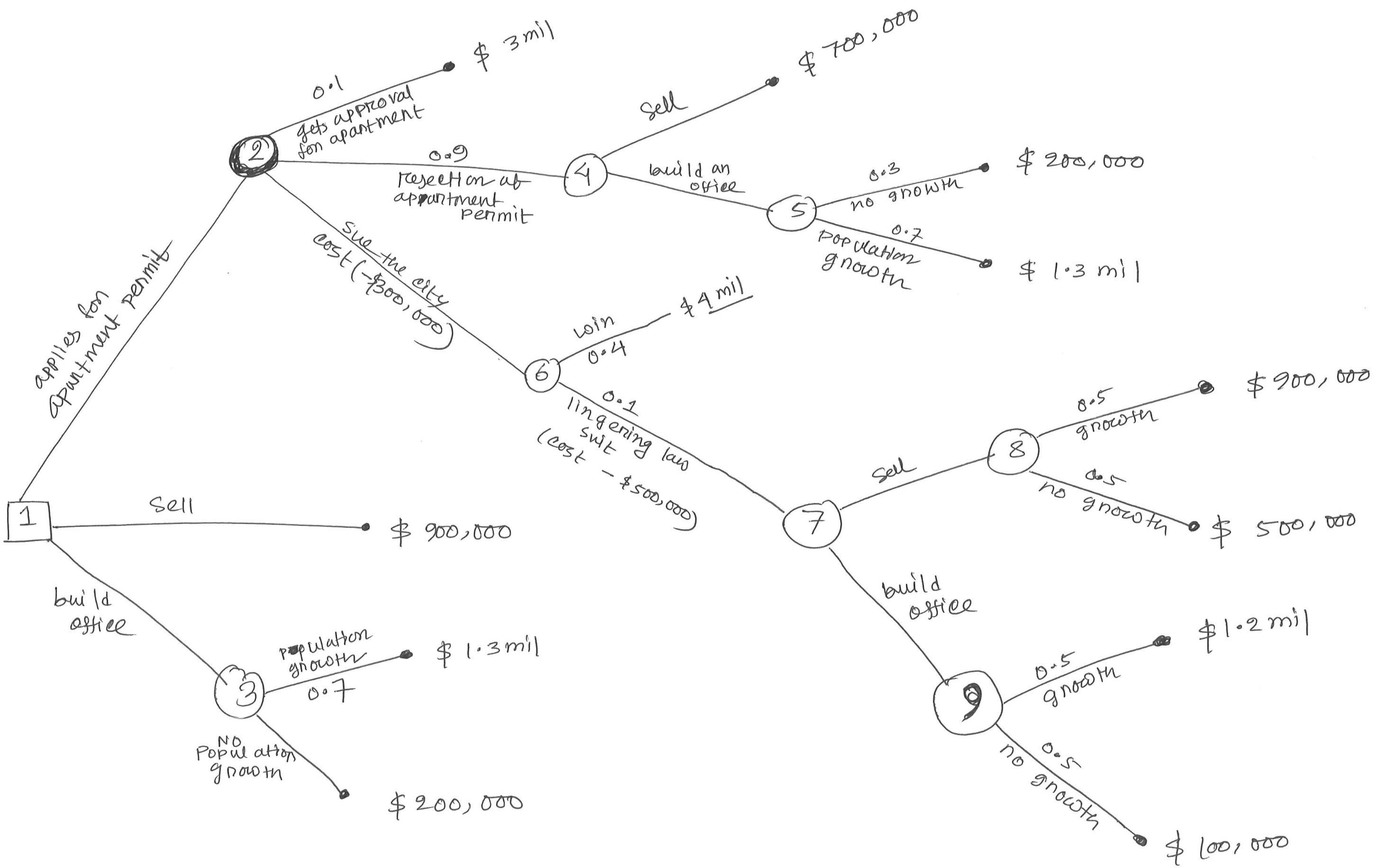
a)



1
applies for apartment permit
Sell
$ 900,000
build office
2
0.1
gets approval for apartment
$ 3mil
0.9
rejection of appartment permit
4
Sell
$ 700,000
build an office
5
0.3
no growth
$ 200,000
0.7
population growth
$ 1.3 mil
sue the city cost (-$300,000)
6
win
0.4
$ 4 mil
0.1
lingering law suit (cost - $500,000)
7
sell
8
0.5
growth
$ 900,000
0.5
no growth
$ 500,000
build office
9
0.5
growth
$ 1.2 mil
0.5
no growth
$ 100,000
3
population growth
0.7
$ 1.3mil
NO Population growth
$ 200,000

