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HOMEWORK 3

JESSE COBB - 3PM SECTION (MON,WED)

1.

$$F_X(k)\begin{cases}0 & k<1\\ 0.4 & 1\le k<2\\ 0.65 & 2\le k<3\\ 0.75 & 3\le k<3.5\\ 0.95 & 3.5\le k<5\\ 1 & k\ge 5\end{cases}$$

a. $S_X = \{1, 2, 3, 3.5, 5\}$

b.

	1	2	3	3.5	5
$p_X(k)$	.4	.25	.10	.20	.05

2. Let $S_X = \{1, 2, 3, 4, 5\}$ and

	1	2	3	4	5
$p_X(k)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{16}$	$\frac{1}{16}$

a. $P(X \le 4) = F_X(4) = \frac{1}{8} + \frac{1}{4} + \frac{3}{8} + \frac{3}{16} = \frac{15}{16} = .9375$
$P(X < 4) = F_X(3) = \frac{1}{8} + \frac{1}{4} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4} = .75$

b. $P(X < 4.12 | X > 1.638) = \frac{P(X \le 4 \cap X \ge 2)}{P(X \ge 2)} = \frac{\frac{13}{16}}{\frac{7}{8}} = \frac{13}{14} \approx .929$

3. $S_X = \{1, 2, 3, 4, 5, 6\}$

$$p_X(k)\begin{cases}ck & k=1,2,3,4,5,6\\ 0 & \text{otherwise}\end{cases}$$

a. $c(1+2+3+4+5+6) = 21c \implies c = \frac{1}{21} \approx .0476$

b. $E[X] = \frac{1}{21} + \frac{4}{21} + \frac{9}{21} + \frac{16}{21} + \frac{25}{21} + \frac{36}{21} = \frac{91}{21} \approx 4.3\bar{3}$

c. $E[X^2] = \frac{1}{21} + \frac{8}{21} + \frac{27}{21} + \frac{64}{21} + \frac{125}{21} + \frac{216}{21} = \frac{441}{21} \approx 21$

d. $\mathrm{SD}(X) = \sqrt{\mathrm{Var(X)}} = \sqrt{21 - \frac{91}{21}^2} \approx 1.49$

e. $E[X] - \mathrm{SD}(X) = 2.843$ and $E[X] + \mathrm{SD}(X) = 5.823$
$P(X \ge 3 \cap X \le 5) = \frac{3}{21} + \frac{4}{21} + \frac{5}{21} = \frac{12}{21} \approx .571$

4. There are five closed boxes on a table. Three of the boxes have nice prizes inside. The other two do not. You open boxes one at a time until you find a prize. Let X be the number of boxes you open.

a.

	1	2	3
$p_X(k)$	$\frac{{}_3P_1}{{}_5P_1} = \frac{3}{5}$	$\frac{{}_2P_1 {}_3P_1}{{}_5P_2} = \frac{3}{10}$	$\frac{{}_2P_2 {}_3P_1}{{}_5P_3} = \frac{1}{10}$

b. $E[X] = \frac{3}{5} + \frac{3}{5} + \frac{3}{10} = \frac{3}{2}$

c. $\mathrm{Var}(X) = E[X^2] - E[X]^2 = (\frac{3}{5} + \frac{6}{5} + \frac{9}{10}) - (\frac{3}{2})^2 = .45$
$\mathrm{SD}(X) = \sqrt{\mathrm{Var}(X)} = \sqrt{.45} \approx .671$

d. $E[g(x)] = \frac{300}{5} + \frac{-100}{10} = 50$

5. $S_X\{0, 1, 2 \ldots\}$

$$p_X(k) \begin{cases} c \cdot (\frac{1}{5})^k & k = 0, 1, 2 \ldots \\ 0 & \text{otherwise} \end{cases}$$

a. $c \sum_k \frac{1}{5^k} = c(1.25) = 1 \implies c = \frac{4}{5} = .8$

b. $P(X \geq 2) = -\frac{4}{5}(\frac{6}{5}) + \frac{4}{5} \sum_k \frac{1}{5^k} = 1 - \frac{4}{5}(\frac{6}{5}) = .04$

c. $P(X = 2|X \geq 2) = \frac{P(X=2)}{P(X \geq 2)} = \frac{\frac{4}{5}\frac{1}{25}}{.04} = \frac{4}{5} = .8$

d. $F_X(x) = \frac{4}{5} \cdot 1 \sum_{x+1} \frac{1}{5^x} = \frac{4}{5}(\frac{1-(\frac{1}{5})^x}{1-\frac{1}{5}}) = 4(\frac{1-(\frac{1}{5})^{x+1}}{4}) = 1 - (\frac{1}{5})^{x+1}$