Lecture-02

Image Processing

standard image processing operators

- point operators
  - manipulate each pixel independently of its neighbors
- neighborhood (area-based) operators
  - new pixel's value depends on a small number of neighboring input values

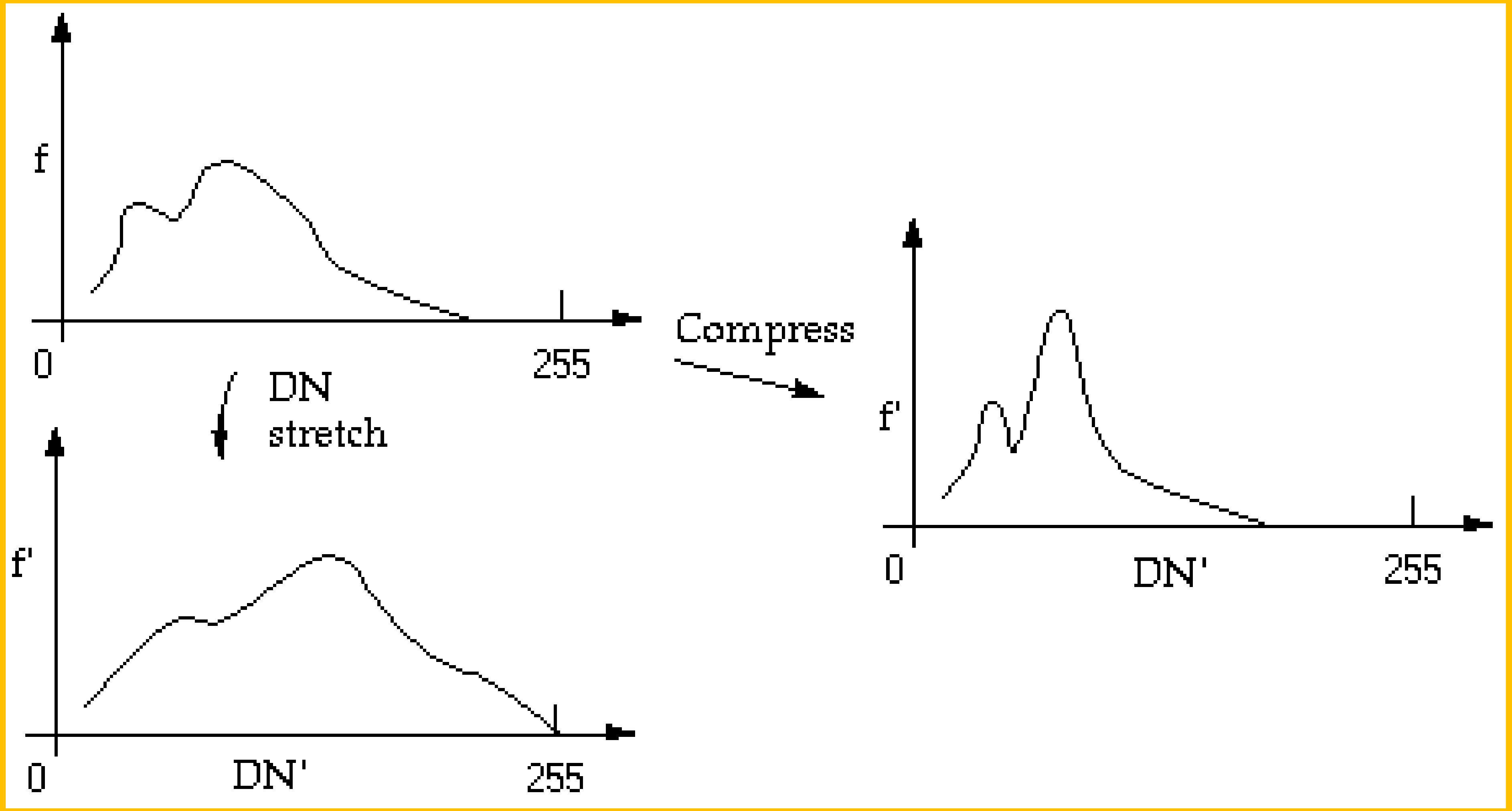
Point Operators-Histogram equalization

- Image Histogram
  - a histogram is the estimation of the probability distribution of a particular type of data.
  - An image histogram is a type of histogram which offers a graphical representation of the tonal distribution of the gray values in a digital image.

