The Aharonov-Bohm Effect

$$K(x', t'; x, t) = \int \exp\left(\frac{i}{\hbar} S[x(t)]\right) \mathcal{D}[x(t)]$$

Everything can be approximated with a Taylor Series Expansion

The Path Integral Formulation

Schrödinger's Formulation stems from Hamiltonian mechanics whereas Feynman's Formulation is tied to Lagrangian mechanics.

The quantum problem is fully solved once the propagator is known.

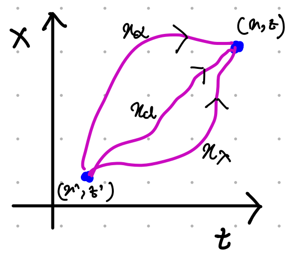
$$|\psi(t=t)\rangle = U(t)|\psi(t=0)\rangle$$

We will use the Lagrangian Formalism to derive the propagator $U(x, t, x', t')$.

$$U(x, t, x', t') = A \sum_{\text{all}} \exp(i S[x(t)]/\hbar)$$

where S is the action for each path $\forall\ (x,t) \rightarrow (x', t')$ and A is a normalisation factor.

For now, we will assume the continuum of paths linking the end points is a discrete set. The correct way to do this, however, would be to use a path integral.



$$Z_\alpha = \exp(i S[x_\alpha(t)]/\hbar)$$

Adding up many numbers w/ complex phases causes destructive interference. However, near the classical path, the functional is stationary - small changes to the path do not change the action to the first order.

$$S = \int_{t_i}^{t_f} \mathcal{L}(x,t)\,\dot{x}(t)\,dt$$

[Proof omitted for now] Small fluctuations around the classical path don't significantly change the action 'S' (to the first order).

Crudely, one says that coherence is lost once the phase differs from the stationary value $S_{cl}/\hbar = S[x_{cl}(t)]/\hbar$ by about π.

How far must we go from x_{cl} until destructive interference sets in?

Phase $\phi = \dfrac{S[x(t)]}{\hbar}$

If $|S - S_{cl}| \gtrsim \hbar$, then $\phi \gtrsim 1$

Coherence: $|S_i - S_{cl}| \lesssim \hbar\pi$

On $\{S_i\}$ contribute meaningfully.

For a macro system, $S_{cl} \sim 1 \text{ erg} \sim 10^{27}\hbar$ (moves over 1 second)

This is an enormous phase (10^{27} spins around the complex plane) $\Rightarrow$ A small deviation from S_{cl} shifts by multiple cycles and the paths cancel

However, for quanta,

$S_{cl} \sim \hbar$, and thus $\pi\hbar$ is a significant chunk of the total action

Approximating $U(t)$ for a free particle

Ignoring all but the classical path gives us an excellent approximation.

$$U(t) = A' \exp(i S_{cl}/\hbar)$$

The classical path for a free particle is a straight line in the x-t plane.

$$x_{cl}(t'') = x' + \frac{x - x'}{t - t'}(t'' - t')$$

We see that $v = \dfrac{x - x'}{t - t'}$

and since $\mathcal{L} = \frac{1}{2} m v^2$ is constant,

$$S_{cl} = \int_{t'}^{t} \mathcal{L}\, dt = \frac{1}{2} m \frac{(x - x')^2}{(t - t')}$$

Thus,

$$U(x, t; x', t') = A' \exp\left(i \frac{1}{2} m \frac{(x - x')^2}{(t - t')} \cdot \frac{1}{\hbar} \right)$$

$(t - t') \to 0$, we use this to find A'

* We know $\delta(x - x') = \lim_{\Delta \to 0} \dfrac{1}{\Delta\sqrt{\pi}} \exp\left(- \dfrac{(x - x')^2}{\Delta^2} \right)$; $\Delta \in \mathbb{C}$

$$\psi(x, t) = \int_{-\infty}^{\infty} dx'\, U(x, t; x', t')\, \psi(x', t')$$

if $t - t' \to 0$ $\Rightarrow$ $t \to t'$

$$\psi(x, t') = \int_{-\infty}^{\infty} dx'\, U(x, t'; x', t')\, \psi(x', t') \Rightarrow U(x, t'; x', t') = \delta(x - x')$$

Evaluating $\delta(x-x') = A' \exp\left(\frac{im}{2\hbar}\frac{(x-x')^2}{(t-t')}\right)$,

$$A' = \sqrt{\frac{m}{2\pi\hbar i\,(t-t')}}$$

$$\therefore\ U(x,t;x',0) = \left(\frac{m}{2\pi\hbar i t}\right) \exp\left(\frac{im\,(x-x')^2}{2\hbar\, t}\right)$$

This method of approximating fails if

- V is not of the form $V = a + bx + cx^2 + d\dot{x} + ex\dot{x}$

  [that $U(t) = A(t)\exp(i S_{cl}/\hbar)$ holds true]

- A may contain arbitrary functions $\$\ f \to 1$ as $t \to 0$.

Path Integral Evaluation of the Free - Particle Propagator

$$U(x_N, t_N; x_0, t_0) = \int_{t_0}^{t_N} e^{\left(iS[x(t)]/\hbar\right)} \mathcal{D}[x(t)]$$

where $\int_{t_0}^{t_N} \mathcal{D}[x(t)]$ represents an integration over all the possible paths between x_N and x_0

Since each path $x(t)$ is defined by $t \in [t_0, t_N]$, there is an uncountable infinity of numbers. Since we cannot integrate over an uncountable set, we discretize time.

$$\#([t_0, t_N]) = N$$

We divide the set $[t_0, t_N]$ into equal slices

$$\varepsilon = \frac{t_N - t_0}{N}$$

where $t_m = t_0 + m\varepsilon$; $m = 0, 1, 2, \ldots, N$

Thus, each discrete path can be represented as a set $\{x_0, x_1, x_2, \ldots, x_N\}$ where $x_m = x(t_m)$

We can thus expand the path integration measure as

$$\int_{t_0}^{t_N} \mathcal{D}[x(t)] = \lim_{N \to \infty} \int_{-\infty}^{+\infty} dx_1 \cdot \int_{-\infty}^{\infty} dx_2 \cdot \ldots \cdot \int_{-\infty}^{\infty} dx_{N-1} \times \text{Normalisation}$$

The action integral can be represented as a Riemann sum

$$S[x(t)] = \int_{t_0}^{t_N} \mathcal{L}(x, \dot{x}, t)\, dt$$

For a free particle, $\mathcal{L} = \frac{1}{2} m \dot{x}^2$

and $\dot{x} \approx \frac{x_{i+1} - x_i}{\varepsilon}$ (since we discretised time)

$$\therefore\ S[x(t)] \approx \sum_{i=0}^{N-1} \mathcal{L}_i \varepsilon = \sum_{i=0}^{N-1} \frac{m}{2} \frac{(x_{i+1} - x_i)^2}{\varepsilon}$$

Putting all these together,

$$U(x_N, t_N; x_0, t_0) = \int_{t_0}^{t_N} \mathcal{D}[x(t)]\, \mathrm{e}^{(i S[x(t)] / \hbar)}$$

$$= \lim_{\substack{N \to \infty \\ \varepsilon \to 0}} A \left\{ \prod_{i=1}^{N-1} \int_{-\infty}^{+\infty} \mathrm{e}^{\left(\frac{i}{\hbar} \frac{m}{2} \sum_{j=0}^{N-1} \frac{(x_{j+1} - x_j)^2}{\varepsilon} \right)} dx_i \right\}$$

Simplifying further,

let $y_i = \sqrt{\frac{m}{2\hbar \varepsilon}}\, x_i$

Then,

$$U = \lim_{N \to \infty} A' \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{\left(-\sum_{i=0}^{N-1} \frac{(y_{i+1}-y_i)^2}{i}\right)} \prod_{i=1}^{N-1} dy_i$$

where $A' = A\left(\frac{2\hbar\varepsilon}{m}\right)^{(N-1)/2}$

Integrating individually, we first consider 'y_1'.

$$\int_{-\infty}^{\infty} e^{\left(-\frac{1}{i}\left\{(y_2-y_1)^2 + (y_1-y_0)^2\right\}\right)} dy_1$$

ignoring calculations (lazy) $= \sqrt{\frac{i\pi}{2}}\, e^{\left(-(y_2-y_0)^2/2i\right)}$

Considering the next part: over 'y_2'.