(b) By analyzing the influence diagram, I think it is better to build an office in the first place. When it's decided to build an office, from the influence diagram it can bea seen that the probability of population growth is 0.7 (70%) and the final gain is $ 1.3 million. I think this decision is much more safe and also the final amount is satisfactory

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Answer to the question NO - 3

3) You have \$1000 to invest in speculative stocks A and B. You are considering investing X dollars in stock A and (1000 − X) dollars in stock B. Investment in stock A has a 0.6 chance at doubling in value and 0.4 chance at being lost. Investment in stock B has a 0.7 chance at doubling in value and 0.3 chance at being lost. Your utility function for a change in wealth, z, is $U(z) = \log(0.0007 * z + 1)$.

(a) What are the elements for Z?

Ans: There are four elements for Z. (if the range, $-1000 \leq Z \leq 1000$)

$Z = \{-1000,\ 2a - 1000,\ 1000 - 2a,\ 1000\}$

(b) Find the optimal value of X in terms of expected utility.

Ans:

Z	−1000	2a − 1000	1000 − 2a	1000
Utility	log(0.3)	log(0.0014a + 0.3)	log(1.7 − 0.0014a)	log(1.7)
Probability	(0.4)(0.3)	(0.6)(0.3)	(0.4)(0.7)	(0.6)(0.7)

Based on the above table, we can compute the expected utility, that is

$$U = 0.12 \log(0.3) + 0.18 \log(0.0014a + 0.3) + $$
$$+ 0.28 \log(1.7 - 0.0014a) + 0.42 \log(1.7)$$

Now, let us look for the value of a which maximizes the expected utility (a^*). We have,

→

$$\frac{dU}{da} = \frac{0.18}{0.0014a+0.3}(0.0014) + \frac{0.28}{1.7-0.0014a}(-0.0014)$$

$$= \frac{0.000252}{0.0014a+0.3} - \frac{0.000392}{1.7-0.0014a}$$

Setting the derivatives equal to zero and evaluating it with a^* leads to the following equation :

$$\frac{0.000252}{0.0014a^* + 0.3} = \frac{0.000392}{1.7 - 0.0014a^*}$$

By solving the above, we obtain, $a = 344.72$

Also, since the second derivative,

$$\frac{d^2U}{da^2} = \frac{0.000252(0.0014)}{(0.0014a+0.3)^2} - \frac{0.000392(0.0014)}{(1.7-0.0014a)^2} < 0$$

So, we can conclude that, $a = 344.72$

This value maximizes the expected utility U.

Question 4:

In decision under ignorance, the maxmin, minimax and other decision criteria focus on the act's best and worst possible outcomes. What is so special about these best and worst outcome? Discuss

Solution:

Decision under ignorance refers to a situation where we are unable to assign probabilities to any of the states. Maxmin, Minimax these decision criteria or principals are used after deletion of dominance.

For maxmin, this refers to a pessimistic decision. The outcome of this principal is based on worst possible outcome of each alternative. Here the agent only thinks about maximizing the minimal value or the worst possible outcome. An example has been stated below that shows states from s_1 to s_4 and actions from a_1 to a_4.

	s_1	s_2	s_3	s_4
a_1	6	9	3	0
a_2	-5	7	4	12
a_3	6	4	5	2
a_4	14	-8	5	7

For action a_1, a_2, a_3, a_4, a_5 the smallest values are 0, -5, 2, and -8 respectively. Since the goal of maxmin is to maximize the minimal value so in this regard a_3 is the most preferred decision.

For minimax, the agent looks at maximum possible outcome. Whichever criteria give the maximum outcome the agent looks out for that criteria. This also uses the concept of a regret table. A regret table is formed after choosing the maximum outcome. If a certain action (**let $\boldsymbol{a_3}$**) has been taken and a state (**let $\boldsymbol{s_1}$**) has happened the regret table is formed as below:

	s_1	s_2	s_3	s_4
a_1	12	8	20	**20**
a_2	10	**15**	16	8
a_3	**30**	6	25	14
a_4	20	4	**30**	10

The maximum value has been marked in bold. After taking action with max value the regret table looks like below:

	s_1	s_2	s_3	s_4
a_1	**18**	7	10	0
a_2	**20**	0	14	12
a_3	0	**9**	5	6
a_4	10	**11**	0	10

This is how best and worst outcomes are chosen in decision principals.