Histogram Equalization

- The purpose of a histogram-based operation is that
  - when a grey-level transformation is made, pixels in the image having a specific range of grey levels can be enhanced or suppressed.
  - This is also called contrast adjustment.
- It can be done using:
  - histogram stretching
  - histogram compression

Histogram Equalization



Linear Histogram Equalization

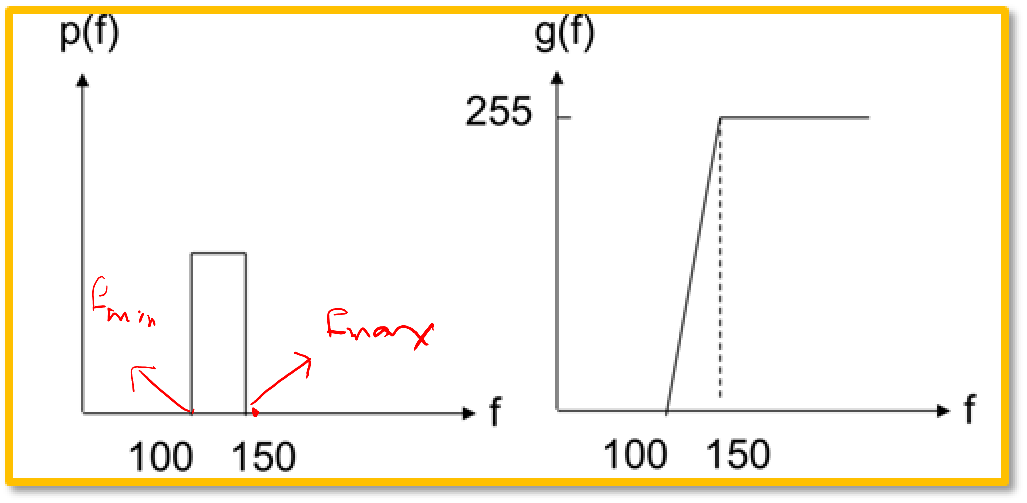
$P_1 = (f_{min}, 0)$

$P_2 = (f_{max}, 255)$

$$\frac{g - g_1}{f - f_1} = \frac{g_2 - g_1}{f_2 - f_1} = m$$

