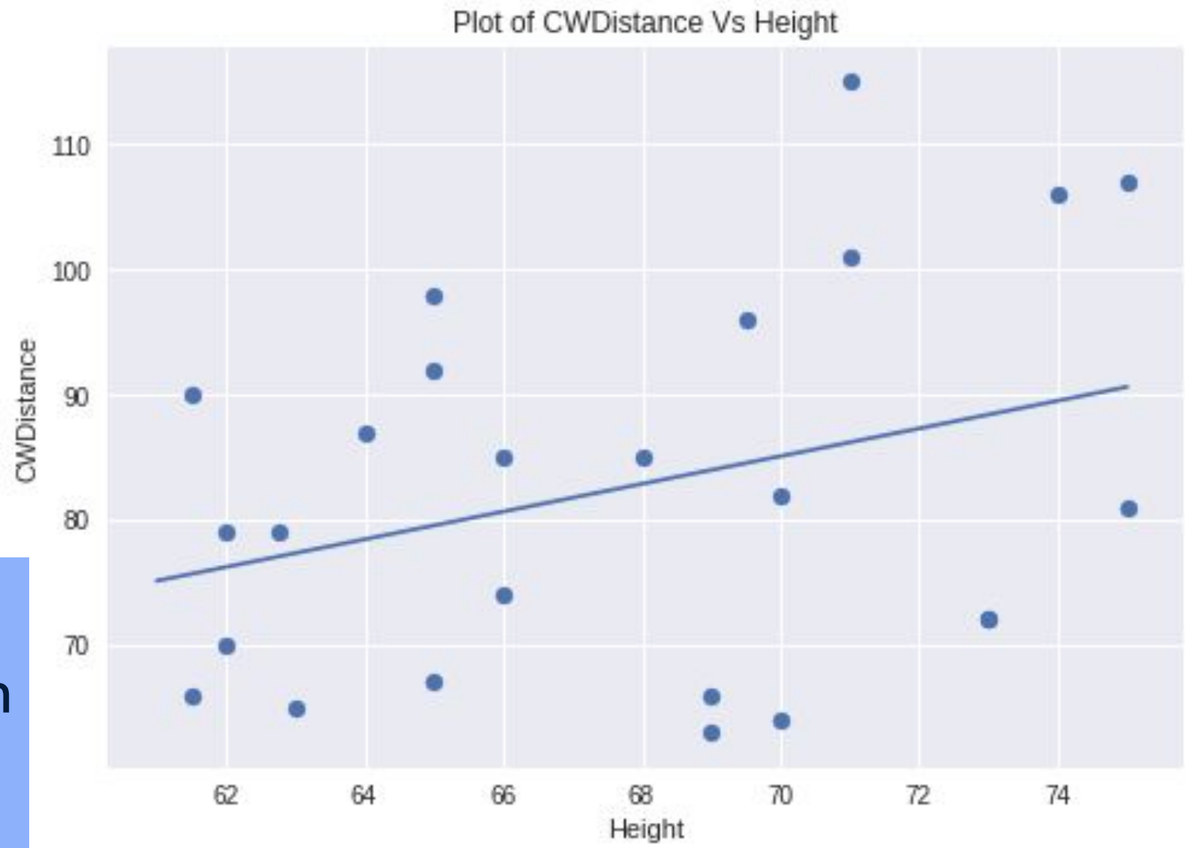
slope: estimated change in y for one unit increase in x