Question 5:

Your doctor suspects that you may have a rare disease that affects about 1 in 50,000. Given that you have the disease the test offered by the doctor will show positive with 0.9 probability. Unfortunately, the test will also show positive with probability 0.01 when applied to a healthy person. What is the probability that you have the disease given that the test is positive? What is your opinion about the test offered by the doctor?

Solution:

To solve this problem I used the concept of Bayes' Theorem. According to this theorem the probability of two events A and B is given as P(A|B)=P(A)P(B|A) / P(B).

In this case,

P(A) = probability that I have the disease = 1/50000 = 0.00002

P(B) = probability that I test positive

P(B|not A) = false positive = probability that I test positive even if I don't have the disease = 0.01

Here,

P(B|A) = 0.9 and P(A) = 0. 00002

Now, P(B) can be derived by conditioning on whether event A does or does not happen.

P(B) = P(B|A) P(A) + P(B|not A) P(not A) = (0.9 * 0.00002) + (0.01 * 0.99998) = 0.0100178

Thus, the ration that we get from Bayes' Theorem is less than 1 percent.

The reasoning behind this result can be given as follows:

I think the disease is so rare that the number of false positives greatly outnumbers the people who truly have the disease. That's why the likelihood that I have the disease is less than 1% from the given explanation above.

Question 6:

Your prior probability that the coin is biased to land heads is 0. Prove that whatever happens in your experiment (or observations in tossing the coin), the posterior probability will always be zero. Discuss the implications.

Solution:

Posterior probabilities are those we ascribe to after updating our beliefs in the face of new evidence. In this question since prior probability is said to be zero because of that no amount of evidence will ever convince otherwise. After all, even the strongest evidence to the contrary will still yield a posterior probability of zero when multiplied by zero.

That's why in this case both prior and posterior probabilities are zero.

Question 7:

Based on Spencer et al. (1990), when you lease 800 phone numbers from AT&T for telemarketing, AT&T uses an optimization model to tell you where you should locate calling centers to minimize your operating costs over a 10-year horizon. To illustrate the model, suppose you are considering seven calling center locations: Boston, NY, Charlotte, Dallas, Chicago, LA and Omaha. You know the average cost (in dollars) incurred if a telemarketing call is made from any of these cities to any region in the country. You also know the hourly wage that you must pay workers in each city. This information is given in table 1 (excel sheet: Q7_tables). Also, assume that an average call requires 4 minutes of labor. You make calls 250 days per year, and the average number of calls made per day to each region of the country is listed in table 3 (excel sheet: Q7_tables). Each calling center can make up to 5000 calls per day. Given this information how can you minimize the discounted cost (at a 10% per year) of running the telemarketing operation for 10 years? Assume that all wage and calling costs are paid at the ends of the respective years.

Excel Note/Hint: Your objective function (total present value of costs) can be calculated as follows in excel:

$$O.Function\ =\ Onetime\ Building\ Cost\ +$$
$$PV(interest\ rate, planning\ horizon(10\ years), -\ sum(annual\ wage\ cost, annual\ calling\ cost))$$

