

question 1:

$$T_n = T_{n/3} + \frac{4}{3} n \log_3 n \quad T(1) = 0$$

$$n = 3^k$$

$$t_k - t_{k-1} = \frac{4}{3} 3^k \log_3(3^k)$$

$$t_k - t_{k-1} = \frac{4}{3} 3^k \cdot k$$

$$\downarrow b=3 \quad d(p(k))=1$$

$$t_k - t_{k-1} \Rightarrow (x-1)^2$$

$$t_k - t_{k-1} \Rightarrow (x-3)^2$$

$$(x-1) - (x-3)^2 \Rightarrow \text{characteristic equation}$$

$$x_1 = 1 \quad x_{2,3} = 3$$

$$t_k = c_1 1^k + c_2 \cdot 3^k + c_3 k \cdot 3^k$$

$$n = 3^k \quad k = \log_3 n$$

$$T_n = c_1 + c_2 n + c_3 \cdot n \cdot \log_3 n$$

$$\rightarrow T_n = T_{n/3} + \frac{4}{3} n \cdot \log_3 n \quad \boxed{T(1) = 0}$$

$$T(3) = T(1) + \frac{4}{3} \cdot 3 \cdot \log_3 3$$

$$T(3) = 0 + 4 \cdot 1 \quad \boxed{T(3) = 4}$$

$$T(9) = T(3) + \frac{4}{3} \cdot 9 \cdot \log_3 9$$

$$T(9) = 4 + 4 \cdot 3 \cdot 2 \quad \boxed{T(9) = 28}$$

$$\rightarrow T_n = c_1 + c_2 n + c_3 n \cdot \log_3 n$$

$$T(1) = c_1 + c_2 = 0$$

$$T(3) = c_1 + 3c_2 + 3c_3 = 4$$

$$T(9) = c_1 + 9c_2 + 18c_3 = 28$$

$$c_1 = -c_2$$

$$2c_2 + 3c_3 = 4$$

$$8c_2 + 18c_3 = 28$$

$$6c_3 = 12 \quad c_3 = 2 \quad c_2 = -1 \quad c_1 = 1$$

$$\boxed{T_n = 1 - n + 2 \cdot n \cdot \log_3 n}$$

question 2:

$n$	$i$	$j$	1st for	2nd for	statement
1	1	1	1	1	1.1
2	1	2	1	2	1.2
3	1, 3	3	2	3	1.3
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
8	1, 3	8	2	8	2.8
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
9	1, 3, 9	9	3	9	3.9
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n$	1, 3, ...	1, 2, ...	$\log_3 n + 1$	$n$	$(\log_3 n + 1) * n$

$$t(n) = (\log_3 n + 1) * n$$

$$O(t(n)) = n \cdot \log n$$