$$m = \frac{255 - 0}{f_{max} - f_{min}}$$

$$\frac{g - 0}{f - f_{min}} = m \Rightarrow g = m \cdot (f - f_{min})$$



Linear Histogram Equalization

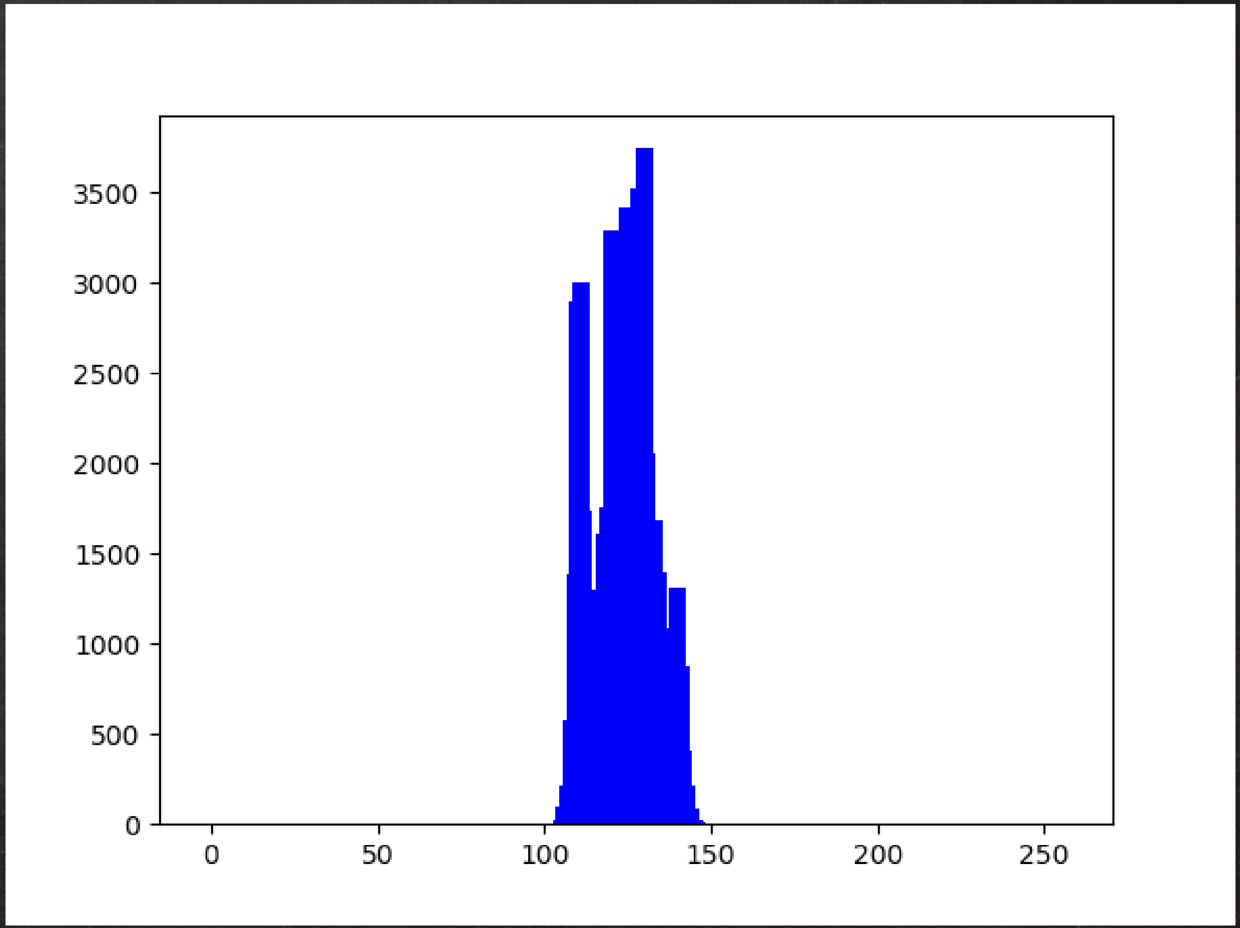
- Let the pixel value $f(x, y) = r$

h(r): number of pixels that has the value of r

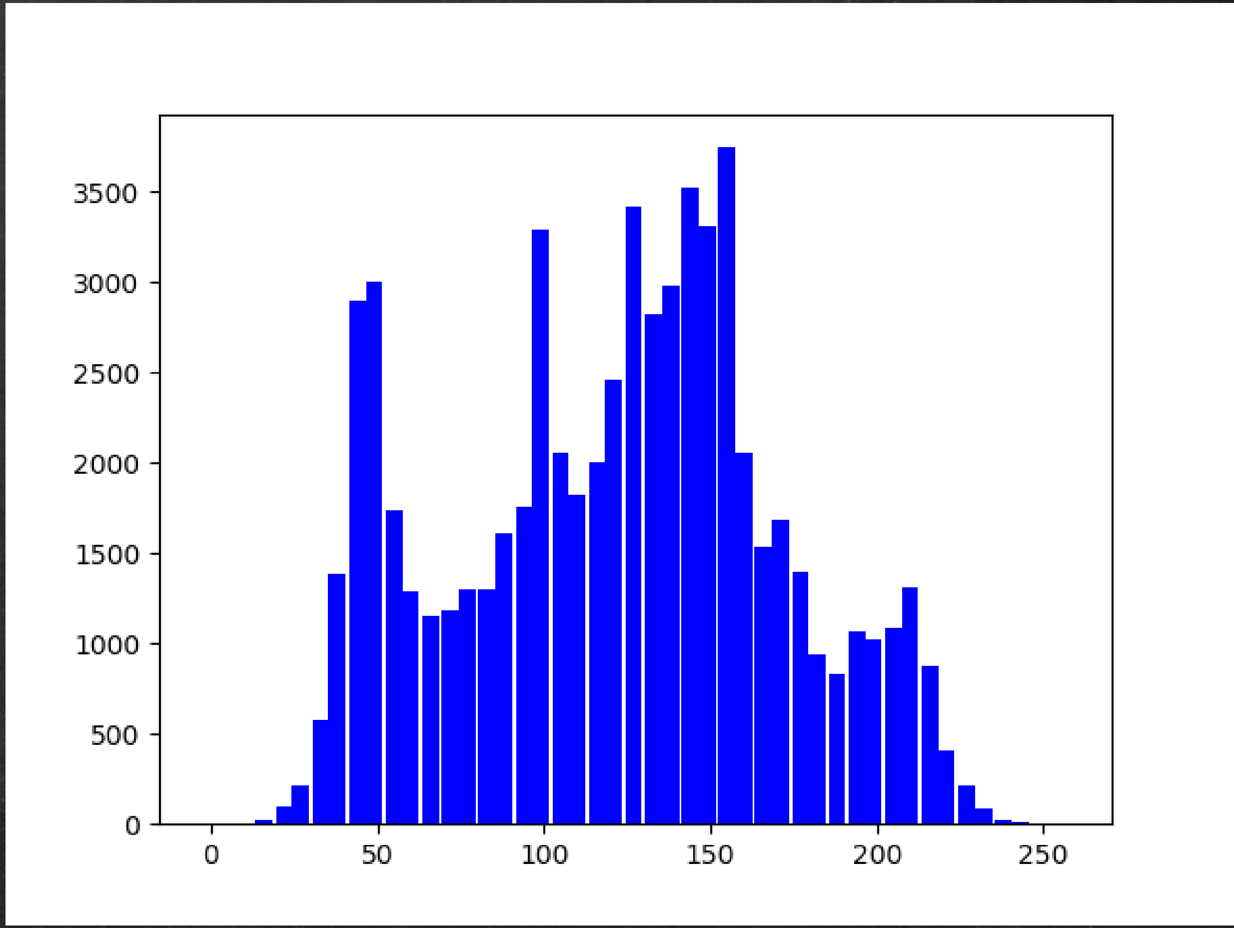
$$m = \frac{255}{(h_{max} - h_{min})}$$

$$h_e(r) = \begin{cases} m \times (h(r) - h_{min}) & h_{min} < h(r) \le h_{max} \\ 255 & h(r) > h_{max} \end{cases}$$

Linear Histogram Equalization



Linear Histogram Equalization



Cumulative Histogram Equalization

- The intensity values in an image can be regarded as random variables that can have any value between 0 and L-1. 255
- This random event has a so called cumulative distribution function (CDF) associated to itself.
- This function describes the likelihood that the random variable will be assigned a value less or equal to a specific value.

Cumulative Histogram Equalization

$$cdf(i \leq t) = \sum_{k=1}^{t} p_k$$

describes the CDF for the probability that a pixel has the intensity equal or lower than t.

