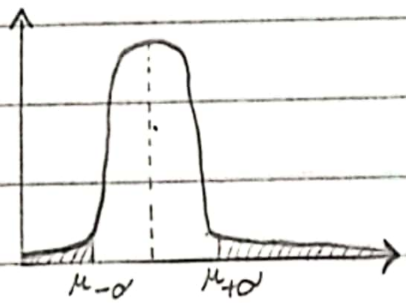
* Probability Distribution:-

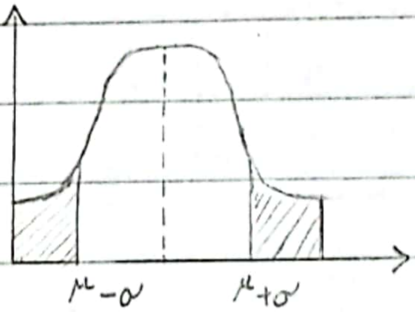
→ Distribution : تعرض القيم المحتملة التي يمكن أن يأخذها المتغير ومدى تكرار حدوثها

← الـ Probabilities تقيس احتمالية الـ outcome

→ Probability Frequency Distribution: تسجل تكرار (Fr) كل قيمة وتقسم على مجموع العناصر عند ← (Finite number of possible outcomes)

population data → "all" the data	Sample data → just a part of it
mean → μ , variance → σ^2	sample mean → $\bar{x}$
Standard Deviation(STD) → σ	sample variance → s^2
	sample STD → s





كلما زاد ازدحام منتصف الـ distribution كلما زادت البيانات التي تقع ضمن تلك الفواصل الزمنية	كلما قلت البيانات في الفواصل الزمنية كلما زاد تشتت البيانات

← توجد علاقة تابعة بين الـ mean والـ variance ⇐ $\sigma^2 = E((Y-\mu)^2) = E(Y^2) - \mu^2$ ←

* Types of Probability Distribution :-

- Finite number of outcomes ⇒ Discrete distributions

مثال حجر نرد (Die) ، رمي عملة نقدية (Heads or tails)؟

- Infinitely many outcomes ⇒ Continous distribution

مثال وقت ، مسافة؟

→ Notation:-

$$X \sim N(\mu, \sigma^2)$$

تتغير على حسب نوع ال distribution

Variable Tilda Type Characteristics

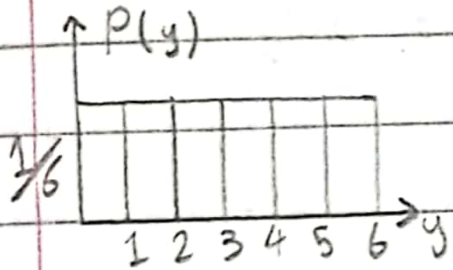
* Discrete Uniform Distribution :-

→ $U(a,b) \Longrightarrow X \sim U(3,7)$

Uniform ← Type, Characteristics

→ ALL outcomes have equal probability

مثال عند رمي حجر نرد ← $P(1)=P(2)=P(3)=P(4)=P(5)=P(6)$



← لا يقدم أي معلومات ذات صلة لأن كل قيم ال outcomes متساوية

$\mu = 3.5$, $\sigma^2 = \frac{105}{36}$

← ال mean وال variance ← غير قابلين للتفسير

← لا يوجد intuition لما يقصدونه

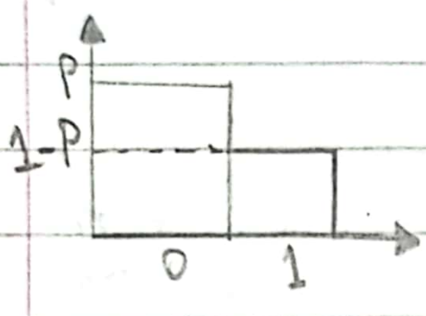
* **Bernoulli Distribution :-**

$X \sim Bern(p)$

↑ Type ↑ probability of success

Quiz
☐ True ☐ False

- 1 Trial , 2 Possible outcomes



← نحدد من الاحتمالات من يكون الـ 0 ومن الـ 1
وعليها يكون الـ $E(X)=p$ أو $E(X)=1-p$

