Alternating current Fundamentals

One of the important reasons for concentrating on the sinusoidal voltage is that it is the voltage generated by utilities throughout the world. Other reasons include its application throughout electrical, electronic, communication, and industrial systems. Sinusoidal ac voltages are available from a variety of sources. The most common source is the typical home outlet, which provides an ac voltage that originates at a power plant; such a power plant is most commonly fueled by water power, oil, gas, or nuclear fission.

Definitions

Waveform: The path traced by a quantity, such as the voltage in Fig. 5.1 (B13.3), plotted as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.

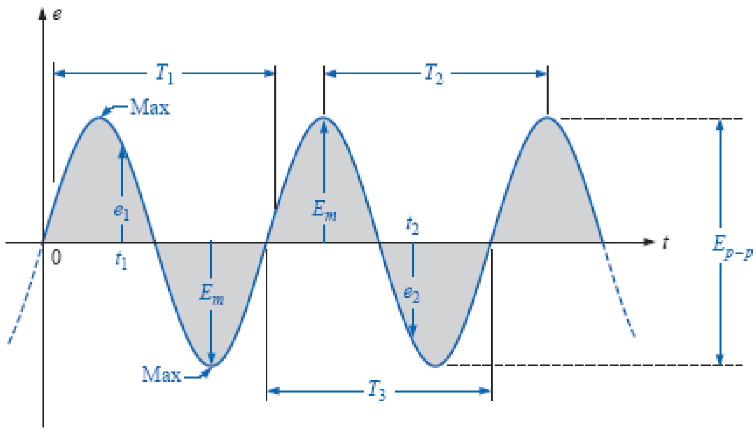


Fig 5.1 (a) Sine wave

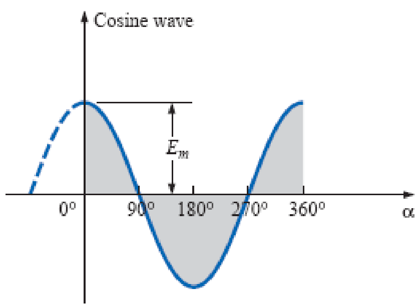


Fig 5.1 (b) Cosine wave

Instantaneous value (e_1):: The magnitude of a waveform at any instant of time.

Peak amplitude (E_m): The maximum value of a waveform as measured from its average or mean value. It is the maximum value, positive or negative, of an alternating quantity.

Peak-to-peak value (E_{P-P}): The maximum value of a waveform from positive to negative peaks.

Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform of Fig. 5.1 is a periodic waveform.

Cycle: One complete set of positive and negative values of alternating quantity is known as a cycle.

Period (T_1 or T_2): The time taken by an alternating quantity to complete one cycle is called its time period T. For example, a 50 Hz alternating current has a time period of 1/50 seconds.

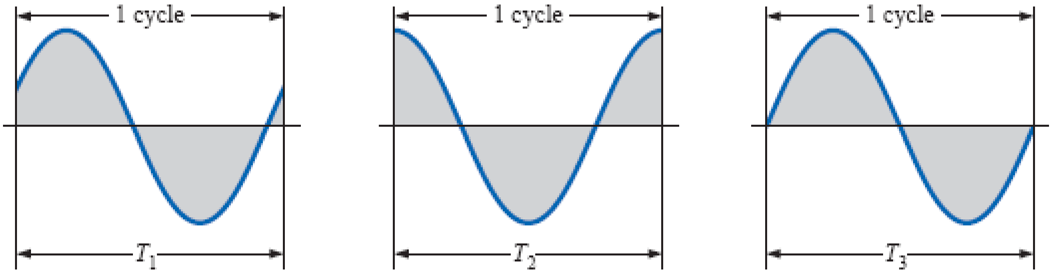


Fig. 5.2: Defining cycle and time period.

Frequency: The number of cycles that occur in 1 s. The unit of frequency is hertz (Hz), where 1 Hz = 1 cycle per second. The frequency of an ac voltage is 50 Hz means, the ac voltage completes 50 full cycles in 1 second. The frequency is given by the reciprocal of the time period of the alternating quantity.

Radio frequency spectrum

3 kHz – 30 kHz	Very low frequency
30 kHz – 300 kHz	Low frequency
300 kHz – 3 MHz	Medium frequency
3 MHz – 30 MHz	High frequency
30 MHz – 300 MHZ	Very high frequency
300 MHz – 3 GHz	Ultra high frequency
3 GHz – 30 GHz	Super high frequency
300 GHz – 300 GHz	Extremely high frequency
300 MHz – 30 GHz	Microwave

Areas of application for specific frequency

FM	88 MHz – 108 MHz
TV (2-6 channels)	54 MHz – 88 MHz
TV (7-13 channels)	174 MHz – 216 MHZ
TV (channels 14 – 83)	470 MHz – 890 MHz
CB	26.9 MHz – 27.4 MHz
Countertop microwave oven	2.45 GHz
Shortwave	1.5 MHz – 30 MHz
Cordless telephone	46 MHz – 49 MHz
Pager VHF	30 MHz – 50 MHz
Pager UHF	405 MHz – 512 MHz

General form of ac current or voltage

The basic mathematical form for sinusoidal waveform is

$$y = A \sin \alpha = A \sin \omega t \quad \ldots \quad (5.1)$$

Here , A_m = amplitude
ω = angular frequency
t = time
α= angular distance
y = instantaneous value

