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HOMEWORK 2

JESSE COBB - 3PM SECTION (MON,WED)

1. A department on campus is forming a committee of 7 people to be chosen from 21 faculty members, 6 staff members, and 5 graduate student council members.
   a. Total combinations: $\binom{(21+6+5)}{7} = \binom{32}{7} = 3365856$
   b. Total combinations: $\binom{21}{3} \cdot \binom{6}{2} \cdot \binom{5}{2} = 199500$
   c. $A =$"Probability committee has at least 2 faculty members."
   $A^C =$"Probability committee has less than 2 faculty members."
   $P(A^C) = \frac{\binom{21}{0}\cdot\binom{11}{7}}{\binom{32}{7}} + \frac{\binom{21}{1}\cdot\binom{11}{6}}{\binom{32}{7}} = \frac{10032}{3365856}$
   $P(A) = 1 - P(A^C) = \frac{3355824}{3365856} \approx .9970$
2. We choose a number from the set $\{1, 2, 3, \ldots 50\}$ uniformly at random and denote this number by X. For each of the following choices decide whether the two events in question are independent or not.
   a. $A = \{X \text{ is even}\} \quad B = \{X \text{ is divisible by } 5\}$
   $P(A) = \frac{1}{2} \quad P(B) = \frac{1}{5}$
   $P(A \cap B) = \frac{1}{10} =$ "X is even and divisible by 5."
   $P(A) \cdot P(B) = \frac{1}{10} = P(A \cap B)$ so A and B are independent.
   b. $C = \{X \text{ has two digits}\} \quad D = \{X \text{ is divisible by } 4\}$
   $P(C) = \frac{41}{50} \quad P(D) = \frac{12}{50}$
   $P(C \cap D) = \frac{1}{5}$
   $P(C) \cdot P(D) = \frac{492}{2500} = .1968 \neq P(C \cap D)$ so C and D are not independent.
   c. $E = \{X \text{ is a prime number}\} \quad F = \{X \text{ has a digit of } 3\}$
   $P(E) = \frac{3}{10} \quad P(F) = \frac{13}{50}$
   $P(E \cap F) = \frac{1}{10}$
   $P(E) \cdot P(F) = \frac{39}{500} = .078 \neq P(E \cap F)$ so E and F are not independent.
3. A restaurant offers three ways of paying the bill: cash, credit card, and mobile payment (e.g., Apple Pay, Google Wallet). At the end of the day there were 200 paid bills. Some of the bills were paid with a mix of cash, credit card, and mobile payment. There were 36 bills paid with cash, 82 bills paid with credit card, and 113 bills paid by mobile payment. There were 8 bills that used cash and credit card, 13 bills that used cash and mobile payment, and 15 bills that used credit card and mobile payment. Five of the bills were paid with all three methods.
   $A = 36 =$"Bill was paid with cash."
   $B = 82 =$"Bill was paid with card."
   $C = 113 =$"Bill was paid with mobile payment."
   a. $W =$"Probability that at least one of the bills was paid for by 2 or more methods.
   $|(A \cap B) \cup (A \cap C) \cup (B \cap C)| = 8 + 13 + 15 - 10 = 26$
   $P(W) = \frac{(26)(200-26)+\binom{26}{2}}{\binom{200}{2}} = .244$

b. X ="Probability that a bill chosen at random and was paid using all 3 methods."
$P(X) = \frac{5}{26} = .192$
c. Y ="Probability that a bill was only paid using a single method."
$P(Y) = \frac{200-26}{200} = .870$
d. Z ="Probability a bill was paid for by mobile payment given it was paid by a single method."
$P(Z) = \frac{113-13-15+5}{200-26} = .517$

4. I have a bag with 3 fair dice: a 4-sided die, a 6-sided die, and a 12-sided die. I reach into the bag, pick one die at random and roll it twice. The first roll is a 3, and the second roll is a 4. What is the probability that I pulled out the 4-sided die?
A ="event of picking the 4-sided die"
B ="event of rolled a 3 and then a 4."
$P(A) = \frac{1}{3} = .333$
$P(B) = (\frac{1}{4})^2\frac{1}{3} + (\frac{1}{6})^2\frac{1}{3} + (\frac{1}{12})^2\frac{1}{3} = .0324$
$P(A \cap B) = (\frac{1}{4})^2 = .0625$
$P(A|B) = \frac{P(B|A)\cdot P(A)}{P(B)} = \frac{(\frac{1}{4})^2\frac{1}{3}}{(\frac{1}{4})^2\frac{1}{3}+(\frac{1}{6})^2\frac{1}{3}+(\frac{1}{12})^2} = .643$
5. BritBox has shows in four genres: drama; mysteries; comedy; and documentaries/lifestyle. ll customers watch at least one genre of shows. 80% of customers watch more than one genre of shows. 50% of customers watch mysteries. Of those who watch more than one genre of shows, 60% watch mysteries.
A ="Event that a person watches more than 1 genre."
A^C ="Event that a person watches only 1 genre."
B ="Event that a person watches mysteries."
B^C ="Event that a person doesn't watch mysteries."
$P(B|A) = .60$
$P(A \cap B) = P(B|A)P(A) = .48$
$P(A \backslash B) = P(A) - P(A \cap B) = .32$
$P(B \backslash A) = P(B) - P(A \cap B) = .02$
$P(A^C \cap B^C) = 1 - P(A \cap B) - P(A \backslash B) - P(B \backslash A) = 1 - .48 - .32 - .02 = .18$
6. Consider rolling two fair, 6-sided dice.
   a. List the elements and calculate the probabilities of the following events
      i. $A = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$
      $P(A) = \frac{1}{6}$
      ii. $B = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(2,6),(3,5),$
      $(4,4),(5,3),(6,2),(3,6),(4,5),(5,4),(6,3),(4,6),(5,5),(6,4)\}$
      $P(B) = \frac{1}{2}$
      iii. $C = \{(1,1),(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(2,6),$
      $(3,5),(4,4),(5,3),(6,2)\}$
      $P(C) = \frac{1}{3}$
   b. $P(A \cap B) = \frac{1}{18} \neq P(A)P(B)$
   Therefore not pairwise independent.
   c. Not mutually independent because they are not pairwise independent.
7. A ="You are infected." $P(A) = .04$
A^C ="You are not infected." $P(A^C) = .96$
X ="Test returns 'infected'."

$Y =$"Test returns 'not infected'."
$Z =$"Test returns 'inconclusive'."
$P(X|A) = .45 \qquad P(X|A^C) = .30$
$P(Y|A) = .30 \qquad P(Y|A^C) = .50$
$P(Z|A) = .25 \qquad P(Z|A^C) = .20$
$P(Z) = P(A)P(Z|A) + P(A^C)P(Z|A^C) = .202$
$P(A|Z) = \frac{P(Z|A)P(A)}{P(Z)} = \frac{(.25)(.04)}{.202} = .0495$