Matrix Chain Multiplication (MCM) – Step-by-Step Solution

Given matrices:
A1 = 40×20
A2 = 20×30
A3 = 30×10
A4 = 10×30

Dimensions array:
dims = [40, 20, 30, 10, 30]

1) Goal
Find the parenthesization of A1 A2 A3 A4 with minimum scalar multiplications.

2) Base Case
For any single matrix Ai:
m[i][i] = 0

3) Chain Length 2
m[1][2] = 40×20×30 = 24000
m[2][3] = 20×30×10 = 6000
m[3][4] = 30×10×30 = 9000

4) Chain Length 3

m[1][3]:
k=1 → 0 + 6000 + 40×20×10 = 14000
k=2 → 24000 + 0 + 40×30×10 = 36000
Minimum = 14000 (k = 1)

m[2][4]:
k=2 → 0 + 9000 + 20×30×30 = 27000
k=3 → 6000 + 0 + 20×10×30 = 12000
Minimum = 12000 (k = 3)

5) Chain Length 4 (m[1][4])
k=1 → 0 + 12000 + 40×20×30 = 36000
k=2 → 24000 + 9000 + 40×30×30 = 69000
k=3 → 14000 + 0 + 40×10×30 = 26000
Minimum = 26000 (k = 3)

6) Final DP Table (m[i][j])

(Only upper triangle shown)
$m[1][2] = 24000$
$m[1][3] = 14000$
$m[1][4] = 26000$
$m[2][3] = 6000$
$m[2][4] = 12000$
$m[3][4] = 9000$

7) Optimal Parenthesization
Using split table:
$s[1][4] = 3$
$s[1][3] = 1$
$s[2][3] = 2$

Optimal order:
((A1 (A2 A3)) A4)

8) Total Cost
6000 + 8000 + 12000 = 26000 scalar multiplications.

Final Answer:
Minimum multiplications = 26000
Optimal parenthesization = ((A1 (A2 A3)) A4)