Problem: Test the consistency and hence solve the following system of linear equations by using Gauss-Jordan elimination method:

$$x + y + z = 1$$
$$x + 3y + 6z = 10$$
$$x + 2y + 3z = 4$$
$$x + 4y + 10z = 19$$

Form an augmented matrix:

$$(A/B) = \begin{bmatrix} 1 & 1 & 1: & 1 \\ 1 & 3 & 6: & 10 \\ 1 & 2 & 3: & 4 \\ 1 & 4 & 10: & 19 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 1 & 1: & 1 \\ 0 & 2 & 5: & 9 \\ 0 & 1 & 2: & 3 \\ 0 & 1 & 4: & 9 \end{bmatrix} \begin{pmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_2 \end{pmatrix}$$

$$\approx \begin{bmatrix} 2 & 0 & -3: & -7 \\ 0 & 0 & -1: & -3 \\ 0 & 0 & 2: & 6 \\ 0 & -1 & 0: & 3 \end{bmatrix} \begin{pmatrix} R_1 \to 2R_1 - R_2 \\ R_2 \to 2R_3 - R_2 \\ R_3 \to R_4 - R_3 \\ R_4 \to R_4 - 2R_3 \end{pmatrix}$$

$$\approx \begin{bmatrix} 2 & 0 & -3: & -7 \\ 0 & -1 & 0: & 3 \\ 0 & 0 & 0: & 0 \\ 0 & 0 & -1: & -3 \end{bmatrix} \begin{pmatrix} R_2 \leftrightarrow R_4 \\ R_3 \to 2R_2 + R_3 \end{pmatrix}$$

$$\approx \begin{bmatrix} 2 & 0 & 0: & 2 \\ 0 & -1 & 0: & 3 \\ 0 & 0 & 1: & 3 \end{bmatrix} \begin{pmatrix} R_1 \to R_1 - 3R_4 \\ R_3 \leftrightarrow -R_4 \end{pmatrix}$$

$$\approx \begin{bmatrix} 1 & 0 & 0: & 1 \\ 0 & 1 & 0: & -3 \\ 0 & 0 & 1: & 3 \end{bmatrix} \begin{pmatrix} R_1 \to \frac{R_1}{2} \\ R_2 \leftrightarrow -R_2 \end{pmatrix}$$

The matrix is in row-echelon form. Therefore rank $\rho(A) = 3$. So the given system is consistent and transform into the linear system we get

$$x \quad = 1$$

y =-3

z=3

This system is in echelon form and it has three equations and three unknowns. So the system has unique solution. Thus the general solution is x=1, y=-3 and z=3

P-2: Solve the following system of linear equations using Gaussian elimination method:

$$\begin{aligned} x+y+z+t &= 2 \\ 2x+3y+4z+5t &= 9 \\ 3x-2y+z+2t &= 9 \\ x+2y-2z+3t &= 5 \end{aligned}$$

Form an augmented matrix (A/B) =

$$\begin{bmatrix} 1 & 1 & 1 & 1:2 \\ 2 & 3 & 4 & 5:9 \\ 3 & -2 & 1 & 2:9 \\ 1 & 2 & -2 & 3:5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1:2 \\ 0 & 1 & 2 & 3:5 \\ 0 & 5 & 2 & 1:-3 \\ 0 & 1 & -3 & 2:3 \end{bmatrix} \begin{pmatrix} R_2 \to R_2 - 2R_1 \\ R_3 \to 3R_1 - R_3 \\ R_4 \to R_4 - R_1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1:2 \\ 0 & 1 & 2 & 3:5 \\ 0 & 0 & 8 & 14:28 \\ 0 & 0 & 5 & 1:2 \end{bmatrix} \begin{pmatrix} R_3 \to 5R_2 - R_3 \\ R_4 \to R_2 - R_4 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1:2 \\ 0 & 1 & 2 & 3:5 \\ 0 & 0 & 4 & 7:14 \\ 0 & 0 & 0 & 62:124 \end{bmatrix} \begin{pmatrix} R_4 \to 5R_3 - 8R_4 \\ R_3 \to \dfrac{R_3}{2} \end{pmatrix}$$

$$\begin{aligned} \Rightarrow x+y+z+t = 2 &\Rightarrow x-1+0+2 = 2 \Rightarrow x = 1 \\ y+2z+3t = 5 &\Rightarrow y+2.0+3.2 = 5 \Rightarrow y = -1 \\ 4z+7t = 14 &\Rightarrow 4z+7.2 = 14 \Rightarrow z = 0 \\ 62t = 124 &\Rightarrow t = 2 \end{aligned}$$