Linear Algebra Questions (Make sure to attend the Online session)

1. Given the matrices:

$$A = \begin{bmatrix} -1 & 23 & 10 \\ 0 & -2 & -11 \end{bmatrix}, \quad B = \begin{bmatrix} -6 & 2 & 10 \\ 3 & -3 & 4 \\ -5 & -11 & 9 \\ 1 & -1 & 9 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 2 & 9 & -5 & 7 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 6 \\ -5 & 2 \end{bmatrix}, \quad E = [3], \quad F = \begin{bmatrix} 3 \\ 5 \\ -11 \\ 7 \end{bmatrix}, \quad G = \begin{bmatrix} -6 & -4 & 23 \\ -4 & -3 & 4 \\ 23 & 4 & 1 \end{bmatrix}$$

a) What is the dimension of each matrix?

A -> 2x3, B -> 4x3, C -> 1x5, D -> 2x2, E -> 1x1, F -> 4x1, G -> 3x3

b) Which matrices are square? D, E, G

c) Which matrices are symmetric? Only G

d) Which matrix has the entry at row 3 and column 2 equal to -11? B

e) Which matrices has the entry at row 1 and column 3 equal to 10? B

f) Which are column matrices? F

g) Which are row matrices? C

h) Find AT,CT,ET,GT. (T -> Transpose)

(h) $A^T = \begin{bmatrix} -1 & 0 \\ 23 & -2 \\ 10 & -11 \end{bmatrix}$, $B^T \rightarrow \begin{bmatrix} -6 & 3 & -5 & 1 \\ 2 & -3 & -11 & -1 \\ 10 & 4 & 9 & 9 \end{bmatrix}$, $C^T = \begin{bmatrix} -3 \\ 2 \\ 9 \\ -5 \\ 7 \end{bmatrix}$, $E^T = [3]$, $G^T = \begin{bmatrix} -6 & -4 & 23 \\ -4 & -3 & 4 \\ 23 & 4 & 1 \end{bmatrix}$

2. A, B, C, D and E are matrices given by:

$$A=\begin{bmatrix}-1 & 1 & -2\\ 0 & -2 & 1\end{bmatrix},\quad B=\begin{bmatrix}-1 & 2 & 0\\ 0 & -3 & 4\\ -1 & -2 & 3\end{bmatrix},\quad C=[-3 \quad 2 \quad 9 \quad -5 \quad 7$$

$$D=\begin{bmatrix}-2 & 6\\ -5 & 2\end{bmatrix},\quad E=\begin{bmatrix}3\\ 5\\ -11\end{bmatrix},\quad F=\begin{bmatrix}-1 & 0 & 2\\ -2 & -3 & 4\\ 1 & 4 & -3\end{bmatrix}$$

Find if possible:

a) AB -> 2x3 . 3x3 Possible
b) BC -> 3x3 . 1x5 Not possible
c) AD -> 2x3 . 2x2 Not possible
d) EF -> 3x1 . 3x3 Not possible
e) FE -> 3x3 . 3x1 Not possible

3. Find the determinant of the matrix M :

$$M=\begin{pmatrix}15 & 10\\ 3 & 2\end{pmatrix} \qquad M=\begin{pmatrix}2 & 3 & 1\\ -1 & 2 & 3\\ 3 & 2 & -1\end{pmatrix}$$

$\boxed{3}$ $M=\begin{bmatrix}15 & 10\\ 3 & 2\end{bmatrix} \Rightarrow |M| = 15\times 2 - 10\times 3 = 0$

$M=\begin{bmatrix}2 & 3 & 1\\ -1 & 2 & 3\\ 3 & 2 & -1\end{bmatrix} \Rightarrow |M| = 2\begin{vmatrix}2 & 3\\ 2 & -1\end{vmatrix} - 3\begin{vmatrix}-1 & 3\\ 3 & -1\end{vmatrix} + 1\begin{vmatrix}-1 & 2\\ 3 & 2\end{vmatrix} = 0$

4. Find the inverse matrix A-1 to the matrix A :

$$A = \begin{pmatrix} -3 & -2 \\ 3 & 3 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\boxed{4} \quad A^{-1}_{2\times2} = \frac{1}{|A|}\begin{bmatrix} 3 & 2 \\ -3 & -3 \end{bmatrix} = \frac{1}{-3\times3-(-2)\times3}\begin{bmatrix} 3 & 2 \\ -3 & -3 \end{bmatrix} = \begin{bmatrix} -1 & -\frac{2}{3} \\ 1 & 1 \end{bmatrix}$$

$$A^{-1}_{3\times3} = \frac{1}{|A|} * \mathrm{adj}(A) \qquad |A| = -1$$

$$C = \begin{bmatrix} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\mathrm{adj}(A) = C^T = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

5. What does it mean if three equations are linearly independent? B

a. Two of the equations can be combined to come up with the third equation.
b. There is no way to combine any two equations to come up with the third equation.
c. The graphical representations of the equations are lines that do not intersect.
d. The graphical representations of the equations are lines that do intersect.

6. Let

$$f(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{x}^\top \boldsymbol{A}\boldsymbol{y} + \boldsymbol{x}^\top \boldsymbol{B}\boldsymbol{x} - \boldsymbol{C}\boldsymbol{y} + \boldsymbol{D}$$

with $\boldsymbol{x} \in \mathbb{R}^M, \boldsymbol{y} \in \mathbb{R}^N$, function $f : \mathbb{R}^M \times \mathbb{R}^N \to \mathbb{R}$.
Compute the dimensions of the matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ for the function so that the mathematical expression is valid.

$$f(x,y) = \underbrace{x^T A y}_{1st} + \underbrace{x^T B x}_{2nd} - \underbrace{C y}_{3rd} + \underbrace{D}_{4th}$$

$x : M \times 1$, $y : N \times 1$

1st term: $x^T A y = (1 \times M) . A . (N \times 1) \Longrightarrow A : M \times N$

2st term: $x^T B x = (1 \times M) . B . (M \times 1) \Longrightarrow B : M \times M$

3rd term: $Cy = C . (N \times 1) \Longrightarrow C : 1 \times N$

4th term: $D \Longrightarrow D : 1 \times 1$

$A : M \times N$, $B : M \times M$, $C : 1 \times N$, $D : 1 \times 1$