Matrix: A system of any mn numbers arranged in a rectangular array of m rows and n columns is called a matrix of order $m \times n$. A matrix is usually denoted by a single capital letter, namely A, B, C, … … or by the symbols $\left[a_{ij}\right]$, $\left(a_{ij}\right)$, $\left\|a_{ij}\right\|$.

The matrix of order $m \times n$ is written as:

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}_{m\times n}$$

Example: $\mathbf{A} = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}_{3\times 3}$ **;** $\mathbf{B} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1}$ **;** $\mathbf{C} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3}$**;** $\mathbf{D} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$ **.**

Question: Distinguish between a matrix and a determinant.

Answer: The differences between a matrix and a determinant are as follows:

Matrix	**Determinant**
1. A matrix cannot be reduced to a single number.	**1**.A determinant can be reduced to a single number.
2. In a matrix, the number of rows may not be equal to the number of columns.	**2**. In a determinant, the number of rows must be equal to the number of columns.
3. An interchange of rows or columns gives a different matrix.	**3**. An interchange of rows or columns gives the same determinant with +*ve* or –*ve* sign.
4.Examples: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$.	**4**. Examples: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$; $\begin{vmatrix} 1 & 2 & 3 \\ -2 & 1 & 0 \\ 2 & 3 & 4 \end{vmatrix}$.

Question: Define Complex Matrix and Conjugate of a Complex Matrix.

Answer: **Complex Matrix**: Any matrix having complex elements is called a complex matrix.

Example: $A = \begin{bmatrix} 2+i & -2i & 3 \\ 2 & 3i & -1 \\ -3 & 1+2i & 2i \end{bmatrix}$.

<u>Conjugate of a Complex Matrix</u>: The matrix obtained from any given matrix ***A*** of order $m \times n$ with complex elements a_{ij}, by replacing its elements by the corresponding conjugate complex numbers is called the complex conjugate or conjugate of A and is denoted by $\overline{A}$.

Example: If $A = \begin{bmatrix} 2+i & -2i & 3 \\ 2 & 3i & -1 \\ -3 & 1+2i & 2i \end{bmatrix}$ then $\overline{A} = \begin{bmatrix} 2-i & 2i & 3 \\ 2 & -3i & -1 \\ -3 & 1-2i & -2i \end{bmatrix}$.

<u>Question:</u> Define real and imaginary matrix.

Answer:<u>Real Matrix:</u> A matrix ***A*** is called real if each element is a real number or it satisfies the relation $A = \overline{A}$.

Example: $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$.

<u>Imaginary Matrix:</u>A matrix ***A*** is called imaginary if each element is imaginary or it satisfies the relation $A = -\overline{A}$.

Example: $A = \begin{bmatrix} i & -2i & 3i \\ 2i & 3i & -i \\ -3i & i & 2i \end{bmatrix}$.

<u>Question:</u> Define rectangular and square matrix.

Answer:<u>Rectangular Matrix</u>:A matrix ***A*** of order $m \times n$ is called a rectangular matrix if the number of rows and the number of columns are not equal***i.e,*** $m \neq n$.

Example: $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$.

<u>Square Matrix</u>:A matrix ***A*** of order $m \times n$ is called a square matrix if the number of rows and the number of columns are equal***i.e,*** $m = n$.

Example: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; $B = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$

<u>Question:</u> Define Horizontal and Vertical matrix.

Answer: **<u>Horizontal Matrix</u>:** A matrix ***A*** of order $m \times n$ is called a horizontalmatrix if the number of rows is less than the number of columns, ***i.e,*** $m < n$**.**

Example: $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 1 \end{bmatrix}$.

<u>Vertical Matrix</u>: A matrix ***A*** of order $m \times n$ is called a horizontalmatrix if the number of rows is more than the number of columns, ***i.e,*** $m > n$**.**

Example: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$.

<u>Question:</u> Define Row and Column matrix.

