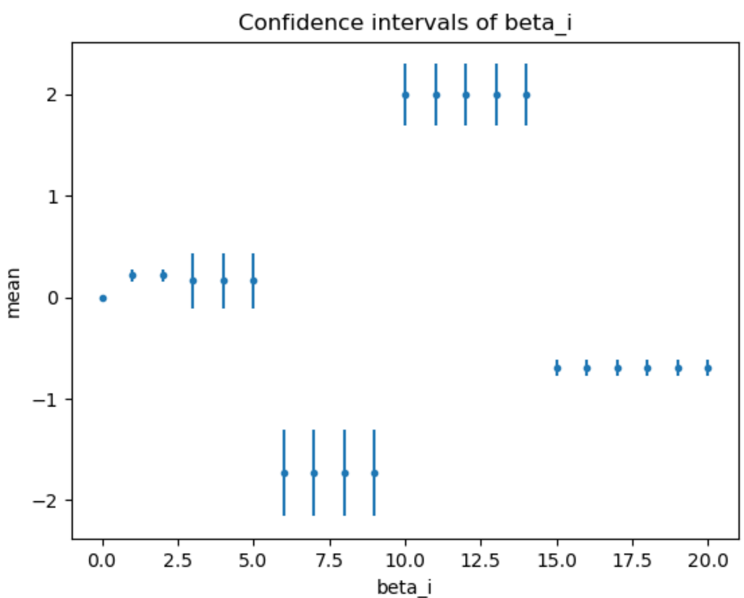
FYS-STK4155 Project 1

Espen Lønes

October 12, 2021

Exercise 1:



Exercise 2:

We have:

$$C(\mathbf{X}, \beta) = \frac{1}{n}\sum_{i=0}^{n-1}(y_i - \tilde{y}_i)^2 = \mathbb{E}[(\mathbf{y} - \tilde{\mathbf{y}})^2]$$

To Derive the wanted equation we use the fact that the variance of $\mathbf{y}$ and ϵ are both σ^2. The mean of ϵ is zero and f is not stochastic for $\tilde{\mathbf{y}}$. And using the more compact notation of expected value, we get.

$$\mathbb{E}[(\mathbf{y} - \tilde{\mathbf{y}})^2] = \mathbb{E}[(\mathbf{f} + \epsilon - \tilde{\mathbf{y}})^2]$$

Then add and subtract $\mathbb{E}[\tilde{\mathbf{y}}]$

$$\mathbb{E}[(\mathbf{y} - \tilde{\mathbf{y}})^2] = \mathbb{E}[(\mathbf{f} + \epsilon - \tilde{\mathbf{y}} + \mathbb{E}[\tilde{\mathbf{y}}] - \mathbb{E}[\tilde{\mathbf{y}}])^2]$$

Then we use the expectation values mentioned above and get.

$$\mathbb{E}[(\mathbf{y} - \tilde{\mathbf{y}})^2] = \mathbb{E}[(\mathbf{y} - \mathbb{E}[\tilde{\mathbf{y}}])^2] + Var[\tilde{\mathbf{y}}] + \sigma^2$$

The tree terms in the equation represent, in order. The square of the bias of the model, the variance of the model and lastly the variance of the error.