← $p>1-p$ ، $p \leftarrow 1$ ، $1-p \leftarrow 0$

→ $E(x) = 1.p + 0.(1-p) = p \Rightarrow$ احتمالية الحدث المراد

→ $\sigma^2 = p(1-p)$

مثال Unfair coin $\Rightarrow P(T) = 0.6$, $T \rightarrow 1$, $H \rightarrow 0 \Rightarrow E(x) = p = 0.6$

$\sigma^2 = p(1-p) = 0.6 * 0.4 = 0.24$

* **Binomial Distribution:-** A sequence of identical Bernoulli events

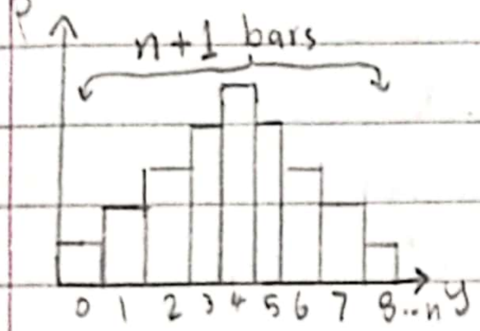
$$B(n,p) \Longrightarrow X \sim B(10, 0.6)$$

type / no. of trials / probability of success in each one

← إذا كان $n=1$ تكون Bernoulli

→ E(Bern) → outcome probability for a single trial

→ E(B) → The number of times we expect to get a specific outcome



→ P(desired outcome) = p

→ P(alternative outcome) = $1-p$

→ $\binom{n}{y} = {}^{n}C_{y}$

مثال للحصول على 2 Tails في 3 Trials ⇐ ${}^{3}C_{2} = 3$

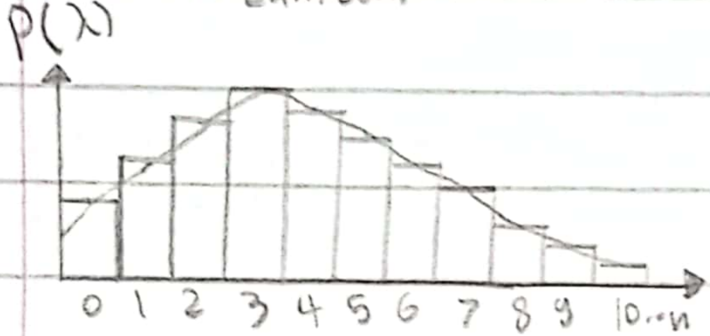
→ $P(y) = {}^{n}C_{y} \cdot p^{y} \cdot (1-p)^{n-y}$

→ $\sigma^2 = E(y^2) - E(y)^2 = n \cdot p \cdot (1-p)$

* **Poisson Distribution** :- The frequency with which an event occurs

$Po(\lambda) \Longrightarrow Y \sim Po(4)$

↑
Lambda



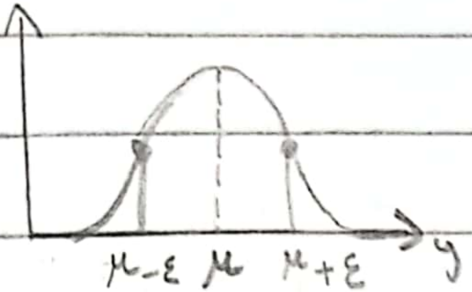
→ $P(y) = \frac{\lambda^{y} e^{-\lambda}}{y!}$

→ $E(y) = y_0 \frac{\lambda^{y_0} e^{-\lambda}}{y_0!} + y_1 \frac{\lambda^{y_1} e^{-\lambda}}{y_1!} + \_\_\_\_ = \lambda$

