$\dot{u}(x,t) = u''(x,t)$

$u(0,t) = u(1,t) = 0$ randkrav

$u(x,0) = f(x)$ initialkrav

$$\dot{u}_i(t) = \frac{u_{i+1}(t) - 2u_{ij}(t) + u_{i-1}(t)}{h^2}$$

$n-1$ likninger for $u_0 = 0$ og $u_n = 0$

$u(x_i, t_j) \approx u_{ij}$

Eksplisitt:

$$u_{i,j+1} + u_{ij} = \frac{k}{h^2} u_{i+1,j} - \frac{k}{h^2} 2u_{i,j} + \frac{k}{h^2} u_{i-1,j}$$

$\lambda = \frac{k}{h^2}$ (faktor)

$$u_{i,j+1} = \lambda u_{i+1,j} - \lambda 2u_{i,j} + \lambda u_{i-1,j} + u_{ij}$$
$$= \lambda u_{i+1,j} + (1-2\lambda) u_{i,j} + \lambda u_{i-1,j}$$

$\boxed{6}$ $\dot{z}(t) y(x) = \alpha z(t) y''(x)$ $\quad | : z(t)\alpha y(x)$

$$\frac{\dot{z}(t)}{\alpha z(t)} = \frac{y''(x)}{y(x)} = k$$

$u(0,t) = u(1,t) = 0$

Randbetingelse

$\dot{z}(t) = k\alpha z(t)$

$y''(x) = k y(x)$

$y''(x) - ky(x) = 0$

$\left.\begin{array}{l} y^2 - ky = 0 \\ \lambda^2 - k = 0 \end{array}\right\}$ Løs $\quad \lambda^2 - k = 0$

$\lambda = \pm\sqrt{k}$

for $k=0$

$\lambda^2=0 \Rightarrow \lambda=\delta$

$y''(x)=0 \Rightarrow y(x)=A_1e^{\delta x}+A_2te^{\delta x}$

$=A_1+A_2x$

$y(x)=A_1+A_2x$

$y(0)=A_1$

$y(1)=\underbrace{A_1}_{0}+A_2=A_2=0$

$y(x)=0+0x=0$ Ingen løsninger

for $k>0$

$\lambda^2=k$

$\lambda=\pm\sqrt{k}$

$y(x)=e^{\delta x}(A_1e^{\sqrt{k}x}+A_2e^{-\sqrt{k}x})$, $\delta=0$

$y(x)=A_1+A_2$

$A_1+A_2=0 \Rightarrow A_1=-A_2$

$y(1)=\underbrace{e^{\delta}}_{\neq 0}(A_2\underbrace{e^{\sqrt{k}}}_{\neq 0}-A_2\underbrace{e^{\sqrt{k}}}_{\neq 0}) \Rightarrow A_2=0$ Ingen løsnger

for $k<0$

$\lambda=\pm\sqrt{k}\sqrt{-1}$

$\lambda=\pm\sqrt{k}i$

$y(x)=A_1e^{(\delta+\sqrt{k}i)x}+A_2e^{(\delta-\sqrt{k}i)x}$ gitt $\delta=0$

$=A_1e^{\sqrt{k}ix}+A_2e^{-\sqrt{k}ix}$

$e^{ix}=\cos x+i\sin x$, $e^{-ix}=\cos x-i\sin x$

Gir $A_1(\cos(\sqrt{k}i)+xi\sin(\sqrt{k}i))+A_2(\cos(\sqrt{k}x)-i\sin(\sqrt{k}x))$

$\Rightarrow \underbrace{(A_1+A_2)}_{B_1}\cos(\sqrt{k}i)+i\underbrace{(A_1-A_2)}_{B_2}\sin(\sqrt{k}i)$

$y(x) = B_1 \cos(\sqrt{k}x) + B_2 \sin(\sqrt{k}x)$

$y(0) = B_1 \cos(0) + 0$

$= B_1$

$y(1) = B_2 \sin(\sqrt{k})$

Setter $\sqrt{k} = n$ $\Rightarrow y(x) = B_n \sin(nx)$

$\Rightarrow k = -n^2$

$u(x,t) = z(t) b_n \sin(nx)$

$\dot{z}(t) = k\alpha\, z(t)$

$= -n^2 \alpha\, z(t)$

Stemmer

$\dot{z}(t) + n^2 \alpha\, z(t) = 0$

$z(t) = e^{-n^2 \alpha t} \Rightarrow \dot{z}(t) = -n^2 \alpha e^{-n^2 \alpha t} = -n^2 \alpha\, z(t)$

Gir $u(x,t) = z(t)\, y(x) = b_n e^{-n^2 \alpha t} \sin(nx)$

$f(x) = \sin(x)$

$b_n = \frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} f(x) \sin\left(\frac{n\pi x}{L}\right) dx = \frac{1}{L} 2 \int_{-\frac{L}{2}}^{\frac{L}{2}} \sin(x) \sin\left(\frac{n\pi x}{L}\right) dx$

$= \underbrace{\frac{2}{L} \frac{1}{(2i)^2}}_{-\frac{1}{2L}} \int_0^{\frac{L}{2}} \left(e^{ix} - e^{-ix}\right)\left(e^{\frac{in\pi x}{L}} - e^{\frac{-in\pi x}{L}}\right) dx$

$= -\frac{1}{2L} \int_0^{\frac{L}{2}} e^{ix(1+\frac{n\pi}{L})} - e^{ix(1-\frac{n\pi}{L})} - e^{-ix(1-\frac{n\pi}{L})} + e^{-ix(1+\frac{n\pi}{L})}$

$= -\frac{1}{2L} \left[\frac{1}{i(1+\frac{n\pi}{L})} e^{ix(1+\frac{n\pi}{L})} - \frac{1}{i(1-\frac{n\pi}{L})} e^{ix(1-\frac{n\pi}{L})} - \frac{1}{-i(1-\frac{n\pi}{L})} e^{-ix(1-\frac{n\pi}{L})} + \frac{1}{-i(1+\frac{n\pi}{L})} e^{-ix(1+\frac{n\pi}{L})} \right]_0^{\frac{L}{2}}$

Siden det blir så langt så hopper jeg over mellomregning

$$-\frac{1}{2L}\left(\frac{1}{i(1+\frac{n\pi}{L})} e^{\frac{iL}{2}+\frac{i\cancel{L}n\pi}{2\cancel{L}}} - \frac{1}{-i(1-\frac{n\pi}{L})} e^{\frac{iL}{2}-\frac{i\cancel{L}n\pi}{2\cancel{L}}} - \frac{1}{-i(1-\frac{n\pi}{L})} e^{-\frac{iL}{2}+\frac{i\cancel{L}n\pi}{2\cancel{L}}}\right.$$

$$\left. + \frac{1}{-i(1+\frac{n\pi}{L})} e^{-\frac{iL}{2}-\frac{i\cancel{L}n\pi}{2\cancel{L}}}\right)$$

Her går det surr for meg men det skal bli

$$e^{-\alpha t}\cdot \sin(\pi x) \qquad \alpha = 1$$

$$= e^{-t}\cdot \sin(\pi x)$$

settes i python