HPS/Pl 125: Problem 1

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HPS/Pl 125, WI '25

January, 2025

In Maudlin's presentation of the EPR argument, the two-particle state

$$|S\rangle = \frac{1}{\sqrt{2}} |z\uparrow\rangle_R |z\downarrow\rangle_L - \frac{1}{\sqrt{2}} |z\downarrow\rangle_R |z\uparrow\rangle_L,$$

in which the spins are anti-aligned is used to show that: ***the results of spin measurements on either particle, along any axis, must already be determined before either particle has its spin measured.***

1 Part a

Q: *Could you produce the analogous argument for the same conclusion using the state $|A\rangle = |z\uparrow\rangle_R |z\downarrow\rangle_L$ in which the spins are aligned?*

Let's attempt to construct the analogous argument for the given state. The original argument hinges on the fact that knowing the spin of one particle along some axis allows us to predict with certainty the spin of the other particle, mathematically shown to hold regardless of the axis along which you measure or predict the spin. The mathematic proof in question is simply to rewrite the z-spin state in terms of the x-spin state and observe the consequences. So allow us to do the same and rewrite $|A\rangle$ in the x-state. We know that $|z\uparrow\rangle = \frac{1}{\sqrt{2}}|x\uparrow\rangle + \frac{1}{\sqrt{2}}|x\downarrow\rangle$, such that

$$|A\rangle = |z\uparrow\rangle_R |z\downarrow\rangle_L = (\frac{1}{\sqrt{2}}|x\uparrow\rangle_R + \frac{1}{\sqrt{2}}|x\downarrow\rangle_R)(\frac{1}{\sqrt{2}}|x\uparrow\rangle_L + \frac{1}{\sqrt{2}}|x\downarrow\rangle_L)$$
$$= \frac{1}{2}(|x\uparrow\rangle_R |x\uparrow\rangle_L + |x\uparrow\rangle_R |x\downarrow\rangle_L + |x\downarrow\rangle_R |x\uparrow\rangle_L + |x\downarrow\rangle_R |\downarrow\rangle_L)$$

Now we can see that, in terms of x-spin, the state $|A\rangle$ is an equal superposition of all possible x-spin states. This means that the EPR argument doesn't apply - given the x-spin of one particle, we can't predict anything about the x-spin of the other particle. Why is this?

Note that the issue is not the alignment of the spins. To prove this, consider instead a superposition of 2 spin-aligned states, and see if the EPR argument holds.

$$|L\rangle = \frac{1}{\sqrt{2}} |z\uparrow\rangle_R |z\uparrow\rangle_L + \frac{1}{\sqrt{2}} |z\downarrow\rangle_R |z\downarrow\rangle_L$$

$$= \frac{1}{\sqrt{2}}((\frac{1}{\sqrt{2}}|x\uparrow\rangle_R + \frac{1}{\sqrt{2}}|x\downarrow\rangle_R)(\frac{1}{\sqrt{2}}|x\uparrow\rangle_L + \frac{1}{\sqrt{2}}|x\downarrow\rangle_L) + (\frac{1}{\sqrt{2}}|x\uparrow\rangle_R - \frac{1}{\sqrt{2}}|x\downarrow\rangle_R)(\frac{1}{\sqrt{2}}|x\uparrow\rangle_L - \frac{1}{\sqrt{2}}|x\downarrow\rangle_L))$$

$$= \frac{1}{\sqrt{2}}|x\uparrow\rangle_R|x\uparrow\rangle_L + \frac{1}{\sqrt{2}}|x\downarrow\rangle_R|x\downarrow\rangle_L$$

and in this aligned-spin state, we again are able to predict the spin of one particle given the spin of the other along any axis. So clearly, it is not the alignment or anti-alignment of the state that does the work here - it is instead the entanglement of the state.

Notice that state $|A\rangle$ is a product state - it can be written as a simple product of 2 z-spin states, meaning that the spins of the two particles are independent of each other. Yes, the spins are aligned, but this alignment is not due to any entanglement, it is just the way the state was constructed. This means that the spins of the two particles are not correlated in any way, and so the EPR argument does not apply to this state.

On the other hand, both $|S\rangle$ and $|L\rangle$ are entangled states. The wave functions do not have the same separability and do not factor into a simple product of the state of the right particle and the left particle. This means that the spins of the two particles are correlated in some way, and the nature of this correlation is what gives rise to the EPR argument.

2 Part b

The EPR argument is an attempt to show that quantum mechanics is an incomplete description of the physical world. The argument proceeds as follows:

1. When making measurements of the spin of two entangled particles, the results of the measurements are correlated - reading the spin of one particle allows us to instantaneously predict with certainty the spin of the other.

2. No information may travel faster than the speed of light.

3. If information about the spin of one particle is not transmitted to the other particle, then the results of the measurements on the two particles must have been determined before the measurements were made.

4. If the measurements were determined before the measurements were made, but couldn't be ascertained from the wave function, then the wave function does not provide a complete description of the physical state of the system.

5. If wave functions do not provide a complete description of the physical state of a system, then quantum mechanics does not provide a complete description of the physical world, and is therefore incomplete.

Notice that the above argument hinges on a key detail. If the wave function provides a complete description of the physical state of a system, then the EPR argument fails, and we may be forced to conclude that the measurement of one spin somehow transmits to the entangled particle the information about the measurement instantaneously - spooky action at a distance. If, on the other hand, the wave function does not provide a complete description of the system, then there must be some other hidden variable that determines the results of the measurement. Therefore, the presence of a hidden variable gains deep significance in attempting to determine whether or not quantum mechanics demands spooky action at a distance. Therefore, someone (Bell) needs to come along to shed light on the possibility of a hidden variable…