Some derivatives

So far in the examples and exercises we have found the following derivatives.

	$f(x)$	f $0(x)$
constant	c	0
linear	$ax + b$	a
	$x2$	$2x$
	$x3$	$3x^2$

Note that the example $f(x) = ax + b$ includes the case $f(x) = x$, which has derivative $f\ 0(x) = 1$.

Since we have seen that

$$\frac{d}{dx}(x) = 1, \qquad \frac{d}{dx}(x^2) = 2x \quad and \qquad \frac{d}{dx}(x^3) = 3x$$

it is natural to conjecture that the derivative of x^n is $nx^{n}{-}^{1}$.

Even more generally, for any *real number a*, including irrational a, the derivative of

$f(x) = x^a$ is $f\ 0(x) = ax^{a}{-}^{1}$.

It is not obvious how to even *define* what it means to raise a number to the power of an irrational number. For instance, 2^3 just means $2\times2\times2$, and $2^{\frac{7}{5}}$ just means $p^5\, 2^7$, but p what does $2\ ^3$ mean? In the module *Exponential and logarithmic functions*, we explore these issues, show how to define x^a precisely for any real number a, and show that the derivative of x^a is $ax^{a}{-}^{1}$.

In summary, the following theorem is true.

Theorem

For any real number a, the derivative of $f(x) = x^a$ is $f\ 0(x) = ax^{a}{-}^{1}$, wherever $f(x)$ is defined.