We see $\sqrt{\frac{i\pi}{2}} \int_{-\infty}^{\infty} e^{\left(-(y_3-y_2)/i\right)} \cdot e^{\left(-(y_2-y_0)^2/2i\right)} dy_2$

$$= \left(\frac{(i\pi)^2}{3}\right)^{1/2} e^{\left(-(y_3-y_0)^2/3i\right)}$$

Deducing a pattern, we conclude

$$\frac{(i\pi)^{(N-1)/2}}{\sqrt{N}} e^{\left(\frac{-(y_N-y_0)^2}{Ni}\right)}$$

Or,

$$\frac{\sqrt{i\pi}^{\,(N-1)}}{\sqrt{N}}\; e^{\left(-\frac{m(x_N - x_0)^2}{2\hbar \varepsilon N i}\right)}$$

Thus,

$$U = A\left(\frac{2\pi\hbar\varepsilon i}{m}\right)^{N/2}\left(\frac{m}{2\pi\hbar i N\varepsilon}\right)^{1/2} e^{\left(\frac{im\,(y_N - y_0)^2}{2\hbar N\varepsilon}\right)}$$

(lots of regrouping w/ exponents)

A free particle has only one prefactor which is $\left(\frac{m}{2\pi i\hbar N\varepsilon}\right)^{1/2}$. We pick up a factor B from each Gaussian Integral $B = \sqrt{\frac{2\pi i\hbar\varepsilon}{m}}$.

$$\therefore AB^N = 1$$

$$A = \left(\frac{m}{2\pi i\hbar\varepsilon}\right)^{-N/2} = \left(\frac{2\pi i\hbar\varepsilon}{m}\right)^{1/2}$$

and $B^{-N} = \left(\frac{m}{2\pi i\hbar\varepsilon}\right)^{-N/2}$

Then $\displaystyle\int \mathcal{D}[x(t)] = \lim_{\substack{\varepsilon\to 0\\ N\to\infty}} \frac{1}{B}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\prod_{i=1}^{N-1}\frac{dx_i}{B}$

Equivalence to the Schrödinger Equation

We know $i\hbar \frac{d}{dt}|\psi(t)\rangle = \mathcal{H}|\psi(t)\rangle$

w/ a finite time evolution,

$$|\psi(\varepsilon)\rangle = U(\varepsilon)|\psi(0)\rangle$$

where $U(\varepsilon) \equiv e^\wedge\left(-\frac{i}{\hbar}\mathcal{H}\varepsilon\right)$

Using Taylor's Expansion,

$$e^\wedge\left(-\frac{i}{\hbar}\mathcal{H}\varepsilon\right) = 1 - \frac{i}{\hbar}\mathcal{H}\varepsilon + O(\varepsilon^2)$$

(More specifically, the MacLaurin Series)

$$\Rightarrow |\psi(\varepsilon)\rangle = \left(1 - \frac{i}{\hbar}\mathcal{H}\varepsilon\right)|\psi(0)\rangle$$

$$\Rightarrow |\psi(\varepsilon)\rangle - |\psi(0)\rangle = \frac{-i\varepsilon}{\hbar}\mathcal{H}|\psi(0)\rangle$$

In the $|x\rangle$ basis,

$$\psi(x,\varepsilon) - \psi(x,0) = \frac{-i\varepsilon}{\hbar}\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi(x,0)$$

(to the first order of ε)

We begin w/

$$\psi(x,\varepsilon) = \int_{-\infty}^{+\infty} U(x,\varepsilon;x')\,\psi(x',0)\,dx'$$

In general, $U(t,t_0) = e^{\left(-\frac{i}{\hbar}\mathcal{H}(t-t_0)\right)}$ → The schrödinger operator

$$U(x,\varepsilon;x') = A\, e^{\left(\frac{i}{\hbar} S(x,\dot{x})\right)}$$

we have already shown $A = \left(\frac{m}{2\pi\hbar i\varepsilon}\right)^{1/2}$

and $S[x(t),\dot{x}(t)] = \int_{t=0}^{\varepsilon} dt\left[\frac{m}{2}\dot{x}^2 - V(x)\right]$

$$= \frac{m}{2}\frac{(x-x')^2}{\varepsilon^2}(\varepsilon-0)^2 - \int_0^{\varepsilon} V(x)\,dt\,;$$

$-\int_0^{\varepsilon} V(x,t)\,dt \approx -\varepsilon V\left(\frac{x+x'}{2},0\right)$; # midpoint rule since any time dependence of V contributes to $O(\varepsilon^2)$

$$\therefore S = \frac{m}{2}\frac{(x-x')^2}{\varepsilon} - \varepsilon V\left(\frac{x+x'}{2},0\right);$$

$$\therefore U(x,\varepsilon;x') = \sqrt{\frac{m}{2\pi\hbar i\varepsilon}}\, e^{\left(\frac{i}{\hbar}\left(\frac{m(x-x')^2}{2\varepsilon} - \varepsilon V\left(\frac{x+x'}{2},0\right)\right)\right)};$$

From $U(\varepsilon) = e^{-\frac{i}{\hbar}\mathcal{H}\varepsilon}$;

Delta normalisation
$\langle p|p'\rangle = 2\pi\hbar\,\delta(p-p')$

$$U(x_f,\varepsilon;x_i) = \langle x_f|U(\varepsilon)|x_i\rangle$$
$$= \langle x_f|e^{\left(-\frac{i}{\hbar}\mathcal{H}\varepsilon\right)}|x_i\rangle$$

We know $\mathbb{1} = \int dp\,|p\rangle\langle p|$

Thus, $\langle x_f | e^{\left(-\frac{i}{\hbar}\mathcal{H}\varepsilon\right)} | x_i\rangle = \int dp \langle x_f|p\rangle \, e^{\left(-\frac{i}{\hbar}\mathcal{H}\varepsilon\right)} \langle p|x_i\rangle$

And $\langle x_f|p\rangle = \dfrac{1}{\sqrt{2\pi\hbar}} e^{\left(ipx_f/\hbar\right)}$; $\langle p|x_i\rangle = \dfrac{1}{\sqrt{2\pi\hbar}} e^{\left(-ipx_i/\hbar\right)}$

$$\Rightarrow \langle x_f | e^{\left(-\frac{i}{\hbar}\mathcal{H}\varepsilon\right)} | x_i\rangle = \frac{1}{2\pi\hbar}\int dp \; e^{\left(\frac{ip}{\hbar}(x_f - x_i)\right)} e^{\left(-\frac{i\varepsilon}{\hbar}\left\{\frac{p^2}{2m} + V(\tilde{x})\right\}\right)}$$

Solving for the Gaussian Integral,

$$= \sqrt{\frac{m}{2\pi i\hbar\varepsilon}} \; e^{\left(\frac{i}{\hbar}\sum_{j=1}^{N} \frac{m}{2\varepsilon}(x_j - x_{j-1})^2 - \varepsilon V(\tilde{x}_{j-1})\right)}$$

We see that the Lagrangian is present in the phase.