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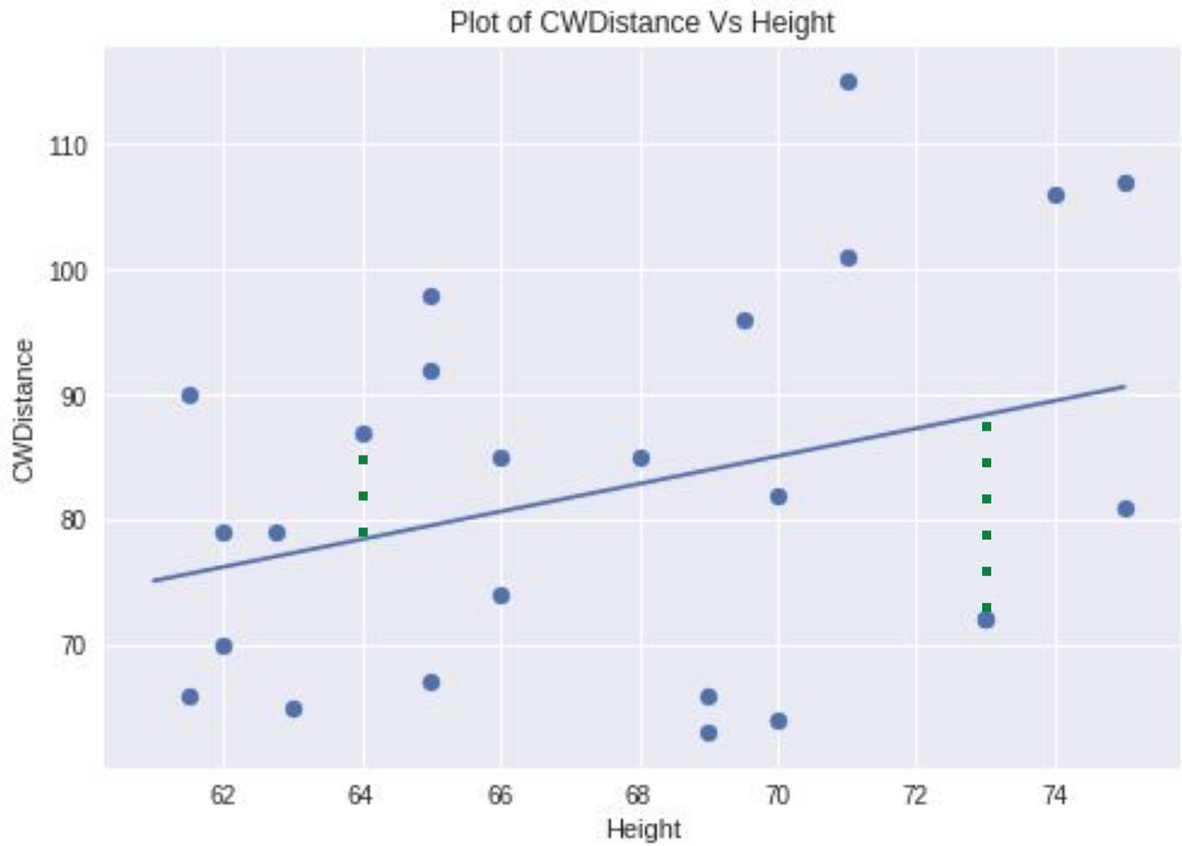