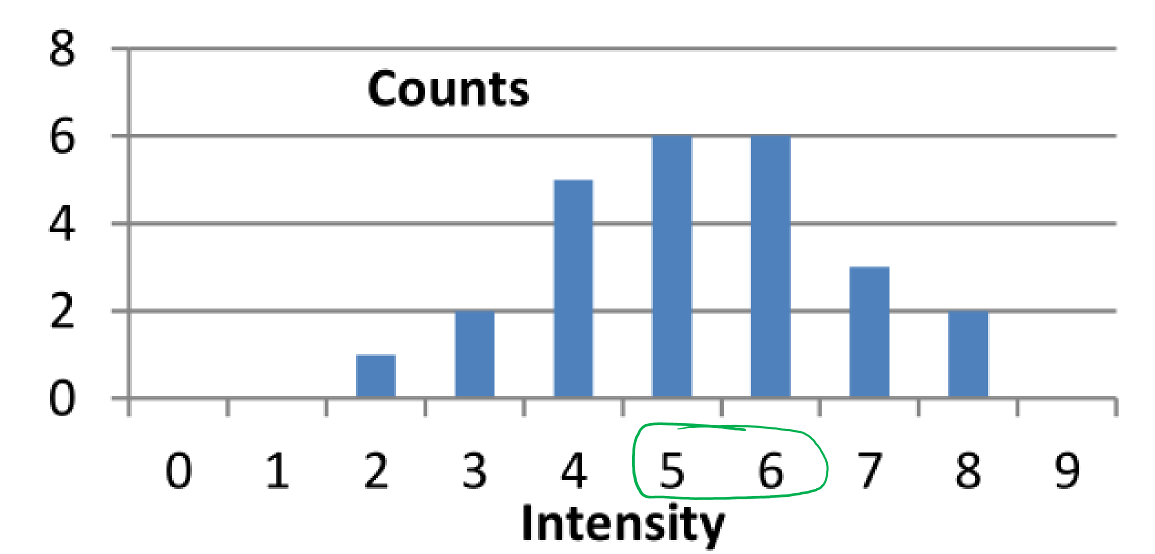
Cumulative Histogram Equalization

- The probability that a random pixel has the intensity value k is the number of pixels with the intensity k divided by the total number of pixels, as shown in the following equation.

$$p_k = \frac{amount\ of\ pixels\ with\ intensity\ k}{total\ number\ of\ pixels}$$