Returning to

$$U(x,\varepsilon;x') = \sqrt{\frac{m}{2\pi\hbar i\varepsilon}} \; e^{\left(\frac{i}{\hbar}\left(\frac{m(x-x')^2}{2\varepsilon} - \varepsilon V\left(\frac{x+x'}{2},0\right)\right)\right)};$$

$$\therefore \psi(x,\varepsilon) = \sqrt{\frac{m}{2\pi\hbar i\varepsilon}} \int_{-\infty}^{\infty} dx' \, e^{\left[\frac{im(x-x')^2}{2\varepsilon\hbar} - \frac{i\varepsilon}{\hbar} V\left(\frac{x+x'}{2},0\right)\right]} \psi(x',0)$$

$e^{\left(\frac{im(x-x')^2}{2\varepsilon\hbar}\right)}$ oscillates rapidly since ε is infinitesimal and $\hbar$ is small.

When such a function multiplies a smooth function like $\psi(x',0)$, the integral vanishes for the most part, due to the random phase of the exp.

Only contribution comes from the stationary phase

$x = x'$;

$\eta = x' - x$

As postulated before, the region of coherence is given by the equation

$$\frac{m\eta^2}{2\varepsilon\hbar} \lesssim \pi$$

$$\Rightarrow |\eta| \lesssim \left(\frac{2\varepsilon\hbar\pi}{m}\right)^{1/2}$$

Basics of Electromagnetism

Gauss' Law

1) $\nabla \cdot \vec{E} = \dfrac{\rho}{\epsilon_0}$ or $\Phi_E = \dfrac{Q}{\epsilon_0}$

$\left[d\Phi_X = \vec{X} \cdot d\vec{A} \right]$

2) $\nabla \cdot \vec{B} = 0 \Rightarrow \Phi_B = 0$

Faraday's Law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(differential form)}$$

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A}$$

$\oint_C \vec{E} \cdot d\vec{l} = \iint_S (\nabla \times \vec{E}) \cdot d\vec{A}$ From Stokes Theorem

$\Rightarrow \iint_S (\nabla \times \vec{E}) \cdot d\vec{A} = -\frac{d}{dt} \iint_S \vec{B} \cdot d\vec{A}$

$\Rightarrow \iint_S (\nabla \times \vec{E}) \cdot d\vec{A} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$

$\Rightarrow (\nabla \times \vec{E}) \cdot d\vec{A} = -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$

A changing magnetic field generates an electric field.

Ampere-Maxwell Law

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{or} \quad \oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d}{dt} \int_S \vec{E} \cdot d\vec{A}$$

Original Ampere's Law: $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ Current through a surface generates a magnetic field around it.

A changing electric field is equivalent to current.

* The Potentials

Electromagnetic fields are described by two potentials :

A scalar potential $\phi(\vec{r}, t)$

A vector potential $\vec{A}(\vec{r}, t)$

where

$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \nabla \times \vec{A}$$

From Gauss' Law,

$$\Phi_B = 0$$

$$\Rightarrow \oint_S \vec{B} \cdot d\vec{A} = 0$$

From Divergence Theorem,

$$\int (\nabla \cdot \vec{B}) \cdot d\vec{A} = 0$$

$$\Rightarrow (\nabla \cdot \vec{B}) = 0$$

We see that B is curl free; from Helmholtz's Decomposition, we can conclude B is purely solenoidal.

$$\therefore \vec{B} = (\nabla \times \vec{A}) \quad \text{where } \vec{A} \text{ is some vector field.}$$

Now from Faraday's Law,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$= -\frac{\partial (\nabla \times \vec{A})}{\partial t} = -\nabla \times \left(\frac{\partial \vec{A}}{\partial t}\right)$$

$$\Rightarrow \nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0$$

The curl of a gradient is always 0.

$$\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$

The curl of a gradient is always 0.

$$\therefore \quad \vec{E} + \frac{\partial \vec{A}}{\partial t} = \nabla \cdot \phi$$

$$\Rightarrow \vec{E} = \nabla \cdot \phi - \frac{\partial \vec{A}}{\partial t}$$

Aharonov - Bohm Effect

We have seen

$$\vec{B} = \nabla \times \vec{A}$$

and $$\vec{E} = \nabla \cdot \phi - \frac{\partial \vec{A}}{\partial t}$$

where $\vec{A} = \vec{A}(\vec{r}, t)$
and $\phi = \phi(\vec{r}, t)$

Consider a long solenoid with magnetic flux Φ_B.

Inside: $\vec{B}_{in} \neq 0$

Outside: $\vec{B}_{out} = 0$

Classically, $\vec{B}_{out} = 0$, so e^- outside will not be affected

However, interference pattern changes w/ Φ_B inside.

The Lagrangian for a charged particle in an EM field is

$$\mathcal{L}(\vec{r}, \dot{\vec{r}}, t) = \frac{1}{2} m \dot{\vec{r}}^2 + q \dot{\vec{r}} \cdot \vec{A}(\vec{r}, t) - q \phi(\vec{r}, t)$$

Canonical momentum:

$$\vec{p} = \frac{\partial \mathcal{L}}{\partial \dot{\vec{r}}} = m\dot{\vec{r}} + q\vec{A}$$

Hamiltonian is computed using the Legendre Transform

$$\mathcal{H} = \vec{p} \cdot \dot{\vec{r}} - \mathcal{L}$$

$$= \vec{p} \cdot \frac{1}{m}(\vec{p} - q\vec{A}) - \mathcal{L}$$

$$\mathcal{H} = \vec{p}\cdot \frac{1}{m}(\vec{p} - q\vec{A}) - \mathcal{L}$$

$$\Rightarrow \mathcal{H} = \vec{p}\cdot \frac{1}{m}(\vec{p} - q\vec{A}) - \left\{\frac{1}{2}m\dot{\vec{r}}^2 + q\dot{\vec{r}}\cdot\vec{A} - q\phi\right\}$$

Trust me, I did the math!

$$\mathcal{H}(\vec{r}, \vec{p}, t) = \frac{1}{2m}(\vec{p} - q\vec{A})^2 + q\phi$$

Outside the solenoid, $\vec{E} = 0$ & $\vec{B} = 0$

Consider the TISE in the x basis,

$$\tilde{\mathcal{H}}(\vec{r}, \vec{p})\,\psi(x) = \tilde{E}\,\psi(x)$$

Since $\vec{E} = 0 \Rightarrow \phi = 0$ $\because$ in the TISE, $-\frac{\partial \vec{A}}{\partial t} = 0$

and $\nabla\times\vec{A} = 0 \not\Rightarrow \vec{A} = 0$

TISE:

$$\left(\frac{1}{2m}(\hat{p} - qA)^2 + \cancel{q\phi}^{\,0}\right)\psi(\vec{x}) = E\,\psi(\vec{x})$$

$$\Rightarrow \left(\frac{1}{2m}(-i\hbar\nabla - qA)^2\right)\psi(\vec{x}) = E\,\psi(\vec{x})$$

We are interested to see how the phase of $\psi(\vec{x})$ changes w/ $\vec{A}$.