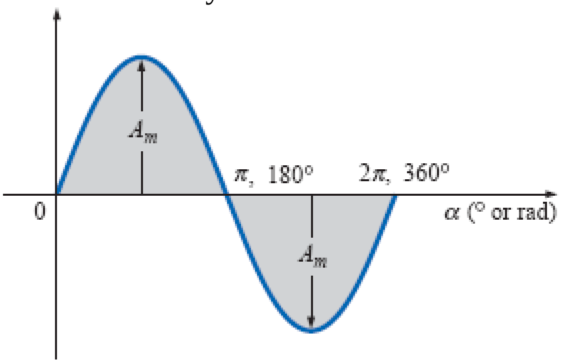


Fig. 5.3 (a)

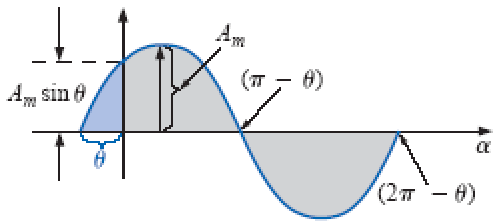


Fig. 5.3 (b)

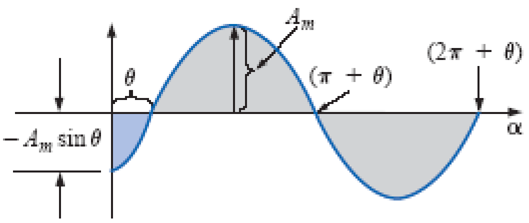


Fig. 5.3 (c)

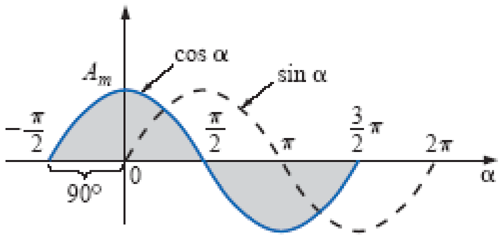


Fig. 5.3 (d)

This equation is valid when the waveform passes through origin. In other word, the wave has the maxima at $\pi/2$, $3\pi/2$, $5\pi/2$, etc. with a zero value at 0, π, 2π, as shown in Fig. 5.3 (a).

If the wave form shifted to the right or left of 0°, the expression becomes

$$y = A \sin \quad = A \sin (\omega t \pm \theta) \quad \ldots \quad \ldots \quad (5.2)$$

If the waveform passes through the horizontal axis with a positive going (increasing with time) slope before 0°, as shown in Fig. 5.3(b) the expression is

$$y = A \sin \quad = A \sin (\omega t + \theta) \quad \ldots \quad \ldots. \quad (5.3)$$

If the waveform passes through the horizontal axis with a positive going (increasing with time) slope after 0°, as shown in Fig. 5.3(c), the expression is

$$y = A \sin \quad = A \sin (\omega t - \theta) \quad (5.4)$$

If the waveform crosses the horizontal axis with a positive-going slope 90° (π/2) sooner, as shown in Fig. 5.3 (d), it is called a cosine wave; that is

$$\sin(\omega t + 90°) = \sin(\omega t + \pi/2) = \cos \omega t$$

or $$\sin \omega t = \cos(\omega t - 90°) = \cos(\omega t - \pi/2)$$

The term lead and lag are used to indicate the relationship between two sinusoidal waveforms of the same frequency plotted on the same set of axes. In Fig. 5.3 (d), the cosine wave is said to lead the sine curve by 90º, and the sine curve is said to lag the cosine curve by 90º. The 90º is referred to as the phase angle between the two waveforms.

Phase and phase difference

By phase of an ac is meant the fraction of the time period of that alternating current which has elapsed since the current last passed through the zero position of reference. For example, the phase of current at point A is T/4 second, where T is the time period or expressed in terms of angle, it is π/2 radians (Fig. 5.3 (d).

In electrical engineering we are more concerned with relative phases or phase differences between different ac, rather than their absolute phases. The term lead and lag are used to indicate the relationship between two sinusoidal waveforms of the same frequency plotted on the same set of axis. In Fig. 5.3 (d), the cosine curve is said to lead the sine curve by 90°, and the sine curve is said to lag the cosine curve by 90°. The 90° is referred to as the phase angle between the two waveforms (out of phase by 90°). The phase angle between two waveforms is measured between those two points on the horizontal axis through which each waveform passes with the same slope. If both the waveform crosses the axis at the same point with the same slope, they are said to be in phase. In other words, if the two ac or emf reach their maximum and zero at the same time as shown in Fig. 5.4, such ac or voltages are said to be in phase with each other. The two voltages will have the equations,

$$e_1 = E_{m1} \sin \omega t \quad \text{and} \quad e_2 = E_{m2} \sin \omega t$$

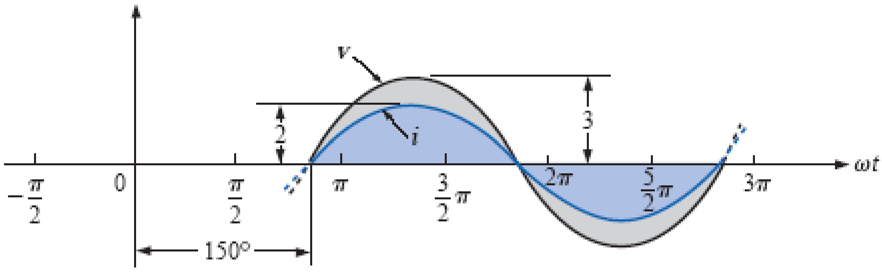


Fig. 5.4.