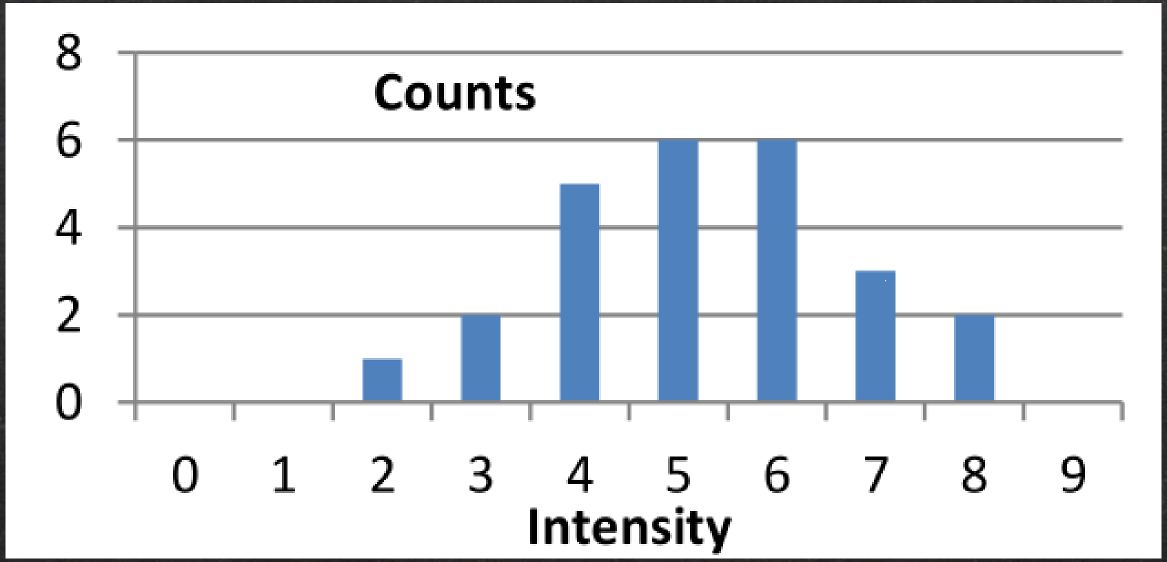
Cumulative Histogram Equalization

6	2	3	4	5
5	5	6	7	5
5	6	4	8	6
4	7	4	3	4
8	5	6	6	7



Cumulative Histogram Equalization

- The goal is to create some transformation s=T(i) that creates a new image with a histogram more resembling



Cumulative Histogram Equalization

6	2	3	4	5
5	5	6	7	5
5	6	4	8	6
4	7	4	3	4
8	5	6	6	7

Intensity	Frequency
0	0
1	0
2	1
3	2
4	5
5	6
6	6
7	3
8	2
9	0

0 — 255 we will map pro.

$P_0 = \frac{0}{25} = 0 \quad P_1 = \frac{0}{25} = 0 \quad P_2 = \frac{1}{25} = 0.04 \quad P_3 = \frac{2}{25} = 0.08 \quad P_4 = \frac{5}{25} = 0.2$

$P_5 = \frac{6}{25} = 0.24 \quad P_6 = \frac{6}{25} = 0.24 \quad P_7 = \frac{3}{25} = 0.12 \quad P_8 = \frac{2}{25} = 0.08 \quad P_9 = \frac{0}{25} = 0$

$$s = T(i) = floor((L-1) * \sum_{k=0}^{i} p_k) = floor(9 * \sum_{k=0}^{i} p_k)$$

Cumulative Histogram Equalization

Intensity	Frequency
0	0
1	0
2	1
3	2
4	5
5	6
6	6
7	3
8	2
9	0

i	T(i)
0	0
1	0
2	0
3	1
4	2
5	5
6	7
7	8
8	9
9	9

P(g) is small

P(i ≤ g)

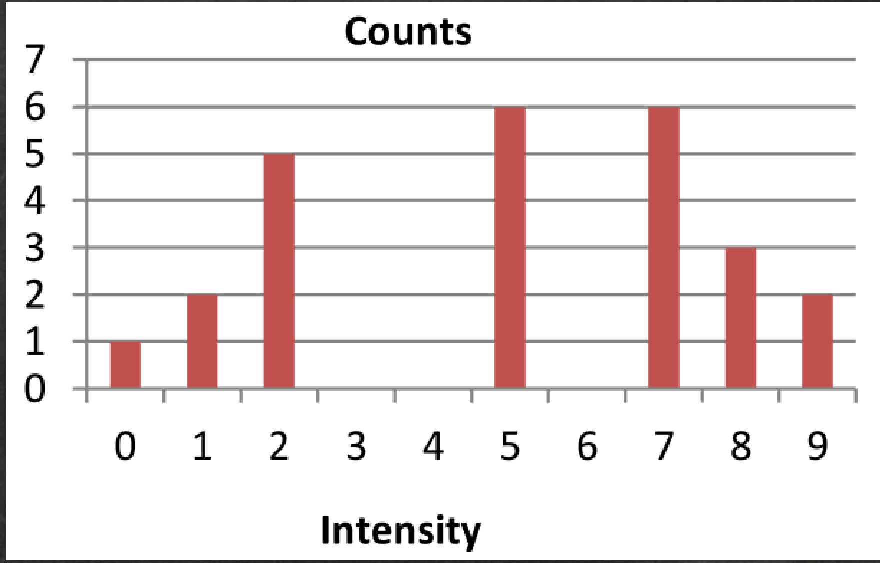
$$P_0 = \frac{0}{25} = 0 \quad P_1 = \frac{0}{25} = 0 \quad P_2 = \frac{1}{25} = 0.04 \quad P_3 = \frac{2}{25} = 0.08 \quad P_4 = \frac{5}{25} = 0.2$$

