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HOMEWORK 1

JESSE COBB - 3PM SECTION (MON,WED)

1. $\Omega = \{1, 2, \ldots, 74, 75\}$

$$A = \{1, 3, \ldots, 73, 75\}$$
$$B = \{15, 25, 35, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 65, 75\}$$
$$C = \{3, 6, \ldots, 72, 75\}$$
$$D = \{19, 28, 37, 46, 55, 64, 73\}$$

a. $A \backslash C =$
$\{1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, 41, 43,$
$47, 49, 53, 55, 59, 61, 65, 67, 71, 73\}$
$P(A \backslash C) = \frac{25}{75} = .3333$

b. $A \cap C \cap D^C =$
$\{3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75\}$
$P(A \cap C \cap D^C) = \frac{13}{75} = .1733$

c. $B \cup C \cup D =$
$\{3, 6, 9, 12, 15, 18, 19, 21, 24, 25, 27, 28, 30, 33, 35,$
$36, 37, 39, 42, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56,$
$57, 58, 59, 60, 63, 64, 65, 66, 69, 72, 73, 75\}$
$P(B \cup C \cup D) = \frac{41}{75} = .5466$

2. Suppose I toss a fair coin, roll a fair 4-sided die, and pick a number at random between 1 and 4 (inclusive), all at the same time.

a. $\Omega = \{(x, y, z) : x \in \{1, 2\}, y \in \{1, 2, 3, 4\}, z \in \{1, 2, 3, 4\}\}$

b. List the elements and calculate the probabilities of the following events

i. $A = \{(2, y, z) : y \in \{1, 2, 3, 4\}, z \in \{1, 2, 3, 4\}\}$
$P(A) = \frac{4 \times 4}{2 \times 4 \times 4} = .50$

ii. $B = \{(x, y, z) : x \in \{1, 2\}, y \in \{2, 3, 4\}, z \in \{1, 2, 3\} : y > z\}$
$P(B) = \frac{2 \times 6}{2 \times 4 \times 4} = .375$

iii. $C = \{(2, y, z) : y \in \{2, 3, 4\}, z \in \{1, 2, 3\} : y > z\}$
$C = A \cap B$
$P(C) = \frac{6}{2 \times 4 \times 4} = .1875$

iv. $D = \{(x, y, z) : (x = 2, y \in \{1, 2, 3, 4\}, z \in \{1, 2, 3, 4\}) \vee (x \in \{1, 2\}, y \in \{1, 2, 3, 4\}, z \in \{1, 2, 3, 4\} : y > z)\}$
$D = A \cup B$
$P(D) = P(A) + P(B) - P(C) = \frac{16+12-6}{32} = .6875$

3. $E = A \cap B, F = A \cap B^C$

a. $E \cap F = (A \cap B) \cap (A \cap B^C) = A \cap (B \cap B^C) = A \cap \varnothing = \varnothing$

b. $E \cup F = (A \cap B) \cup (A \cap B^C) = A \cap (B \cup B^C) = A \cap \Omega = A$

4. Every day a kindergarten class chooses randomly one of the 50 state flags to hang on the wall, without regard to previous choices. We are interested in the flags that are chosen on the five days next week.

a. $A =$ "Arizona's flag on Monday, Michigan's flag on Tuesday, California's flag on Wednesday, Ohio's flag on Thursday, and North Carolina's flag on Friday."
$P(A) = (\frac{1}{50})^5 = 3.2 \times 10^{-9}$

b. $B =$ "California's flag will be hung at least once next week."
$P(B) = \binom{5}{1}(\frac{1}{50})(\frac{49}{50})^4 + \binom{5}{2}(\frac{1}{50})^2(\frac{49}{50})^3 + \binom{5}{3}(\frac{1}{50})^3(\frac{49}{50})^2 + \binom{5}{4}(\frac{1}{50})^4(\frac{49}{50}) + (\frac{1}{50})^5$
$= 5(\frac{1}{50})(\frac{49}{50})^4 + 10(\frac{1}{50})^2(\frac{49}{50})^3 + 10(\frac{1}{50})^3(\frac{49}{50})^2 + 5(\frac{1}{50})^4(\frac{49}{50}) + (\frac{1}{50})^5 = .0961$

c. $D = A$ but a state flag can only be chosen once a week
$P(D) = 1/(50)_5 = \frac{1}{50\times49\times48\times47\times46} = 3.93 \times 10^{-9}$
$E = B$ but a state flag can only be chosen once a week
$P(E) = \binom{5}{1}(\frac{1}{50})(\frac{49}{50})(\frac{48}{50})(\frac{47}{50})(\frac{46}{50}) = .0814$

5. a. $\Omega = \{(x, y, z) : x \in \{G, Y, R\}, y \in \{G, Y, R\}, z \in \{G, Y, R\}\}$
$A = \{(x, y, z) : x \in \{G, Y, R\}, y \in \{G, Y, R\}, z \in \{G, Y, R\} : x \neq y, x \neq z, y \neq z\}$

b. $P(A) = \frac{\binom{3}{1}\binom{4}{1}\binom{5}{1}}{\binom{12}{3}} = \frac{3\times4\times5}{\frac{12\times11\times10}{3\times2}} = \frac{3}{11} = .2727$