Let a wave function in the absence of $\vec{A}$ be

$$\psi_0(\vec{x}) = R\exp(iS_0/\hbar) \quad \text{(free particle solution)}$$

Now we introduce $\vec{A}$; in a region where $\vec{B} = \nabla\times\vec{A} = 0$

$\because \nabla\times\vec{A} = 0$, using Helmholtz decomposition we may say

locally, $\vec{A} = \nabla\Lambda(\vec{x})$

Now we consider the schrödinger Equation

$$i\hbar \frac{\partial \psi}{\partial t} = \tilde{\mathcal{H}} \psi$$

$$\Rightarrow i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m}\left(-i\hbar \nabla - q\vec{A}\right)^2 \psi + q\phi\psi$$

We define a gauge transformation

$$\vec{A}' = \vec{A} \pm \nabla\Lambda \qquad \text{where } \Lambda = \Lambda(\vec{r}, t)$$

$$\phi' = \phi \mp \frac{\partial \Lambda}{\partial t}$$

$$\psi' = \psi \circ e^{\pm\left(iq\Lambda/\hbar\right)}$$

This gauge transformation ensures schrödinger equation invariance.

In our region,

$$\vec{A} = \nabla\Lambda \quad \text{(pure gauge)}$$

$$\phi = 0$$

$$\vec{B} = 0$$

We carry out the transformations

$$\vec{A}' = \nabla\Lambda - \nabla\Lambda = 0$$

$$\phi' = 0 + \frac{\partial \Lambda}{\partial t}$$

$$\psi' = \psi_0 e^{\left(-\frac{i}{\hbar} q\right)}$$

Now we consider two paths indexed w/ i, on either side of the Solenoid.

Locally, on each branch

$$\vec{A}_i = \nabla \Lambda_i \ , \quad \phi = 0$$

Thus, $\vec{A}_i' = \vec{A}_i - \nabla \Lambda_i = 0$

$$\phi' = \phi + \frac{\partial \Lambda_i}{\partial t}$$

$$\mathcal{L} = \frac{1}{2} m \dot{\vec{x}} + q \dot{\vec{r}} - q\phi$$

$$= \frac{1}{2} m \dot{\vec{x}} + q \dot{x}^{\nu} A_{\nu} \quad \text{where } A_{\nu} = (\phi, -\vec{A})$$

$$^{\vec{B}}\Delta \varphi_{AB} = \frac{q}{\hbar} \iint_{\Sigma} (\nabla \times A_{\nu})\, d\Sigma = \frac{q \Phi_B}{\hbar}$$

$$^{\vec{E}}\Delta \varphi_{AB} = -q \oint \phi(t)\, dt$$

Gravitational Aharonov-Bohm Effect

The spacetime metric of GEM is derived from linear perturbations in flatspace $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ (weak field approximation).

From $R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$, we get $\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$ where $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}(\eta^{\mu\nu} h_{\mu\nu})$

$$ds^2 = -c^2\left(1 - 2\frac{\Phi}{c^2}\right)dt^2 - \frac{4}{c}(\vec{A}\cdot dx)\,dt + \left(1 + 2\frac{\Phi}{c^2}\right)\delta_{ij}\,dx^i dx^j$$

Derivation involves Green's retarded function

We may demonstrate that in the Newtonian limit, Φ reduces to the Newtonian gravitational potential.

$$\mathcal{L} = -mc\int d\tau \qquad \text{where } d\tau^2 \overset{\text{def}}{=} -\frac{ds^2}{dt^2}$$

$$= -mc\int \frac{ds}{dt}$$

Expand sqrt to $O(1/c^2)$, $\vec{A} = 0$

$$\frac{ds^2}{dt^2} = -c^2\left(1 - 2\frac{\Phi}{c^2}\right) + 0 + \left(1 + 2\frac{\Phi}{c^2}\right)\delta_{ij} v^i v^j$$

$$= c^2\left\{-1 + \frac{2\Phi}{c^2} + \frac{v^2}{c^2} + 2\Phi\frac{v^2}{c^4}\right\}$$

Ignoring $O(1/c^4)$, $\frac{ds^2}{dt^2} = c^2\left(-1 + \frac{v^2}{c^2} + \frac{2\Phi}{c^2}\right)$

We know that if x is small, $\sqrt{1 \pm x} \approx 1 \pm \frac{x}{2}$

$$\frac{ds^2}{dt^2} = -c^2\left\{1 - \left(\frac{v^2}{c^2} + \frac{2\Phi}{c^2}\right)\right\}.$$

$$\frac{ds}{dt} = c\left(1 - \frac{v^2}{2c^2} + \frac{\Phi}{c^2}\right)$$

Then $\mathcal{L} = -mc\int d\tau$

$$\Rightarrow \mathcal{L} = -mc^2\left(1 - \frac{v^2}{2c^2} + \frac{\Phi}{c^2}\right)$$

$$= -mc^2 + \frac{mv^2}{2} + m\Phi$$

$\mathcal{L}$ discards consts,

$$\Rightarrow \mathcal{L} = \frac{mv^2}{2} + m\Phi$$

We see that far from a spinning mass,

$$\Phi \sim \frac{Gm}{r}$$

$$\vec{A} \sim \frac{G}{c} \frac{\vec{L} \times \vec{x}}{r^3}$$

$L =$ angular momentum

$r = |x|$

In Newtonian Gravity, the only source is mass. In GR, all components of the momentum-energy tensor $T_{\mu\nu}$ are sources for gravity.

T^{00} : energy density $\equiv$ mass

T^{0i} : momentum density $\equiv$ mass current

T^{ij} : stresses and pressures

Spinning objects have mass currents.

We thus see a direct analogy to Maxwell's equations.

$$\vec{E} = -\nabla\Phi - \frac{1}{2c} \frac{\partial}{\partial t} \vec{A}$$

$$\vec{B} = \nabla \times \vec{A}$$

The weak field split :

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right)$$

In a weak field, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\Rightarrow g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} - O(h^2)$$

Consider $(\eta^{\mu\nu} + k^{\mu\nu})$ & $(\eta_{\mu\nu} + h_{\mu\nu})$

Then $(\eta^{\mu\alpha} + k^{\mu\alpha})(\eta_{\alpha\nu} + h_{\alpha\nu}) = \delta^{\mu}_{\nu}$

$$\Rightarrow \eta^{\mu\alpha}\eta_{\alpha\nu} + \eta^{\mu\alpha} h_{\alpha\nu} + k^{\mu\alpha}\eta_{\alpha\nu} + O(h^2) = \delta^{\mu}_{\nu}$$

$$\Rightarrow \cancel{\delta^{\mu}_{\nu}} + \eta^{\mu\alpha} h_{\alpha\nu} + k^{\mu\alpha}\eta_{\alpha\nu} = \cancel{\delta^{\mu}_{\nu}}$$

$$\Rightarrow \eta^{\mu\alpha} h_{\alpha\nu} = -k^{\mu}_{\nu}$$

$$\Rightarrow h^{\mu}_{\nu} = -k^{\mu}_{\nu}$$

$$\Rightarrow h^{\mu\nu} = -k^{\mu\nu} \quad \because k^{\mu\nu} = \eta^{\nu\alpha} k^{\mu}{}_{\alpha}$$