Answer: **<u>Row Matrix</u>:** A matrix ***A*** is called a row matrix row vector if it contains only one row.

Example: $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$.

<u>Column Matrix</u>: A matrix ***A*** is called a column matrix or column vector if it contains only one column.

Example: $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

<u>Question:</u> Define Null matrix and Identity matrix.

Answer: **<u>Null Matrix</u>:** Amatrix, rectangular or square, each of whose elements is zero is called a **Zero matrix** or **Null matrix** and is denoted by ***O***.

Example: $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

<u>Identity Matrix</u>: A square matrix whose elements $a_{ij} = 0 \text{ when } i \neq j$ and $a_{ij} = 1 \text{ when } i = j$ is called the **Identity matrix or Unit matrix** and is denoted by ***I*** or ***U.***

Example: $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

<u>Question:</u> Define Diagonal matrix and Scalar matrix.

Answer:<u>Diagonal Matrix</u>:A square matrix whose elements $a_{ij} = 0 \; when \; i \neq j$ is called a **Diagonal matrix**. The elements $a_{ij} \; when \; i = j$ are known as diagonal elements and the line along which they lie is known as the **principal diagonal** or **leading diagonal**.

Example: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

<u>Scalar Matrix</u>: If in a square matrix ***A*** all the diagonal elements are equal to a nonzero element ***a***and all the remaining elements are equal to zero then it is called a **Scalar matrix.**

Example: $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

<u>Question:</u> Define Triangular, Upper Triangular and Lower Triangular matrices.

Answer:<u>Triangular Matrix</u>:A square matrix whose elements $a_{ij} = 0 \; when \; i > j \; or \; i < j$ is called a **Triangular matrix**.

Example: $A = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 5 & 7 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$

<u>Upper Triangular Matrix</u>:A square matrix***A***whose elements $a_{ij} = 0$ when $i > j$ is called an **upper triangular matrix**.

Example: $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & 7 \end{bmatrix}$

Lower Triangular Matrix: A square matrix ***A*** whose elements $a_{ij} = 0 \quad i < j$ is called a **lower triangular matrix**.

Example: $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 4 & 3 & 2 & 0 \\ 5 & 4 & 5 & 7 \end{bmatrix}$

Question: Define sub-matrix of a matrix.

Answer: A matrix, which is obtained from a given matrix by deleting any number of rows and columns or both, is called a **sub-matrix** of the given matrix.

Example:If $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 0 & 7 & 5 & 2 \end{bmatrix}$ be a matrix then $\begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 0 & 7 & 5 \end{bmatrix}; \begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix}$ etc. are its sub-matrices.

Question:Define minor of a matrix.

Answer: Let ***A*** be an $m \times n$ matrix. The determinant of the square sub-matrix of order $r \times r$ obtained by deleting $(m-r)$ rows and $(n-r)$ columns from ***A***, is called a **minor** of order *r* of ***A***.

Example: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ then $\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}; \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}; \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix}$ are minors of order 2 of ***A***.

Question:Define equality of matrices.

Answer: Two matrices $A = \left[a_{ij}\right]$ and $B = \left[b_{ij}\right]$ are said to equal iff

- They are of the same dimensions ***i.e,*** they are of the same order.
- The elements in the corresponding positions of the two matrices are same.

Question:Define order or dimension of a matrix.

Answer: The **order** or **dimension** of a matrix is given by stating the number of rows and the number of columns in the matrix.

Example: $A=\begin{bmatrix}1 & 2 & -1 & 3\\ 4 & 3 & -2 & 0\\ -2 & 1 & 0 & 4\end{bmatrix}$ is a matrix of order 3×4.

Question:Explain the addition and subtraction of matrices.

Answer: Addition: If ***A*** and ***B*** be two matrices of order $m\times n$ given by $A=\left[a_{ij}\right]$ and $B=\left[b_{ij}\right]$, then the matrix $A+B$ is defined as the matrix each element of which is the sum of the corresponding elements of ***A*** and ***B*** i.e. $A+B=\left[a_{ij}+b_{ij}\right]$, where $i=1,2,3,.....,m$ and $j=1,2,3,.....,n$.

