=

HOMEWORK 4

JESSE COBB - 3PM SECTION (MON,WED)

1. 12% of vehicle sales in 2022 were electric vehicles, 32% were hybrid, and the remaining 56% were gas. A surveyor goes from parking lot to parking lot, recording the fuel option of each car one at a time. The surveyor is a bit forgetful and sometimes records the same car twice.
   a. $X \sim \text{Bin}(15, .12)$
   $P(X = 4) = \binom{15}{4}(.88)^{11}(.12)^4 \approx .06937$
   b. $X \sim \text{Geom}(.32)$
   $P(X = 8) = (.68)^7(.32) \approx .02151$
   c. $X \sim \text{NegBin}(4, .56)$
   $P(X = 16) = \binom{15}{3}(.44)^{12}(.56)^4 \approx .002356$
   d. $X \sim \text{NegBin}(4, .56)$
   $E[X] = \frac{r}{p} = \frac{4}{.56} \approx 7.143$
   e. $X \sim \text{Bin}(25, .32)$
   $\text{Var}(X) = np = (25)(.32)(.68) = 5.44$
2. $A =$"Infected"
   $B =$"Tested Infected"
   $B_6 =$"Tested infected for 6 out of 20 tests"
   $P(A) = .08$
   $P(B|A) = .75 \quad P(B|A^c) = .10$
   $P(B) = P(B|A)P(A) + P(B|A^c)P(A^c) = (.75)(.08) + (.10)(.92) = .152$
   $X \sim \text{Bin}(20, P(B) = .152)$
   $P(B_6) = P(X = 6) = \binom{20}{6}(.152)^6(.848)^{14} \approx 0.04753$
   $Y \sim \text{Bin}(20, P(B|A) = .75)$
   $P(B_6|A) = P(Y = 6) = \binom{20}{6}(.75)^6(.25)^{14} \approx 0.00002570$
   $P(A|B_6) = \frac{P(B_6|A)P(A)}{P(B_6)} = \frac{(0.00002570)(.08)}{0.04752} \approx 0.00004326$
3. A bag contains 17 red marbles and 8 blue marbles. A friend reaches in and selects 5 marbles at random, with replacement. For each red marble drawn you win \$1; for each blue marble you lose \$1. Let W denote your net winnings.
   $X \sim \text{Bin}(5, \frac{17}{25})$
   a. $p_X(k) = \binom{5}{k}(\frac{17}{25})^k(\frac{8}{25})^{5-k}$
   b. $E[X] = np = (5)(\frac{17}{25}) = 3.4$
   c. $X \sim \text{HyperGeom}(25, 17, 5)$
   $p_X(k) = \frac{\binom{17}{k}\binom{8}{5-k}}{\binom{25}{5}}$
4. When a certain basketball player takes his first shot in a game he succeeds with probability $\frac{1}{2}$. If he misses his first shot, he loses confidence and his second shot will go in with probability $\frac{1}{3}$. If he misses his first 2 shots then his third shot will go in with probability $\frac{1}{4}$. His success probability goes down further to $\frac{1}{5}$

after he misses his first 3 shots. If he misses his first 4 shots then the coach will remove him from the game.

a. $p_X(k) = \begin{cases} \frac{1}{2} & k=0 \\ \frac{1}{6} = (\frac{1}{2})(\frac{1}{3}) & k=1 \\ \frac{1}{12} = (\frac{1}{2})(\frac{2}{3})(\frac{1}{4}) & k=2 \\ \frac{1}{20} = (\frac{1}{2})(\frac{2}{3})(\frac{3}{4})(\frac{1}{5}) & k=3 \\ \frac{1}{5} = (\frac{1}{2})(\frac{2}{3})(\frac{3}{4})(\frac{4}{5}) & k=4 \end{cases}$

b. $E[X] = 0 \cdot (\frac{1}{2}) + 1 \cdot (\frac{1}{6}) + 2 \cdot (\frac{1}{12}) + 3 \cdot (\frac{1}{20}) + 4 \cdot (\frac{1}{5}) \approx 1.28\bar{3}$

5. $X \sim \text{DiscUnif}\{1, 2, \ldots\}$
$p_X(k) = \frac{c}{k(k+1)}$ where $c > 0$
   a. $1 = c \sum_k \frac{1}{k(k+1)} = c \implies c = 1$
   b. $E[X] = \sum_k \frac{x_k}{k(k+1)} = \sum_k \frac{k}{k(k+1)} = \sum_k \frac{1}{k+1} = \text{DNE}$