$(\eta k)^{\mu}_{\nu} = \eta^{\mu\alpha} k_{\alpha\nu}$ 1st index

$(\eta k)^{\mu}_{\nu} = k^{\mu}_{\alpha} \eta^{\alpha\nu}$ 2nd index

Looking at the first term,

$$\frac{1}{2} g^{\rho\sigma} \partial_\mu g_{\sigma\nu}$$

$$\Rightarrow = \frac{1}{2} (\eta^{\rho\sigma} - h^{\rho\sigma}) \partial_\mu (\eta_{\sigma\nu} + h_{\sigma\nu})$$

$$= \frac{1}{2} \left(\eta^{\rho\sigma} \partial_\mu \eta_{\sigma\nu} + \eta^{\rho\sigma} \partial_\mu h_{\sigma\nu} + - h^{\rho\sigma} \partial_\mu \eta_{\sigma\nu} - h^{\rho\sigma} \partial_\mu h_{\sigma\nu} \right)$$

$[\partial \eta = 0]$ $\because$ η has no 'x' dependence

$$= \frac{1}{2} \left(\eta^{\rho\sigma} \partial_\mu h_{\sigma\nu} \right)$$

We have shown Φ to be the Newtonian potential in the classical limit.

Then, far from a spinning mass, in velocities $v << c$,

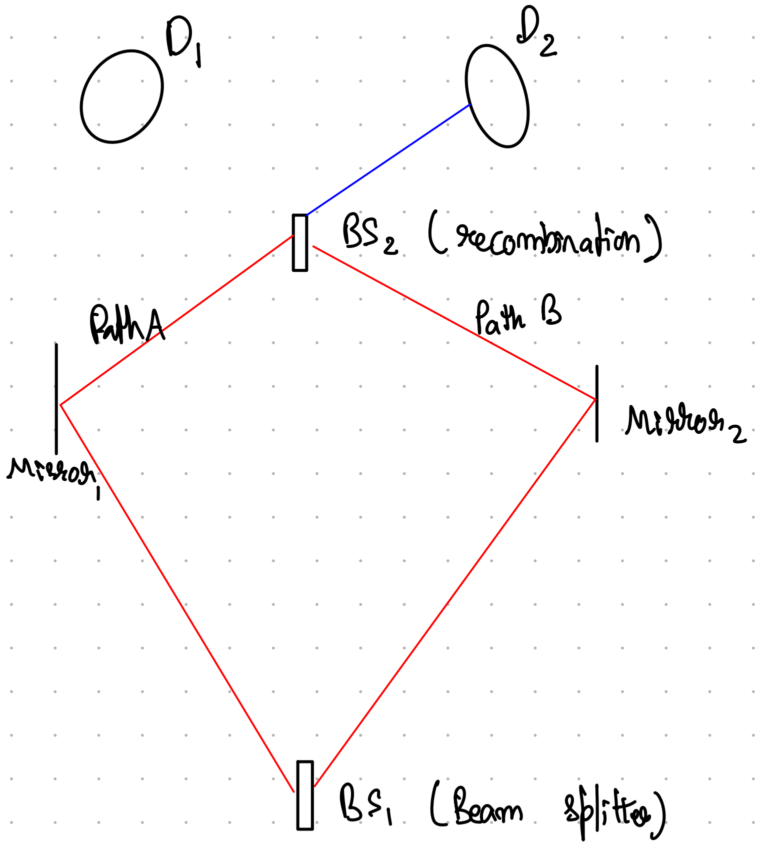
$$\Phi(r) = -\frac{Gm}{r}$$

On the Role of Potentials in the AB Effect

By Lev Vaidman

① The Electric AB effect

Consider two charged particles that have their net field cancel at a point, where the test e^- resides.



About Interferometers:

Such a device

i) Splits an incoming wave

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle)$$

ii) Brings about changes to each path

In a Mach-Zehnder Interferometer, two paths are introduced to a phase shift.

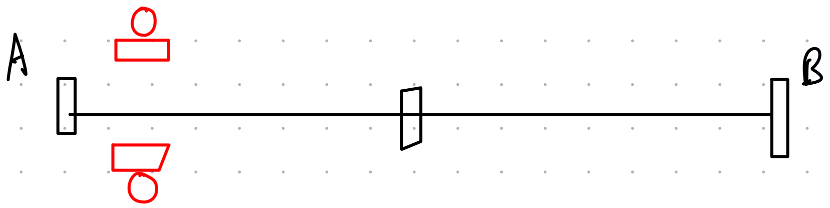
$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(e^{(i\phi_A)}|A\rangle + e^{(i\phi_B)}|B\rangle\right)$$

iii) Recombine the paths (M-Z uses "mirrors"), w/ a probability of detection in M-Z being

$$P(D_1) = \cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$P(D_2) = \sin^2\left(\frac{\Delta\phi}{2}\right)$$

Consider a 1D interferometer



To quantize the two charges, we describe the system at $t=0$ as

$$|\psi(t=0)\rangle = |\psi_e\rangle \otimes |\psi_o\rangle_c$$

$$\mathcal{H}_{net} = |R\rangle\langle R|_e \otimes \mathcal{H}_R + |L\rangle\langle L|_e \otimes \mathcal{H}_L$$

where $|x\rangle\langle x|$ are projectors to the x states.

$$(|u\rangle\langle u|)|v\rangle \equiv \langle u|v\rangle|u\rangle$$

We know $\mathcal{H}_R = 0$ $\because$ no interaction on the right.

$$\mathcal{H}_L = \mathcal{H}_c$$

In the schrödinger formalism,

$$\mathcal{K}(T) = e^{\left(-\frac{i}{\hbar}\mathcal{H}T\right)},$$

Thus,

$$\mathcal{K} = |R\rangle\langle R|_e \otimes e^{\left(\frac{-i}{\hbar}\mathcal{H}_R T\right)} + |L\rangle\langle L|_e \otimes e^{\left(\frac{-i}{\hbar}\mathcal{H}_L T\right)}$$

$$\Rightarrow \mathcal{K}(T) = |R\rangle\langle R|_e \otimes \mathbb{I}_c + |L\rangle\langle L|_e \otimes \mathcal{K}_c(T)$$

where $\mathcal{K}_c(T) = e^{\left(\frac{-i}{\hbar}\mathcal{H}_c T\right)}$

We start w/

$$|\psi_e\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$$

Two particles m and Q are placed symmetrically on a perpendicular axis at equal large distances from the mirror A, w/ equal initial velocities toward mirror A.

At distance r, Q-particles bounce back due to mirrors, causing them to spend a time T near these mirrors.

The e^- spends long time $\mathcal{T}$ near its mirrors.

We chose $T < \mathcal{T}$ so that Q-particles are near when e^- wave packets are near their respective mirrors.

We can thus say that the potential felt by the e^- for time T is

$$V(r) = -2\frac{keQ}{r}$$

This is purely coloumbic

We may neglect V when Q-particles are far away

$$r \to \infty \Rightarrow V(r) \to 0$$

To determine the phase difference of the two states we consider their propagators.