Root-Mean-Square (R.M.S.)

The rms value of an alternating current is given by that steady (dc) current which when flowing through a given circuit for a given time produces the same **heat** as produced by the alternating current when flowing through the same circuit for the same time. It is also known as the effective value of the alternating current. RMS value can be computed using either mid-ordinate method or analytical method. Here we shall use analytical method.

Standard form of ac is $i = I_m \sin \omega t = I_m \sin \theta$.

The mean of the squares of the instantaneous values of current over one complete cycle is (even the value over half a cycle will do), $= \int [(i^2\, d\theta)/2\pi]$

and the square root of this value is,

$$= \sqrt{[(i^2\, d\theta)/2\pi]}$$

Hence, the rms value of ac is,

$$I_{\text{eff}} = \sqrt{\frac{\int_0^T i^2(t)\, dt}{T}}$$

which yields,

$$I_{rms} = I_{eff} = I_m/\sqrt{2} = 0.707\, I_m.$$

Hence, rms value of ac = 0.707 X maximum value of current. The rms value of an ac is of considerable important in practice, because the ammeters and voltmeters record the rms value of alternating current and voltage respectively. Unless indicated otherwise, the values of the given current and voltage are always the rms values.

Average value

The average value I_{av} of an ac is expressed by that steady current which transfers across any circuit the same charge as is transferred by that ac during the same time. In case of a symmetrical ac (i.e. one whose two half cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half cycle only. But in the case of an unsymmetrical ac (like half wave rectified current) the average value must always be taken over the whole cycle.

$$I_{av} = \int [(i\, d\theta)/2\pi\,] \quad \text{here integration limit is 0 to } \pi.$$

Therefore, $I_{av} = I_m/(\pi/2)$ = twice the maximum current / π

Average value of current = 0.637 X Maximum value. RMS value is always greater than the average value.

Form factor

It is defined as the rstio, K_f = (rms value)/(average vakue) = 0.707 I_m/ 0.637 I_m = 1.11

Crest or Peak or Amplitude factor

It is defined as the ratio, K_a = (Maximum value)/(rms value) = $I_m/(I_m/\sqrt{2})$ = $\sqrt{2}$ = 1.414 (for sinusoidal ac only).

Example 5.1: Determine the angular velocity of a sine wave having a frequency of 60 Hz.

$$\omega = 2\pi f = (2\pi)(60\text{ Hz}) \cong \mathbf{377\ rad/s}$$

Example 5.2: Determine the frequency and Time period of a sine wave having angular velocity of 500 rad/s.

Since $\omega = 2\pi/T$,

$$T = \frac{2\pi}{\omega} = \frac{2\pi\text{ rad}}{500\text{ rad/s}} = \frac{2\pi\text{ rad}}{500\text{ rad/s}} = \mathbf{12.57\ ms}$$

and $$f = \frac{1}{T} = \frac{1}{12.57 \times 10^{-3}\text{ s}} = \mathbf{79.58\ Hz}$$

Example 5.3: Determine the average value and rms value of the sine wave shown in Fig. 5.5.

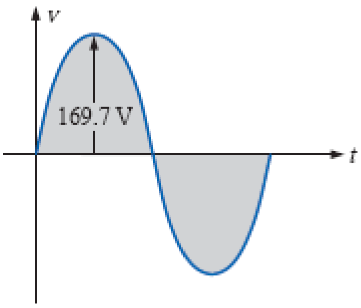


Fig. 5.5.

V_{av} = (0.637) (V_m) = (0.637) (169.7 V) = 108 V
V_{rms} = (0.707) (V_m) = (0.707) (169.7 V) = 120 V

AC through resistance, inductance and capacitance

a) **AC through ohmic resistance alone**

Let us consider the circuit shown in Fig. 5.6(a). Let the applied voltage is given by

$$v = V_m \sin \omega t. \qquad (5.5)$$

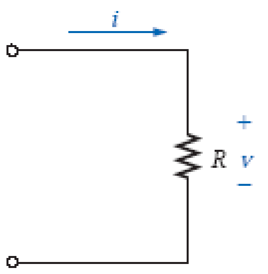


Fig. 5.6 (a)

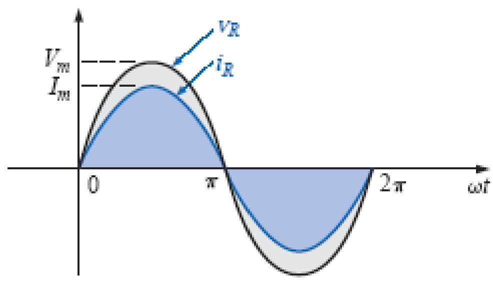


Fig. 5.6 (b)

Let R = ohmic resistance, i = instantaneous current. Obviously, the applied voltage has to supply ohmic voltage drop only. Hence, for equilibrium, v = iR. Therefore

For $v = V_m \sin \omega t$, (5.5)

$$iR = V_m \sin \omega t$$

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

Current, i is maximum when sin ωt is unity. Hence $\boxed{I_m = \frac{V_m}{R}}$

Hence $i = I_m \sin \omega t$ (5.6)

Comparing Eqs. (5.5) and (5.6), we find that the alternating voltage and current are in phase with each other as shown in Fig. 5.6 (b). Therefore,

for a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.

