HOMEWORK 9

JESSE COBB - 3PM SECTION (MON, WED)

1. a.
$$P(X < m) = P(X - 30 < m - 30) = .99$$
 $P(X - 30 \ge m - 30) = .01$
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 $P(X - 30 \ge m - 30) \le \frac{2^2}{(m - 30)^2} = .01$
 $(m - 30)^2 = 400 \implies m - 30 = 20 \implies m = 50$
b. $P(X < m) = P(Z < \frac{m - (30)}{\frac{2}{10}}) = \Phi(\frac{m - (30)}{\frac{2}{2}}) = .99$
 $\frac{10m - 300}{2} = 2.33 \implies 10m = 304.66 \implies m = 30.466$
2. $f_{X,Y}(x,y) = \begin{cases} \frac{\lambda}{y} \cdot e^{-\lambda y} & 0 < x < y < \infty \\ 0 & \text{otherwise} \end{cases}$
a. $\int_0^\infty \int_0^y \frac{\lambda}{y} \cdot e^{-\lambda y} dx = \lambda \cdot e^{-\lambda y} dy = -e^{-\lambda y} \Big|_0^\infty = 1$
b. $f_Y(y) = \int_0^y \frac{\lambda}{y} \cdot e^{-\lambda y} dx = \lambda \cdot e^{-\lambda y}$
c. $f_{X|Y}(x|y) = \frac{\lambda}{\lambda} e^{-\lambda y} = \frac{1}{y}$
d. $E[X] = \int_0^\infty E[X|Y] f_Y(y) dy = \int_0^\infty f_Y(y) \int_0^y x e^{-\lambda y} dx dy$

$$\frac{1}{y} = \frac{\lambda}{0} \cdot \frac{10}{0} \cdot \frac{2}{0} \cdot \frac{4}{0} = \frac{1}{0} \cdot \frac{16}{0} = \frac{4}{0} \cdot \frac{16}{0} = \frac{4}{0} = \frac{1}{0} \cdot \frac{16}{0} = \frac{1}{0}$$

e. $P(Y < 1|X = \frac{1}{2}) = \frac{3}{(\frac{3}{2})^2} \int_0^1 y dy = \frac{2}{3}y^2]_0^1 = \frac{2}{3}$

f. $E[Y|X = \frac{1}{2}] = \frac{4}{3} \int y^2 dy = \frac{4}{9} y^3]_0^1 = \frac{4}{9}$