We see that $K_R(T) \cdot K_C(T) = e^{\wedge}\left(\frac{-i}{\hbar} \mathcal{H}_c T\right)$

From coloumb's law, we get $V(r) = -\frac{kc Q}{r}$

Henceforth we may ignore k (scale it to 1)

$$\mathcal{H}_c = 2 \cdot \mathcal{H}_{\text{individual charges}}$$
$$= -\frac{2eQ}{r} + \text{neglected } T \quad \because \quad e^- \text{ is stationary}$$

$$\mathcal{H}_R = 0$$

$$\therefore \Delta\varphi = \left\{-\frac{1}{\hbar}\left(-\frac{2eQk}{\hbar}\right)T\right\} - 0$$

$$= \frac{2eQk}{\hbar} T$$

We see that the e- does not exhibit the behavior of a particle @ a region w/ an electric field.

Although the e- is in $\vec{E} = 0$, the Q-particles do experience different forces in each branch, and this affects the e- that is entangled w/ the charges

During the time T that Q-particles reside near the mirror, they develop a small shift of position.

We say that the e^- is near the mirror if $x \in [0, d]$ and $d \ll r$.

Since $d \ll r$ we approximate δv to be constant (and small).

$$V(r) = -\frac{keQ}{r} \xrightarrow{\text{Scale}} -\frac{eQ}{r} = T_e \text{ (kinetic energy)}$$

$$T_e = \delta\left(\frac{1}{2} m v^2\right)$$

$$= \frac{1}{2} m (v + \delta v)^2 - \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \left(\cancel{v^2} + \overset{\sim 0}{\cancel{\delta v^2}} + 2v\delta v - \cancel{v^2}\right)$$

$$\simeq m v \delta v$$

Since the e^- stays for time T, $\delta x = T\delta v \Rightarrow \delta v = \frac{\delta x}{T}$

Revisiting the equivalence of $V(r)$ and T_e and solving for δv,

$$-\frac{keQ}{r} = m v \delta v$$

$$\Rightarrow \frac{\delta x}{T} = \frac{-keQ}{rmv}$$

$$\Rightarrow \delta x = \frac{-keQ}{rmv} T$$

Coherence

Formally, coherence refers to the presence of off-diagonal terms in a system's density matrix - terms that reflect phase relationships between states.

$$\underset{\sim}{\rho} = |\psi\rangle\langle\psi|$$

or $\underset{\sim}{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ # mixed state

where $p_i = P(|\psi_i\rangle)$

For example, if $|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$

the $\rho = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ In basis $\{|1\rangle, |2\rangle\}$ # $|1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

In a matrix like

$$\underset{\sim}{\rho} = \begin{bmatrix} \rho_{LL} & \rho_{LR} \\ \rho_{RL} & \rho_{RR} \end{bmatrix}$$

diagonal terms represent the probabilities of being in states $|L\rangle$ or $|R\rangle$.

The off diagonal terms encode quantum coherence - the phase relationship between paths.

If OFD terms go to zero (decoherence), the system behave like a classical mixture.

$$\rho = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \Rightarrow \text{no coherence} \Rightarrow \text{no interference}$$

So when does coherence survive?

Consider $|\psi_0\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$ and a pointer in state $|\Phi_0\rangle$.

If path $|i\rangle$ is followed, the pointer evolves to state $|\Phi_i\rangle$.

Thus the entangled system is expressed as

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle \otimes |\Phi_1\rangle + |2\rangle \otimes |\Phi_2\rangle\right)$$

$$\underset{\sim}{\rho}_{tot} = |\psi\rangle\langle\psi| = \frac{1}{2}\sum_{i,j=\{L,R\}} |i\rangle \otimes |\Phi_i\rangle \langle j| \otimes \langle \Phi_j|)$$

$$= \frac{1}{2}\sum_{i,j} |i\rangle\langle j| \otimes |\Phi_i\rangle\langle\Phi_j|$$

$$\underset{\sim}{\rho}_{sys} = \mathrm{Tr}\left(|\psi\rangle\langle\psi|\right)$$

w/ any orthonormal pointer basis $|\nu\rangle$, # $|\Phi_i\rangle$ need not explicitly be in $|\nu\rangle$

$$\mathrm{Tr}(\underset{\sim}{\rho}_{tot}) = \sum_{\nu} \left(\mathbb{I}_{sys} \otimes \langle\nu|\right) \underset{\sim}{\rho}_{tot} \left(\mathbb{I}_{sys} \otimes |\nu\rangle\right)$$

$$= \frac{1}{2}\sum_{i,j}\sum_{\nu} |i\rangle\langle j| \otimes \langle\nu|\Phi_i\rangle\langle\Phi_j|\nu\rangle$$

$$\because \sum_{\nu} \langle\nu|\nu\rangle = \mathbb{1}$$

$$= \frac{1}{2}\sum_{i,j} |i\rangle\langle j| \sum_{\nu} \langle\Phi_j|\Phi_i\rangle |\nu\rangle\langle\nu|$$

we can do this because they're all numbers.

$$= \frac{1}{2}\sum_{i,j} \langle\Phi_j|\Phi_i\rangle |i\rangle\langle j|$$

In the $\{|L\rangle, |R\rangle\}$ basis, $|L\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|R\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\underset{\sim}{\rho}_{sys} = \mathrm{Tr}\left(\underset{\sim}{\rho}_{tot}\right) = \frac{1}{2}\sum_{i,j} \langle \Phi_j | \Phi_i \rangle |i\rangle\langle j| \quad = \frac{1}{2}\left(\langle \Phi_L | \Phi_R \rangle |R\rangle\langle R| + \ldots\right)$$

$$|R\rangle\langle R| = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & \langle \Phi_R | \Phi_L \rangle \\ \langle \Phi_L | \Phi_R \rangle & 1 \end{bmatrix}$$

Let the pointers, in this context be $|Q_R\rangle$ and $|Q_L\rangle$

We use a Gaussian to represent $\langle x | Q_L \rangle = Q_L(x) = \dfrac{1}{\sqrt{2\pi(\Delta x)^2}} \exp\left(\dfrac{-x^2}{2(\Delta x)^2}\right)$
where Δx is the intrinsic position uncertainty.

If δx is the position shift of the pointer's (source charge's) wave packet center.

Thus $Q_R(x) = Q_L(x - \delta x)$

We compute the overlap $\langle Q_R | Q_L \rangle = \displaystyle\int_{-\infty}^{+\infty} dx\, Q_L^*(x - \delta x)\, Q_L(x)$

complete the square

$$= \exp\left(\frac{-\delta x^2}{8\Delta x^2}\right)$$

To achieve coherence we need $\langle Q_R | Q_L \rangle \approx 1$

$$\therefore \; -\frac{\delta x^2}{8\Delta x^2} \approx 0 \Rightarrow \underline{\underline{\delta x \ll \Delta x}}$$

We know de Broglie's wavelength λ has a waveform

$$\Psi(x) = A e^{(ikx)} \quad \text{where} \quad k = \frac{2\pi}{\lambda}$$

A shift of δx implies

$$\Delta \varphi_{uni} = k\delta x = \frac{2\pi\,\delta x}{\lambda}$$

For two charges, $\Delta\varphi_{bi} = 2\left(\frac{2\pi}{\lambda}\,\delta x\right) = \Delta\varphi$