Power: Instantaneous power, $p = v\,i = V_m \sin \omega t \times I_m \sin \omega t$

$$= V_m I_m \sin^2 \omega t \qquad \text{(shown in Fig.5.8.)}$$

$$= (V_m I_m/2)(1 - \cos 2\omega t) = V_m I_m/2 - (V_m I_m/2)\cos 2\omega t \; \ldots \qquad (5.7)$$

Power consists of a constant part $V_m I_m/2$ and a fluctuating part $(V_m I_m/2) \cos 2\omega t$ of frequency double that of voltage and current waves. For a complete cycle, the average value of $(V_m I_m/2) \cos 2\omega t$ is zero.

Hence, power for the whole cycle is

$$P = (V_m I_m/2) = (V_m/\sqrt{2}) I_m/\sqrt{2})$$

$$P = V \times I \text{ watt}$$

Where V = rms value of applied voltage and I = rms value of the current.

It is seen from Fig. 5.7 that no part of the power cycle becomes negative at any time. In other words, in a purely resistive circuit, power is never zero. This is so because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive.

EXAMPLE 14.1 The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 Ω. Sketch the curves for v and i.

a. $v = 100 \sin 377t$

b. $v = 25 \sin(377t + 60°)$

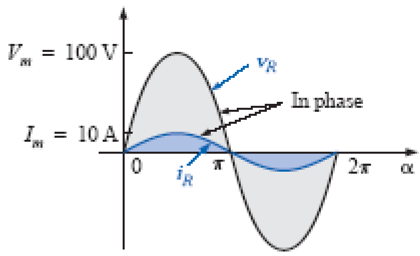
Solutions:

a. Eq. (14.2): $I_m = \dfrac{V_m}{R} = \dfrac{100\text{ V}}{10\ \Omega} = 10\text{ A}$

(v and i are in phase), resulting in

$$\mathbf{i = 10 \sin 377t}$$

The curves are sketched in Fig. 14.13.

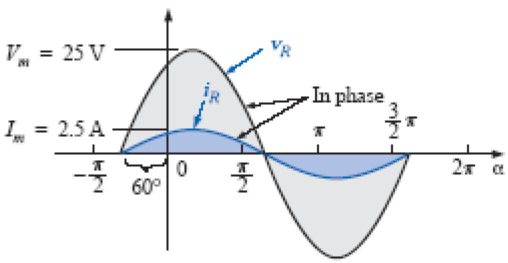


b. Eq. (14.2): $I_m = \dfrac{V_m}{R} = \dfrac{25\text{ V}}{10\ \Omega} = 2.5\text{ A}$

(v and i are in phase), resulting in

$$\mathbf{i = 2.5 \sin(377t + 60°)}$$

The curves are sketched in Fig. 14.14.



EXAMPLE 14.2 The current through a 5-Ω resistor is given. Find the sinusoidal expression for the voltage across the resistor for $i = 40 \sin(377t + 30°)$.

Solution: Eq. (14.3): $V_m = I_m R = (40\text{ A})(5\ \Omega) = 200\text{ V}$

(v and i are in phase), resulting in

$$\mathbf{v = 200 \sin(377t + 30°)}$$

(b) **AC through pure inductance alone**

Whenever an alternating voltage is applied to a purely inductive coil, as shown in Fig. 5.8 (a), a back emf is produced due to the self inductance of the coil. The back emf at every step, opposes the rise or fall of current through the coil. As there is no ohmic voltage drop, the applied voltage has to overcome this self induced emf only.

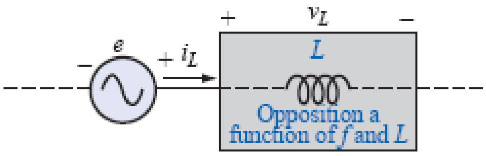


Fig. 5.8 (a)

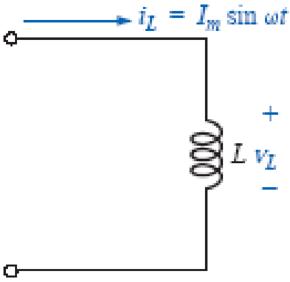


Fig. 5.8 (b)

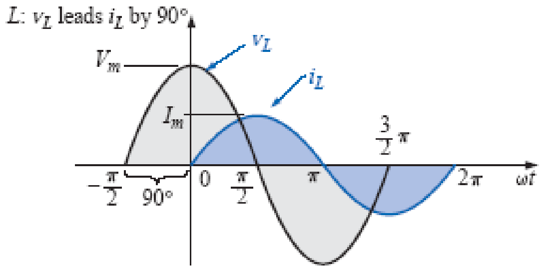


Fig. 5.8 (c)

Fig. 5.8 (d)

So at every step $\quad v = L\,(di/dt)$

Now, $\quad v = V_m \sin \omega t$

$$V_m \sin \omega t = L\,(di/dt)$$

$$di = (V_m/L)\ \sin \omega t\ dt$$

Integrating both sides, $\quad i = (V_m/L)\ \ \sin \omega t\ dt$

$$= (V_m/\omega L)\,(-\cos \omega t) \quad = (V_m/\omega L) \cos \omega t$$

$$= (V_m/\omega L) \sin(\omega t - \pi/2) \ = (V_m/X_L)\sin(\omega t - \pi/2)$$

Maximum value of i is $I_m = (V_m/\omega L)$ when $\sin(\omega t - \pi/2)$ is unity.