Example: If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ *and* $B=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$ then $A+B=\begin{bmatrix}1+2 & 2+3\\ 3+4 & 4+5\end{bmatrix}=\begin{bmatrix}3 & 5\\ 7 & 9\end{bmatrix}$.

Subtraction: If ***A*** and ***B*** be two matrices of order $m\times n$ given by $A=\left[a_{ij}\right]$ and $B=\left[b_{ij}\right]$, then the matrix $A-B$ is defined as the matrix each element of which is obtained by subtracting the elements of***B***from the corresponding elements of ***A***i.e. $A-B=\left[a_{ij}-b_{ij}\right]$, where $i=1,2,3,.....,m$ and $j=1,2,3,.....,n$.

Example: If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ *and* $B=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$ then $A-B=\begin{bmatrix}1-2 & 2-3\\ 3-4 & 4-5\end{bmatrix}=\begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}$.

Question:Define conformable for addition and subtraction.

Answer: If the two matrices ***A*** and ***B***are of the same order, then only their addition and subtraction is possible and these matrices are said to be **Conformable** for **Addition** or **Subtraction**.

Question:Explain the multiplication of matrices.

Answer: If ***A*** and ***B*** be two matrices such that the number of columns in ***A*** is equal to the number rows in ***B*** i.e. if $A=\left[a_{ij}\right]$and $B=\left[b_{jk}\right]$ are $m\times n$, $n\times p$ matrices then the product of the matrices ***A*** and ***B*** denoted by ***AB*** is defined as matrix

$$C=\left[c_{ik}\right]$$

$$=\sum_{j=1}^{n}a_{ij}b_{jk}$$

In the matrix product ***AB***, the matrix ***A*** is called the pre-multipliers and ***B*** is called the post- multipliers.

Example: If $A=\begin{bmatrix}1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 2 & 3\end{bmatrix}$ then $AB=\begin{bmatrix}1+4 & 0+6\end{bmatrix}=\begin{bmatrix}5 & 6\end{bmatrix}$.

<u>Question:</u>Define transpose of a matrix.

Answer: The matrix obtained from any given matrix ***A*** by interchanging its rows and columns is called its transpose. The transpose of ***A,*** is denoted by A^t *or* A'**.**

Example: If $A=\begin{bmatrix}1 & 2\\ 0 & 1\\ 1 & -2\end{bmatrix}, then\ A^t=\begin{bmatrix}1 & 0 & 1\\ 2 & 1 & -2\end{bmatrix}$.

<u>Question:</u>Define commutative and anti-commutative matrices.

Answer: <u>Commutative matrices</u>: If ***A***and ***B*** are two square matrices such that ***AB***=***BA***, then ***A*** and ***B*** are called **commutative matrices** or are said to **commute.**

<u>Anti-commutative matrices</u>: If ***A***and ***B*** are two square matrices such that $AB=-BA$, then ***A*** and ***B*** are called **anti-commutative matrices** or are said to **anti-commute.**

NOTE: The transpose of the product of two matrices is equal to the product of their transpose in reverse order, i.e. $(AB)^t=B^tA^t$**.**

<u>Question:</u>Define determinant of a square matrix.

Answer: The determinant whose elements are exactly the same as those of a square matrix***A,*** is called the **determinant of the square matrix *A***and denoted by$|A|$**.**

Example: If $A=\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix};\ then\ |A|=\begin{vmatrix}1 & -1\\ 0 & 2\end{vmatrix}$.

<u>Question:</u>Define singular and non-singular matrix.

Answer: <u>Singular matrix</u>: A square matrix***A***is said to be a **singular matrix**, if the determinant of***A***is zero**, i.e.**$|A|=0$**.**

Example: Let $A=\begin{bmatrix}1 & -2\\ -1 & 2\end{bmatrix}$**;** then$|A|=\begin{vmatrix}1 & -2\\ -1 & 2\end{vmatrix}=0$.