$$P_5 = \frac{6}{25} = 0.24 \quad P_6 = \frac{6}{25} = 0.24 \quad P_7 = \frac{3}{25} = 0.12 \quad P_8 = \frac{2}{25} = 0.08 \quad P_9 = \frac{0}{25} = 0$$

$$s = T(i) = floor((L-1) * \sum_{k=0}^{i} p_k) = floor(9 * \sum_{k=0}^{i} p_k)$$

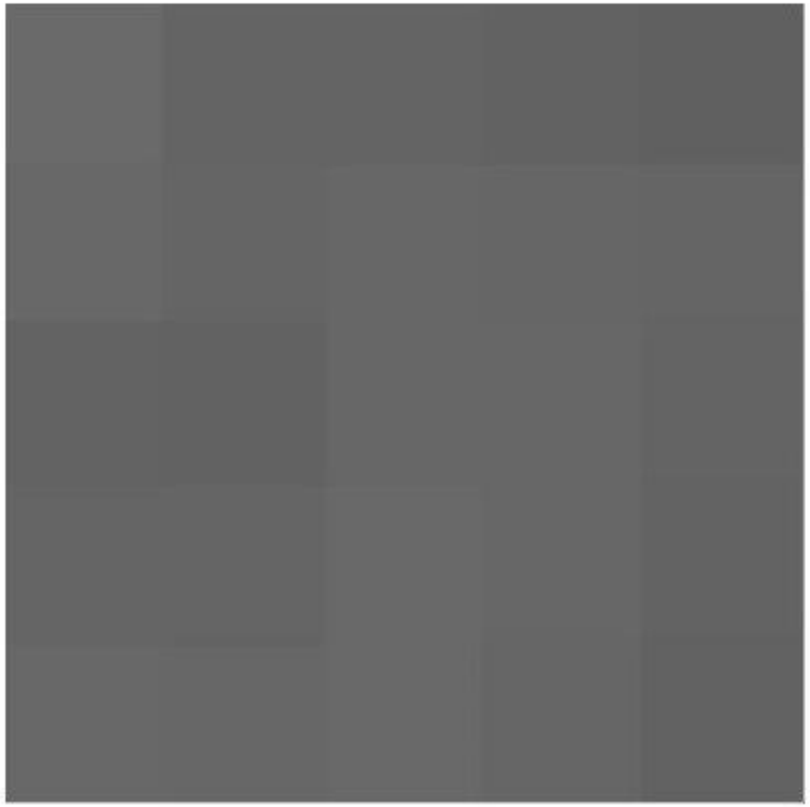
Cumulative Histogram Equalization

7	0	1	2	5
5	5	7	8	5
5	7	2	9	7
2	8	2	1	2
9	5	7	7	8



Convolution

105	102	100	97	96
103	99	103	101	102
101	98	104	102	100
99	101	106	104	99
104	104	104	100	98

