The Aharonov - Bohm Effect

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The Potential Formulation of Electromagnetic Fields

$$\begin{aligned}\vec{B} &= \nabla \times \vec{A} \\ \vec{E} &= -\nabla\phi - \frac{\partial \vec{A}}{\partial t}\end{aligned}$$

Gauge Freedom in the Potential Formulation

$$\begin{aligned}\vec{A}' &= \vec{A} + \nabla\chi \\ \phi' &= \phi - \frac{\partial \chi}{\partial t}\end{aligned}$$

$$\begin{aligned}\vec{B}' &= \nabla \times \vec{A}' = \nabla \times (\vec{A} + \nabla\chi) = (\nabla \times \vec{A}) + \cancelto{0}{\nabla \times (\nabla\chi)} = \vec{B} \\ \vec{E}' &= -\nabla\phi' - \frac{\partial \vec{A}'}{\partial t} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t} + \cancelto{0}{(\partial_t \nabla\chi - \nabla\partial_t \chi)} = \vec{E}.\end{aligned}$$

System Propagator

$$\mathcal{K}(x_b, t_b; x_a, t_a) = \int \mathcal{D}[x(t)] e^{\frac{i}{\hbar} S[x(t)]}$$

$$S = \int \mathcal{L} \, \mathrm{d}t$$

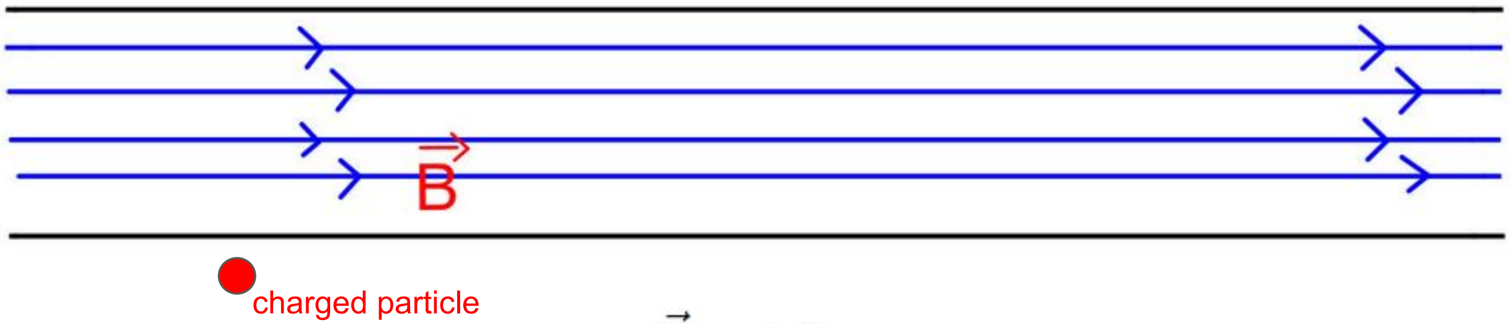
The Lagrangian for a charged particle in an EM field:

$$\mathcal{L} = \frac{1}{2} m \dot{\vec{x}}^2 + q \dot{\vec{x}} \cdot \vec{A} - q\phi$$

$$\mathcal{K}(t) = \mathcal{K}_0 \int_0^t \mathcal{D}[x(t)] \exp\left(\frac{i}{\hbar} S_{\text{interactive}} \right)$$

The Magnetic AB Effect

Infinitely Long Solenoid



$$\vec{B}_{\text{in}} \neq 0$$
$$\vec{B}_{\text{out}} = 0$$

$$\vec{E}_{\text{out}} = 0$$

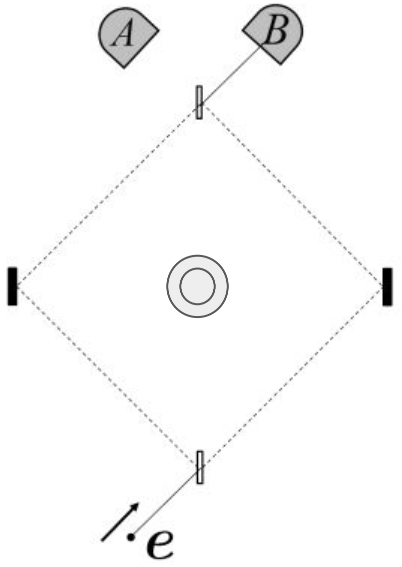
The Magnetic AB Effect

Define a 4-potential,

$$A_\mu = (\phi, -\vec{A})$$

$$\mathcal{K}_j = \mathcal{K}_0 \int_0^t \mathcal{D}[x(t)] \exp\left(\frac{iq}{\hbar}\int_{C_j} \mathrm{d}x^\mu A_\mu\right)$$