<u>Singular matrix</u>: A square matrix***A***is said to be a **non-singular matrix**, if the determinant of***A***is not zero**, i.e.**$|A|\neq 0$**.**

Example: Let $A=\begin{bmatrix}1 & 2\\ -1 & 2\end{bmatrix}$; then $|A|=\begin{vmatrix}1 & 2\\ -1 & 2\end{vmatrix}\neq 0.$

Question: **Define Symmetric and Skew-Symmetric matrix.**

Answer: **Symmetric matrix:** A square matrix $A=\left[a_{ij}\right]$ is said to be a **symmetric matrix** if $a_{ij}=a_{ji}$ for all values of i and j **i.e.** $A^t=A$.

Example: Let $A=\begin{bmatrix}a & h & g\\ h & b & f\\ g & f & c\end{bmatrix}$; then $A^t=\begin{bmatrix}a & h & g\\ h & b & f\\ g & f & c\end{bmatrix}=A$.

Here $\boldsymbol{A}$ is A symmetric matrix.

Skew-Symmetric matrix: A square matrix $A=\left[a_{ij}\right]$ is said to be a **skew-symmetric matrix** if $a_{ij}=-a_{ji}$ for all values of i and j **i.e.** $A^t=-A$.

Example: Let $A=\begin{bmatrix}0 & h & g\\ -h & 0 & f\\ -g & -f & 0\end{bmatrix}$; then $A^t=\begin{bmatrix}0 & -h & -g\\ h & 0 & -f\\ g & f & 0\end{bmatrix}=-\begin{bmatrix}0 & h & g\\ -h & 0 & f\\ -g & -f & 0\end{bmatrix}=A$.

Here $\boldsymbol{A}$ is a skew- symmetric matrix.

NOTE: Every square matrix can be expressed uniquely as the sum of a symmetric matrix and a skew-symmetric matrix.

- Let $\boldsymbol{A}$ be a square matrix of order n and A^T be the transpose of $\boldsymbol{A}$. Then we can write,

  $$A=\frac{1}{2}\left(A+A^T\right)+\frac{1}{2}\left(A-A^T\right)$$

  where

  $\frac{1}{2}\left(A+A^T\right)=$ Symmetric matric.

  $\frac{1}{2}\left(A-A^T\right)=$ Skew- symmetric matrix.

Question: **Define Hermitian and Skew-Hermitian matrix.**

Answer: **Hermitian matrix:** A square complex matrix $A=\left[a_{ij}\right]_{n\times n}$ is said to be a Hermitian matrix if it is equal to the transpose of its conjugate complex i.e. if $A=\left(\overline{A}\right)^t$.

Example: $A=\begin{bmatrix}l & \alpha+i\beta\\ \alpha-i\beta & m\end{bmatrix}$

Skew-Hermitian matrix: A square complex matrix $A=\left[a_{ij}\right]_{n\times n}$ is said to be a Skew-Hermitian matrix or anti-Hermitian matrix if $\left(\overline{A}\right)^{t}=-A$.

Example: $A=\begin{bmatrix} i & 1+i \\ -1+i & 0 \end{bmatrix}$

Question: **Define Unitary matrix.**

Answer: **Unitary matrix:** A square complex matrix $A=\left[a_{ij}\right]_{n\times n}$ is said to be a Unitary matrix if $AA^{*}=A^{*}A=I$ where $A^{*}=\left(\overline{A}\right)^{t}$.

Example: $A=\dfrac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & -1 \end{bmatrix};\ B=\dfrac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

Question: **Define Normal matrix.**

Answer: **Normal matrix:** A square complex matrix $A=\left[a_{ij}\right]_{n\times n}$ is said to be a Normal matrix if $AA^{*}=A^{*}A$ where $A^{*}=\left(\overline{A}\right)^{t}$.