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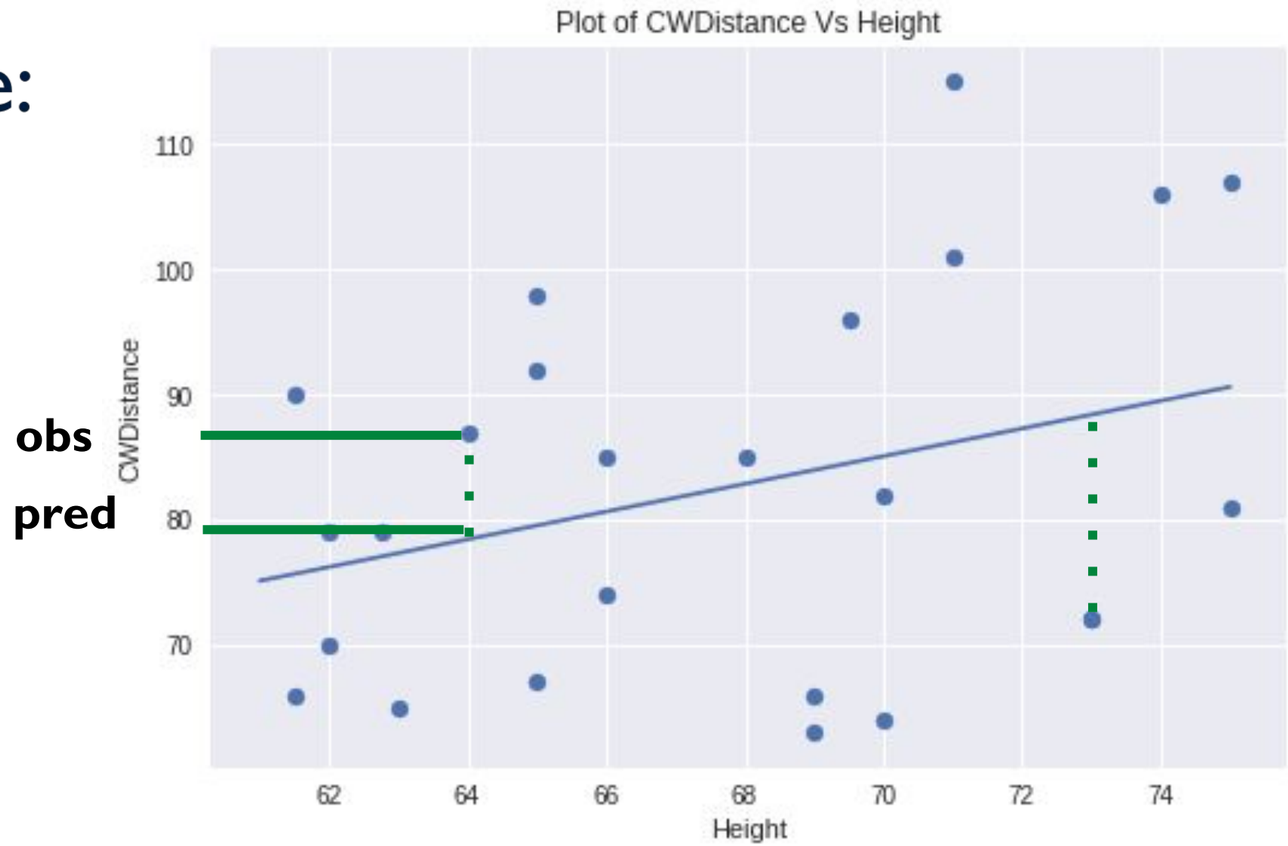