→ $\mu = \sigma^2 = \lambda$

* Normal Distribution:-

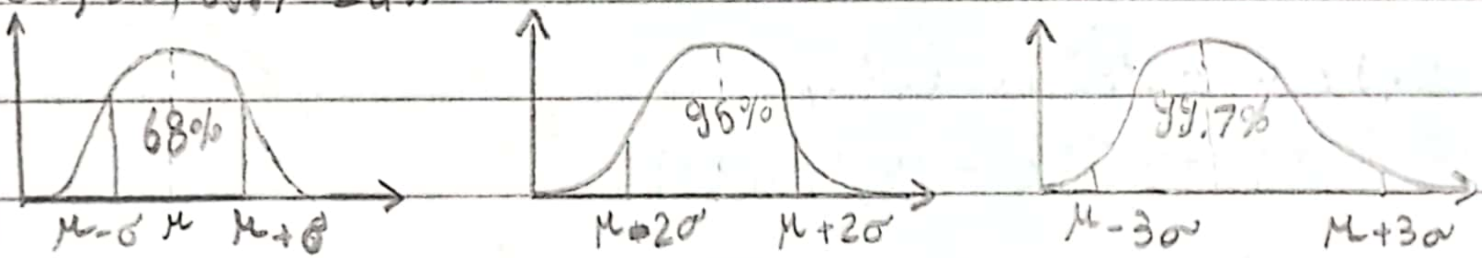
$$X \sim N(\mu, \sigma^2)$$



$$E(x) = \mu$$

$$Var(x) = \sigma^2 = E(x^2) - E(x)^2$$

* 68, 95, 99.7 Law



• distribution: توزيع

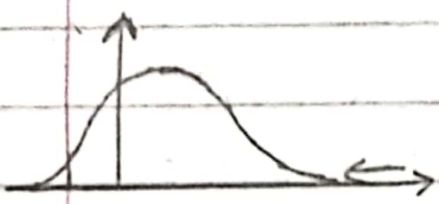
* Standard Normal Distribution:-

→ Transformation:- طريقة لتغيير كل عنصر في الdistribution للحصول على distribution جديد بصفات مشابهة

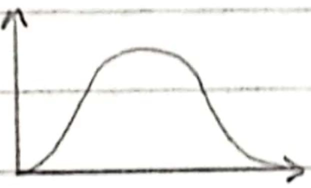
← للdistribution Normal يمكننا استخدام $(+ - \times \div)$ بدون تغيير نوع الdistribution

← ماذا يحدث للgraph عند استخدام:

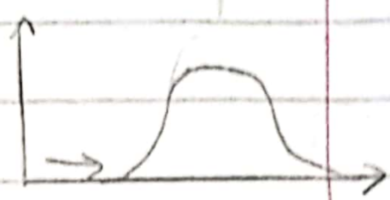
Addition and Subtraction



$y=f(x-3)$

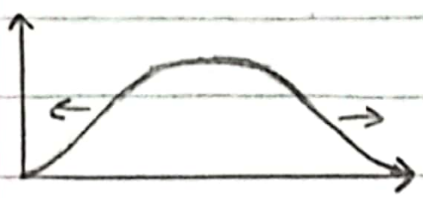


$y=f(x)$

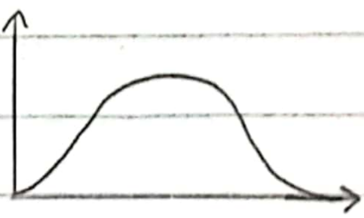


$y=f(x+3)$

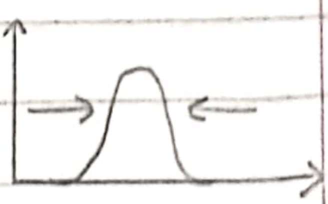
Multiplication and Division



$y=f(\frac{x}{c})$



$y=f(x)$



$y=f(x.c)$

if $1>c>0 \Rightarrow \frac{x}{1/2} = x.2 =$

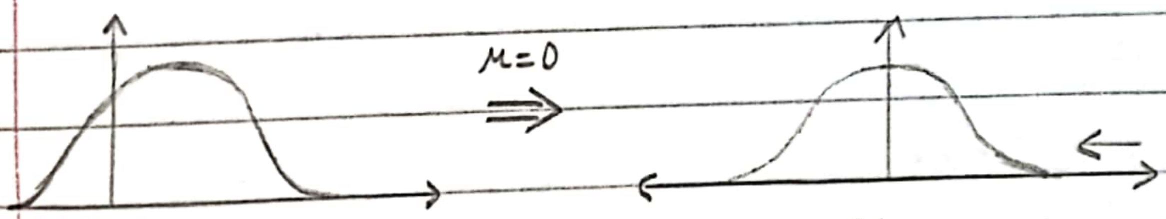
→ Standardizing :- نوع خاص من الـ transfomation وبه نجعل القيمة المرتقبة (expected value) تساوي صفر والـ variance يساوي 1

