Task 1: What are the fields in computer science that use non-linear algebra?

1. **Computer Graphics:** Non-linear algebra is crucial for rendering realistic images and animations. Techniques like transformation matrices and non-linear transformations are used to manipulate and animate objects in 3D space. In computer graphics, consider a 3D animation of a character jumping. The character's position and orientation are continuously changing over time, and these transformations involve non-linear algebra operations to achieve smooth and realistic motion.

2. **Machine Learning and Deep Learning:** Neural networks and other machine learning models often involve non-linear activation functions, such as sigmoid, ReLU, or tanh. Solving for weights and biases in these networks can involve non-linear algebraic equations. In deep learning, a neural network uses non-linear activation functions like the Rectified Linear Unit (ReLU). The activation function introduces non-linearity, which allows the network to learn complex patterns and relationships in data.

3. **Computer Vision:** Image processing and computer vision tasks often require solving non-linear equations to estimate parameters, such as camera calibration, feature detection, and image registration. Image stitching is a computer vision task that involves estimating the transformation parameters (translation, rotation, scaling) that align multiple images into a single panorama. Solving for these parameters requires non-linear algebraic methods.

4. **Optimization:** Many computer science problems involve optimizing non-linear objective functions, such as in data fitting, resource allocation, or game theory. Non-linear optimization techniques like gradient descent are used. Consider portfolio optimization in finance, where you want to maximize returns while managing risk. This problem involves non-linear optimization to find the right allocation of assets to achieve your objectives.

5. **Robotics:** Robot motion planning and control involve non-linear equations due to the non-linear dynamics of robots. Non-linear algebra is used to model and control these systems. In robot control, calculating the joint angles necessary to move a robotic arm to a specific position involves solving non-linear equations due to the non-linear kinematics of the robot.

6. **Cryptography:** Cryptographic algorithms, like elliptic curve cryptography, rely on the properties of non-linear algebraic structures for secure communication and data protection. In elliptic curve cryptography, the security relies on the difficulty of solving the discrete logarithm problem, which involves non-linear algebraic structures over elliptic curves.

7. **Simulation:** Simulating physical systems often involves solving non-linear equations to model complex behaviours accurately. This applies to physics simulations, fluid dynamics, and more. In fluid dynamics simulations, the Navier-Stokes equations describe the behaviour of fluids. These equations are non-linear and require numerical methods to approximate solutions.

8. **Natural Language Processing (NLP):** Some NLP tasks, especially those related to syntactic or semantic analysis, may involve non-linear algebraic operations when dealing with complex linguistic structures. Parsing natural language sentences to create syntactic or semantic parse trees often involves non-linear algebraic operations when handling complex linguistic rules and structures.

9. **Bioinformatics:** Analysing biological data often requires solving non-linear equations in tasks like protein folding, molecular dynamics, and genetic modelling. Protein folding simulations

use non-linear equations to model the interactions between atoms and molecules to predict the 3D structure of proteins, a fundamental problem in bioinformatics.

10. **Control Systems:** Control theory, used in various engineering and computer science applications, often involves non-linear models for controlling dynamic systems. In aerospace, controlling the flight of an aircraft involves non-linear control systems to handle varying conditions, such as turbulence and changes in altitude.

Task 2:

Probability sampling means that every member of the population has a chance of being selected. It is mainly used in quantitative research. If you want to produce results that are representative of the whole population, probability sampling techniques are the most valid choice.

There are four main types of probability sample:

1. Simple random sampling

In a simple random sample, every member of the population has an equal chance of being selected. Your sampling frame should include the whole population. To conduct this type of sampling, you can use tools like random number generators or other techniques that are based entirely on chance.

2. Systematic sampling

Systematic sampling is like simple random sampling, but it is usually slightly easier to conduct. Every member of the population is listed with a number, but instead of randomly generating numbers, individuals are chosen at regular intervals.

3. Stratified sampling

Stratified sampling involves dividing the population into subpopulations that may differ in important ways. It allows you draw more precise conclusions by ensuring that every subgroup is properly represented in the sample.

