Past problems #1

Jimmy
Marijó
Laika

Departamental otoño 2022

1.- $|-2x^2+3x+2| \geq -x+2$

· factorizamos

$$|\underbrace{(x-2)}_{A}\underbrace{(2x+1)}_{B}| \geq -x+2 \Rightarrow$$

· cuatro casos

1.- $A^+ B^+$
2.- $A^- B^+$
3.- $A^+ B^-$
4.- $A^- B^-$

$\Rightarrow$

caso #1:

$(x-2)(2x+1) \geq -x+2$

lo mismo con diferente signo

$$\Rightarrow \quad -(2x+1) \geq 1$$
$$2x+1 \leq -1$$
$$2x \leq -2$$
$$x \leq -1$$

caso #2:

$$\cancel{(-x+2)}(2x+1) \geq \cancel{-x+2}$$
$$2x+1 \geq 1$$
$$2x \geq 0$$
$$x \geq 0$$

$\Rightarrow$

caso #3:

$$(x-2)(-2x-1) \geq -x+2$$
$$-(-2x-1) \geq 1$$
$$2x+1 \geq 1$$

igual que #2

$\Rightarrow$

caso #4:

$$(-x+2)(-2x-1) \leq -x+2$$
$$-2x-1 \leq 1$$

igual a #1

$$\Rightarrow \quad \therefore x \in (-\infty,-1] \cup [0,\infty)$$

2.- $g(x) = 3f(-x^2) - f(3-4x^2)$ si $D_f: x \in [-1,0]$

podemos ver que $g(x) = (h \circ f)(x)$ donde $h(x) = 3x - x$, sabemos que $h(x)$ es un polinomio $\therefore D_h: x \in \mathbb{R}$, así que no nos preocupamos por $h(x)$, necesitamos que lo que esté adentro de $f(x)$ esté entre $[-1,0]$, esto implica

$$-1 \leq -x^2 \leq 0 \quad y \quad -1 \leq 3-4x^2 \leq 0$$

$$-1 \leq -x^2 \leq 0$$
$$0 \leq x^2 \leq 1$$
$$\pm 0 \leq x \leq \pm 1$$

$$0 \leq x \leq 1 \quad y \quad -1 \leq x \leq 0$$

$$-4 \leq -4x^2 \leq -3$$
$$3 \leq 4x^2 \leq 4$$
$$3/4 \leq x^2 \leq 1$$
$$\frac{\pm\sqrt{3}}{\pm 2} \leq x \leq \pm 1$$
$$\frac{\sqrt{3}}{2} \leq x \leq 1 \quad y \quad -1 \leq x \leq -\frac{\sqrt{3}}{2}$$

$$\therefore D_g : x \in \left[-1, -\frac{\sqrt{3}}{2}\right] \cup \left[\frac{\sqrt{3}}{2}, 1\right]$$

3.- $$\lim_{x \to 0} \frac{x^2 - 5x\sin(3x)}{x^2}$$

$$\lim_{x\to 0}\left(\frac{\cancel{x^2}}{\cancel{x^2}} - \frac{5\cancel{x}\sin(3x)}{x^{\cancel{2}}}\right) = \lim_{x\to 0} 1 - \lim_{x\to 0}\frac{5\sin(3x)}{x} = 1 - 5\lim_{x\to 0}\frac{\sin(3x)}{x}\cdot\frac{3}{3} = 1-15\lim_{x\to 0}\frac{\sin(3x)}{3x} = 1-15=-14$$

4.- $\left| \frac{1}{|x|} - 2 \right| < 1$

$g(|x|) = \frac{1}{|x|} - 2 \rightarrow |g(|x|)| < 1 \Rightarrow$ 4 casos

- $g(x)$
- $-g(x)$
- $g(-x)$
- $-g(-x)$

1) $\frac{1}{x} - 2 < 1$

$\frac{1}{x} < 3$

$x > \frac{1}{3}$

2) $2 - \frac{1}{x} < 1$

$-\frac{1}{x} < -1$

$\frac{1}{x} > 1$

$x < 1$

3) $\frac{1}{-x} - 2 < 1$

$-\left(\frac{1}{x} + 2\right) < 1$

$\frac{1}{x} + 2 > -1$

$\frac{1}{x} > -3$

$x < -\frac{1}{3}$

4) $2 - \frac{1}{-x} < 1$

$2 + \frac{1}{x} < 1$

$\frac{1}{x} < -1$

$\frac{1}{x} < -1$

$x > -1$

$\therefore \; x \in \left(-1, -\frac{1}{3}\right) \cup \left(\frac{1}{3}, 1\right)$

