Matrix chain Multipication

$A_1 = 40 \times 20$

$A_2 = 20 \times 30$

$A_3 = 30 \times 10$

$A_4 = 10 \times 30$

a) Construct the DP table for Matrix chain Multipuation

b) Find the minimum number of scaler multipication

c) Show the optimal parenthization.

(a)

	1	2	3	4
1	0			
2		0		
3			0	
4				0

Here

$m[i][j] = 0$

so for our own chain $m[1][1] = m[2][2] = m[3][3] = m[4][4] = 0$.

Now,

$m[1][2]$: cost of $(A_1, A_2) = 40 \times 20 \times 30 = 24000$.

$m[2][3]$: cost of $(A_2, A_3) = 20 \times 30 \times 10 = 6000$.

$m[3][4]$: cost of $(A_3 A_4) = 30 \times 10 \times 30 = 9000$.

So,

$m[1][2] = 24000$

$m[2][3] = 6000$

$m[3][4] = 9000.$

For $m[1][3]$

$\therefore$ Split at $K=1$ $(A_1)(A_2A_3)$

$$\text{Cost} = m[1][1] + m[2][3] + 40 \times 20 \times 10$$

$$= 0 + 6000 + 8000$$

$$= 14000.$$

Split at $K=$ $(A_1A_2)(A_3)$

$$= m[1][2] + m[3][3] + 40 \times 30 \times 10$$

$$= 24000 + 0 + 12000$$

$$= 36000.$$

Minimum is 14000, with split at $K=1$, so

$m[1][3] = 14000$, $S[1][3] = 1.$

Now computing $m[2][4]$ for A_2, A_3, A_4.

$K=2$: $(A_2)(A_3A_4)$.

$$\text{Cost} = m[2][2] + m[3][4] + 20 \times 30 \times 30$$

$$= 0 + 9000 + 18000 = 27000.$$

$K=3$: $(A_2A_3)A_4$

$$\text{Cost} = m[2][3] + m[4][4] + 20 \times 10 \times 30$$

$$= 6000 + 0 + 6000$$

$$= 12000.$$

Minimum is 12000 with split $K=3$ so $m[2][4] = 12000$.

$s[2][4] = 3$.

Full chain length 4: $m[1][4]$

Try all splits for $A_1(A_2A_3A_4)$ [$\because$ Here $K=1$]

$$\text{Cost} = m[1][1] + m[2][4] + 40 \times 20 \times 30$$

$$= 0 + 12000 + 24000 = 36000.$$

Minimum is 36000 with split at $K=3$ so

$m[1][4] = 36000$, $s[1][4] = 3$.

~~Now computing m[2][4] for A_2, A_3, A_4~~

For $K=2$; $(A_1A_2)(A_3A_4)$

$$Cost = 24000 + 9000 + 40 \times 30 \times 30$$

$$= 24000 + 9000 + \cancel{18000}\ 36000$$

$$= 69000,$$

For $K=3$: $(A_1A_2A_3)(A_4)$

$$Cost = m[1][3] + m[4][4] + 40 \times 10 \times 30$$

$$= 26000,$$

~~S[1][2] =~~

~~Split at K=2 : $(A_1A_2)(A_3)$~~

S[i][j] (split K).

S[1][2] = 1 . S[1][3] = 1 S[1][4] = 3

S[2][3] = 2 S[2][4] = 3, S[3][4] = 3

We use S to build parenthesis,

S[1][4] = 3 → split into $(A_1A_2A_3)$ and (A_4)

for left parts S[1][3] = 1 split into (A_1) and (A_2A_3) and S[2][3] = 2 so A_2A_3 is grouped.

So, the optimal order is

$(A_1(A_2A_3)A_4)$

or written more readably

$((A_1 (A_2 \cdot A_3) \cdot A_4)$.

So now we can write the answers.

(a)

	1	2	3	4
1	0	24000	14000	36000
2		0	6000	12000
3			0	9000
4				0

(b) 26000.

(c) $(A_1(A_2A_3)A_4)$.