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HOMEWORK 7

JESSE COBB - 3PM SECTION (MON,WED)

1. $f(x,y) = \begin{cases} 4e^{-(x+4y)} & x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases}$

   a. $f_X(x) = 4\int_0^\infty e^{-x-4y}dy = -[e^{-x-4y}]_0^\infty = -[0 - e^{-x}] = e^{-x}$
   $f_Y(y) = 4\int_0^\infty e^{-x-4y}dx = -4[e^{-x-4y}]_0^\infty = -4[0 - e^{-4y}] = 4e^{-4y}$

   b. X and Y are independent as their bounds have no relation and $f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$.

2.

		x		
		0	2	4
	0	0.02	0.04	0.04
y	1	0.10	0.25	0.15
	2	0.08	0.16	0.16

   a. $P(X = 2Y) = P(X = 0, Y = 0) + P(X = 2, Y = 1) + P(X = 4, Y = 2)$
   $= 0.02 + 0.25 + 0.16 = 0.43$

   b. $E[XY] = 2(0.25) + 4(0.15) + 4(0.16) + 8(.16) = 3.02$

   c.

	0	2	4
$p_X(x)$	0.20	0.45	0.35

   $E[X] = 2(.45) + 4(.35) = 2.30$

   d.

	0	1	2
$p_Y(y)$	0.10	0.50	0.40

   $E[Y] = 1(.50) + 2(.40) = 1.30$

   e. $\text{Cov}(X,Y) = E[XY] - E[X]E[Y] = 3.02 - (2.30)(1.30) = 0.03$
   $\rho_{X,Y} = \text{Corr}(X,Y) = \frac{\text{Cov}(X,Y)}{\sigma_X \cdot \sigma_Y}$
   $\sigma_X = \sqrt{7.40 - (2.30)^2} = \sqrt{2.11}$
   $\sigma_Y = \sqrt{2.10 - (1.30)^2} = \sqrt{0.41}$
   $\rho_{X,Y} = \frac{0.03}{\sqrt{2.11}\sqrt{0.41}} \approx 0.0323$

   f. X and Y are not independent as their Covariance is nonzero.

3. $f_X(x) = \begin{cases} \frac{3}{8}x^2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$

   $x \le T \le 2x$

   $f_{X,T}(x,t) = \frac{\frac{3}{8}x^2}{2x-x} = \frac{3}{8}x$

   $f_T(t) = \begin{cases} \int_{\frac{t}{2}}^{t} \frac{3}{8}x dx = \frac{3}{16}[t^2 - \frac{t^2}{4}] = \frac{9t^2}{64} & 0 \le t \le 2 \\ \int_{\frac{t}{2}}^{2} \frac{3}{8}x dx = \frac{3}{16}[4 - \frac{t^2}{4}] = \frac{3(16-t^2)}{64} & 2 < t \le 4 \end{cases}$

   $P(T \ge 3) = \int_3^4 \frac{3}{4} - \frac{3t^2}{64}dt = [\frac{3t}{4} - \frac{t^3}{64}]_3^4 = 3 - 1 - \frac{9}{4} + \frac{27}{64} = 2 - \frac{117}{64} = \frac{11}{64} \approx .1719$

4. $f(x,y) = \begin{cases} 24xy & 0 \le x \le 1, 0 \le y \le 1, 0 \le x + y \le 1 \\ 0 & \text{otherwise} \end{cases}$

a. $\int_0^1 \int_0^{1-x} 24xydydx = \int_0^1 12x(1-x)^2dx = 12\int_0^1 x - 2x^2 + x^3dx$
$= 12[\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4}]_0^1 = 12[\frac{6}{12} - \frac{8}{12} + \frac{3}{12}] = 6 - 8 + 3 = 1$

b. $E[X] = \int_0^1 \int_0^{1-x} 24x^2ydydx = \int_0^1 12x^2(1-x)^2dx = 12\int_0^1 x^2 - 2x^3 + x^4dx$
$= 12[\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}]_0^1 = 12[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}] = 12[\frac{10}{30} - \frac{15}{30} + \frac{6}{30}] = \frac{12}{30} = \frac{2}{5}$

c. $E[X] = \int_0^1 \int_0^{1-y} 24xy^2dxdy = \int_0^1 12y^2(1-y)^2dy = 12\int_0^1 y^2 - 2y^3 + y^4dy$
$= 12[\frac{y^3}{3} - \frac{y^4}{2} + \frac{y^5}{5}]_0^1 = 12[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}] = 12[\frac{10}{30} - \frac{15}{30} + \frac{6}{30}] = \frac{12}{30} = \frac{2}{5}$

d. $E[XY] = \int_0^1 \int_0^{1-x} 24x^2y^2dydx = \int_0^1 8x^2(1-x)^3dx$
$= 8\int_0^1 x^2 - 3x^3 + 3x^4 - x^5dx = 8[\frac{x^3}{3} - \frac{3x^4}{4} + \frac{3x^5}{5} - \frac{x^6}{6}]_0^1$
$= 8[\frac{1}{3} - \frac{3}{4} + \frac{3}{5} - \frac{1}{6}] = 4[\frac{1}{3} - \frac{3}{2} + \frac{6}{5}] = 2[\frac{10}{15} - \frac{45}{15} + \frac{36}{15}] = \frac{2 \cdot 1}{15} = \frac{2}{15}$

e. $\mathrm{Cov}(X,Y) = \frac{2}{15} - (\frac{2}{5})^2 = \frac{2}{15} - \frac{4}{25} \approx -0.026\bar{6}$

f. X and Y are not independent as their Covariance is nonzero.