Causal Data Analysis
Study Questions – Solution Key

Short Questions

1. **Statement:** "In experimental data ATE will always be equal to ITE." (Assume that the randomization was done correctly.)

   Answer: *False / it depends.*

   Let the individual potential outcomes be:

   $$Y_i(1) \quad \text{and} \quad Y_i(0),$$

   and the *individual treatment effect* (ITE):

   $$\tau_i = ITE_i = Y_i(1) - Y_i(0).$$

   The *average treatment effect* is:

   $$ATE = \mathbb{E}[Y(1) - Y(0)] = \mathbb{E}[\tau_i].$$

   In randomized experiments, random assignment ensures that

   $$D_i \perp (Y_i(0), Y_i(1)),$$

   therefore the difference in sample means between treated and control units identifies the ATE:

   $$\widehat{ATE} = \overline{Y}_1 - \overline{Y}_0.$$

   However, we can never observe both $Y_i(1)$ and $Y_i(0)$ for the same individual. Hence the ITE is never identified, only its expectation (the ATE) is.

   $ATE = ITE$ holds only under *treatment effect homogeneity*:

   $$\tau_i = \tau \quad \forall i.$$

In general, treatment effects are heterogeneous, so the statement is false except in special cases.

2. **Statement:** "In experimental data, a simple regression conveys the same information as a simple mean comparison between the treated and control groups."

   Answer: *True (under the usual setup).*

   Consider the simple OLS regression:

   $$Y_i = \alpha + \beta D_i + u_i,$$

   where D_i is a binary treatment (1=treated, 0=control). Under OLS:

   $$\hat{\beta} = \overline{Y}_1 - \overline{Y}_0,$$

   i.e., the difference between treated and control sample means.

   The test statistics and p-values are identical to those from a two-sample t-test.

   Therefore, with a constant term and only the treatment dummy as regressor, the regression and the mean comparison contain exactly the same information.

3. **Statement:** "Exact matching produces the same estimated effect as running an OLS regression with controls for the variables which were used in the exact matching."

   Answer: *Generally false / it depends.*

   Exact matching keeps only those treated–control pairs whose covariates match *exactly*. The treatment effect is then an average difference computed on the matched sample.

   In contrast, OLS:

   $$Y_i = \alpha + \tau D_i + X_i'\gamma + u_i$$

   uses all observations and imposes a linear functional form. OLS may also extrapolate into regions with poor or no overlap.

   The two methods differ in:

   - the sample used (matching discards some observations),
   - weighting (OLS assigns weights based on variance and covariate distribution),
   - functional form assumptions (OLS imposes linearity; matching does not).

Only in special cases (same sample, same weighting, fully interacted specification) will the two methods coincide.

4. **Statement:** "A simple OLS regression will always provide biased estimates."

   Answer: *False.*

   OLS is unbiased under the standard exogeneity assumption:

$$\mathbb{E}[u_i \mid X_i] = 0.$$

   In experimental data, this often holds due to random assignment. In observational data it may fail, but not always. Hence the statement is false.

Long Questions

1. **What is the ideal way to measure the causal effect of x on y (ITE), and how to do this in reality (ATE)? Define the two measures and provide the conditions when ATE=ITE. Give an example when they are not equal.**

   The *ideal* causal effect is the *individual treatment effect*:

   $$ITE_i = Y_i(1) - Y_i(0).$$

   This is fundamentally unobservable because we never see both potential outcomes for the same individual.

   In practice, we estimate the *average treatment effect*:

   $$ATE = \mathbb{E}[Y(1) - Y(0)].$$

   In randomized experiments, the ATE is identified by comparing sample means.

   When is ATE = ITE?

   When the treatment effect is homogeneous:

   $$Y_i(1) - Y_i(0) = \tau \quad \forall i.$$

   Example when they differ

   Suppose high-ability individuals benefit more from a training program:

   $$\tau_i = \begin{cases} 5, & \text{high ability} \\ 1, & \text{low ability} \end{cases}$$

   Then ITE varies by person, but ATE is only the average of these values.

2. **Explain the three types of endogeneity which can arise in observational data. Draw the causal map and give an example for each type.**

   The three types:

   (a) **Omitted variable bias (confounding).** A third factor affects both X and Y:

   $$X \leftarrow U \rightarrow Y.$$

   Example: ability affects both education (X) and earnings (Y).

(b) **Reverse causality.** Causality runs both ways:

$$X \to Y \quad \text{and} \quad Y \to X.$$

Example: health and income mutually influence each other.

(c) **Measurement error in X.** Observed X is measured with noise:

$$X^{obs} = X^{true} + \varepsilon.$$

Example: self-reported income contains random errors, biasing OLS estimates of its effect on consumption.

3. **An analyst runs the regression:**

$$quality_i = \alpha + \beta \cdot family_i,$$

where quality is rated 1–5 and family is a dummy (1 if family-owned). The OLS regression with controls gives $\beta = -0.10^{**}$, $N = 8440$; exact matching on the same controls gives $\beta = -0.16^{**}$, $N = 1207$.

- **Meaning of β:** The estimated causal (or associational) effect of being family-owned on quality. A negative coefficient means family firms have lower average quality scores.
- **Why different N?** OLS uses all observations; exact matching keeps only treated–control pairs with identical covariates, discarding cases without exact matches. Therefore matching yields a smaller sample.
- **Why is β different across methods?** Because:
  - the sample is different,
  - the weighting is different (OLS imposes variance-based weights),
  - matching avoids functional form assumptions while OLS assumes linearity,
  - OLS may extrapolate outside regions with support.
- **Limits of matching; example of bias:** Matching relies on overlap. If treated units exist only in covariate regions where no controls exist, matched estimates become biased or impossible. Example: if all large firms are treated and all small firms are controls, matching cannot separate treatment effects from firm size.

4. **Describe how to do propensity score matching. Explain the differences between this method and exact matching.**

   Steps of PSM:

   (a) Estimate the propensity score:

   $$p(X_i) = \Pr(D_i = 1 \mid X_i),$$

   typically via logit or probit.

   (b) Match treated and control units with similar $p(X)$ values.

   (c) Check balance (standardized differences).

   (d) Estimate the treatment effect on the matched sample.

   Differences from exact matching:

   - Exact matching requires identical covariate values; PSM matches on a single scalar index.
   - Exact matching suffers severely from curse of dimensionality; PSM does not.
   - PSM ensures similarity in predicted probabilities, not exact covariates.
   - PSM typically keeps a larger sample than exact matching.