5. $D(g \circ f) \rightarrow f(x) = \frac{1}{\sqrt{(3-x)x}}$ y $D_g = \left(\frac{1}{\sqrt{2}}, \infty\right)$

1ra cosa: $x \neq 0$ $\quad$ $3-x \neq 0$ $\quad$ $(3-x)x > 0$

$x \neq 3$ $\quad$ $\therefore \; x > 0$ y $x < 3$ $\quad$ $\therefore$ $D_{g \circ f}$ es $\left(\frac{1}{\sqrt{2}}, 3\right)$

6. $\lim_{x \to \frac{\pi}{4}} \dfrac{\operatorname{sen} x - \cos x}{1 - \tan x}$

$$\lim_{x \to \frac{\pi}{4}} \frac{\operatorname{sen} x - \cos x}{1 - \frac{\operatorname{sen} x}{\cos x}} = \lim_{x \to \frac{\pi}{4}} \frac{\operatorname{sen} x - \cos x}{\frac{\cos x - \operatorname{sen} x}{\cos x}} = \lim_{x \to \frac{\pi}{4}} \frac{\frac{\operatorname{sen} x - \cos x}{1}}{\frac{\cos x - \operatorname{sen} x}{\cos x}}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos x (\operatorname{sen} x - \cos x)}{\cos x - \operatorname{sen} x} = \lim_{x \to \frac{\pi}{4}} \frac{\cos x \cancel{(\operatorname{sen} x - \cos x)}}{-\cancel{(\operatorname{sen} x - \cos x)}} = \lim_{x \to \frac{\pi}{4}} -\cos(x)$$

$$= -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

7. $\lim_{x \to -1^+} \dfrac{|x^2 - 1|}{-1 + \sqrt{-x}}$

$$\lim_{x \to -1^+} \frac{1 - x^2}{-1 + \sqrt{-x}} \left(\frac{\sqrt{-x} + 1}{\sqrt{-x} + 1}\right) = \lim_{x \to -1^+} \frac{(1 - x^2)(\sqrt{-x} + 1)}{-x - 1} =$$

$$\lim_{x \to -1^+} \frac{\cancel{(1 + x)}(1 - x)(\sqrt{-x} + 1)}{-\cancel{(1 + x)}} = \lim_{x \to -1^+} (1 - x)(\sqrt{-x} + 1) = (1 + 1)(\sqrt{1} + 1) = 4$$

8.- $\lim_{x\to\infty} \left(x + \sqrt{x^2 - 7x + 1}\right)$

$$\lim_{x\to\infty} \left(x + \sqrt{x^2 - 7x + 1}\right)\left(\frac{x - \sqrt{x^2 - 7x + 1}}{x - \sqrt{x^2 - 7x + 1}}\right) = \lim_{x\to\infty} \frac{\cancel{x^2} - \cancel{x^2} + 7x - 1}{x - \sqrt{x^2 - 7x + 1}}$$

$$= \lim_{x\to\infty} \frac{7x - 1}{x - \sqrt{x^2 - 7x + 1}} = \lim_{x\to\infty} \frac{7 - \frac{1}{x}}{1 - \sqrt{1 - \frac{7}{x} + \frac{1}{x^2}}} = \frac{7}{1 - \sqrt{1}} = \frac{7}{0} = \infty$$

$$\bar{o} \quad \underbrace{\lim_{x\to\infty} x}_{\infty} + \underbrace{\lim_{x\to\infty} \sqrt{x^2 - 7x + 1}}_{\infty}$$