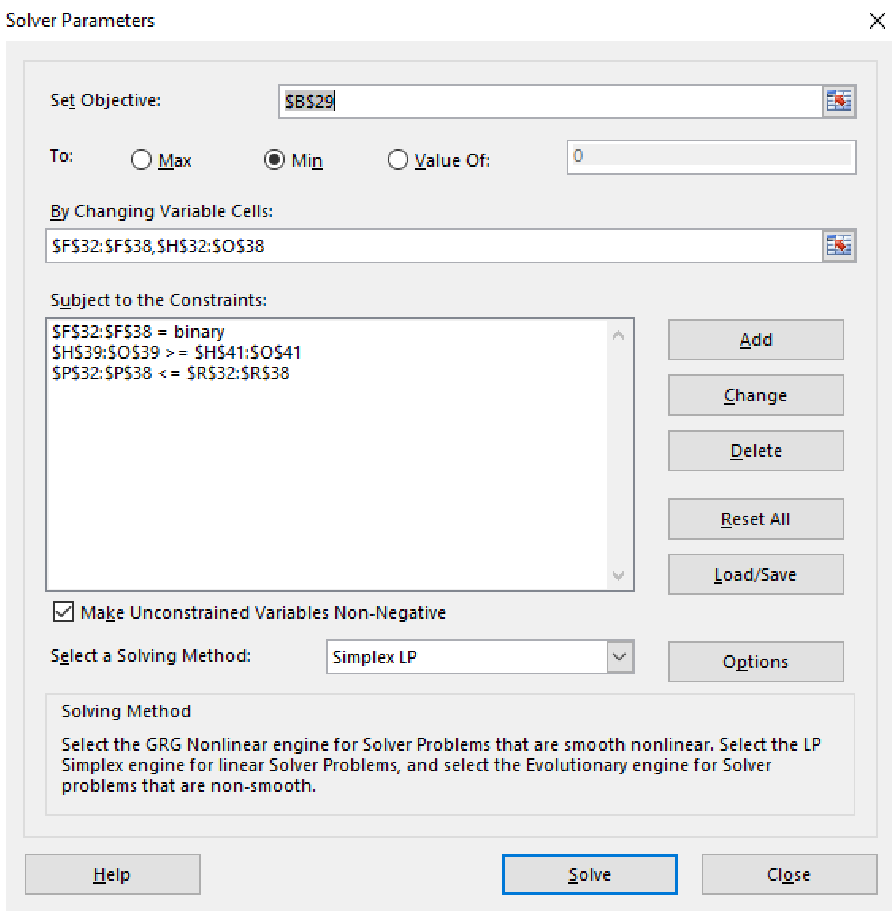
PV is an Excel Function!!

Solution:

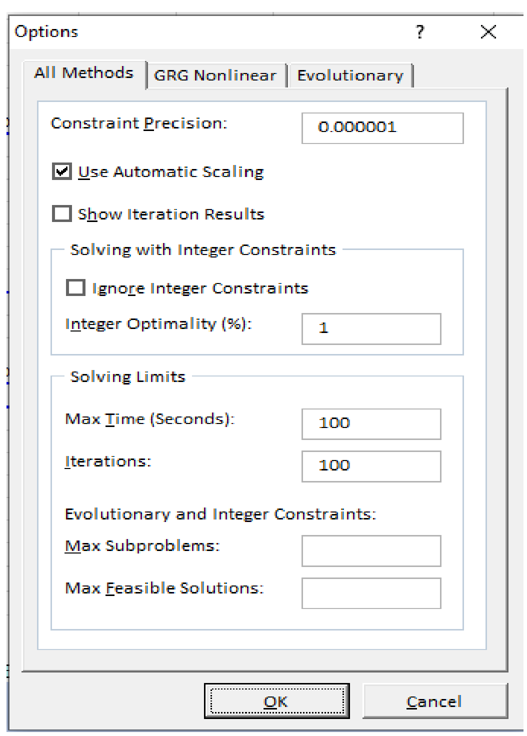
According to given details it is necessary to identify the optimal solution of where the call center is to be located to minimize the total calling cost.

In this exercise there are 56 + 7 = 63 decision variables.

Solver parameter:



Solver Options:



The optimized spreadsheet looks like below: (The excel file has been added to the main folder with detailed explanation of mathematics)

25	**Monetary summary**	
26	Annual wage cost	$3,616,667
27	Annual calling cost	$4,275,000
28	Building cost	$0
29	Total present value of costs	$48,490,875
30		

		call centers	New England	Middle Atlantic	Southeast	Southwest	Great Lakes	Plains	Rocky Mountains	Pacific	Total calls		Logical bound
32	0	Boston	1000	0	0	0	0	0	0	0	1000	<=	5000
33	0	New York	0	2000	0	0	0	0	0	0	2000	<=	5000
34	0	Charlotte	0	0	2000	0	0	0	0	0	2000	<=	5000
35	0	Dallas	0	0	0	2000	0	0	0	0	2000	<=	5000
36	0	Chicago	0	0	0	0	3000	0	0	0	3000	<=	5000
37	0	LA	0	0	0	0	0	0	0	2000	2000	<=	5000
38	0	Omaha	0	0	0	0	0	1000	2000	2000	5000	<=	5000
39		Made to region	1000	2000	2000	2000	3000	1000	2000	4000			
40			>=	>=	>=	>=	>=	>=	>=	>=			
41		Required	1000	2000	2000	2000	3000	1000	2000	4000			
42													

The answer report has been attached with the excel sheet for reference.