RSA Example

4- Using the Euclidean algorithm, find the multiplicative inverse of
a. 1234 mod 4321
c. 550 mod 1769

In a public-key system using RSA, you intercept the cipher text C=10 sent for a user whose public key is e=5, n=35. What is the plaintext M?

-Assume that Bob has public RSA *key* ($n = 65$, $e = 5$). Show that Bob's private key is ($d = 29$)
- Alice wants to send the message $m = 11$ to Bob. She encrypts the message using Bob's public key. What is the value of the ciphertext that Alice sends to Bob?
- David has also sent an encrypted message to Bob. The ciphertext value that Bob receives from David is 19. Showing all your working, use Bobs key to decrypt this ciphertext and recover the value of David's message.

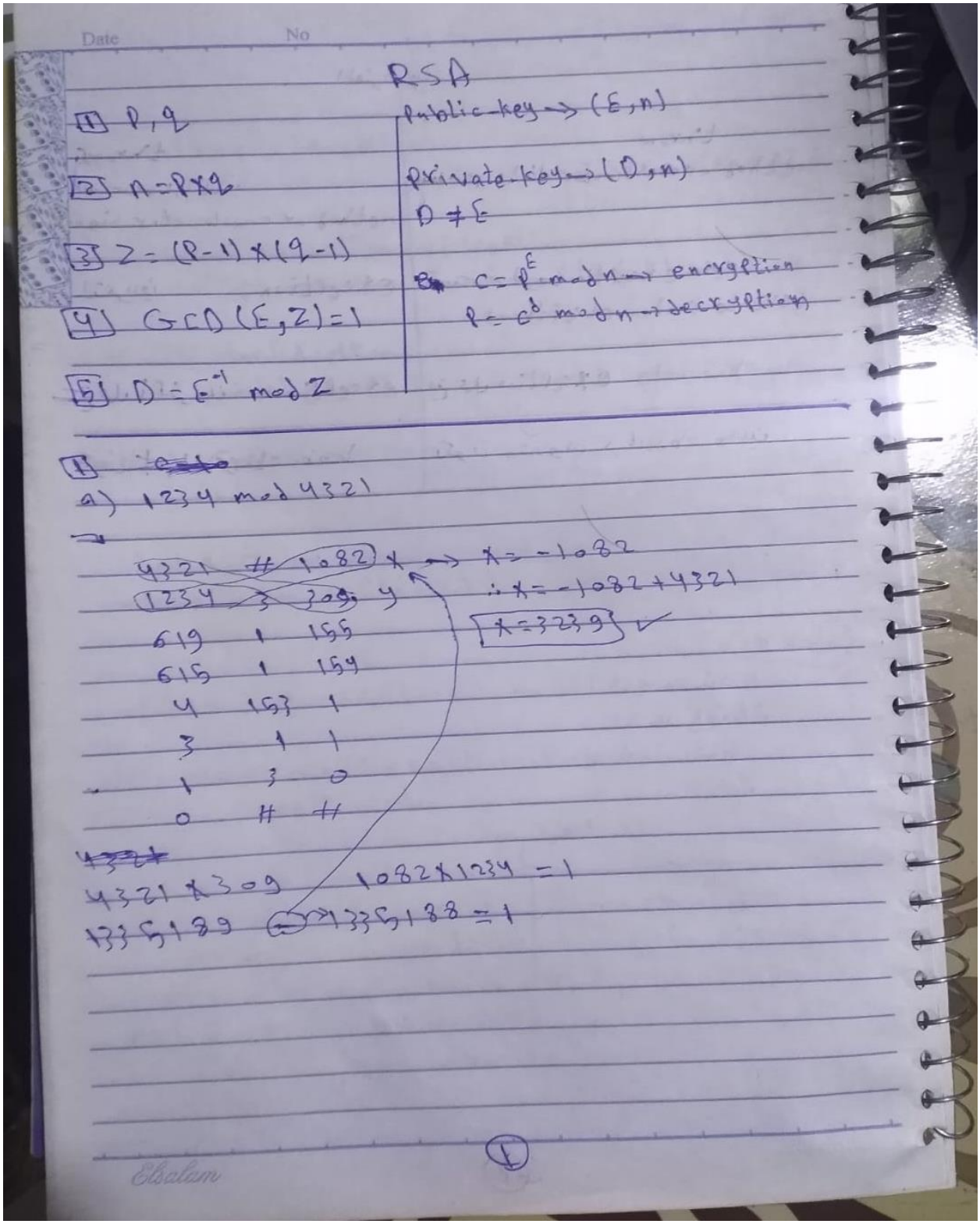
Question Three

What is $11^{-1} \pmod{29}$? Show your work.?

Question one

1- Bob has public RSA *key* ($n = 65$, $e = 5$) Show that Bob's private key is ($d = 29$)
2-What is difference between rule based anomaly

Answer:


RSA
Public key → (E, n)
Private key → (D, n)
D ≠ E
C = P^E mod n → encryption
P = C^D mod n → decryption
1) P, q
2) n = P × q
3) Z = (P-1) × (q-1)
4) GCD (E, Z) = 1
5) D = E^-1 mod Z
a) 1234 mod 4321
4321 = 1082 x → x = -1082
1234 = 309 y
∴ x = -1082 + 4321
x = 3239
619 1 155
615 1 154
4 153 1
3 1 1
1 3 0
0 # #
4321 × 309 1082 × 1234 = 1
1335189 1335188 = 1
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$550 \bmod 1769$

1769	#	550	x
550	3	171	y
119	4	37	
74	1	23	
45	1	14	
29	1	9	