Hence the equation of the current becomes, $\quad i = I_m \sin(\omega t - \pi/2) \quad \ldots \quad (5.8)$

So, we find that if applied voltage is represented by $v = V_m \sin \omega t$, then current flowing in a purely inductive circuit is given by $i = I_m \sin(\omega t - \pi/2)$.

Clearly, the current lags behind the applied voltage by a quarter cycle (as shown in Fig. 5.8 (c) or the phase difference between is $\pi/2$ with voltage leading. Vectorially the voltage and current are shown in Fig. 5.8 (d), where voltage has been taken along the reference axis. We have seen that $I_m = (V_m/\omega L) = (V_m/X_L)$. Hence ωL plays the part of resistance. It is called the **inductive reactance** X_L of the coil and is given in ohms if L is in henry and ω is in radian/second.

Now, $X_L = \omega L = 2 \pi f L$ ohm. X_L directly depends on frequency of the voltage. Higher the value of f, greater the reactance offered and vice versa.

Power: Instantaneous power $= p = vi = V_m \sin \omega t \,.\, I_m \sin(\omega t - \pi/2)$

$$= V_m I_m \sin \omega t \,.\, \sin(\omega t - \pi/2)$$

$$= - V_m I_m \sin \omega t \,.\, \cos \omega t$$

$$= - (V_m I_m/2) \sin 2\omega t \qquad (5.9)$$

Power for whole cycle is $P = - (V_m I_m/2) \sin 2\omega t \; dt = 0$

It is also clear that the average demand of power from the supply for a complete cycle is zero. Here again it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of instantaneous power is $V_m I_m/2$.

EXAMPLE 14.3 The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

a. $i = 10 \sin 377t$
b. $i = 7 \sin(377t - 70°)$

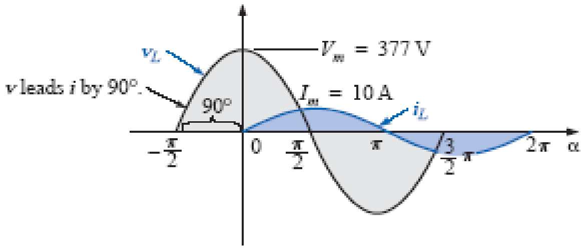
Solutions:

a. Eq. (14.4): $X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.7 \ \Omega$
Eq. (14.5): $V_m = I_m X_L = (10 \text{ A})(37.7 \ \Omega) = 377 \text{ V}$

and we know that for a coil v leads i by 90°. Therefore,

$$v = 377 \sin(377t + 90°)$$

The curves are sketched in Fig. 14.15.



b. X_L remains at 37.7 Ω.

$V_m = I_m X_L = (7\text{ A})(37.7\ \Omega) = 263.9\text{ V}$

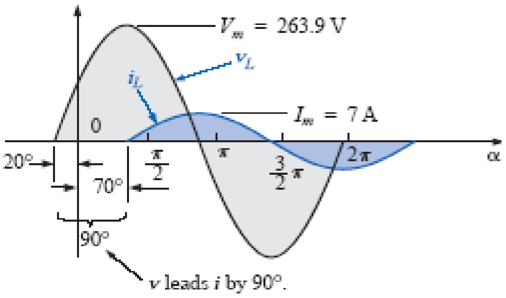
and we know that for a coil v leads i by 90°. Therefore

$$v = 263.9 \sin(377t - 70° + 90°)$$

and

$$\mathbf{v = 263.9 \sin(377t + 20°)}$$

The curves are sketched in Fig. 14.16.



EXAMPLE 14.4 The voltage across a 0.5-H coil is provided below. What is the sinusoidal expression for the current?

$$v = 100 \sin 20t$$

Solution:

$$X_L = \omega L = (20\text{ rad/s})(0.5\text{ H}) = 10\ \Omega$$

$$I_m = \frac{V_m}{X_L} = \frac{100\text{ V}}{10\ \Omega} = 10\text{ A}$$

and we know that i lags v by 90°. Therefore,

$$\mathbf{i = 10 \sin(20t - 90°)}$$

(c) **AC through Capacitance only**

Whenever an alternating voltage is applied to the plates of a capacitor, as shown in Fig. 5.9 (a), the capacitor is charged first in one direction and then in the opposite direction.

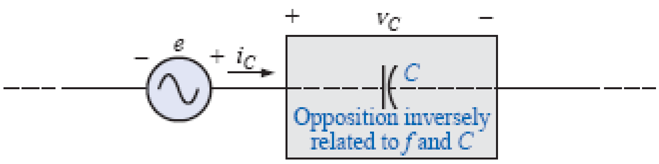


Fig. 5.9 (a)

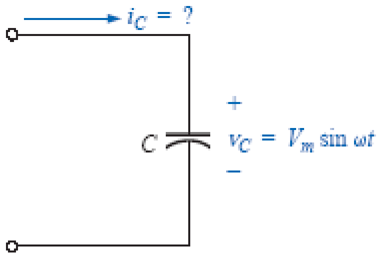


Fig. 5.9 (b)

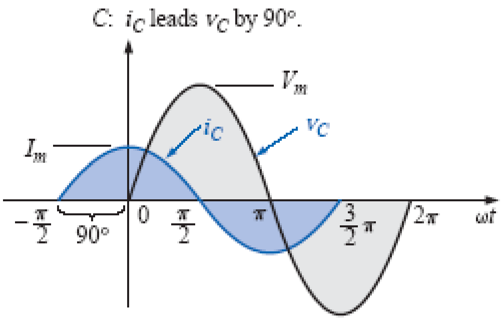


Fig. 5.9 (c)

Fig. 5.9 (d)

Let

v_C = potential difference developed between the plates at any instant and
q = charge on plates at that instant

Then $q = Cv_C$, where C is the capacitance

If the applied voltage is $v = V_m \sin \omega t$, then

$$q = C V_m \sin \omega t$$

Now, current is given by the rate of flow of charge.

