Linear Algebra Questions (Make sure to attend the Online session)

1. Given the matrices:

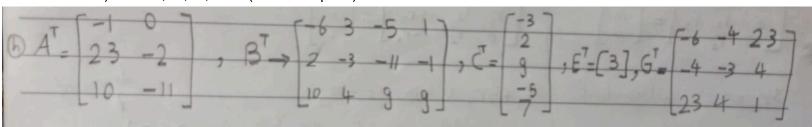
$$A = \begin{bmatrix} -1 & 23 & 10 \\ 0 & -2 & -11 \end{bmatrix}, \quad B = \begin{bmatrix} -6 & 2 & 10 \\ 3 & -3 & 4 \\ -5 & -11 & 9 \\ 1 & -1 & 9 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 2 & 9 - 5 & 7 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 6 \\ -5 & 2 \end{bmatrix}, \quad E = \begin{bmatrix} 3 \\ 5 \\ -11 \\ 7 \end{bmatrix}, \quad G = \begin{bmatrix} -6 & -4 & 23 \\ -4 & -3 & 4 \\ 23 & 4 & 1 \end{bmatrix}$$

a) What is the dimension of each matrix?

A -> 2x3, B -> 4x3, C -> 1x5, D -> 2x2, E -> 1x1, F -> 4x1, G -> 3x3

- b) Which matrices are square? D, E, G
- c) Which matrices are symmetric? Only G
- d) Which matrix has the entry at row 3 and column 2 equal to -11? B
- e) Which matrices has the entry at row 1 and column 3 equal to 10? B
- f) Which are column matrices? F
- g) Which are row matrices? C
- h) Find AT,CT,ET,GT. (T -> Transpose)



2. A, B, C, D and E are matrices given by:

$$A = egin{bmatrix} -1 & 1 & -2 \ 0 & -2 & 1 \end{bmatrix}, \quad B = egin{bmatrix} -1 & 2 & 0 \ 0 & -3 & 4 \ -1 & -2 & 3 \end{bmatrix}, \quad C = egin{bmatrix} -3 & 2 & 9 & -5 & 7 \ \end{bmatrix}$$
 $D = egin{bmatrix} -2 & 6 \ -5 & 2 \end{bmatrix}, \quad E = egin{bmatrix} 3 \ 5 \ -11 \end{bmatrix}, \quad F = egin{bmatrix} -1 & 0 & 2 \ -2 & -3 & 4 \ 1 & 4 & -3 \end{bmatrix}$

Find if possible:

- a) AB -> 2x3 . 3x3 Possible
- b) BC -> 3x3 . 1x5 Not possible
- c) AD -> 2x3 . 2x2 Not possible
- d) EF -> 3x1 . 3x3 Not possible
- e) FE -> 3x3 . 3x1 Not possible

3. Find the determinant of the matrix M:

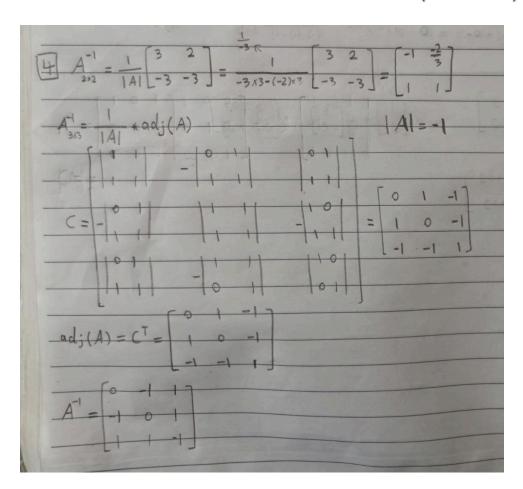
$$M = \begin{pmatrix} 15 & 10 \\ 3 & 2 \end{pmatrix} \qquad M = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 3 & 2 & -1 \end{pmatrix}$$

$$\frac{3}{M} = \begin{bmatrix} 15 & 10 \\ 3 & 2 \end{bmatrix} \Rightarrow |M| = |15 \times 2 - |0 \times 3| = 0$$

$$\frac{2}{M} = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \end{bmatrix} \Rightarrow |M| = 2 \begin{bmatrix} 2 & 3 \\ 2 & -1 \end{bmatrix} - 3 \begin{bmatrix} -1 & 3 \\ 3 & -1 \end{bmatrix} + 1 \begin{bmatrix} -1 & 2 \\ 3 & 2 \end{bmatrix} = 0$$

4. Find the inverse matrix A-1 to the matrix A:

$$A = \begin{pmatrix} -3 & -2 \\ 3 & 3 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$



- 5. What does it mean if three equations are linearly independent? B
 - a. Two of the equations can be combined to come up with the third equation.
 - b. There is no way to combine any two equations to come up with the third equation.
 - c. The graphical representations of the equations are lines that do not intersect.
 - d. The graphical representations of the equations are lines that do intersect.

$$f(x, y) = x^{\top} A y + x^{\top} B x - C y + D$$

with $\boldsymbol{x} \in \mathbb{R}^M, \boldsymbol{y} \in \mathbb{R}^N$, function $f : \mathbb{R}^M \times \mathbb{R}^N \to \mathbb{R}$.

Compute the dimensions of the matrices A, B, C, D for the function so that the mathematical expression is valid.

 $f(x,y) = x^{T}Ay + x^{T}Bx - Cy + D$ $x : M \times 1, y : M \times 1$ $1 \times t \text{ term} : x^{T}Ay = (1 \times M) \cdot A \cdot (N \times 1) \implies A : M \times N$ $2 \times t \text{ term} : x^{T}Bx = (1 \times M) \cdot B \cdot (M \times 1) \implies B : M \times M$ $3 \text{ rd term} : Cy = C \cdot (N \times 1) \implies C : 1 \times N$ $4 \text{ th term} : D \implies D : 1 \times 1$ $A : M \times N, B : M \times M, C : 1 \times N, D : 1 \times 1$