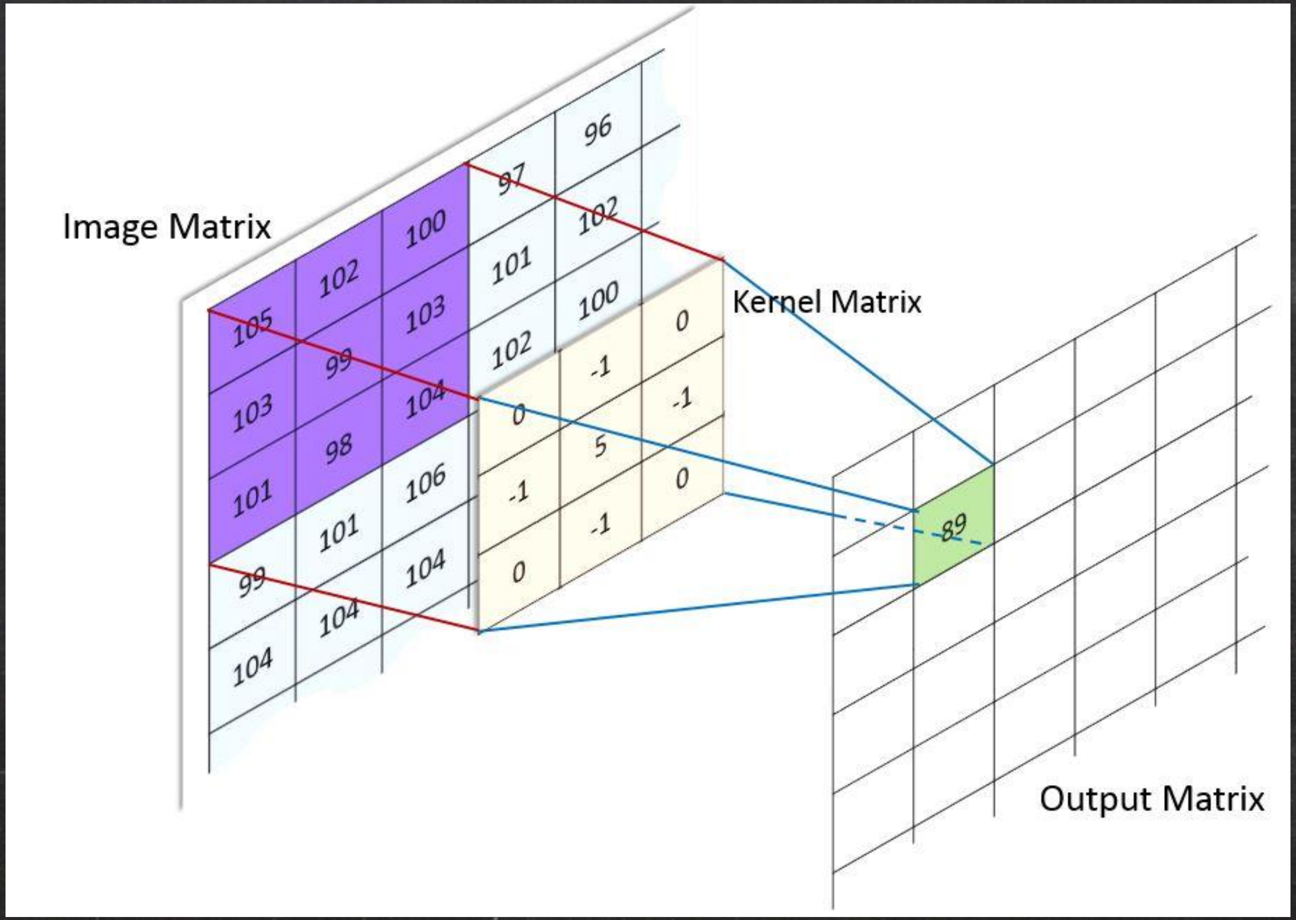


Convolution

- Let's start with the sharpening kernel which is defined as:

$$k = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Convolution



Convolution

0	0	0	0	0	0
0	105	102	100	97	96
0	103	99	103	101	102
0	101	98	104	102	100
0	99	101	106	104	99
0	104	104	104	100	98

Image Matrix

Kernel Matrix

0	-1	0
-1	5	-1
0	-1	0

210	89	111		

Output Matrix

$$\begin{aligned}&0*0+105*-1+102*0\\&+0*-1+103*5+99*-1\\&+0*0+101*-1+98*0 = 210\end{aligned}$$

linear operation

Convolution

- Given a filter kernel H, the convolution of the kernel with image F is an image R. The i, j'th component of R are given by:

$$R_{ij} = \sum_{u,v} H_{i-u,j-v} R_{u,v}$$

Smoothing by Averaging

$$\text{Kernel} = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 1 \\ 6 & 1 & 5 \\ 1 & 4 & 4 \end{bmatrix} * \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 3$$

Convolution