We have shown $\delta x = \frac{-K_e Q}{\hbar M v} T$

$$\therefore \Delta\varphi_{bi} = \frac{4\pi}{\lambda} \; \frac{-K_e Q}{\hbar M v} T$$

and from de Broglie's law, $\lambda = \frac{h}{Mv}$

$$\therefore \Delta\varphi_{bi} = \frac{2(2\pi)\cancel{Mv}}{h} \cdot \frac{-K_e Q}{\hbar \cancel{Mv}} T$$

$$= \frac{-2K_e Q}{\cancel{h}\,\hbar} T = \Delta\varphi_{AB}$$

We have computed the AB phase by summing local Coulomb interactions on quantised source charges.

$$\mathcal{L} = \frac{1}{2} m \dot{\vec{x}} + q \dot{\vec{x}} \cdot \vec{A} - q\phi$$

$$= \frac{1}{2} m \dot{x}^2 + q \dot{x}^{\nu} A_{\nu} \quad \text{where } A_{\nu} = (\phi, -\vec{A})$$

We chose a region $\vec{E} = 0$, $\vec{B} = 0$

and $\vec{E} = -\nabla \cdot \phi - \frac{\partial \vec{A}}{\partial t}$

$\vec{B} = \nabla \times \vec{A}$

We chose a gauge in which $\vec{A} = 0$

$\Rightarrow \vec{B} = 0$

and $\vec{E} = -\nabla \cdot \phi$

$$\mathcal{L} = \frac{1}{2} m \dot{\vec{x}} - q\phi$$

$$S = \int \mathcal{L}\, dt$$

$$S_{int} = \int -q\,\phi(t)\, dt$$

$$K(t) = K_0 \int \mathcal{D}[x(t)] \exp\left(\frac{i}{\hbar} S_{int}\right)$$

where $K_0 = \int \mathcal{D}[x(t)] \exp\left(\frac{i}{\hbar} S_{free}\right)$

$$\Rightarrow K(t) = K_0 \int \mathcal{D}[x(t)] \exp\left(\frac{i}{\hbar} \int -q\phi(t)\, dt\right)$$

$$\therefore \vec{E}\ \Delta\varphi_{AB} = -\frac{q}{\hbar} \int \phi(t)\, dt$$

$\therefore \quad \vec{E} \; \Delta \varphi_{AB} = -\frac{q}{\hbar} \int \phi(t)\, dt$

$\vec{E} = \nabla \cdot \phi(t)$

In this gauge we may treat ϕ to be the coloumbic potential $U(r) = -\frac{kQ}{r}$

In Vaidman's setu, the e^- $(q = e)$ spends a long time stationary and charges Q and Q spend time T near the e^-.

We approximate $\phi = \phi(t)$ and remove time dependence.

$$\therefore K(T) = K_0 \int \mathcal{D}[x(t)] \exp\left(\frac{iq}{\hbar} \phi \int_0^T dt\right)$$

$$= K_0 \int \mathcal{D}[x(t)] \exp\left(2 \frac{ie}{\hbar} kQT\right)$$

Vaidman's System

$*$ $q = e$

multiply system by 2 $(2Q)$

$$\therefore \vec{E} \; \Delta \varphi_{AB} = \frac{2 keQ}{\hbar} T$$

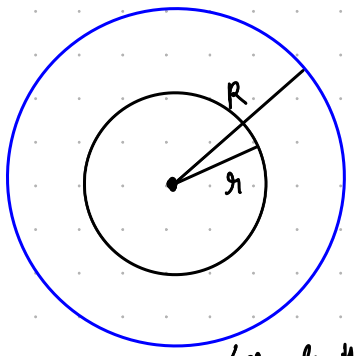
which matches Vaidman's result

$$\Delta \varphi = \left\{ -\frac{1}{\hbar} \left(-\frac{2eQk}{\hbar} \right) T \right\} - 0$$

$$= \frac{2eQk}{\hbar} T$$

The Magnetic AB effect

Consider a solenoid



Large length L

charges $+Q$ and $-Q$ are homogeneously present on the solenoid surfaces

Surface velocity $= v$

e^- velocity $= u$

We have previously shown that $\Delta\varphi_{AB} = \frac{q}{\hbar}\Phi_{AB}$

We know $\Phi_B = \int \vec{B}\cdot d\vec{A}$

The charge density of one surface $\sigma = \frac{Q}{2\pi r L}$

Surface current $I_\sigma = \sigma u = \frac{Qu}{2\pi r L}$

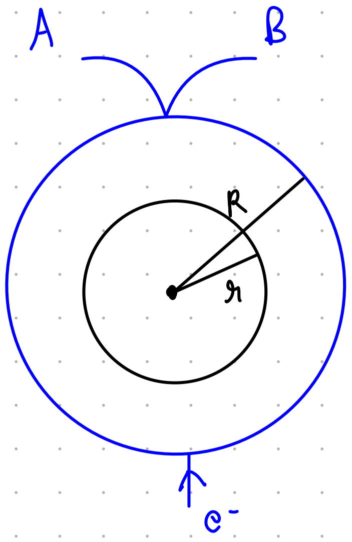
$$B = \mu_0 I_\sigma$$

Thus, $\Phi = \int \vec{B}\cdot d\vec{A} = \mu_0 \frac{Qu}{2\pi r L}\cdot \pi r^2$

$$\Phi = \cancel{2}\cancel{\pi}\cancel{r^2}\cdot\frac{\mu_0 Q u}{\cancel{2}\cancel{\pi r^2} L} = \frac{\mu_0 Q u}{L}$$

and $\Delta\varphi_{AB} = \frac{e}{\hbar}\Phi_{AB} = \frac{\mu_0 e Q u}{L\hbar}$

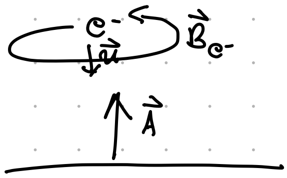
We assume $r \ll R \ll L$



Before entering the circular trajectory the e^- moves towards the axis of the solenoid and thus provides zero total flux through any cross section of the solenoid.

① As e^- approaches axis of solenoid radially

$$\Phi_{e^-} = 0 \quad \because \quad \vec{B}_{e^-} \perp \vec{A}_{surface}$$



② In circular motion,

$$I_{e^-} = \frac{e}{T} \qquad \text{where } T = \text{time period}$$

$$T = \frac{2\pi R}{u} \Rightarrow I_{e^-} = \frac{eu}{2\pi R}$$

From the Biot-Savart Law

$$\vec{B}_z(z) = \frac{\mu_0}{4\pi} I \int_0^{2\pi} R\,d\theta \frac{(\hat{\theta} \times (\vec{z} - R\hat{r})) \cdot \hat{z}}{(R^2+z^2)^{3/2}} = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$$

here $$\vec{B}_z(z) = \frac{\mu_0 e u R^2}{2(R^2+z^2)^{3/2}\, 2\pi R} = \frac{\mu_0 e u R}{4\pi (R^2+z^2)^{3/2}}$$

$$\int dA = \pi r^2 \qquad \text{flux of the solenoid}$$

$$\therefore \Phi_z(z) = \frac{\mu_0 e u R r^2}{4(R^2+z^2)^{3/2}}$$

The e^- exerts a electromotive force on the charged solenoids changing their angular velocity.

Considering an infinitesimal charge dQ to the impulse is Φ_z

$$\Phi_z(z) = \frac{\mu_0 e u R r^2}{4(R^2+z^2)^{3/2}}$$

From Faraday's Law, we know

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi}{dt}$$

$$\Rightarrow E(2\pi r) = -\frac{d}{dt}\Phi$$

and a force $\vec{F} = dQ \cdot \vec{E}$.