$E(x) = 0 \quad , \quad Var(x) = 1$

← الـ distribution الناتج من الـ Standarization لـ Normal distribution يسمى Standard Normal Distribution

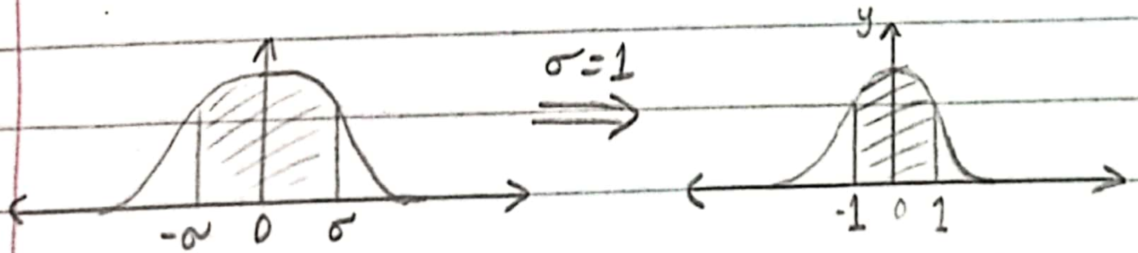
→ How to standarize?

أولاً نجعل منتصف (mean) الـ graph يساوي صفر



$y = f(x)$ $\qquad\qquad$ $y = f(x-\mu)$

ثانياً نتأكد أن STD (σ) يساوي 1 عن طريق قسمة كل قيمة على σ



$y = f(x-\mu)$ $\qquad\qquad$ $y = f\left(\frac{x-\mu}{\sigma}\right)$

$$\left.\begin{array}{l} Z \sim N(0,1) \\ Y \sim N(\mu, \sigma^2) \end{array}\right\} \Rightarrow Z = \frac{Y-\mu}{\sigma}$$ معادلة الـStandarization

← تنفيذ المعادلة على أي Normal Distribution يحولها إلى Normal Standard Distribution

$$\rightarrow Z = \frac{Y-\mu}{\sigma} \Rightarrow Y \rightarrow Z$$

$$\rightarrow Y = \mu + 2.3\sigma \Rightarrow z = 2.3$$

← تحتاج كمية بيانات كبيرة

→ Risk of outliers drastically affecting our analysis

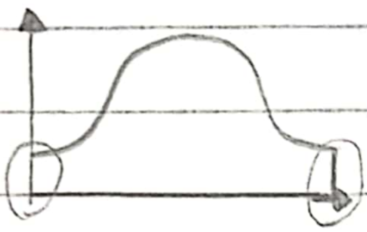
* **Student's T Distribution:-** A small sample size approximation of a Normal Distribution

$t(K) \Rightarrow Y \sim t(3)$

$\uparrow$ Degrees of freedom

→ Certain characteristics + Sufficient data = Normal distribution

→ Certain characteristics + ~~sufficient data~~ = Student's T distribution



→ if $K > 2 \Rightarrow E(Y) = \mu$, $Var(Y) = \dfrac{S^2 \cdot K}{K-2}$

* Application:-

① Frequently used when conducting stastical analysis

② Hypothesis testing with Limited data

③ CDF table (T-table)

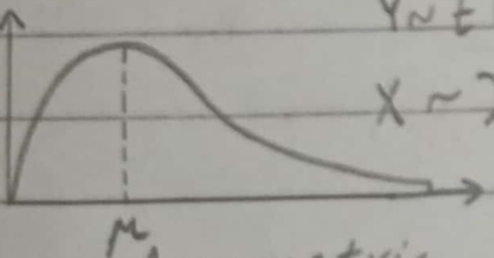
* Chi-Squared Distribution:-

$$\chi^2(K) \Rightarrow Y \sim \chi^2(3)$$

* Application:-
  1. Few events in real Life
  2. Statistical analysis → Hypothesis testing
     → Computing confidence intervals
  3. Goodness of Fit