$$\begin{aligned} {}^{\vec{B}}\Delta\varphi_{\mathrm{AB}} &= \frac{q}{\hbar}\iint_\Sigma (\nabla \times A_\mu)\, \mathrm{d}\Sigma \\ &= \frac{q}{\hbar}\Phi_B \end{aligned}$$



Y. Aharonov and D. Bohm. “Significance of Electromagnetic Potentials in the Quantum Theory”. In: The Physical Review (1959)

The Electric AB Effect

Choose a gauge where

$$\vec{A} = 0$$

$$\begin{aligned} \vec{B} &= \nabla \times \vec{A} \\ \vec{E} &= -\nabla\phi - \frac{\partial \vec{A}}{\partial t} \end{aligned}$$

$$^{\vec{E}}\Delta\varphi_{\mathrm{AB}} = \frac{q}{\hbar}\int_{t_a}^{t_b} \mathrm{d}t\, \phi(t)$$

Y. Aharonov and D. Bohm. “Significance of Electromagnetic Potentials in the Quantum Theory”. In: The Physical Review (1959)

Non Local?

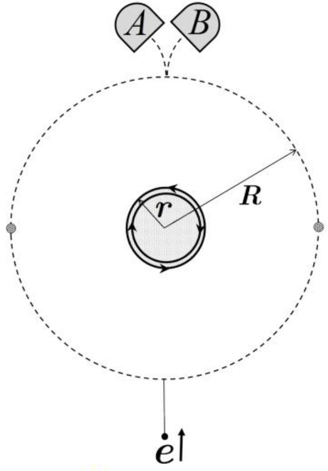
$$^{\vec{B}}\Delta\varphi_{\mathrm{AB}} = \frac{q}{\hbar}\Phi_B$$

$$^{\vec{E}}\Delta\varphi_{\mathrm{AB}} = \frac{q}{\hbar}\int_{t_a}^{t_b} \mathrm{d}t\ \phi(t)$$

- Non-local theory for quantum interactions?
- Present local theory - new interpretation for potentials?
- Local interactions seemingly non-local?

Localising in the AB effect

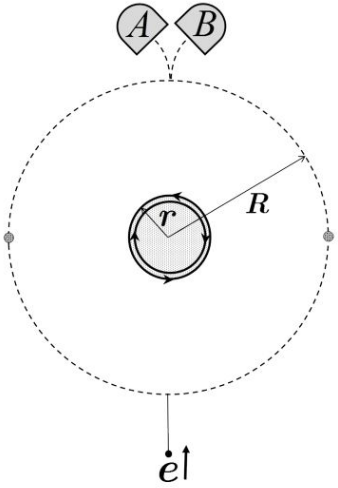
System 1: Magnetic AB Effect



$$|\Psi_B(t=0)\rangle = \frac{1}{\sqrt{2}}\big(|R\rangle_e + |L\rangle_e\big) \otimes |\Psi_0\rangle_c$$

Lev Vaidman. “Role of potentials in the Aharonov–Bohm effect”. In: Physical Review A (2012)

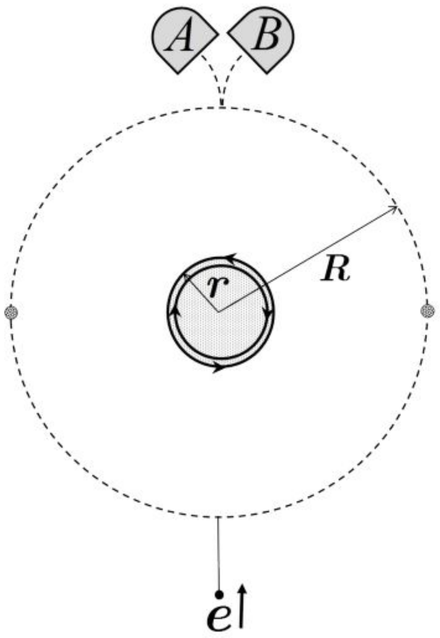
Vaidman's Magnetic AB Phase



$$\mathcal{K}_B(t) = |R\rangle \langle R|_e \otimes \mathcal{K}_R(t) + |L\rangle \langle L|_e \otimes \mathcal{K}_L(t)$$