So, $i_C = dq/dt = d/dt(C V_m \sin \omega t) = \omega C V_m \cos \omega t$

Fig. 5.11

Or, $i_C = (V_m /(1/\omega C)) \cos \omega t = (V_m /(1/\omega C)) \sin (\omega t + \pi/2)$

Obviously $I_m = (V_m /(1/\omega C)) = V_m/X_c$

$$i_C = I_m \cos \omega t = I_m \sin (\omega t + \pi/2) \quad \dots \quad \dots \qquad (5.10)$$

The denominator $X_c = 1/\omega C$, is known as **capacitive reactance** and is in ohms if C is in farad and ω in radian/s.

So, if the applied voltage is represented by $v = V_m \sin \omega t$, then current flowing in a purely capacitive circuit is given by $i = I_m \sin(\omega t + \pi/2)$. Hence the current in a pure capacitor leads its voltage by a quarter cycle as shown in Fig. 5.9 (c) or phase difference between its voltage and current is $\pi/2$ with the current leading. Vector representation is given in Fig. 5.9 (d).

Power: Instantaneous power

$$p = v_C i_C = V_m \sin \omega t . I_m \sin(\omega t + \pi/2).$$
$$= V_m I_m \sin\omega t \cos \omega t$$
$$= ½ V_m I_m \sin 2\omega t. \quad ... \qquad \dots \qquad (5.11)$$

Power for the whole cycle = $½ V_m I_m \sin 2\omega t \, dt = 0$

So, in a purely capacitive circuit, the average power demand from power supply is zero. Also power wave is a sine wave of frequency double that of the voltage and current waves. Maximum value of the instantaneous power is = $V_m I_m/2$.

EXAMPLE 14.5 The voltage across a 1-μF capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.

$$v = 30 \sin 400t$$

Solution:

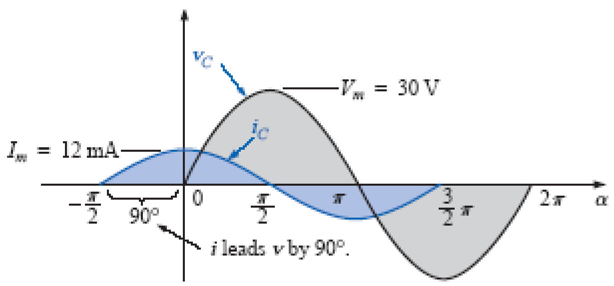
Eq. (14.6): $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(400 \text{ rad/s})(1 \times 10^{-6} \text{ F})} = \dfrac{10^6 \ \Omega}{400} = 2500 \ \Omega$

Eq. (14.7): $I_m = \dfrac{V_m}{X_C} = \dfrac{30 \text{ V}}{2500 \ \Omega} = 0.0120 \text{ A} = 12 \text{ mA}$

and we know that for a capacitor i leads v by 90°. Therefore,

$$\mathbf{i = 12 \times 10^{-3} \sin(400t + 90°)}$$

The curves are sketched in Fig. 14.17.



EXAMPLE 14.6 The current through a 100-μF capacitor is given. Find the sinusoidal expression for the voltage across the capacitor.

$$i = 40 \sin(500t + 60°)$$

Solution:

$$X_C = \frac{1}{\omega C} = \frac{1}{(500 \text{ rad/s})(100 \times 10^{-6} \text{ F})} = \frac{10^6 \ \Omega}{5 \times 10^4} = \frac{10^2 \ \Omega}{5} = 20 \ \Omega$$

$$V_m = I_m X_C = (40 \text{ A})(20 \ \Omega) = 800 \text{ V}$$

and we know that for a capacitor, v lags i by 90°. Therefore,

$$v = 800 \sin(500t + 60° - 90°)$$

and $\quad$ $\mathbf{v = 800 \sin(500\mathit{t} - 30°)}$

Power in ac networks

Average power or Active power, Reactive power and power factor

Average power is the power delivered by the source and dissipated or consumed in the load. Sometimes this is also known as **Active power**. For any load in a sinusoidal ac network, the voltage across the load and the current through the load will vary in a sinusoidal nature.