$Y \sim t(K) \Rightarrow Y^2 \sim \chi^2(K)$

$X \sim \chi^2(K) \Rightarrow \sqrt{X} \sim t(K)$



Asymmetric

→ A table of Known values: N, T

→ $E(X) = K$

→ $Var(X) = 2K$

* **Conditional Probability:-** احتمال حدوث B بالنسبة لـ A

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

مثال

$s = \{1,2,3,4,5,6\}$, $A = \{1,3,5\}$, $B = \{1,2\}$

$P(A \cap B) = \frac{1}{6}$, $P(A) = \frac{1}{2}$

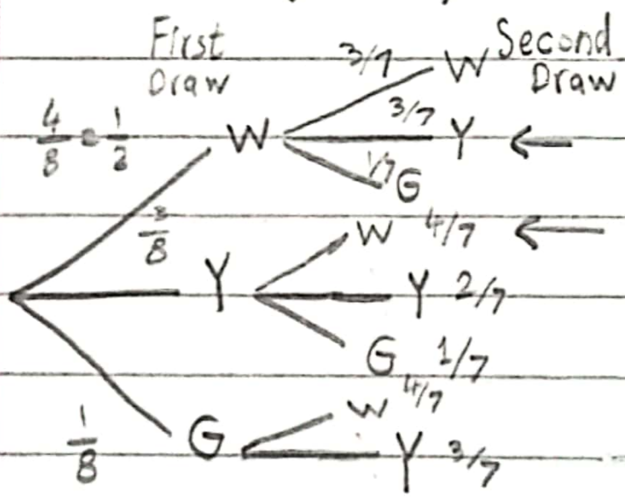
$P(B/A) = \frac{1}{6} * 2 = \frac{1}{3}$

→ **product Rule:-**

$$P(A \cap B) = P(A) \cdot P(B/A)$$

→ **Tree Diagram:-**

$W = 4$, $Y = 3$, $G = 1$ — مثال احتمال سحب W و Y



$$P(w \cap y) = P(Y_{1st}) \cdot P(W_{2nd}/y_{1st}) + P(W_{1st}) \cdot P(y_{2nd}/W_{1st})$$

$$= \frac{3}{8} \times \frac{4}{7} + \frac{1}{2} \times \frac{3}{7}$$

$$= \frac{3}{7}$$

Independent Events:-

$$P(B/A) = P(B)$$

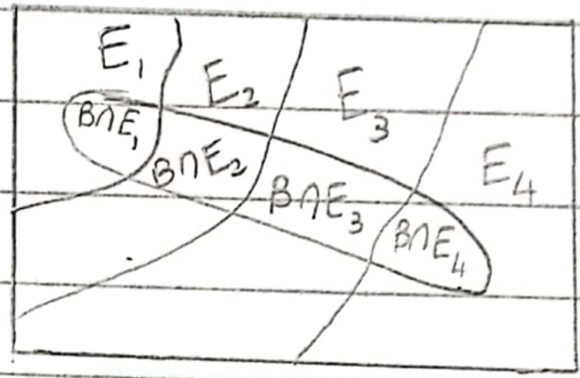
$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(E_1 \cap E_2 \cap E_3 \cap \dots \cap E_n) = P(E_1) \cdot P(E_2) \cdot P(E_3) \cdot \dots \cdot P(E_n)$$

Law of Total Probability:-

$$P(B) = P(B \cap E_1) + P(B \cap E_2) + \dots + P(B \cap E_K)$$

$$= P(B/E_1) P(E_1) + P(B/E_2) P(E_2) + \dots + P(B/E_K) \cdot P(E_K)$$



$$B = (B \cap E_1) \cup (B \cap E_2) \cup (B \cap E_3) \cup (B \cap E_4)$$

* Bayes' Theorem:-

$$P(A/B) = \frac{P(B/A) * P(A)}{P(B)}$$

$$P(B/A) = \frac{P(A/B) * P(B)}{P(A)}$$

$$P(A_j/B) = \frac{P(A_j).P(E/A_j)}{P(E)} = \frac{P(A_j).P(E/A_j)}{P(A_1).P(E/A_1) + P(A_2).P(E/A_2) + \dots + P(A_n).P(E/A}$$