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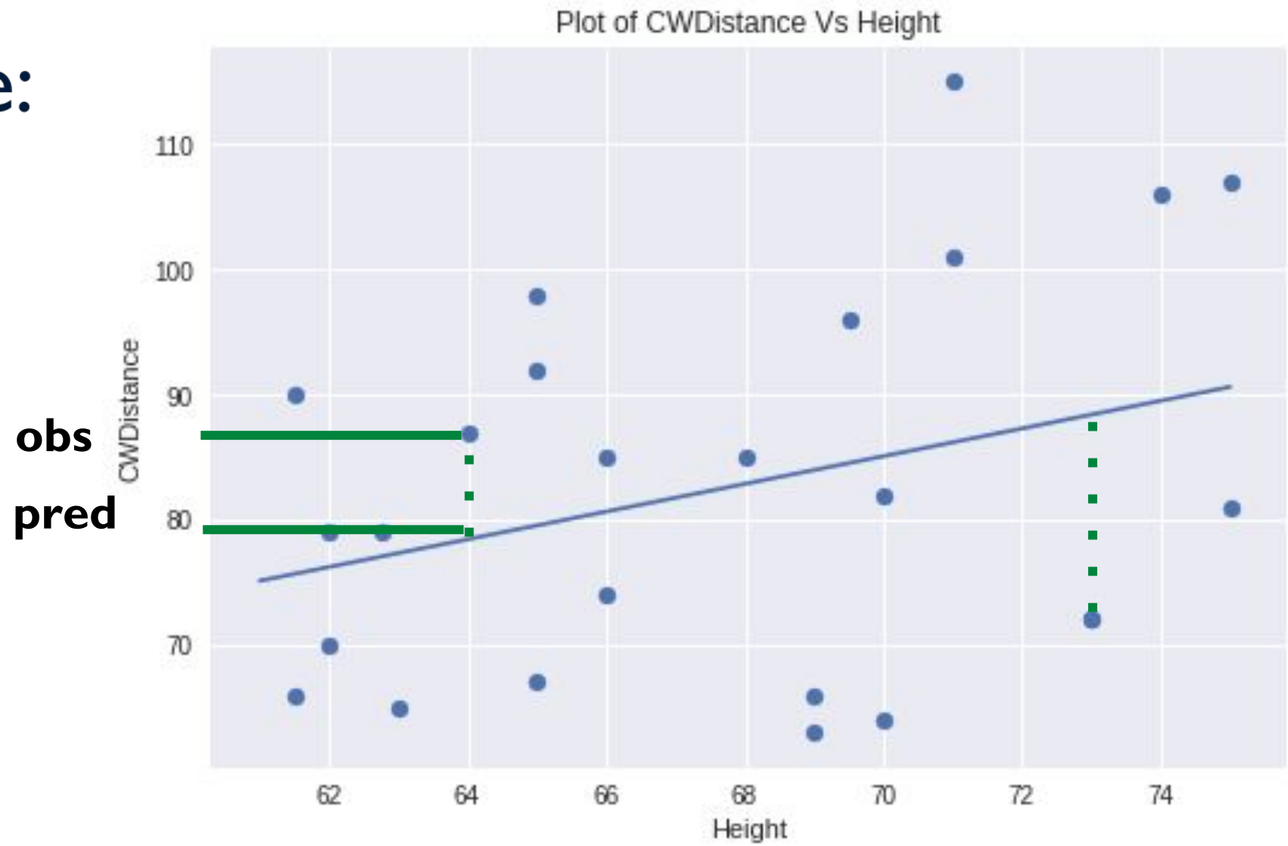
Goal:
Find line that minimizes total squared (observed) error
Least Squares Regression



Best Fitting Line

Predicted CWDist
= 7.5518 + 1.1076(height)

OLS Regression Results

Dep. Variable:	CWDistance	R-squared:	0.106
Model:	OLS	Adj. R-squared:	0.067
Method:	Least Squares	F-statistic:	2.734
Date:	Mon, 26 Nov 2018	Prob (F-statistic):	0.112
Time:	05:06:55	Log-Likelihood:	-101.36
No. Observations:	25	AIC:	206.7
Df Residuals:	23	BIC:	209.2
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	7.5518	45.412	0.166	0.869	-86.391	101.494
Height	1.1076	0.670	1.653	0.112	-0.278	2.493

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slope: estimated change in y for one unit increase in x

We would estimate that an adult who is **one inch taller** than another adult would have a **CW distance** that is **1.1 inch longer**, *on average*.

Making Predictions

What would you predict the cartwheel distance to be for an adult who is 64 inches tall?

Predicted CWDist = 7.5518 + 1.1076(height)

PAUSE HERE to provide time for IVQ

Making Predictions

What would you predict the cartwheel distance to be for an adult who is 64 inches tall?

$$\text{Predicted CWDist} = 7.5518 + 1.1076(\text{height})$$
$$= 7.5518 + 1.1076(64)$$
$$= 78.4382 \sim 78.4 \text{ inches}$$

Making Predictions

What would you predict the cartwheel distance to be for an adult who is 64 inches tall?

$$\begin{aligned}\text{Predicted CWDist} &= 7.5518 + 1.1076(\text{height}) \\ &= 7.5518 + 1.1076(64) \\ &= 78.4382 \sim 78.4 \text{ inches}\end{aligned}$$

We would also **estimate the mean cartwheel distance for all adults who are 64 inches tall** to be 78.4 inches

Observed Errors (Residuals)

64 inch tall adult had cartwheel distance of 87 inches

What is the **observed error (residual)** for this adult?