To use this sampling method, you divide the population into subgroups (called strata) based on the relevant characteristic (e.g., gender identity, age range, income bracket, job role). Based on the overall proportions of the population, you calculate how many people should be sampled from each subgroup. Then you use random or systematic sampling to select a sample from each subgroup.

4. Cluster sampling

Cluster sampling also involves dividing the population into subgroups, but each subgroup should have similar characteristics to the whole sample. Instead of sampling individuals from each subgroup, you randomly select entire subgroups. If it is practically possible, you might include every individual from each sampled cluster. If the clusters themselves are large, you can also sample individuals from within each cluster using one of the techniques above. This is called multistage sampling. This method is good for dealing with large and dispersed populations, but there is more risk of error in the sample, as there could be substantial differences between clusters. It’s difficult to guarantee that the sampled clusters are representative of the whole population.

In a non-probability sample, individuals are selected based on non-random criteria, and not every individual has a chance of being included. This type of sample is easier and cheaper to access, but it has a higher risk of sampling bias. That means the inferences you can make about the population are weaker than with probability samples, and your conclusions may be more limited. If you use a non-probability sample, you should still aim to make it as representative of the population as possible. Non-probability sampling techniques are often used in exploratory and qualitative research. In these types of research, the aim is not to test a hypothesis about a broad population, but to develop an initial understanding of a small or under-researched population.

1.Convenience sampling

A convenience sample simply includes the individuals who happen to be most accessible to the researcher.

This is an easy and inexpensive way to gather initial data, but there is no way to tell if the sample is representative of the population, so it can't produce generalizable results. Convenience samples are at risk for both sampling bias and selection bias.

Example: Convenience sampling

You are researching opinions about student support services in your university, so after each of your classes, you ask your fellow students to complete a survey on the topic. This is a convenient way to gather data, but as you only surveyed students taking the same classes as you at the same level, the sample is not representative of all the students at your university.

2. Voluntary response sampling

Similar to a convenience sample, a voluntary response sample is mainly based on ease of access. Instead of the researcher choosing participants and directly contacting them, people volunteer themselves (e.g. by responding to a public online survey).

Voluntary response samples are always at least somewhat biased, as some people will inherently be more likely to volunteer than others, leading to self-selection bias.

Example: Voluntary response sampling

You send out the survey to all students at your university and a lot of students decide to complete it. This can certainly give you some insight into the topic, but the people who responded are more likely to be those who have strong opinions about the student support services, so you can't be sure that their opinions are representative of all students.

3. Purposive sampling

This type of sampling, also known as judgement sampling, involves the researcher using their expertise to select a sample that is most useful to the purposes of the research.

It is often used in qualitative research, where the researcher wants to gain detailed knowledge about a specific phenomenon rather than make statistical inferences, or where the population is very small and specific. An effective purposive sample must have clear criteria and rationale for inclusion. Always make sure to describe your inclusion and exclusion criteria and beware of observer bias affecting your arguments.

Example: Purposive sampling

You want to know more about the opinions and experiences of disabled students at your university, so you purposefully select a number of students with different support needs in order to gather a varied range of data on their experiences with student services.

4. Snowball sampling

If the population is hard to access, snowball sampling can be used to recruit participants via other participants. The number of people you have access to “snowballs” as you get in contact with more people. The downside here is also representativeness, as you have no way of knowing how representative your sample is due to the reliance on participants recruiting others. This can lead to sampling bias.

Example: Snowball sampling

You are researching experiences of homelessness in your city. Since there is no list of all homeless people in the city, probability sampling isn’t possible. You meet one person who agrees to participate in the research, and she puts you in contact with other homeless people that she knows in the area.

5. Quota sampling

Quota sampling relies on the non-random selection of a predetermined number or proportion of units. This is called a quota.

You first divide the population into mutually exclusive subgroups (called strata) and then recruit sample units until you reach your quota. These units share specific characteristics, determined by you prior to forming your strata. The aim of quota sampling is to control what or who makes up your sample.