Lev Vaidman. "Role of potentials in the Aharonov–Bohm effect". In: Physical Review A (2012)

Vaidman's Magnetic AB Phase

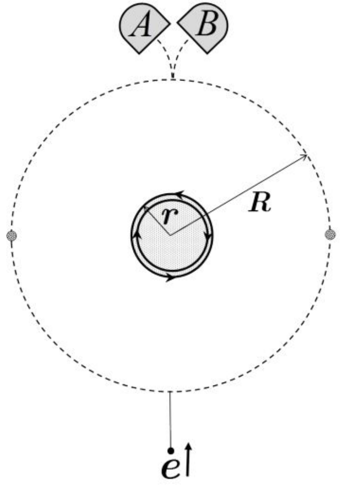


$$\vec{B}_z(z) = \frac{\mu_0}{4\pi} I \int_0^{2\pi} \mathrm{d}\theta \, \frac{R\big(\hat{\theta} \times \big(\vec{z} - \vec{R}\big)\big)}{\big(R^2 + z^2\big)^{3/2}} = \frac{\mu_0 I R^2}{2\big(R^2 + z^2\big)^{3/2}}$$

$$\Phi_z(z) = \int \vec{B} \cdot \mathrm{d}\vec{A} = \frac{\mu_0 e u R}{4\pi\big(R^2 + z^2\big)^{3/2}} \cdot \pi r^2 = \frac{\mu_0 e u R r^2}{4\big(R^2 + z^2\big)^{3/2}}$$

Lev Vaidman. "Role of potentials in the Aharonov–Bohm effect". In: Physical Review A (2012)

Vaidman's Magnetic AB Phase



$$\Delta p = \int F \, \mathrm{dt} = \mathrm{d}Q \int \mathrm{dt} \cdot \frac{-1}{2\pi r} \frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

$$\delta x = \frac{1}{M} \frac{\pi R}{u} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_0^{\Phi_z} \frac{Q}{L} \, \mathrm{d}\Phi_z \, \mathrm{d}z$$

$$\delta x = \frac{\mu_0 Q e r}{4ML}$$

$$\Delta\varphi = k(\delta x)$$

Lev Vaidman. "Role of potentials in the Aharonov–Bohm effect". In: Physical Review A (2012)

Vaidman's Magnetic AB Phase

Phase Entanglement

$$|\Psi_B(T)\rangle \;=\; \mathcal{K}_B(T)\,|\Psi_B(0)\rangle = \frac{1}{\sqrt{2}}\big(\,|R\rangle_e \otimes \mathcal{K}_R\,|\Psi_0\rangle_c + |L\rangle_e \otimes \mathcal{K}_L\,|\Psi_0\rangle_c\big)$$

$$|\Psi_B(T)\rangle = \frac{1}{\sqrt{2}}\big(e^{(i\varphi_R)}\,|R\rangle_e + e^{(i\varphi_L)}\,|L\rangle_e\big) \otimes |\Psi_0\rangle_c$$

Lev Vaidman. “Role of potentials in the Aharonov–Bohm effect”. In: Physical Review A (2012)

Vaidman's Magnetic AB Phase

$$^{B}\Delta\varphi_{\mathrm{local}} = 4\frac{Mv}{\hbar}\frac{\mu_0 Qer}{4ML} = \frac{e}{\hbar}\left(\frac{\mu_0 Qvr}{L}\right)$$

