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HOMEWORK 5

JESSE COBB - 2PM SECTION

3.2.9 d. $\mathbb{Z}_7 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}\}$

3.2.10 c. Congruent to 2(mod 4) and congruent to 8(mod 6).
$14 = 2(\text{mod } 4)$ and $14 = 8(\text{mod } 6)$
$-10 = 2(\text{mod } 4)$ and $-19 = 8(\text{mod } 6)$

3.4.1 a. $6 + 6$ in $\mathbb{Z}_7$
$\bar{6} + \bar{6} = \bar{12} = \bar{5}$

i. 2^{25} in $\mathbb{Z}_7$
$\bar{2^{25}} = \bar{2^3}^8\bar{2} = \bar{1}^8\bar{2} = \bar{2}$

j. 5^{23} in $\mathbb{Z}_7$
$\bar{5^{23}} = \bar{25}^{11}\bar{5} = \bar{4^{11}}\bar{5} = \bar{2^{22}}\bar{5} = \bar{2^3}^7\bar{10} = \bar{1}^7\bar{3} = \bar{3}$

k. 4^{44} in $\mathbb{Z}_7$
$\bar{4^{44}} = \bar{2^{88}} = \bar{2^3}^{29}\bar{2} = \bar{1}^{29}\bar{2} = \bar{2}$

l. 2^{26} in $\mathbb{Z}_7$
$\bar{2^{25}} = \bar{2^3}^8\bar{2}^2 = \bar{1}^8\bar{2}^2 = \bar{4}$

3.4.7 a. $238 + 496 - 44$ in $\mathbb{Z}_9$
$\bar{238} + \bar{496} - \bar{44} = \bar{690} = \bar{90} + \bar{450} + \bar{90} + \bar{45} + \bar{15} = \bar{15} = \bar{6}$

4.1.1 c. $R = \{(1,2), (2,1)\}$
R is a function from A to B
$A = \{1,2\}, B = \mathbb{N}$ or $\mathbb{Z}$

e. $R = \{(x,y) \in \mathbb{N}^2 : x \leq y\}$
R is not a function on $\mathbb{N}$
Since $1 \leq 2$ and $1 \leq 1$ so $f(1) = 1, 2$

f. $R = \{(x,y) \in \mathbb{Z}^2 : y^2 = x\}$
R is not a function on $\mathbb{Z}$
Since $1 = (-1)^2 = (1)^2$ so $f(1) = -1, 1$
and $x \geq 0$ since $y^2 \geq 0$ so $\text{Dom}(R) \neq \mathbb{Z}$

i. $R = \{(a,3), (b,2), (c,1)\}$
R is not a function from A to B
Since $A = \{a,b,c,d\}$ so $\text{Dom}(R) \neq A$

4.1.2 $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = \pm\sqrt{x}$ is not a function as $\mathbb{R} \neq \text{Dom}(R)$ since $\sqrt{x}$ is only defined when $x \geq 0$. Furthermore the function almost always has 2 outputs for every input, an example being $f(1) = -1, 1$ which deviates from the rules of a function.

4.1.7 *Proof.* Let $f : A \to B$ and $g : C \to D$ be functions. Now suppose $\text{Dom}(f) = \text{Dom}(g)$ and for all $x \in \text{Dom}(f), f(x) = g(x)$.
($\subseteq$): Let $(x,y) \in f$ this means that $f(x) = y$. Since $f(x) = g(x) \implies g(x) = y$ this means $(x,y) \in g$. Thus $f \subseteq g$.
($\subseteq$): Let $(x,y) \in g$ this means that $g(x) = y$. Since $f(x) = g(x) \implies f(x) = y$ this means $(x,y) \in f$. Thus $g \subseteq f$.

Thus we've shown that $f = g$ by double containment, if $\mathrm{Dom}(f) = \mathrm{Dom}(g)$ and for all $x \in \mathrm{Dom}(f), f(x) = g(x)$. $\square$

4.1.13 $f : \mathbb{Z} \to \mathbb{Z}_6$

a. $f(3) = \{\ldots -3, 3, 9, 15 \ldots\} = \{6k + 3 : k \in \mathbb{Z}\}$

b. Image of $6 = f(6) = f(0) = \bar{0} = \{6k : k \in \mathbb{Z}\}$

c. A pre-image of $\bar{3} = 3$

d. All pre-images of $\bar{1} = 6k + 1$ where $k \in \mathbb{Z}$

4.1.17 $\overline{\overline{A}} = m, \overline{\overline{B}} = n$

a. A function f from A to B has n^m possible forms.

b. A function f with only one element in the domain from A to B has n possible forms.

4.1.18 a. *Proof.* Let $f : A \to B$ where xTy iff $f(x) = f(y)$ where T is a relation on A. Let $x \in A$. Since f is a function each value of x maps to to a single value so $f(x) = f(x)$. Thus xTx so T is reflexive. Let $x, y \in A$. Assume xTy so that $f(x) = f(y)$. This implies $f(y) = f(x)$ so yTx. Then T is symmetric. Now let $x, y, z \in A$ and xTy and yTz so that $f(x) = f(y)$ and $f(y) = f(z)$. This implies $f(x) = f(z)$ so xTz. Then T is transitive. Thus we've proved T to be an equivalence relation on A. $\square$

b. $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = x^2$.

$\bar{0} = \{0\} \qquad \bar{2} = \{4\} \qquad \bar{4} = \{16\}$

4.2.1 g.

4.2.2 b.

4.2.3 b.

4.2.6

4.2.7

4.3.1 b.

d.

e.

h.

4.3.2 b.

d.

e.

h.

4.3.5

4.3.6

4.3.7

4.3.8