16	1	5	
13	1	4	
3	4	1	
1	3	0	
0	#	#	

$\therefore x = 550$ ✓

$-1769 \times 171 \oplus 550 \times 550 = 1$

(2) $c = 10$, $e = 5$, $n = 35$, $m = ??$

$n = p \times q$ $\quad p \times q = 35$ $\quad p = 5$, $q = 7$

$z = (p-1)(q-1) = 4 \times 6 = 24$

Decryption $\Rightarrow$ $c^d \bmod n$

$\therefore D = 6^{-1} \bmod z \longrightarrow 5 \bmod 24$

24	#	5	x
5	4	1	y
4	1	1	
1	4	0	
0	#	#	

$-24 + 25 = 1$ $\quad \therefore x = 5 = d$

$10^5 \bmod 35 = 5$

$\therefore m = 5$

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Public key (n=55, e=5)

Bob ← Alice

Private key (d=29)

$Pt_{m=11}$

Encryption

$C = Pt^e \bmod n = 11^5 \bmod 55$

$\therefore C = 46$

David → Bob

C = 19

d = 29

Decryption

$Pt = C^d \bmod n \Rightarrow 19^{29} \bmod 55$

$29 = 1+2+2+4+4+8+8$

$(19^1 \bmod 55) * (19^2 \bmod 55) * \ldots \bmod 55$

$(19 * 36 * 36 * 61 * 61 * 16 * 16 * 16) \bmod 55$

$\therefore Pt = 54$

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4) $11^{-1} \pmod{29}$

29	#	8	x
11	2	3	y
7	1	2	
4	1	1	
3	1	1	
1	3	0	
0	#	#	

$29x3 \quad 11x8 = 1$

$-87 + 88$

$\therefore x = 8$

5) $n = 55, e = 5$ $\quad d = 29$ → How??

$\therefore n = p \times q \rightarrow p = 5, q = 13$

$z = (p-1)(q-1) \rightarrow 4 \times 12 = 48$

$d = e^{-1} \bmod z \rightarrow 5 \bmod 48$

48	#	19	x
5	9	2	y
3	1	1	
2	1	1	
1	2	0	
0	#	#	

$48x2 \quad 5x19 = 1$

$96 - 95 = 1$

$x_2 = -19$ $\quad \therefore x = -19 + 48 = 29$

$\therefore d = 29$ ✓

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