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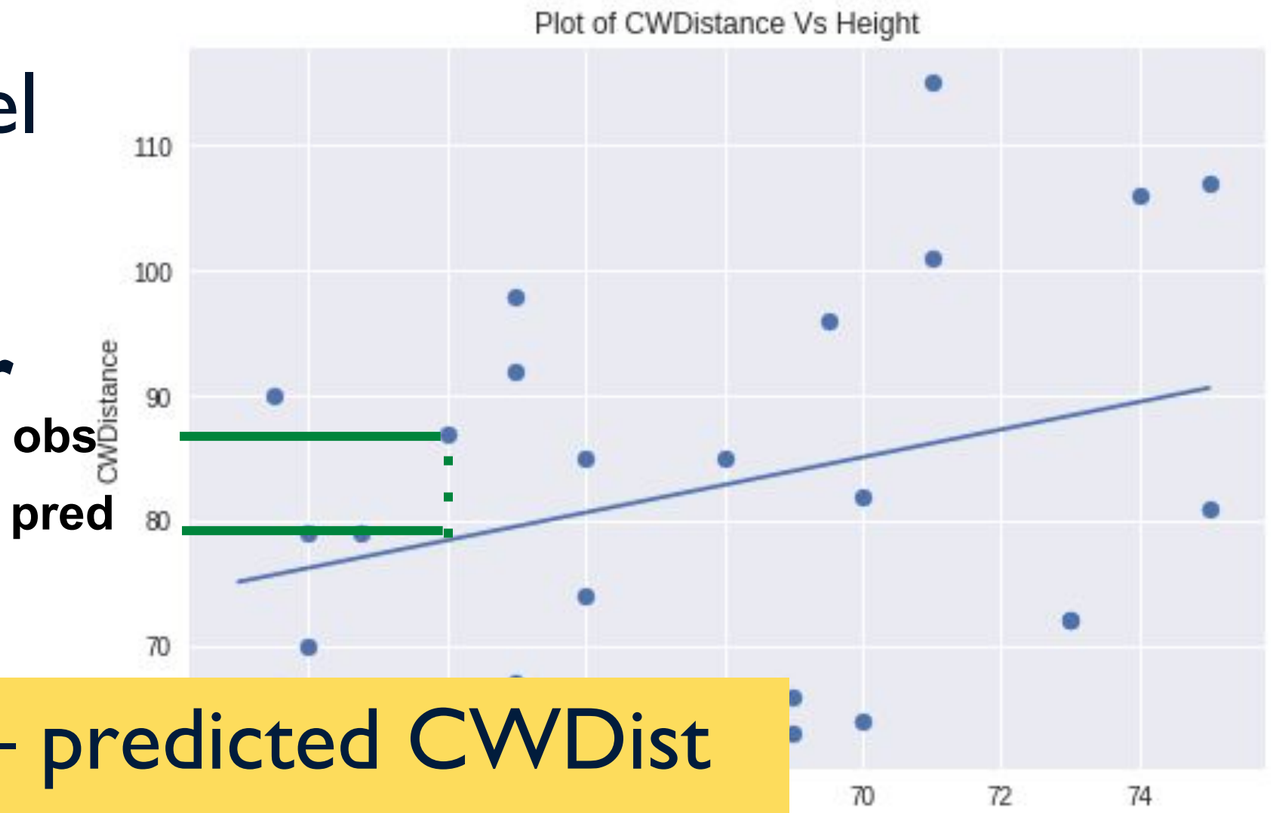

Residual = observed CWDist – predicted CWDist

= 87 inches – 78.4 inches = 8.6 inches

What's Next?

Now that we have worked with the **descriptive** side of regression, we turn to **drawing inferences** from regression:

- **Assessing significance** of the relationship
- **Checking** underlying **assumptions**
- **Extending** regression model to include **more predictors**