Confirming Results

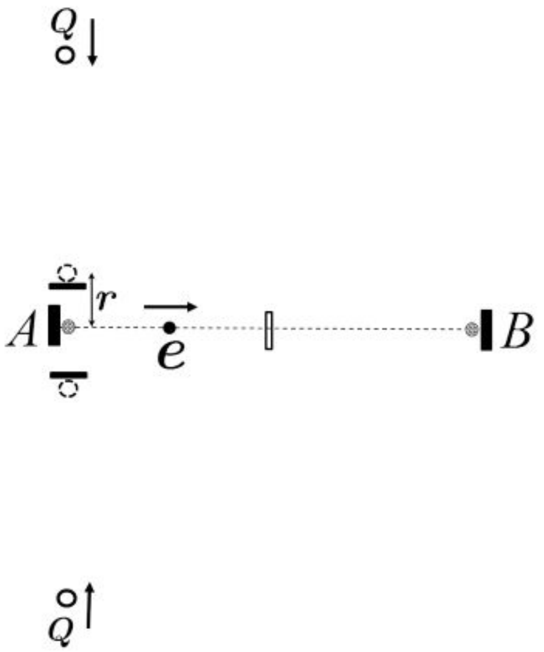
$$^{\vec{B}}\Delta\varphi_{\mathrm{AB}} \quad = \quad \frac{q}{\hbar}\Phi_B$$

$$\Phi_B = \int \vec{B}\cdot \mathrm{d}\vec{A} = (\mu_0 \mathcal{I}_e)(\pi r^2)$$

$$^{B}\Delta\varphi_{\mathrm{AB}} = \frac{e}{\hbar}\frac{\mu_0 Qvr}{L}$$

Lev Vaidman. "Role of potentials in the Aharonov–Bohm effect". In: Physical Review A (2012)

Vaidman's Electric AB Phase



1. A electron wave packet splits into two wave packets, where each wave packet approaches mirror A and B.
2. Each electron wave packet spends a time $\tau > T$ at its respective mirror before being reflected. The charged particles reside near the electron during this time, and the electron resides in a region where electric fields from both particles cancel out, thus availing it a field-free region.
3. The wave packets, once reflected interfere coherently.

$$|\Psi_E(0)\rangle = |\Psi_e\rangle \otimes |\Psi_0\rangle_Q$$

$$|\Psi_e\rangle = \frac{1}{\sqrt{2}}\left(|A\rangle_e + |B\rangle_e \right)$$

$$\mathcal{K}_E(t) = |A\rangle \langle A|_e \otimes \mathcal{K}_Q(t) + |B\rangle \langle B|_e \otimes \mathbb{I}_Q$$

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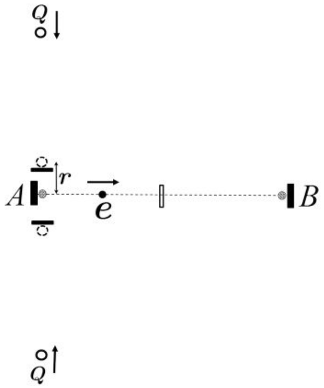
Vaidman’s Electric AB Phase

Comparing Energies



$$\mathcal{T}_e = V(x = r) = -\frac{k_c e Q}{r}$$

$$\delta x = T \delta v.$$

$$\delta x = -\frac{k_c e Q}{r M v} T.$$

Lev Vaidman. “Role of potentials in the Aharonov–Bohm effect”. In: Physical Review A (2012)

Vaidman's Electric AB Phase

$$\Delta\varphi = k(\delta x)$$

$${}^{E}\Delta\varphi_{\text{local}} = -2\frac{k_c e Q}{\hbar r}T.$$

Confirming Results

$$\text{Coulombic potential } U(x) = \frac{-k_c Q}{x}$$

$${}^{E}\Delta\varphi_{\text{AB}} = \frac{e}{\hbar}\cdot -2\frac{k_c Q}{r}\int_0^T dt = -2\frac{k_c e Q}{\hbar r}T.$$

Lev Vaidman. "Role of potentials in the Aharonov–Bohm effect". In: Physical Review A (2012)

Shortcomings of Vaidman’s Localisation

- How does one assume the velocity of the charge to be constant?
- Local interactions with remote fluxes

Y. Aharonov, Eliahu Cohen, and Daniel Rohrlich. “Nonlocality of the Aharonov–Bohm Effect”. In: Physical Review A (2016).