

## Matrix chain Multiplication

$$A_1 = 40 \times 20$$

$$A_2 = 20 \times 30$$

$$A_3 = 30 \times 10$$

$$A_4 = 10 \times 30$$

$$000f[3] = [3][4]m$$

$$000d[2] = [2][3]m$$

$$000l[1] = [1][2]m$$

- a) construct the DP table for Matrix chain multiplication  
 b) Find the minimum number of scalar multiplication  
 c) show the optimal parenthesization.

$$01X02X03P + [2][3]m + [1][2]m = 7200$$

(a)

	1	2	3	4
1	0	<del>0008 + 000d + 0 =</del>		
2		0	<del>0004l =</del>	
3			<del>(L3)(C4) = N to Hq2</del>	
4				<del>0</del>

$$01X02X03P + [2][3]m + [1][2]m = 0$$

$$000L40 + 000d8 =$$

Here

$$m[i][j] = 0$$

$$000d8 =$$

so for our own chain  $m[1][1] = m[2][2] = m[3][3]$

$$1 = [2][2] \quad 000H = [2][m[4][4]] = 0.$$

Now,

$$m[1][2] : \text{cost of } (A_1, A_2) = 40 \times 20 \times 30 = 24000.$$

$$m[2][3] : \text{cost of } (A_2, A_3) = 20 \times 30 \times 10 = 6000.$$

$$m[3][4] : \text{cost of } (A_3, A_4) = 30 \times 10 \times 30 = 9000.$$

~~multiple segments shortest path~~

So,

$$m[1][2] = 24000$$

$$m[2][3] = 6000$$

$$m[3][4] = 9000$$

$$0.5 \times 0.5 = 0A$$

$$0.6 \times 0.5 = 1A$$

$$0.1 \times 0.6 = 0A$$

$$0.6 \times 0.1 = 1A$$

~~multiple segments shortest path cost 90 at distance 10~~

For  $m[1][3]$  reduce to minimum of both (d)

∴ Split at  $k=1$  ( $A_1$ ) ( $A_2 A_3$ ) ~~at distance 10~~  $\Rightarrow$  cost 10

$$\text{Cost} = m[1][1] + m[2][3] + 40 \times 20 \times 10$$

$$= 0 + 6000 + 8000$$

$$= 14000.$$

Split at  $k=1$  ( $A_1 A_2$ ) ( $A_3$ )

$$= m[1][2] + m[3][3] + 40 \times 30 \times 10$$

$$= 24000 + 0 + 12000$$

$$= 36000.$$

$$O = [6][3]m$$

$[e][c]m = [e][c]$  Minimum is 14000 with split at  $k=1$   $\Rightarrow$  ~~as~~

$$O = [6][3]m \quad m[1][3] = 14000, \quad S[1][3] = 1.$$

$$0.000P_2 = 0.6 \times 0.5 \times 0.5 = (0A, 1A) \text{ for } \text{to } O : [6][3]m$$

$$0.000P_1 = 0.1 \times 0.6 \times 0.5 = (0A, 0A) \text{ for } \text{to } O : [6][3]m$$

$$0.000P_3 = 0.6 \times 0.1 \times 0.5 = (1A, 1A) \text{ for } \text{to } O : [6][3]m$$

Now computing  $m[2][4]$  for  $A_2, A_3, A_4$ .

$$k=2 : (A_2)(A_3A_4)$$

$$\text{Cost} = m[2][2] + m[3][4] + 20 \times 30 \times 30$$
$$= 0 + 9000 + 18000 = 27000$$

$$k=3 : (A_2A_3)A_4$$

$$\text{Cost} = m[2][3] + m[4][4] + 20 \times 10 \times 30$$
$$= 6000 + 0 + 6000$$
$$= 12000$$

Minimum is 12000 with split  $k=3$  no  $m[2][4] = 12000$ .

$$S[2][4] = 3$$

Full chain length = 4,  $m[1][4]$

Try all splits for  $A(A_2A_3A_4)$  [∴ Here  $k=1$ ]

$$\text{Cost} = m[1][1] + m[2][4] + 40 \times 20 \times 30$$
$$= 0 + 12000 + 24000 = 36000$$

(+) Area ( $A_2A_3A_4$ ) area data 4 -  $S[1][4]$

Minimum is 36000 with split at  $k=3$  no

$$S[1][4] = 3$$

Now computing  $m[2][4]$  for  $A_2, A_3, A_4$

For  $k=2$ ;  $(A_1 A_2)(A_3 A_4)$

$(A_1 A_2) (A_3 A_4) : \delta = N$

$$\text{Cost} = 24000 + 9000 + 90 \times 30 \times 30$$

$$= 24000 + 9000 + 18000 \times 30 + 0 =$$

$$= 69000,$$

$(A_1 A_2) (A_3 A_4) : \delta = N$

For  $k=3$ ;  $(A_1 A_2 A_3)(A_4)$

$$\text{Cost} = m[1][3] + m[4][4] + 90 \times 10 \times 30$$

$$0000 + 0000 =$$

$$= 26000,$$

$\dots 0000 =$

$\text{Cost} = [S[1][3]]$  on  $\delta = N$  folgt hier  $0000$  & mindestens

split at  $k=2$ ;  $(A_1 A_2)(A_3)$

$S[1][3]$  (split k)

$\dots \delta S = [A_1 A_2]$

$$S[1][3] = 1 \quad S[1][3] = 1 \quad S[1][4] = 3$$

$$S[2][3] = 2 \quad S[2][4] = 3 \quad S[3][4] = 3$$

$$00 \times 00 \times 00 + [P] [S] [m] + [U] [J] [m] = 2000$$

④ We use  $S$  to build parenthesis

$S[1][4] = 3 \rightarrow$  split into  $(A_1 A_2 A_3)$  and  $(A_4)$

on  $\delta = N$  folgt hier  $0000$

for left parts  $S[1][3] = 1$  split into  $(A_1)$  and

$(A_2 A_3)$  and  $S[2][3] = 2$  no  $A_2 A_3$  is grouped

So, the optimal order is

$$(A_1(A_2A_3)A_4)$$

or written more readably

$$(A_1(A_2 \cdot A_3) \cdot A_4).$$

So now we can write the answers.

④

	1	2	3	4
1	0	24000	14000	36000
2		0	6000	12000
3			0	9000
4				0

⑤ 26000.

⑥  $(A_1(A_2A_3)A_4)$ .

