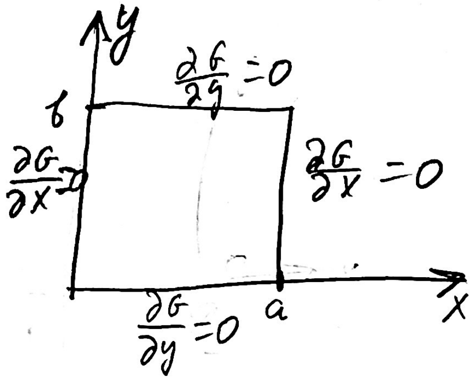
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)G = -\delta(x-x')\,\delta(y-y')$$

↳ why place a negative on right hand side?



$$G = \sum_n g_n(y)\,\overset{\cos}{\cancel{\sin}}\left(\frac{m\pi}{a}x\right)$$

$$\frac{\partial^2}{\partial x^2} = -\sum_n \left(\frac{m\pi}{a}\right)^2 g_n(y)\sin\left(\frac{m\pi}{a}x\right)$$

$$\sum_n\left(\frac{d^2}{dy^2}-\left(\frac{m\pi}{a}\right)^2\right)g_n(y)\sin\left(\frac{m\pi}{a}x\right) = -\delta(x-x')\,\delta(y-y')$$

$$\int_0^a \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}x\right)dx = \begin{cases}\frac{2}{a}; & m=n\\ 0; & m\neq n\end{cases}$$

$$\left(\frac{d^2}{dy^2}-\left(\frac{m\pi}{a}\right)^2\right)g_n(y) = -\overset{2}{\underset{a}{\cancel{\tfrac{a}{2}}}}\sin\left(\frac{m\pi}{a}x'\right)\delta(y-y')$$

(1) $\left.\frac{dg_n}{dy}\right|_{y=0}=0$

(2) $\left.\frac{dg_n}{dy}\right|_{y=b}=0$

$$\int_{y'-\varepsilon}^{y'+\varepsilon}\left(\frac{d^2}{dy^2}g_n - \underbrace{\left(\frac{m\pi}{a}\right)^2 g_n}_{\downarrow 0}\right)dy = -\frac{a}{2}\sin\left(\frac{m\pi}{a}x'\right)$$

why does this term go to zero? Because g_n continuous?

(4) $\left.\frac{dg_n}{dy}\right|_{y=y'+} - \left.\frac{dg_n}{dy}\right|_{y=y'-} = -\frac{a}{2}\sin\left(\frac{m\pi}{a}x'\right)$

(3) $g_n|_{y=y'+} = g_n|_{y=y'-}$

→ why choose hyperbolic $\cosh(\frac{m\pi}{c}y)$? and not something like $\sin(\frac{m\pi}{a}y)$

$$y > y' : \quad g_n = C_1 \overset{\cos}{\cancel{\sinh}}\left(\frac{m\pi}{a}(y-b)\right) \qquad (2)$$

$$y < y' : \quad g_n = C_2 \overset{\cos}{\cancel{\sinh}}\left(\frac{m\pi}{a}y\right) \qquad (1)$$

$$(3) \Rightarrow \quad C_1 \sinh\left(\frac{m\pi}{a}(y'-b)\right) = C_2 \sinh\left(\frac{m\pi}{a}y'\right) \Rightarrow C_2 = C_1 \frac{\sinh(\frac{m\pi}{a}(y'-b))}{\sinh(\frac{m\pi}{a}y')}$$

$$(4) \Rightarrow \quad \frac{m\pi}{a}\left(C_1 \cosh\left(\frac{m\pi}{a}(y'-b)\right) - C_2 \cosh\left(\frac{m\pi}{a}y'\right)\right) = -\frac{a}{2}\sin\left(\frac{m\pi}{a}x'\right)$$

$$C_1\left(\cosh\left(\frac{m\pi}{a}(y'-b)\right) - \frac{\sinh(\frac{m\pi}{a}(y'-b))}{\sinh(\frac{m\pi}{a}y')}\cosh\left(\frac{m\pi}{a}y'\right)\right) = -\frac{a^2}{2m\pi}\sin\left(\frac{m\pi}{a}x'\right)$$

$$C_1\Big(\underbrace{\sinh\left(\frac{m\pi}{a}y'\right)\cosh\left(\frac{m\pi}{a}(y'-b)\right) - \sinh\left(\frac{m\pi}{a}(y'-b)\right)\cosh\left(\frac{m\pi}{a}y'\right)}_{W}\Big) =$$

$$= -\frac{a^2}{m\pi}\sinh\left(\frac{m\pi}{a}y'\right)\sin\left(\frac{m\pi}{a}x'\right)$$

$$C_1 = -\frac{a^2}{m\pi}\sinh\left(\frac{m\pi}{a}y'\right)\sin\left(\frac{m\pi}{a}x'\right)\frac{1}{W}$$

$$C_2 = -\frac{a^2}{m\pi}\sinh\left(\frac{m\pi}{a}(y'-b)\right)\sin\left(\frac{m\pi}{a}x'\right)\frac{1}{W}$$

$$\boxed{\begin{aligned}
y > y' : \quad g_n &= -\overset{\cancel{a^2}}{\underset{2/m\pi}{}}\ \overset{\cos}{\cancel{\sinh}}\left(\frac{m\pi}{a}(y-b)\right)\overset{\cos}{\cancel{\sinh}}\left(\frac{m\pi}{a}y'\right)\frac{1}{W}\overset{\cos}{\cancel{\sin}}\left(\frac{m\pi}{a}x'\right)\\
y < y' : \quad g_n &= -\overset{\cancel{a^2/m\pi}}{\underset{2/m\pi}{}}\ \overset{\cos}{\cancel{\sinh}}\left(\frac{m\pi}{a}y\right)\overset{\cos}{\cancel{\sinh}}\left(\frac{m\pi}{a}(y'-b)\right)\frac{1}{W}\overset{\cos}{\cancel{\sin}}\left(\frac{m\pi}{a}x'\right)\\
W &= \overset{\cos}{\cancel{\sinh}}\left(\frac{m\pi}{a}y'\right)\overset{\sin}{\cancel{\cosh}}\left(\frac{m\pi}{a}(y'-b)\right) - \overset{\sin}{\cancel{\sinh}}\left(\frac{m\pi}{a}y'\right)\overset{\cos}{\cancel{\cosh}}\left(\frac{m\pi}{a}(y'-b)\right)
\end{aligned}}$$