Notation for the derivative

We have introduced two different notations for the derivative. Both are standard, and it is necessary to be proficient with both.

The first notation is to write $f\,O(x)$ for the derivative of the function $f\,(x)$. This functional notation was introduced by Lagrange, based on Isaac Newton's ideas. The dash in $f\,O(x)$ denotes that $f\,O(x)$ is *derived* from $f\,(x)$.

The other notation is to write $\frac{dy}{dx}$. This notation refers to the instantaneous rate of change of y with respect to x, and was introduced by Gottfried Wilhelm Leibniz, one of the discoverers of calculus. (The other discoverer was Isaac Newton. In fact, the question of who discovered calculus first was historically a point of great controversy. For more details, see the *History and applications* section.)

If we have a function $f\,(x)$ and its graph $y = f\,(x)$, then the derivative $f\,O(x)$ is the gradient tangent of $y = f\,(x)$, which is also the instantaneous rate of change $\frac{dy}{dx}$. Thus the notations are equivalent:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = fo(x).$$

For example, we calculated earlier that the derivative of x^2 was $2x$. Thus $y = x^2$ implies $\frac{dy}{dx} = 2x$.

Alternatively, $f\,(x) = x^2$ implies $\mathrm{f}O(x) = 2x$. The two notations express the same result.

Another common usage of Leibniz notation is to consider $\frac{dy}{dx}$ as an *operator*, meaning

differentiate with respect to x'. So

$\frac{dy}{dx} = \frac{d}{dx}(y)$

means: take y, and differentiate with respect to x. An alternative way to denote differentiation of x^2 using Leibniz notation would be

$\frac{d}{dx}(X^2)$ = 2X

At secondary school level, a ‘*d*’ or ‘*dx*’ or ‘*dy*’ has no meaning in itself; it only makes sense as part of a $\frac{d}{dx}$ or $\frac{dy}{dx}$ or similar.[1] The *d*’s do not cancel.

The two types of notation each have their advantages and disadvantages. It is also common to mix notation, and we will do so in this module. For example, we can write

$$\frac{d}{dx} f(x) = f'(x).$$