Mandatory 2

IN5450

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Power estimation from classical and adaptive beamforming

- R
- Classical / DAS
- Capon's / MV
- Eigenvalues / vectors (signal+noise / noise spaces)
- Eigenvalue
- MUSIC

The problem

- 10 element ULA array, with $\lambda/2$ spacing.
- Two incoherent signals at -10 and 0 degrees, with added spatially white noise.
- SNR is 0 for both sources.
- 100 time samples per element.
- Data generated from (fixed) 'generate_data.m'

Spatial correlation matrix R

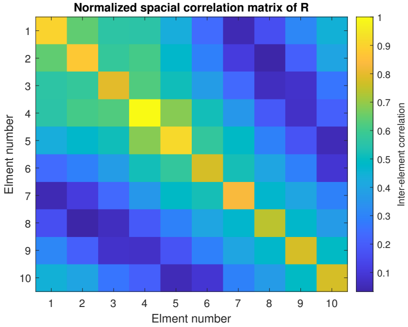
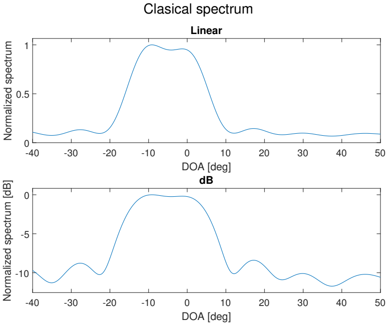


Figure: Caption

$R = x \cdot x^H$

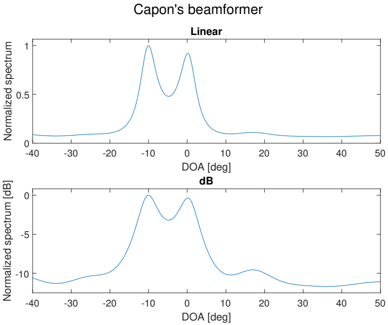
Correlation is a statistical relationship between two variables.

Classical / DAS beamformer



$$P(w) = w^H R w$$
$$max_w \; E(w^H R w) = max_w \; w^H (E(R)) w \rightarrow w_{bf} = \frac{a(\theta)}{\sqrt{a(\theta)^H a(\theta)}}; \quad |w| = 1$$
$$P_{DAS}(\theta) = \frac{a^H R a}{a^H a}$$
$$a_j = exp(j * -kd * sin(\theta * pi/180) * (j-1)); \quad j = 1, ..., 10$$

Capon's / Minimum variance beamformer

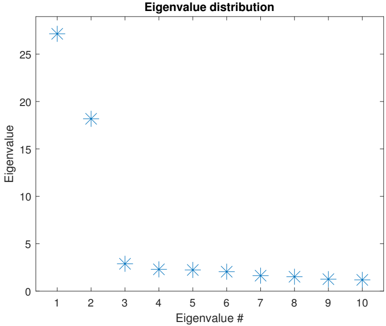


$$P(w) = w^H R w$$
$$min_w \ E(w^H R w); \quad w^H a(\theta) = 1$$
$$w_{CAP} = \frac{R^{-1} a}{a^H R^{-1} a}$$

$$P_{MV}(\theta) = \frac{1}{a^H R^{-1} a}$$

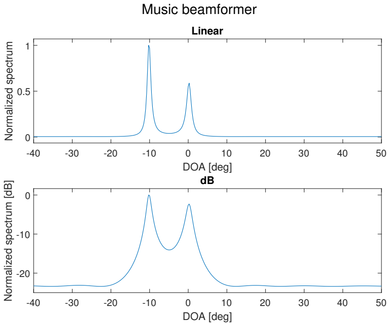
Eigenvalues of R

$R = U \Lambda U^H$



Based on the signal and noise model we know that the eigenvectors corresponding to the largest eigenvalues span the signal+noise space. While the smallest span the noise space.
Largest != signal. But signal vectors are linear combinations.