An impulse (Δ momentum)

$$\Delta p = \int F\,dt = dQ \int dt \cdot -\frac{1}{2\pi r}\frac{d\Phi}{dt}$$

$$= \frac{-dQ}{2\pi r}\int d\Phi(z) = -\frac{dQ\,\Phi}{2\pi r}$$

$\Delta p = m\Delta v$

$$\Rightarrow d(\delta v) = \frac{1}{m}\Delta p = \frac{1}{m}\left(+\frac{dQ\,\Phi}{2\pi r}\right)$$

we drop the sign

$$\Rightarrow \delta v = \frac{1}{m}\int_{-L/2}^{L/2} dQ\,\frac{\Phi}{2\pi r}$$

where $dQ = Q\frac{dz}{L}$

$$\Rightarrow \delta v = \frac{1}{m}\int_{-L/2}^{L/2} dQ \, \frac{\Phi}{2\pi r} \quad \text{where } dQ = Q\frac{dz}{L}$$

$$= \frac{1}{m}\int_{-L/2}^{L/2} \Phi \, \frac{1}{2\pi r} \, \frac{Q}{L}\, dz$$

and $$\Phi = \frac{\mu_0 e u R r^2}{4(R^2+z^2)^{3/2}}$$

$$\therefore \delta v = \frac{\mu_0}{4\pi} \cdot \frac{u Q e r}{m R L}$$

and $\delta x = \delta v \, T$

and $T = \frac{\pi R}{u}$ $\because$ the e^- only goes halfway around the circumference.

$$\Rightarrow \delta x = \delta v \left(\frac{\pi R}{u}\right)$$

$$\therefore \delta x = \frac{\mu_0}{4\cancel{\pi}} \cdot \frac{\cancel{u} Q e r}{m \cancel{R} L} \cdot \frac{\cancel{\pi}\cancel{R}}{\cancel{u}}$$

$$= \frac{\mu_0}{4} \cdot \frac{Q e r}{m L}$$

$$k = \frac{2\pi}{\lambda}$$

$$\delta\varphi = k\,\delta x$$

From de Broglie's law,

$$\lambda = \frac{h}{mv} \Rightarrow \delta\varphi = \frac{mv}{h} \cdot 2\pi \, \delta x$$

$$\Delta\varphi_{AB} = \cancel{4}\,\frac{\cancel{m}v}{\hbar} \, \frac{\mu_0}{\cancel{4}} \, \frac{Qer}{\cancel{m}L}$$

In branch L, both cylinder get a $+\delta x$ and in R they get a $(-\delta x)$

$\therefore$ we have a term 4

$$\Delta \varphi_{AB} = 4 \frac{mv}{\hbar} \frac{\mu_0}{4} \frac{Qe r}{m L}$$

$$= \frac{\mu_0 v Q e r}{\hbar \; L}$$

$$= \frac{e}{\hbar} \left(\frac{\mu_0 v Q r}{L} \right)$$

$$* \quad \frac{4\pi}{c} = \mu_0$$

$$\Phi = \int B \cdot dA$$

$$= B \; (\pi r^2)$$

$$B = \mu_0 I_{e^-}$$

$$\text{and } I_{e^-} = \frac{Qv}{2\pi r L}$$

$$\Rightarrow \Phi = \frac{\mu_0 Q v}{2L} r$$

$$\Phi_{net} = 2\Phi$$

$$= \frac{\mu_0 Q v}{L} r$$

AB phase explained locally :)

Explaining the e- picking up the phase of the cylinders.

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}} \left(|L\rangle_e + |R\rangle_e \right) \otimes |\Psi_0\rangle_c$$

where $|\Psi_0\rangle_c$ is the initial state of the cylinders.

$$\mathcal{K}(t) = |R\rangle\langle R|_e \otimes \exp\left(\frac{i}{\hbar} \mathcal{H}_R t\right) +$$

$$|L\rangle\langle L|_e \otimes \exp\left(\frac{i}{\hbar} \mathcal{H}_L t\right)$$

$$= |R\rangle\langle R|_e \otimes \mathcal{K}_R + |L\rangle\langle L| \otimes \mathcal{K}_L$$

and $\mathcal{K}$ is defined as

$$|\psi(t)\rangle = \mathcal{K}(t) |\psi(0)\rangle$$

and from Vaidman's explaination, we find $\mathcal{K} |\Psi_0\rangle_c = \exp i\varphi_{AB} |\Psi_0\rangle_c$

$$|\psi(t)\rangle = \mathcal{K}(t)\,|\psi(0)\rangle$$

$$= \mathcal{K}(t)\left[\frac{1}{\sqrt{2}}\left(|R\rangle_e + |L\rangle_e\right) + |\Psi_0\rangle_c\right]$$

$$= \left(|R\rangle\langle R|_e \otimes \mathcal{K}_R + |L\rangle\langle L| \otimes \mathcal{K}_L\right)$$

$$\left[\frac{1}{\sqrt{2}}\left(|R\rangle_e + |L\rangle_e\right) + |\Psi_0\rangle_c\right]$$

$$= \frac{1}{\sqrt{2}}\left(|R\rangle_e \otimes \mathcal{K}_R\,|\Psi_0\rangle_c + |L\rangle_e \otimes \mathcal{K}_L\,|\Psi_0\rangle_c\right)$$

$$\mathcal{K}_R = \exp(i\,\varphi_{AB})$$

$$\therefore\ |\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|R\rangle_e + |L\rangle_e\right) \otimes \exp(i\,\varphi_{AB})\,|\Psi_0\rangle_c$$

$$= \frac{1}{\sqrt{2}}\left(\exp(i\,\varphi_{AB})\,|R\rangle_e + \exp(i\,\varphi_{AB})\,|L\rangle_e\right) \otimes |\Psi_0\rangle$$

The cylinders return to their original states; the electron paths pick up the phase differences instead.

Vaidman claims the source entangles w/ the charged particle, a semi classical explaination.

Aharonov's arguments against

→ Vaidman's local interpretation suggests that all fluxes locally affect the system and thus entangle w/ the e^-. Even fluxes outside our interferometer.
This disagrees w/ experiment — phase depends on fluxes enclosed by the path of the e^-

→ why is the e^- velocity constant? (According to quantum mechanics).

we have a Φ_B and a $\vec{B}$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt};$$

but $\vec{B}$ is constant, and so is $dA = \pi r^2$

$\Rightarrow E = 0$

$\therefore \Phi_B$ is const

$$\Delta P = m\, d(\delta v) = \frac{\Phi\, dQ}{2\pi r}$$

Let us Consider the Lorentz Force

$$dF = dQ\,(E + \vec{v} \times \vec{B})$$

We show $E = 0$

and we know $\vec{v} \perp \vec{B}$

$\Rightarrow dF = dQ\, vB$

$$\Delta p = \int_t dF\, dt = dF \int dt \qquad \int dt = T = \frac{2\pi r}{v}$$

$= dQ\, vB \left(\frac{2\pi r}{v}\right) \equiv$ The time the e^- is in orbit.

$\Phi = B\pi r^2$

$$\Rightarrow \Delta P = dQ\, \frac{V}{r}\, \frac{r}{2} = \Delta P_{AB}\, \frac{V}{r}\, \pi$$