Example: $A=\begin{bmatrix} 2+3i & 1 \\ i & 1+2i \end{bmatrix}$

Question: **Define Idempotent matrix.**

Answer: **Idempotent matrix:** A square matrix $A=\left[a_{ij}\right]_{n\times n}$ is said to be an Idempotent matrix if $A^{2}=A$.

Example: $A=\begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

Question: **Define Periodic matrix.**

Answer: **Periodic matrix:** A square matrix $A=\left[a_{ij}\right]_{n\times n}$ is said to be a Periodic matrix if $A^{k+1}=A$, where k is a positive integer.

Example: $A=\begin{bmatrix} 1 & -2 & -6 \\ -3 & 2 & 9 \\ 2 & 0 & -3 \end{bmatrix}$ is a periodic matrix of period 2.

Question: **Define Involutory matrix.**

Answer: **Involutory matrix:** A square matrix $A=\left[a_{ij}\right]_{n\times n}$ is said to be an Involutory matrix if $A^2=I$.

Example: $A=\begin{bmatrix} 4 & 3 \\ -5 & -4 \end{bmatrix}$; $B=\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

Question: **Define Nilpotent matrix.**

Answer: **Nilpotent matrix:** A square matrix $A=\left[a_{ij}\right]_{n\times n}$ is said to be a Nilpotent matrix of order m, if $A^m=O$ and $A^{m-1}\neq O$, where m is a positive integer and O is the null matrix. If m is the least positive integer such that $A^m=O$, then m is called the index of the nilpotent matrix ***A.***

Example: $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is a Nilpotent matrix of index 2.

Question: **Define an orthogonal matrix.**

Answer: **Orthogonal matrix:** A square matrix $A=\left[a_{ij}\right]_{n\times n}$ is said to be an orthogonal matrix if $AA^t=I$, where I is an identity matrix and A^t is the transposed matrix of ***A***.

Example: $A=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$; $B=\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

Question: **Define Trace or Spur of a matrix.**

Answer: Let *A* be a square matrix of order *n*. Then the sum of the elements of *A* lying along the principal diagonal is called the trace of *A*. We shall write trace of *A* as tr.*A*.

Example: Let $A=\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

So that tr.*A*= $\cos\theta+\cos\theta=2\cos\theta$.

Problem-01: If $A=\begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$ and $B=\begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}$ then find $A+B$ and $2A-B$.

Solution: The given matrices are,

$$A=\begin{pmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{pmatrix} \text{ and } B=\begin{pmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{pmatrix}$$

Now, $A+B=\begin{pmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{pmatrix}+\begin{pmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{pmatrix}$

$$=\begin{pmatrix}1+3 & 2-1 & -3+2\\ 5+4 & 0+2 & 2+5\\ 1+2 & -1+0 & 1+3\end{pmatrix}$$

$$=\begin{pmatrix}4 & 1 & -1\\ 9 & 2 & 7\\ 3 & -1 & 4\end{pmatrix}$$

Again, $2A-B=2\begin{pmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{pmatrix}-\begin{pmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{pmatrix}$

$$=\begin{pmatrix}2 & 4 & -6\\ 10 & 0 & 4\\ 2 & -2 & 2\end{pmatrix}-\begin{pmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{pmatrix}$$

$$=\begin{pmatrix}2-3 & 4+1 & -6-2\\ 10-4 & 0-2 & 4-5\\ 2-2 & -2-0 & 2-3\end{pmatrix}$$

$$=\begin{pmatrix}-1 & 5 & -8\\ 6 & -2 & -1\\ 0 & -2 & -1\end{pmatrix}.$$

Problem-02: If $A=\begin{pmatrix}1 & 4 & 0\\ 2 & 5 & 0\\ 3 & 6 & 0\end{pmatrix}$, $B=\begin{pmatrix}3 & 2 & 1\\ 1 & 2 & 3\\ 4 & 5 & 6\end{pmatrix}$ and $C=\begin{pmatrix}3 & 2 & 1\\ 1 & 2 & 3\\ 7 & 8 & 9\end{pmatrix}$ then find AB and BC.