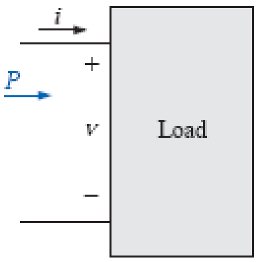


Fig. 5.10 (a)

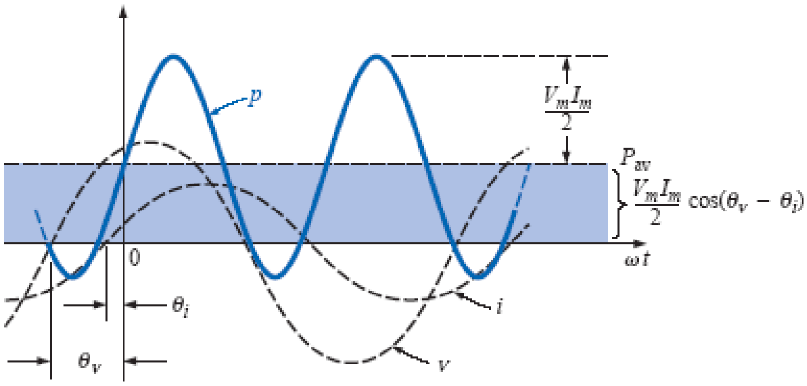


Fig. 5.10 (b)

If we take the general case depicted in Fig. 5.10 (a) and

Let, $v = V_m \sin(\omega t + \theta_v)$ and $i = I_m \sin(\omega t + \theta_i)$

Then power is defined by

$$p = vi = V_m \sin(\omega t + \theta_v) I_m \sin(\omega t + \theta_i)$$
$$= V_m I_m \sin(\omega t + \theta_v) \sin(\omega t + \theta_i)$$

Using the trigonometric identity

$$\sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2}$$

the function $\sin(\omega t + \theta_v) \sin(\omega t + \theta_i)$ becomes

$$\sin(\omega t + \theta_v) \sin(\omega t + \theta_i)$$
$$= \frac{\cos[(\omega t + \theta_v) - (\omega t + \theta_i)] - \cos[(\omega t + \theta_v) + (\omega t + \theta_i)]}{2}$$
$$= \frac{\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)}{2}$$

so that

$$p = \left[\overbrace{\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)}^{\text{Fixed value}}\right] - \left[\overbrace{\frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i)}^{\text{Time-varying (function of } t)}\right] \tag{5.12}$$

A plot of v, i and p on the same set of axes is shown in Fig. 5.10 (b).The second factor in this equation is a cosine wave with an amplitude of V_m $I_m/2$ and a frequency twice that of the voltage or current. The average value of this item is zero over one cycle, producing no net transfer of energy in any one direction.

Equation (5.12) can be rewritten as

$$p = VI\cos(\theta_v - \theta_i)(1 - \cos 2\omega t) + VI\sin(\theta_v - \theta_i)(\sin 2\omega t) \tag{5.13}$$

Let $(\theta_v - \theta_i) = \theta$,

Therefore, $$p = VI\cos\theta - VI\cos\theta\,(\cos 2\omega t) + VI\sin\theta\,(\sin 2\omega t) \tag{5.14}$$

The first term has a constant magnitude (no time dependence) and therefore provides some net transfer of energy. This term is referred to as the average power. The average power or real power or active power is the power delivered to and dissipated by the load. The angle $(\theta_v - \theta_i)$ is the phase angle between v and i. Since $\cos(-\alpha) = \cos\alpha$, the magnitude of average power delivered is independent of whether v leads i or i leads v.

So average power, $$P = [(V_m I_m/2)\cos\theta \quad \text{W} \tag{5.15}$$

$$P = VI\cos\theta$$

For a purely **resistive** network $\theta = 0$, so that $P = V_m I_m/2 = V I = I^2R = V^2/R$ W

For a purely **inductive** network $\theta = 90°$, so that $P = V_m I_m/2 \cos 90° = 0$ W

For a purely **capacitive** network $\theta = 90°$, so that $P = V_m I_m/2 \cos 90° = 0$ W

The average power dissipated by an ideal inductor or capacitor is zero watts.

EXAMPLE 14.10 Find the average power dissipated in a network whose input current and voltage are the following:

$$i = 5 \sin(\omega t + 40^\circ)$$
$$v = 10 \sin(\omega t + 40^\circ)$$

Solution: Since v and i are in phase, the circuit appears to be purely resistive at the input terminals. Therefore,

$$P = \frac{V_m I_m}{2} = \frac{(10\text{ V})(5\text{ A})}{2} = \mathbf{25\ W}$$

or $$R = \frac{V_m}{I_m} = \frac{10\text{ V}}{5\text{ A}} = 2\ \Omega$$

and $$P = \frac{V_{\text{eff}}^2}{R} = \frac{[(0.707)(10\text{ V})]^2}{2} = \mathbf{25\ W}$$

or $$P = I_{\text{eff}}^2 R = [(0.707)(5\text{ A})]^2(2) = \mathbf{25\ W}$$

Reactive Power

The power required by a reactive load in an ac circuit is called **reactive power**. The term **VI sin θ** in Equation (5.14), the peak value of third term, produces no net transfer of energy, is known as Reactive power (Q). **Q = VI sin θ.** The unit of reactive power (Q) is Volt-Ampere Reactive (VAR).

Although no net power transfer occurs, during the positive half cycle of the term, energy will be required to supply to the reactive element inductor or capacitor and in the next negative half cycle power will be returned back. In other words, although power will not be dissipated, but power will be borrowed in one half cycle and returned back in the next half cycle. So generating plant must supply this reactive power. Reactive power is also a **cost factor** and must be on consumers.

