## **Quiz Question**

Which of the following is not accurate about entropy? B

- a. The more knowledge one has, the less entropy
- b. The less knowledge one has, the less entropy
- c. The less knowledge one has, the more entropy

## **Quiz Question**

Which function will help us turn products into sums? C

- a. sin
- b. cos
- c. log
- d. exp

We can see that this is a large number since this set has a lot of entropy. In the more general case with m red balls and n blue balls, this is the formula.

$$Entropy = -\frac{m}{m+n}log_2(\frac{m}{m+n}) - \frac{n}{m+n}log_2(\frac{n}{m+n})$$

This is the general formula for entropy when the balls can be of two colors.

What is the entropy for a bucket with a ratio of four red balls to ten blue balls? Input your answer to at least three decimal places.

Enter your response here

Answer: Entropy =  $-(4/14)*\log 2(4/4+10/14) - (10/14)*\log 2(10/14)$ = 0.12455985407487205

## Multi-class Entropy

Last time, you saw this equation for entropy for a bucket with m red balls and n blue balls:

$$Entropy = -\frac{m}{m+n}log_2(\frac{m}{m+n}) - \frac{n}{m+n}log_2(\frac{n}{m+n})$$

We can state this in terms of probabilities instead for the number of red balls as  $p_1$  and the number of blue balls as  $p_2$ :

$$p_1 = \frac{m}{m+n}$$

$$p_2 = \frac{n}{m+n}$$

$$Entropy = -p_1 \log_2(p_1) - p_2 \log_2(p_2)$$

This entropy equation can be extended to the multi-class case, where we have three or more possible values:

$$Entropy = -p_1 \log_2(p_1) - p_2 \log_2(p_2) - ... - p_n \log_2(p_n) = -\sum_{i=1}^n p_i \log_2(p_i)$$

The minimum value is still 0 when all elements are of the same value. The maximum value is still achieved when the outcome probabilities are the same, but the upper limit increases with the number of different outcomes. (For example, you can verify the maximum entropy is 2 if there are four different possibilities, each with a probability of 0.25.)

If we have a bucket with eight red balls, three blue balls, and two yellow balls, what is the entropy of the set of balls? Input your answer to at least three decimal places.

Enter your response here

Answer: Entropy =  $-(8/13)*\log 2(8/13) - (3/13)*\log 2(3/13) - (2/13)*\log 2(2/13)$ = 1.3346791410515946