Solution: The given matrices are,

$$A=\begin{pmatrix}1 & 4 & 0\\ 2 & 5 & 0\\ 3 & 6 & 0\end{pmatrix}, B=\begin{pmatrix}3 & 2 & 1\\ 1 & 2 & 3\\ 4 & 5 & 6\end{pmatrix} \text{ and } C=\begin{pmatrix}3 & 2 & 1\\ 1 & 2 & 3\\ 7 & 8 & 9\end{pmatrix}$$

Now, $AB = \begin{pmatrix} 1 & 4 & 0 \\ 2 & 5 & 0 \\ 3 & 6 & 0 \end{pmatrix} \begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

$$= \begin{pmatrix} 1.3+4.1+0.4 & 1.2+4.2+0.5 & 1.1+4.3+0.6 \\ 2.3+5.1+0.4 & 2.2+5.2+0.5 & 2.1+5.3+0.6 \\ 3.3+6.1+0.4 & 3.2+6.2+0.5 & 3.1+6.3+0.6 \end{pmatrix}$$

$$= \begin{pmatrix} 3+4+0 & 2+8+0 & 1+12+0 \\ 6+5+0 & 4+10+0 & 2+15+0 \\ 9+6+0 & 6+12+0 & 3+18+0 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 10 & 13 \\ 11 & 14 & 17 \\ 15 & 18 & 21 \end{pmatrix}.$$

Again, $BC = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{pmatrix}$

$$= \begin{pmatrix} 3.3+2.1+1.7 & 3.2+2.2+1.8 & 3.1+2.3+1.9 \\ 1.3+2.1+3.7 & 1.2+2.2+3.8 & 1.1+2.3+3.9 \\ 4.3+5.1+6.7 & 4.2+5.2+6.8 & 4.1+5.3+6.9 \end{pmatrix}$$

$$= \begin{pmatrix} 9+2+7 & 6+4+8 & 3+6+9 \\ 3+2+21 & 2+4+24 & 1+6+27 \\ 12+5+42 & 8+10+48 & 4+15+54 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & 18 & 18 \\ 26 & 30 & 34 \\ 59 & 66 & 73 \end{pmatrix}.$$

Problem-03: Express $A = \begin{pmatrix} -1 & 2 & 3 \\ 0 & 1 & -2 \\ 4 & -5 & -3 \end{pmatrix}$ as the sum of a Symmetric and Skew-symmetric matrices.

Solution: The given matrix is,

$$A = \begin{pmatrix} -1 & 2 & 3 \\ 0 & 1 & -2 \\ 4 & -5 & -3 \end{pmatrix}$$

The transpose of $\boldsymbol{A}$ is $A^t = \begin{pmatrix} -1 & 0 & 4 \\ 2 & 1 & -5 \\ 3 & -2 & -3 \end{pmatrix}$

The symmetric matrix of $\boldsymbol{A}$ is,

$$\frac{1}{2}\left(A + A^t\right) = \frac{1}{2}\left\{\begin{pmatrix} -1 & 2 & 3 \\ 0 & 1 & -2 \\ 4 & -5 & -3 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 4 \\ 2 & 1 & -5 \\ 3 & -2 & -3 \end{pmatrix}\right\}$$

$$= \frac{1}{2}\begin{pmatrix} -2 & 2 & 7 \\ 2 & 2 & -7 \\ 7 & -7 & -6 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 1 & \frac{7}{2} \\ 1 & 1 & -\frac{7}{2} \\ \frac{7}{2} & -\frac{7}{2} & -3 \end{pmatrix}$$

