HOMEWORK 6

JESSE COBB - 2PM SECTION

5.1.1 *Proof.* Let A be a set then $A \approx A$ since there exists a bijection $I_A : A \to A$ given by $I_A(x) = x$. Thus $\approx$ is a reflexive relation.
Let A and B be sets and $A \approx B$ so there exists a bijection $f : A \to B$. Then by definition there exists an inverse bijection $f^{-1} : B \to A$ so $B \approx A$. Thus $\approx$ is a symmetric relation.
Now Let A,B, and C be sets and let $A \approx B$ and $B \approx C$ so that there exists bijections $f : A \to B$ and $g : B \to C$. By definition there exists a bijection $g \circ f : A \to C$ that is a composite of two bijections. This implies $A \approx C$ so $\approx$ is a transitive relation.
Thus we've shown $\approx$ to be an equivalence relation as it is reflexive, symmetric, and transitive. $\square$

5.1.2 a. $A = \{1, 2, 4, 8, 16, 32, 64, 128, 256, 512\} \approx \mathbb{N}_{10}$
$f : \mathbb{N}_{10} \to A$ where $f(n) = 2^{n-1}$ so $\overline{\overline{A}} = 10$

c. $B = \{x \in \mathbb{Z} : x^2 < 11\} \approx \mathbb{N}_7$
$g : \mathbb{N}_7 \to B$ where $g(n) = \begin{cases} \frac{n}{2} & 2 \mid n \\ -\frac{n-1}{2} & 2 \nmid n \end{cases}$ so $\overline{\overline{B}} = 7$

d. $C = \{(x, y) \in \mathbb{N} : x + y < 6\} \approx \mathbb{N}_{10}$
$h : \mathbb{N}_{10} \to C$ where $h(1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$
$= (1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)$ so $\overline{\overline{C}} = 10$

5.1.13 *Proof.* $\square$

5.1.14

5.2.2 a.
b.

5.2.3 c.
e.
f.

5.2.4 a.
b.

5.3.1

5.3.4

5.3.13 a.

5.4.1 a.
b.
c.
d.

5.4.5