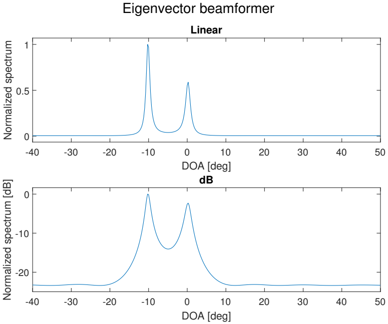
MUSIC beamformer with 2 sources



$R = U_s \Lambda_s U_s^H + U_n \Lambda_n U_n^H$
$U_n^H a(\theta) = 0, \quad \theta \in (1, ..., M),$ (But we have estimates)
2 Sources $\rightarrow$ We use 8 of 10 eigenvectors.
$\Pi = U_n U_n^H$ Orthogonal projector onto noise subspace
$P_{MUSIC} = \frac{a^H a}{a^H \Pi a}$

Eigenvector beamformer with 2 sources

Again we use 8 of 10 eigenvectors.



Similar to MUSIC and uses noise space eigenvalue matrix as well

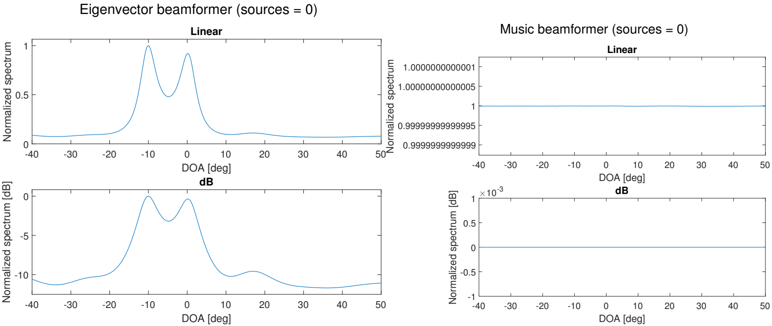
$$P_{EV} = \frac{a^H a}{a^H U_n \Lambda_n^{-1} U_n^H a}$$

Diagonal elements of Λ_n^{-1}

$0.3469, 0.4354, 0.4481, 0.4883, 0.6131, 0.6589, 0.7917, 0.8437$

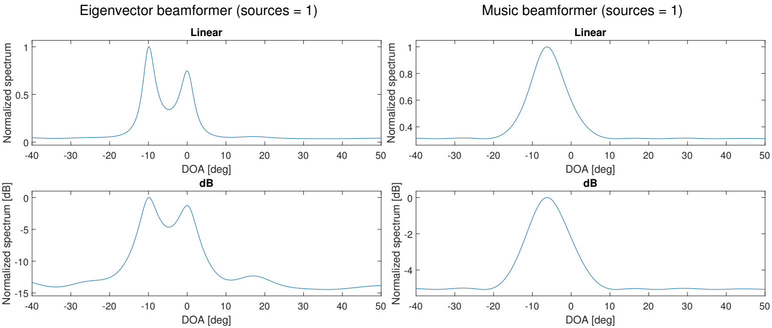
Assuming 0 sources

0 Sources $\rightarrow$ We use 10 of 10 eigenvectors.

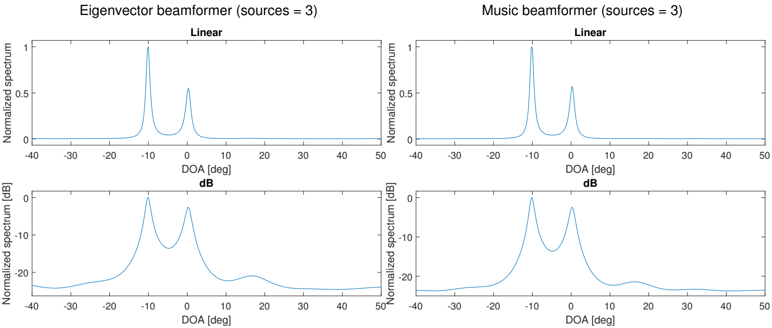


For 0 sources, $U_n \Lambda_n^{-1} U_n^H = R^{-1}$. So it is like the MV beamformer.
MUSIC whitens noise, 0 sources corresponds to only noise.

Assuming 1 source



Assuming 3 sources



References

- H. Krim, M. Viberg, Two decades of array signal processing research – The parametric approach, IEEE Signal Processing Magazine, pp. 67–94, July 1996, `http://dx.doi.org/10.1109/79.526899`