The skew-symmetric matrix of $\boldsymbol{A}$ is,

$$\frac{1}{2}\left(A - A^t\right) = \frac{1}{2}\left\{\begin{pmatrix} -1 & 2 & 3 \\ 0 & 1 & -2 \\ 4 & -5 & -3 \end{pmatrix} - \begin{pmatrix} -1 & 0 & 4 \\ 2 & 1 & -5 \\ 3 & -2 & -3 \end{pmatrix}\right\}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & 2 & -1 \\ -2 & 0 & 3 \\ 1 & -3 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & -\frac{1}{2} \\ -1 & 0 & \frac{3}{2} \\ \frac{1}{2} & -\frac{3}{2} & 0 \end{pmatrix}$$

Therefore, $A=\begin{pmatrix} -1 & 1 & \frac{7}{2} \\ 1 & 1 & -\frac{7}{2} \\ \frac{7}{2} & -\frac{7}{2} & -3 \end{pmatrix}+\begin{pmatrix} 0 & 1 & -\frac{1}{2} \\ -1 & 0 & \frac{3}{2} \\ \frac{1}{2} & -\frac{3}{2} & 0 \end{pmatrix}$.

Problem-04: Express $A=\begin{pmatrix} 1 & 0 & 6 \\ 4 & 4 & -1 \\ 0 & -9 & 6 \end{pmatrix}$ as the sum of a Symmetric and Skew-symmetric matrices.

Solution: The given matrix is,

$$A=\begin{pmatrix} 1 & 0 & 6 \\ 4 & 4 & -1 \\ 0 & -9 & 6 \end{pmatrix}$$

The transpose of ***A*** is $A^t=\begin{pmatrix} 1 & 4 & 0 \\ 0 & 4 & -9 \\ 6 & -1 & 6 \end{pmatrix}$

The symmetric matrix of ***A*** is,

$$\frac{1}{2}\left(A+A^t\right)=\frac{1}{2}\left\{\begin{pmatrix} 1 & 0 & 6 \\ 4 & 4 & -1 \\ 0 & -9 & 6 \end{pmatrix}+\begin{pmatrix} 1 & 4 & 0 \\ 0 & 4 & -9 \\ 6 & -1 & 6 \end{pmatrix}\right\}$$

$$=\frac{1}{2}\begin{pmatrix} 2 & 4 & 6 \\ 4 & 8 & -10 \\ 6 & -10 & 12 \end{pmatrix}$$

$$=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & -5 \\ 3 & -5 & -6 \end{pmatrix}$$

The skew-symmetric matrix of ***A*** is,

$$\frac{1}{2}\left(A-A^t\right)=\frac{1}{2}\left\{\begin{pmatrix} 1 & 0 & 6 \\ 4 & 4 & -1 \\ 0 & -9 & 6 \end{pmatrix}-\begin{pmatrix} 1 & 4 & 0 \\ 0 & 4 & -9 \\ 6 & -1 & 6 \end{pmatrix}\right\}$$

$$=\frac{1}{2}\begin{pmatrix} 0 & -4 & 6 \\ 4 & 0 & 8 \\ -6 & -8 & 0 \end{pmatrix}$$

$$=\begin{pmatrix} 0 & -2 & 3 \\ 2 & 0 & 4 \\ -3 & -4 & 0 \end{pmatrix}$$

Therefore, $A=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & -5 \\ 3 & -5 & 6 \end{pmatrix}+\begin{pmatrix} 0 & -2 & 3 \\ 2 & 0 & 4 \\ -3 & -4 & 0 \end{pmatrix}$.

Exercise: 1. Find the Symmetric and Skew-symmetric parts of the matrix

$$A=\begin{pmatrix} 1 & 2 & 4 \\ 6 & 8 & 1 \\ 3 & 5 & 7 \end{pmatrix}.$$

2. Find the Symmetric and Skew-symmetric parts of the matrix

$$A=\begin{pmatrix} 1 & 1/3 & 1 & 4 \\ 1 & -1 & 0 & -1 \\ -3 & 0 & -2/5 & 6 \\ 1 & -1 & -1 & 1 \end{pmatrix}.$$