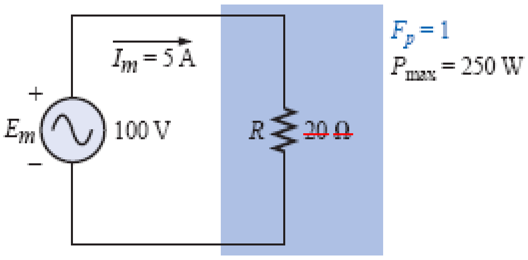
Apparent Power

From dc network analysis, it seems, P = VI (Voltage X Current), with no concern on load. We have also seen that P.F., cos θ has a pronounced effect on power dissipated. Product of Voltage and Current is not always the power delivered, but it (VI) is a power rating of significant usefulness in description and analysis of sinusoidal network and in the maximum rating of electrical components and systems. It (VI) is called **apparent power** (S). Its unit is Volt-Amperes (VA).

Power factor

In Eq.5.12, $P = (V_m\ I_m/2)\cos\theta$, the factor $\cos\theta$ has a significant control on the delivered power. No matter how large the voltage or current, if $\cos\theta = 0$, the power is zero. If $\cos\theta = 1$, the power delivered is maximum. Since it has such control, it is called power factor and is defined by

$$\textbf{Power factor} = P_F = \cos\theta = P/VI \qquad (5.13)$$



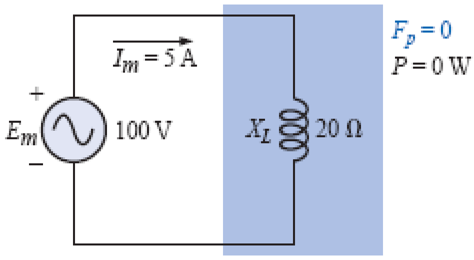


Fig. 5.11 (a) Fig. 5.11 (b)

For a purely resistive load such as the one shown in Fig. 5.11 (a), phase angle is 0° and Power Factor,

$$PF = \cos\theta = \cos 0° = 1,$$

The power delivered is maximum of $(V_m\ I_m/2)\cos\theta$ = (100 V)(5 A) (Cos 0°) = 250 W.

In case of a purely inductive or capacitive load PF = cos θ = cos90° = 0, the power delivered is then minimum of 0 W.

For situations where the load is a combination of resistive and reactive elements, the power factor will vary between 0 and 1. The more resistive the total impedance, the closer the power factor is to 1; the more reactive the total impedance, the closer the power factor is to 0.

In terms of the average power and the terminal voltage and current ,

$$\boxed{F_p = \cos\theta = \frac{P}{V_{\text{eff}} I_{\text{eff}}}}$$

The term leading and lagging are often written in conjunction with the power factor. They are defined by the current through the load. If the current leads the voltage across a load, the load has a leading power factor. If the current lags the voltage across the load, the load has a lagging power factor. In other words,

capacitive networks have leading power factors, and inductive networks have lagging power factors.

Example 5.10: Determine the power factors of the loads shown in Fig. 5.12 and indicate whether they are leading or lagging.

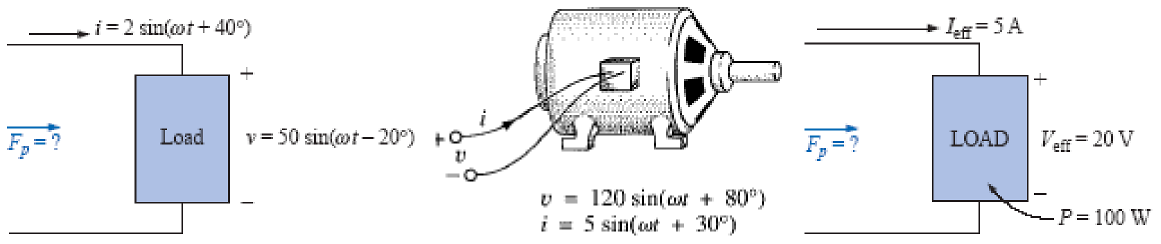


Fig. 5.12 (a) Fig. 5.12 (b) Fig. 5.12 (c)

a. $F_p = \cos\theta = \cos|40° - (-20°)| = \cos 60° = \mathbf{0.5\ leading}$

b. $F_p = \cos\theta\,|80° - 30°| = \cos 50° = \mathbf{0.6428\ lagging}$

c. $F_p = \cos\theta = \dfrac{P}{V_{eff}I_{eff}} = \dfrac{100\text{ W}}{(20\text{ V})(5\text{ A})} = \dfrac{100\text{ W}}{100\text{ W}} = 1$

The load is resistive, and F_p is neither leading nor lagging.

Average power or active power

During the operation of electrical networks and systems there is always some net transfer of energy. **Average power is the power delivered by the source and dissipated or consumed in the load.** Average power, P = VI cos θ .

For pure resistive network, $P = VI = I^2R$ and for pure inductive or capacitive network P = 0.

Reactive Power

The power required by a reactive load in an ac circuit is called **reactive power**. Reactive power Q = VI sin θ.

Apparent Power

Product of voltage and current, VI, is called apparent power (S).

Power factor

The cosine of the phase angle between the voltage and current is known as power factor. That is power factor = cos θ = P/VI

Phasor

A radius vector having a constant magnitude is called a phasor, when applied to electric circuit. Phasor diagram shows at a glance the magnitude and phase relations.

Impedance

How much an ac circuit elements impede or control the flow of charges through the network is called impedance of the circuit.

Inductive reactance

How much an ac circuit elements impede or control the level of ac current through an inductor is called inductive reactance of the circuit.

Capacitive reactance

How much an ac circuit elements impede or control the flow of ac current through a capacitor is called capacitive reactance of the circuit.

Reactance

How much an ac circuit elements impede or control the flow of charges through an inductor or a capacitor is called reactance of the circuit.