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HOMEWORK 5

JESSE COBB - 3PM SECTION (MON,WED)

1. $f_X(x) = \begin{cases} \frac{c}{x^3} & x \geq 2 \\ 0 & \text{otherwise} \end{cases}$
   a. $c\int_2^\infty x^{-3}dx = -c[\frac{1}{2x^2}]_2^\infty = -c[0 - \frac{1}{8}] = \frac{c}{8} = 1 \implies c = 8$
   b. $P(1 \leq X \leq 4) = P(2 \leq X \leq 4)$
   $= 8\int_2^4 x^{-3}dx = -4[x^{-2}]_2^4 = -4[\frac{1}{16} - \frac{4}{16}] = \frac{3}{4} = .75$
   c. $P(X \geq 5 | X \geq 3) = \frac{P(X \geq 5 \cap X \geq 3)}{P(X \geq 3)} = \frac{P(X \geq 5}{P(X \geq 3)}$
   $P(X \geq 5) = 8\int_5^\infty x^{-3} = -4[x^{-2}]_5^\infty = \frac{4}{25}$
   $P(X \geq 3) = 8\int_3^\infty x^{-3} = -4[x^{-2}]_3^\infty = \frac{4}{9}$
   $P(X \geq 5 | X \geq 3) = \frac{4*9}{4*25} = \frac{9}{25}$
   d. $E[X] = 8\int_2^\infty x^{-2}dx = -8[x^{-1}]_2^\infty = -8[0 - \frac{1}{2}] = 4$
   e. $\text{Var}(X) = E[X^2] - E[X]^2 = 8\int_2^\infty x^{-1}dx - 16 = 8\ln|x|]_2^\infty - 16 = 8(\ln(\infty) - \ln(2)) - 16 = \text{DNE}$
   f. $F_X(x) = \int_2^x 8k^{-3}dk = -4k^{-2}]_2^x = 1 - 4x^{-2}$
2. Exponential Distribution with $E[X] = 90 \implies \lambda = \frac{1}{90}$, $X \sim \text{Exp}(\frac{1}{90})$.
   a. $F_X(k) = 1 - e^{-\lambda k}$
   $F_X(100) - F_X(50) = e^{-\frac{50}{90}} - e^{-\frac{100}{90}} = e^{-\frac{5}{9}} - e^{-\frac{10}{9}} \approx .2446$
   b. 95^{th} Percentile:$P(X \geq t) = .05 = F_X(\infty) - F_X(t) = e^{-\frac{t}{90}} - e^{-\infty}$
   $.05 = e^{-\frac{t}{90}} \implies -\ln(\frac{1}{20}) = \frac{t}{90} \implies t = 90\ln(20) \approx 269.6$
   Any amount of time greater than or equal to 269.6 minutes is in the 95^{th} percentile.
   c. $P(X \leq 120) = F_X(120) = 1 - e^{-\frac{120}{90}} = 1 - e^{-\frac{4}{3}} \approx .7364$
   d. $Y \sim \text{Poisson}(\frac{1}{90})$
   $E[Y] = \frac{1}{90}(480) = 5.3\bar{3}$
   e. $W \sim \text{Poisson}(E[Y])$
   $P(W = 4) = \frac{E[Y]^4}{4!}e^{-E[Y]} \approx .1628$
3. $X \sim \text{Bin}(1000, 0.0035)$
   a. $P(X \geq 2) = 1 - P(X \leq 2) = 1 - (P(X = 1) + P(X = 0))$
   $= 1 - (\binom{1000}{1}(0.0035)(1 - 0.0035)^{999} + (1 - 0.0035)^{1000}) \approx .8646$
   b. $E[X] = np = 1000(0.0035) = 3.5 = \lambda$
   $Y \sim \text{Poisson}(3.5)$
   $P(Y \geq 2) = 1 - P(Y \leq 2) = 1 - (P(Y = 1) + P(Y = 0))$
   $= 1 - e^{-3.5}(3.5^1 + 3.5^0) \approx .8641$
4. $X \sim \text{Unif}[18, 22]$
   a. $P(X > 20.5) = P(22 > X) - P(20.5 > X) = \frac{22 - 20.5}{22 - 18} = .375$
   b. $Y \sim \text{Bin}(100, .375)$
   $$P(Y \geq 25) = 1 - P(Y < 25) = 1 - \sum_{i=0}^{24} \binom{100}{i} (.375)^i (.625)^{100-i}$$

5. $X > 0 \implies P(X > 0) = 1$

$$\begin{aligned} \int_0^\infty [1 - F_X(x)]dx &= \int_0^\infty \int_x^\infty f_X(t)dtdx \\ &= \int_0^\infty \int_0^t f_X(t)dxdt \\ &= \int_0^\infty xf_X(t)]_0^t dt \\ &= \int_0^\infty tf_X(t)dt \end{aligned}$$